**LITRUCHER NO. 1**

**BINARY NUMBERS&NUMBER-BASE CONVERSIONS**

**1ST Year**

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**Digital Logic**

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A decimal number such as 7392 represents a quantity equal to 7 thousands plus 3 hundreds, plus 9 tens, plus 2 units. The thousands, hundreds, etc. are powers of 10 implied by the position of the coefficients. To be more exact, 7392 should be written as

7 X 103 + 3 X 102 + 9 X 101 + 2 x 10°

However, the convention is to write only the coefficients and from their position deduce the necessary powers of 10. In general, a number with a decimal point is represented by a series of coefficients as follows:

a5a4a3a2a1a0.a-1a-2a-3

The aj coefficients are one of the ten digits (0, 1,2, ... ,9), and the subscript valuej gives the place value and, hence, the power of 10 by which the coefficient must be multiplied.

105a5 + 104a4 + 103a3 + 102a2 + lO1a1 + 100ao + 1O-1a-1 + 1O-2a-2 + 1O-3a-3

The decimal number system is said to be of base, or radix, 10 because it uses ten digits and the coefficients are multiplied by powers of 10. The binary system is a different number system. The coefficients of the binary numbers system have two possible values: 0 and 1. Each coefficient aj is multiplied by 21. For example, the decimal equivalent of the binary number 11010.11 is 26.75, as shown from the multiplication of the coefficients by powers of 2:

I X 24 + I X 23 + 0 X 22 + 1 X 21+ 0 x 2° + I x 2 -1 + 1 X 2-2 = 26.75

In general, a number expressed in base-r system has coefficients multiplied by powers of r:

an.rn + an-1.rn-1 +........+ a2.r2 + a1.r + a0 + a-1.r-1+ a-2.r-2+……+a-m.r-m

The coefficients aj range in value From 0 to r -1. To distinguish between numbers of different bases, we enclose the coefficients in parentheses and write a subscript equal to the base used (except sometimes for decimal numbers, where the content makes it obvious that it is decimal). An example of a base-5 number is

(4021.2)5 = 4 X 53 + 0 X 52 + 2 X 51 + 1 X 50 + 2 X 5-1 = (511.4)10

that coefficient values for base 5 can be only 0, 1, 2, 3, and 4.The octal number system is a base‐8 system that has eight digits: 0, 1, 2, 3, 4, 5, 6, 7. An example of an octal number is 127.4. To determine its equivalent decimal value, we expand the number in a power series with a base of 8:

(127.4)8=1\*82+2\*81+7\*80+4\*8-1 = (87.5)10

Note that the digits 8 and 9 cannot appear in an octal number.

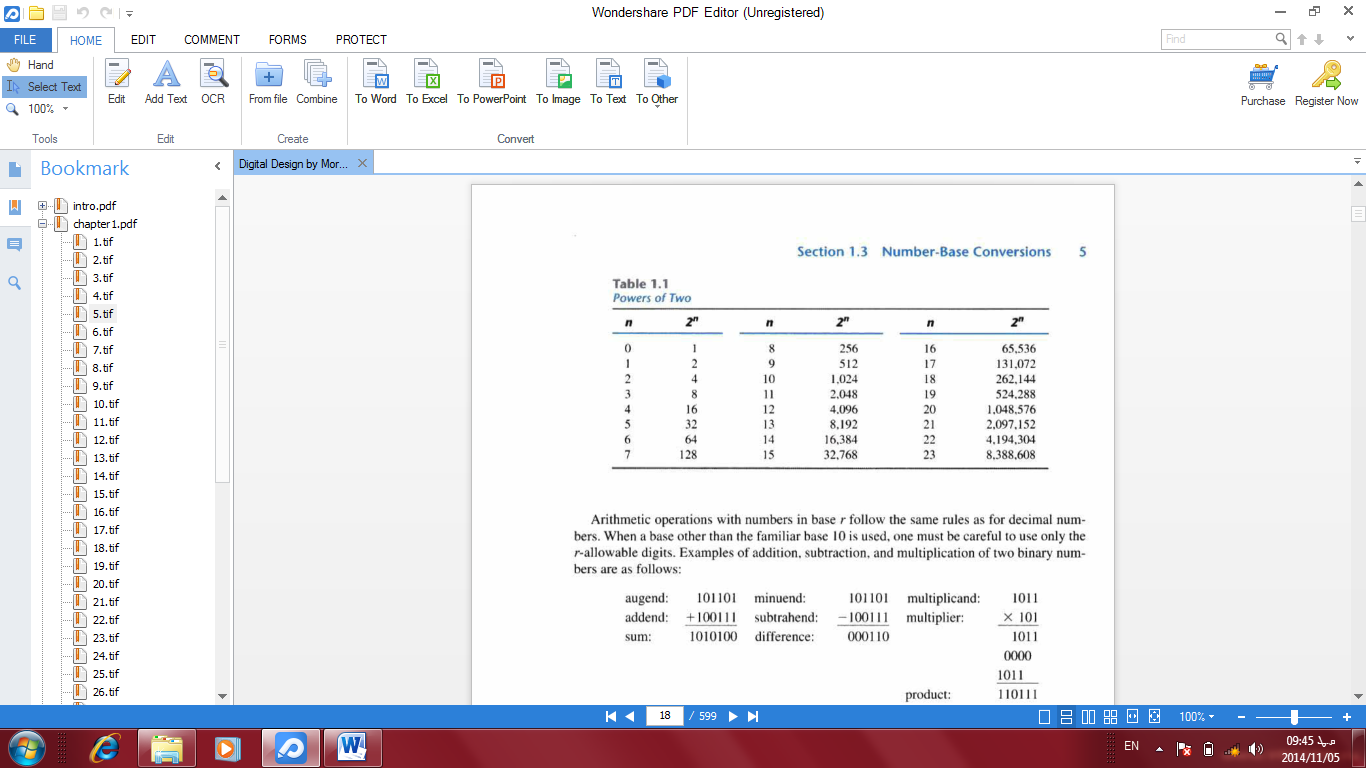
It is customary to borrow the needed r digits for the coefficients from the

Alphabetare used to supplement the 10 decimal digits when the base of the number is greater than 10. For example, in the hexadecimal (base‐16) number system, the first 10 digits are borrowedfrom the decimal system. The letters A, B, C, D, E, and F are used for the digits 10, 11, 12, 13, 14, and 15, respectively. An example of a hexadecimal number is

