Subject:Logic Circuits

Lecturer:Hayder S.Hameed



.

2016-2017

* <u>Numbers Systems</u>

The main factor that specifies the type of a system is the base of that system, for example, the binary number has a base equal to (2), ternary number has base equal to (3), quaternary has base equal to (4)... etc. The main systems that will be studied in this subject are:

1- Decimal Numbers

The digits of this system are [0,1,2,3,4,5,6,7,8, and 9], and the base of it is (10), to express any number in this system, multiply all the coefficients of this number by the base of a positive powers for the real numbers and negative powers for the fractional numbers. The following examples illustrate how to express the numbers in decimal system.

Ex1/

Ex3/

 23.053_{10}

The real part $23 = 3 \times 10^{0} + 2 \times 10^{1}$ 3 + 20 = 23

The fractional part $0.023 = 0 \times 10^{-1} + 5 \times 10^{-2} + 3 \times 10^{-3}$ $0 \quad 0.05 \quad 0.003 = 0.053$

This means that the number is (23.053)

........

Subject:Logic Circuits

Lecturer:Hayder S.Hameed



2016-2017

2- Binary Numbers

This system consists of two digits only, which are (0 & 1), the base of it is (2). To represent the numbers of this system, the digits or coefficients of the number must be multiplied by (2) with positive power for real part and negative power for fractional part. There are two important abbreviations that must be considered, which are [Least Significant Bit (LSB) and Most Significant Bit (MSB)]

11001110011

MSB LSB

Ex4/

Ex5/

$$0.01011_{2} = 0 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4} + 1 \times 2^{-5}$$

= 0 + 0.25 + 0 + 0.0625 + 0.03125
= 0.34375

Ex6/

 110.01_{2}

Real part $110 = 1 \times 2^{2} + 1 \times 2^{1} + 0 \times 2^{0}$ 4 + 2 + 0 = 6Fractional part $0.01 = 0 \times 2^{-1} + 1 \times 2^{-2}$

This means that the number is $(6.25)_{10}$

Lecturer:Hayder S.Hameed



Lecture: 1,2

2016-2017

It is found that the results are decimal numbers, and the same results are obtained from all systems, if the coefficients of numbers are multiplied by their bases with positive power for real part and negative power for fractional part.

3- Octal number

The digits of this system are [0, 1, 2, 3,4,5,6, and 7], and the base of it is (8). The following examples illustrate how the numbers in this system can be expressed.

Ex7/

Ex8/

$$0.406_8 = 4 \times 8^{-1} + 0 \times 8^{-2} + 6 \times 8^{-3}$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad 0.5 + 0 + 0.01171875 \cong 0.512$$

Ex9/

456.157₈

Real part $456 = 4 \times 8^2 + 5 \times 8^1 + 6 \times 8^0$ 4 + 6 = 302

Fractional part $0.157 = 1 \times 8^{-1} + 5 \times 8^{-2} + 7 \times 8^{-3}$ $0.125 + 0.078125 + 0.013671875 \cong 0.217$

4- Hexadecimal Number

The digits of this system are [0,1,2,3,4,5,6,7,8,9,A,B,C,D,E, and F], and the base of it is (16). To represent the numbers of Hexadecimal system, the coefficients of the Hexadecimal number are multiplied by

Lecturer:Hayder S.Hameed

Subject:Logic Circuits



2016-2017

(16) with positive power for real part and negative power for fractional part. Note that (A =10, B =11, C=12, D=13, E = 14, F=15).

Ex10/

Ex11/

$$FA7_{16} = F \times 16^{2} + A \times 16^{1} + 7 \times 16^{0}$$

$$0.41_{16} = 4 \times 16^{-1} + 1 \times 8^{-2}$$

$$0.25 + 0.015625 \cong 0.266$$

Ex12/ 4B6.2E 16

Real part
$$4B6_{16} = 4 \times 16^2 + B \times 16^1 + 6 \times 16^0$$

 $\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$
 $1024 + 176 + 6 = 1206$

* <u>Conversion between systems</u>

1- Binary to Decimal and Vice Versa

To understand the conversion of the numbers from binary system to the decimal and from decimal to binary see the following examples:

Ex13/

 $110111001_{2} = (?)_{10}$ $1 \times 2^{8} + 1 \times 2^{7} + 0 \times 2^{6} + 1 \times 2^{5} + 1 \times 2^{4} + 1 \times 2^{3} + 0 \times 2^{2} + 0 \times 2^{1} + 1 \times 2^{0} = 441_{10}$

There is another method to perform this conversion, which illustrated below 1 1 1 1 0 0 1 0 1 $2^{8}+2^{7}+2^{6}+2^{5}+2^{4}+2^{3}+2^{2}+2^{1}+2^{0}=441_{10}$ Or 1 0 0 0 1 1 1 1 1 $256 + 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 = 441_{10}$ 4

Lecturer:Hayder S.Hameed

Subject:Logic Circuits



.

2016-2017

The above conversion is performed by putting the following sequence $(2^N \dots 2^3 2^2 2^l + 2^0)$ under the binary number, where *N* equal to the MSB of the binary nos.

Ex14/

 $34_{10} = (?)_2$

	Operation	Results	Remainder	LSB
number				
34	34/2	17	0	Read in the following direction
17	17/2	8	1	Read in the following direction
8	8/2	4	0	
4	4/2	2	0	
2	2/2	1	0	
1	1/2	0	1	+
				MSB

This mean that the binary number is $(100010)_2$

Another method can be used to convert number from decimal to binary system by writing the following sequence:

(..... 256 128 64 32 16 8 4 2 1), it must be noted that the largest number of this sequence is less than the given decimal number and this sequence results from $(2^N \dots 2^3 2^2 2^l + 2^0)$, for example $(34)_{10}$ the number in the previous sequence which is smaller than it is (32) therefore the conversion sequence will be

Which is the same as the result was obtained from the first method. For fractional decimal numbers, the number is multiplied by (2) and the result digit after the separator is the first binary digit after the separator in binary system and repeat the multiplication again and until the digits after

Subject:Logic Circuits

Lecturer:Hayder S.Hameed



2016-2017

the separator equal to zero if this results does not occur we continue for some digits after separator of the binary number.

Ex15/ convert the following decimal number $(0.875)_{10}$ and (26.024) to the binary system.

Sol: $0.875 \times 2 = 1.75$ $0.75 \times 2 = 1.5$ $0.75 \times 2 = 1.0$ $0.5 \times 2 = 1.0$ For (26.024) The real part is convert as follow $16 \ 8 \ 4 \ 2 \ 1$ $(1 \ 1 \ 0 \ 1 \ 0)_2$ For fractional part $0.24 \times 2 = 0.48$

 $0.48 \times 2 = 0.96$

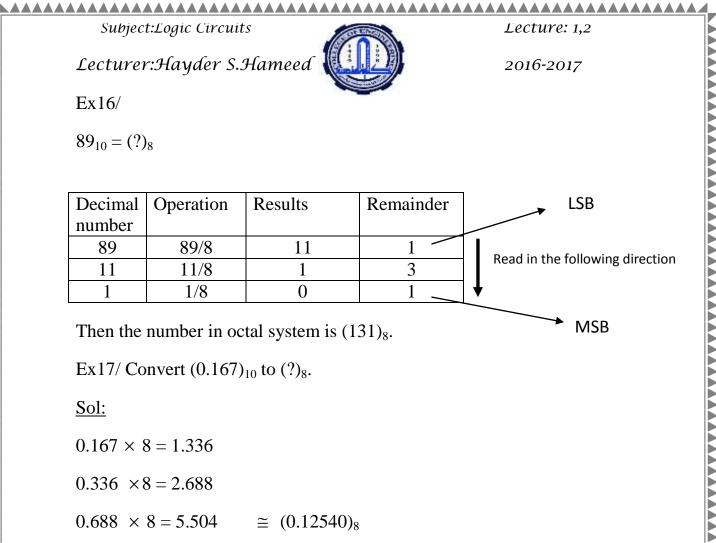
 $0.96 \times 2 = 1.92 \cong (0.001 \ 1 \ 1)_2$

 $0.92 \times 2 = 1.84$

 $0.84 \times 2 = 1.68$ And so on

2- Octal to decimal and vice versa

The second conversion that must be considered is that octal to decimal and vice versa. The following two examples illustrate the conversion between these two systems.



1

3

1

Read in the following direction

MSB

11

1

0

 \cong (0.12540)₈

89

11

1

Sol:

 $0.167 \times 8 = 1.336$

 $0.336 \times 8 = 2.688$

 $0.688 \times 8 = 5.504$

 $0.504 \times 8 = 4.032$

655.325

Real part

 $(6 \ 5 \ 5)_8$

Fractional part

89/8

11/8

1/8

Ex17/ Convert $(0.167)_{10}$ to $(?)_8$.

 $0.032 \times 8 = 0.256$ And so on

Ex18/ Convert $(655.325)_8$ to $(?)_{10}$

Sol: Solving with weights method

 $\rightarrow (6 \times 8^2 + 5 \times 8^2 + 5 \times 8^2) = (429)_{10}$

 $(3 \times 8^{-1} + 2 \times 8^{-2} + 5 \times 8^{-3} \cong (0.41602)_{10}$

Then the number in octal system is $(131)_8$.

7

Lecturer:Hayder S.Hameed



...........

2016-2017

MSB

3- Hexadecimal to decimal and vice versa

To convert from hexadecimal to decimal the real coefficients of hexadecimal number are multiplied by (16) with positive power and fractional coefficients are multiplied by (16) with negative power. The conversion from decimal to hexadecimal we use either division method or weights method for real numbers and multiplying by (16) for fractional numbers.

Ex19/ Convert (9810)₁₀ to (?)₁₆

Sol:

Decimal	Operation	Results	Remainder	LSB
number				
9810	9810/16	613	2	
613	613/16	38	5	Read in the following direction
38	38/16	2	6	↓ ↓
2	2/16	0	2	·

Ex20/ Convert (E768.EF6)₁₆ to (?)₁₀

Sol:

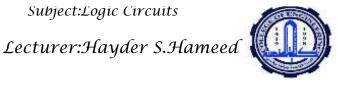
 $E \times 16^3 + 7 \times 16^2 + 6 \times 16^1 + 8 \times 16^0$. $E \times 16^{-1} + F \times 16^{-2} + 6 \times 16^{-2} = (59240.93506)$.

Ex21/ Convert (4076.1796875)₁₀ to (?)

Real part (4076)		(4076)	Fractional part (0.1796875)	
256	16	1	$0.1796875 \times 16 = 2.875$	
(F	Е	C) ₁₆	$0.875 \times 16 = 14$	(2E)

4- Binary to Octal and vice versa

To convert any number from binary to octal system, each three digits are taken together and find the equivalent number in octal system according to the following table:



Lecture: 1,2

2016-2017

Octal digits	Binary numbers	Octal digits	Binary numbers
0	000	4	100
1	001	5	101
2	010	6	110
3	011	7	111

Ex22/ Convert (1101111)₂ to (?)

Sol:

It must be started from the (LSB) toward (MSB) as follow

(001 101 111) **↓** 5 **▼**(1 $7)_{8}$

Ex23/ Convert (4037)₈ to (?)₂

Sol:

(4	0	3	7)
ŧ	¥	¥	↓
(100	000	011	$(111)_2$

5- Hexadecimal to Binary and vice versa

To convert any number from binary to Hexadecimal system, each four digits are taken together and find the equivalent number in Hexadecimal system according to the following table:

Hexadecimal	Binary	Hexadecimal	Binary	Hexadecimal	Binary
0	0000	6	0110	12	1100(C)
1	0001	7	0111	13	1101(D)
2	0010	8	1000	14	1110(E)
3	0011	9	1001	15	1111(F)
4	0100	10	1010(A)		
5	0101	11	1011(B)		

Subject:Logic Circuits Lecturer:Hayder S.Hameed



2016-2017

Ex24/ Convert (10011011010010)₂ to (?)₁₆

Sol:

0010	0110	1101	0010
¥	¥	¥	ŧ
(2	6	D	2) ₁₆

Ex25/ Convert (E767)₁₆ to (?)₂

Sol:

E	7	6	7
¥	¥	¥	¥
(1110	0111	0110	0111) ₂

* <u>Operation on systems</u> 1- Addition

The following examples illustrate the addition operation for all systems. For binary system the following rules must be noted

(0+0=0, 1+0=1, 0+1=1, 1+1=10)

Ex26/ Solve the following

110101	110110	111101
+110110	101101	001010
1101011	+111001	111000
	10011100	+000100
		10000011

Ex27/ Solve the following in octal system

357	42667	355677
+ 276	+ 65777	+ 76746
655	130666	454645

Lecturer:Hayder S.Hameed



...........

Lecture: 1,2

2016-2017

It must be noted that the addition in this system is the same as decimal system except that the carry in octal system is (8).

Ex28/ Add the following hexadecimal numbers

AB2F	FE17
+12EC	+DBC
BE1B	10BD3

The carry in this system is (16).

There is important note for the addition of octal system, which state that when the result of adding two digits is equal or larger than to (8) then (8) is subtracted from the result of adding and the result of subtraction will represent the final result and the carry is (8) which equivalent to (one) in the next order as in decimal system, for example:

4577

+5776

12575

The previous note can be applied to the hexadecimal system except that the base of system will be (16) for example

EF

+FFA

10E9

Subject:Logic Circuits

Lecturer:Hayder S.Hameed



2016-2017

2- Subtraction

For binary system the following rules must be considered {0-0 = 0, 1-1 = 0, 1-0 = 1, 0-1 = not possible except when another digit (1) be taken from neighboring}.

Ex29/ subtract the following numbers

11011	111101	1000000
-10110	-110001	- 110111
00101	001100	001001

Ex30/ Subtract the following numbers

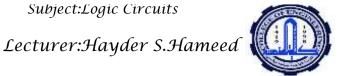
750	711	4321
- 76	- 377	- 2567
652	312	1532

Ex31/Subtract the following hexadecimal numbers

AB21	32FB
- 2EF	- FEF
A832	230C

3- Multiplication

Ex32/ 111	110101	000000
× 110	× 110101	110101
000	110101	+ 110101
111	000000	101011111001
+ 111	110101	
101010		
	12	



Lecture: 1,2

.

2016-2017

The multiplication in octal system is illustrated in the following example.

Ex32/		1.	Multiply (3) by (7) = 21 and subtract (8 \times 2) from (21) = 5, (2) is chosen to
			be the larger number multiplied by (8) lead to number equal or smaller
57			than (21), now (5) is the result and (2) is the carry.
× 23		2.	Multiply (2) by (7) plus (2) = 16 and subtract (8 \times 2) from (16) = 0, (0) is the
	-		result and (2) is the carry.
205		3.	The result will be (205) of the first multiplication.
+137		4.	Repeat the first three steps to obtain the second result (137).
	-	5.	Add them by push the second result one space to obtain the result of
1575			multiplication operation.
Ex33/	762		
2	× 73		
_	2726)	
_			

71306

Follow the same points to perform the multiplication in hexadecimal system. To understand this operation see the following example.

Ex34/	B2	3EC
_	× F3	\times A2D
	216	32FC
+2	A6E	7D8
I	48F6	+ 2738
		27E87C

Subject:Logic Circuits Lecture: 1,2 Lecturer:Hayder S.Hameed 2016-2017

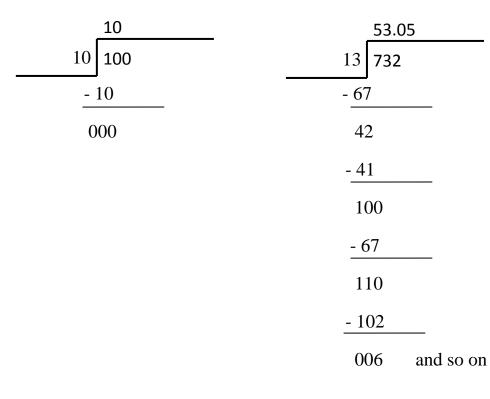
4- Division

An important remark (if the result of division operation is real and has not carry, then there is no problem but if there is a carry then we put a (,) as in decimal system.

Ex35/

1000	1	100100.01
101 1010	101	11 1101101
- 101		- 11
00001	01	00011
- 1	01	- 11
00000	000	000000100
		11
		1

Ex36/ Solve the following in octal system



Subject:Logic Circuits	Lecture: 1,2
Lecturer:Hayder S.Hameed	2016-2017
Ex37/ Solve	
9	<u>8C</u>
32 1C2	138 ABC1
- 1C2	- 9C0
000	FC1
	- EA0
	1210
	- 1110
	0010
	- 39
	100

* <u>Complements of Numbers systems</u>

Each system has two complements according to (base-1)'s complement and base's complement of that system.

1- Binary Numbers

One's Complement for binary system can be obtained by inverting zero to one and one to zero. Two's complement represents (*one's complement* + 1)

2- Octal Numbers

In this system 7'S complement is obtained by subtracting the number from(7, 77, 777, ..., *etc.*) While 8'S complement is obtained by adding one to the 7'S complement.

Subject:Logic Circuits

Lecturer:Hayder S.Hameed



2016-2017

3- Decimal Numbers

There are 9'S and 10'S complements in the decimal system, 9'S complement is obtained by subtract decimal number from (9, 99, 99, ..., *etc*), while 10'S complement is (1+9'S complement).

4- Hexadecimal numbers

This system has 15'S & 16'S complements which can be obtain as in the previous systems. In general to find the (2'S, 8'S, 10'S, and 16'S) can be found from the following equation:

 $(N)_{r,c} = r^n - N$

Where n is number of digits in the integer portion of N, and r is the radix of the number.

Ex38/ Find (1'S, 2'S, 7'S, 8'S, 9'S, 10'S, 15'S, 16'S) complements for $(10011101)_2$.

Sol/ For the number $(10011101)_2$

1'S = (01100010), 2'S = 1'S + 1 $\Box S = (01100011)$

Since $(10011101)_2 = (157)_{10}$ then $10'S = 10^3 - 157 = (843)_{10}$

& $9'S = 843 - 1 = (842)_{10}$

 $(10011101)_2 = (235)_8 \longrightarrow 8'S = 8^3 - (235) = (543)_8$

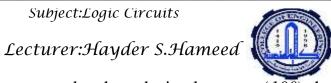
And $7'S = 543 - 1 = (542)_8$

 $(10011101)_2 = (9D)_{16}$ \longrightarrow $16'S = 16^2 - 9D = (63)_{16}$

In addition, $15'S = (62)_{16}$.

An important not $8^3 = 512$ must be converted to octal system (1000) then subtract (235) from (1000) in octal system, and $16^2 = 256$ must be

16



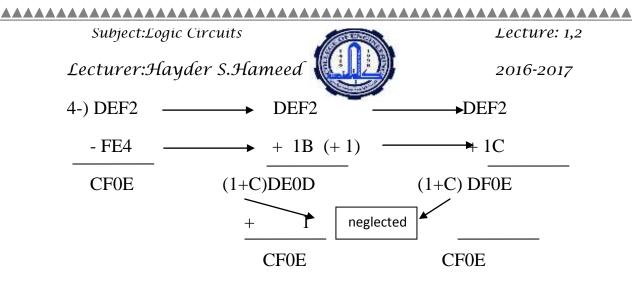
Lecture: 1,2

2016-2017

converted to hexadecimal system (100) then subtract (9D) from (100) in hexadecimal.

Ex39/ Subtract the following numbers using the principle of complement.

1-) ₂ 111011	, 2-) ₈ 3457	3-) ₁₀ 349	4-) ₁₆ DEF	2
- 100011	- 456	- 67	FE4	4
Sol:- 1-)				
111011 -	→11	1011 or		→ 11011
-100011	1'S + 01	1100 (+1)	2'S	+ 011101
011000	10	10111	(1)011000
	+		neglected	
	C	11000		011000
2-) 3457		3457		→ 3457
- 456	7'S	+ 321 (+	1) <u>8'S</u>	+ 322
3001	(1+3	3)4000		(1+3)4001
			neglected	
	_	3001		3001
3-) 349		349		→ 349
- 67	9'S	+ 32 (+ 1)) 10'S	+ 33
282	(1+2	2)381		(1+2)382
	+	1	neglected	
		282		282



* <u>Codes</u>

Generally, there are two types of codes which are:

1- Weighted

There are numerous weighted codes. Among these are 2421 and 84-2-1 code, by using weighted code the corresponding decimal digits easily determined by adding the weights associated with 1 in the code group.

Decimal	Bcd codes	2421	84-2-1	7421	6311
digits	8421				
0	0000	0000	0000	0000	0000
1	0001	0001	0111	0001	0001
2	0010	0010	0110	0010	0011
3	0011	0011	0101	0011	0100
4	0100	0100	0100	0100	0110
5	0101	1011	1011	0101	0111
6	0110	1100	1010	0110	1000
7	0111	1101	1001	1000	1001
8	1000	1110	1000	1001	1011
9	1001	1111	1111	1010	1100

Subject:Logic Circuits

......

Lecturer:Hayder S.Hameed



2016-2017

Lecture: 1,2

2- Non-Weighted

An example of a non-weighted code is the excess-3 code where digit codes is obtained from heir binary equivalent after adding 3. Thus the code of a decimal 0 is 011, that of 6 is 001, etc.

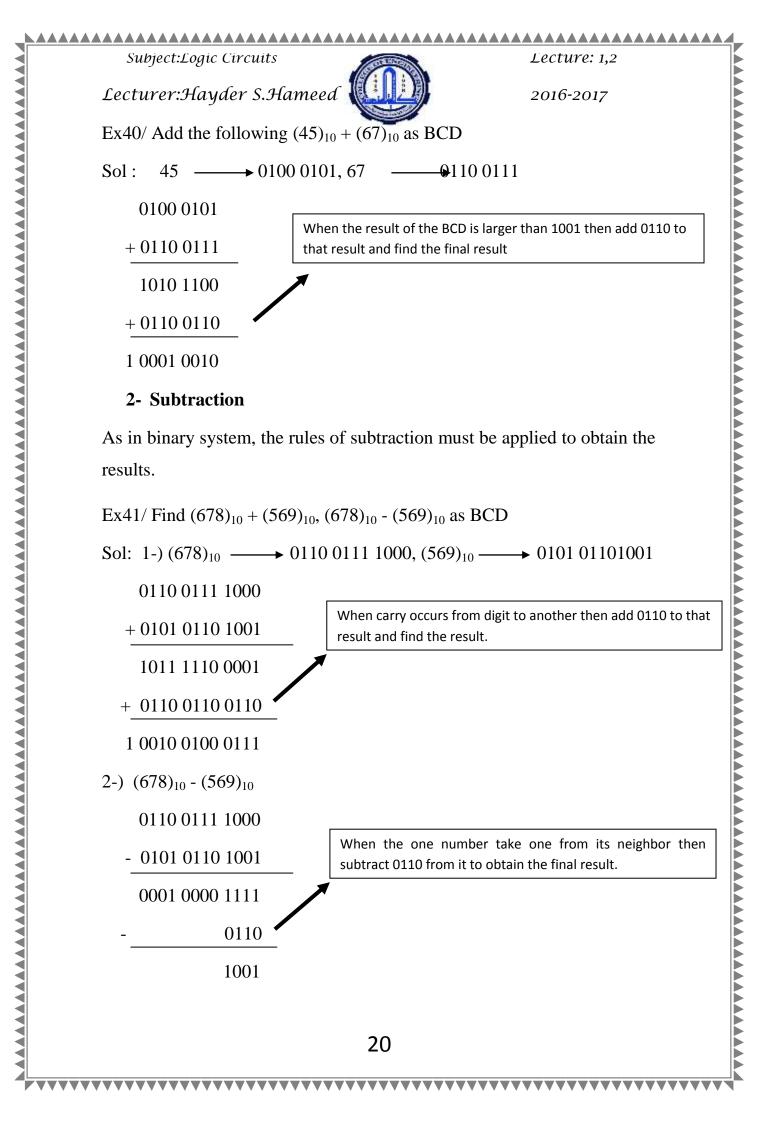
* <u>Binary Coded Decimal (BCD)</u>

In this code, four-bit binary number represents each decimal digit. BCD is a way to express each of the decimal digits with a binary code. In the BCD, with four bits we can represent sixteen numbers (0000 to 1111), but in BCD code only first ten of these are used (0000 to 1001). The remaining six code combinations i.e. (1010 to 1111) are invalid in BCD.

The advantages of this code are that it is very similar to decimal system. In addition, it needs to remember binary equivalent of decimal numbers (0 to 9) only. On the other hand, there are some disadvantages of this code which are the addition and subtraction of BCD have different rules. The BCD arithmetic is little more complicated. BCD needs more number of bits than binary to represent the decimal number. Therefore, BCD is less efficient than binary.

✤ <u>Operation of BCD codecs</u> 1- Addition

When add any numbers in BCD system, it must be noted that the result must not be larger than (1001).



Subject:Logic Circuits

Lecturer:Hayder S.Hameed



2016-2017

✤ <u>Excess-3 codes</u>

The Ex-3 code is also called as XS-3 code. It is non-weighted code used to express decimal numbers. The Excess-3 code words are derived from the 8421 BCD code words adding $(0011)_2$ or $(3)_{10}$ to each code word. The following table convert the decimal digits to EX-3

Decimal digits	Excess-3 codes
0	0011
1	0100
2	0101
3	0110
4	0111
5	1000
6	1001
7	1010
8	1011
9	1100

* **Operation of Ex-3 codes**

1- Addition

Ex42/ Solve the following $(87)_{10} + (45)_{10}$ as Ex-3 code

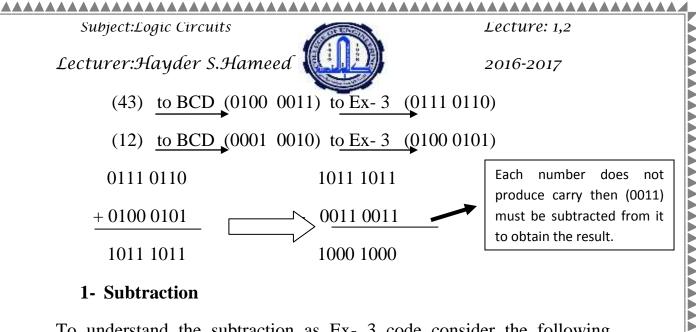
Sol: (87) to BCD (1000 0111) to Ex-3 (1011 1010)

(45) to BCD (0100 0101) to Ex-3 (0111 1000)

1011 1010	1 0011 0010	Ead
		car
+ 0111 1000	0011 0011	(OC res
1 0011 0010	1 0110 0101	

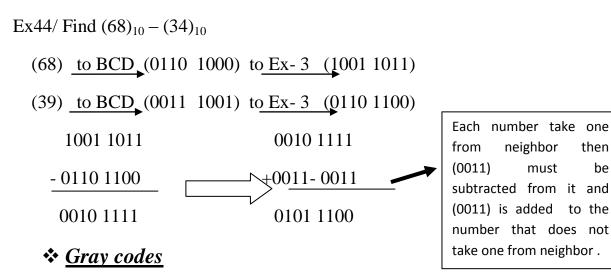
Each number produce carry must be added to (0011) to obtain the result. ___________

Ex43/ Solve the following $(43)_{10} + (12)_{10}$ as Ex-3 code



1- Subtraction

To understand the subtraction as Ex- 3 code consider the following example



An n-bit Gray code, also called the reflected binary code. Today, Gray codes are widely used to facilitate error correction in digital communications such as digital terrestrial television and some cable TV systems.

* Conversion from Binary to Gray and vice versa

The first gray digits is the same as the first binary digits, other bits of gray codes is obtained by adding each pairs of binary bits with each other. While when convert from gray to binary the first digit in binary is the same as the first digits in gray and other digits of binary number can be obtained by adding each binary digits with the next gray digits.

Subject:Logic Circuits Lecturer:Hayder S.Hameed $(2)_{G}$ Ex45/ Convert $(1\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 1)_{B}$ to $(?)_{G}$ Sol: + + + + + + + +

 $(1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1)_{B}$ $(1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1)_{B}$ $(1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1)_{G}$

Ex46/ Convert $(1\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 1\ 1)_{G}$ to $(?)_{B}$

Sol:

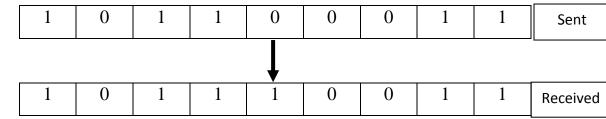
 $1 \ 0 \ 1 \ 0 \ 1 \ 1_{G}$ (1 0 0 0 0 0 0 1 1 $1)_{2}$ 1 1 (1

* <u>Error detecting/correcting codes</u>

Interferences can change the timing and shape of the signal. If the signal is carrying binary encoded data, such changes can alter the meaning of the data. These errors can be divided into two types: Single-bit error and Burst error.

1- Single-bit Error

The term single-bit error means that only one bit of given data unit (such as a byte, character, or data unit) is changed from 1 to 0 or from 0 to 1.

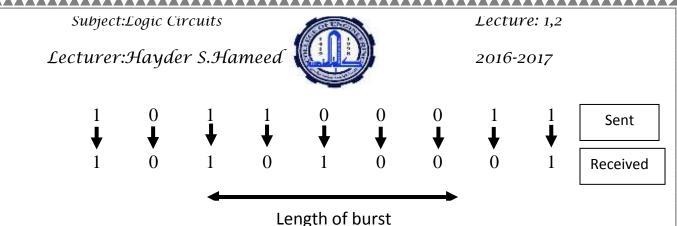


Only five bit changes from 0 to 1.

2- Burst Error

The term burst error means that two or more bits in the data unit have changed from 0 to 1 or vice-versa. The length of the burst error is measured from the first corrupted bit to the last corrupted bit. Some bits in between may not be corrupted.

2016-2017



The simplest way to correct this error is simple parity check in its two types (even and odd) parity,

1- Odd parity

One convention, called odd parity, specifies that the parity bit will be added to the incorrect received bits so that the total number of 1 bits (including the parity bit) is odd. For example

Data 1 1 0 1 0 1 1 1

Data + Odd parity 1 1 1 0 1 0 1 1 1

2- Even parity

An alternate convention, called even parity, sets the parity bit so that the total number of 1 bits (including the parity bit) is even. For example

Data 1 1 0 1 0 0 0 1

Data + Even parity 0 1 1 0 1 0 0 0 1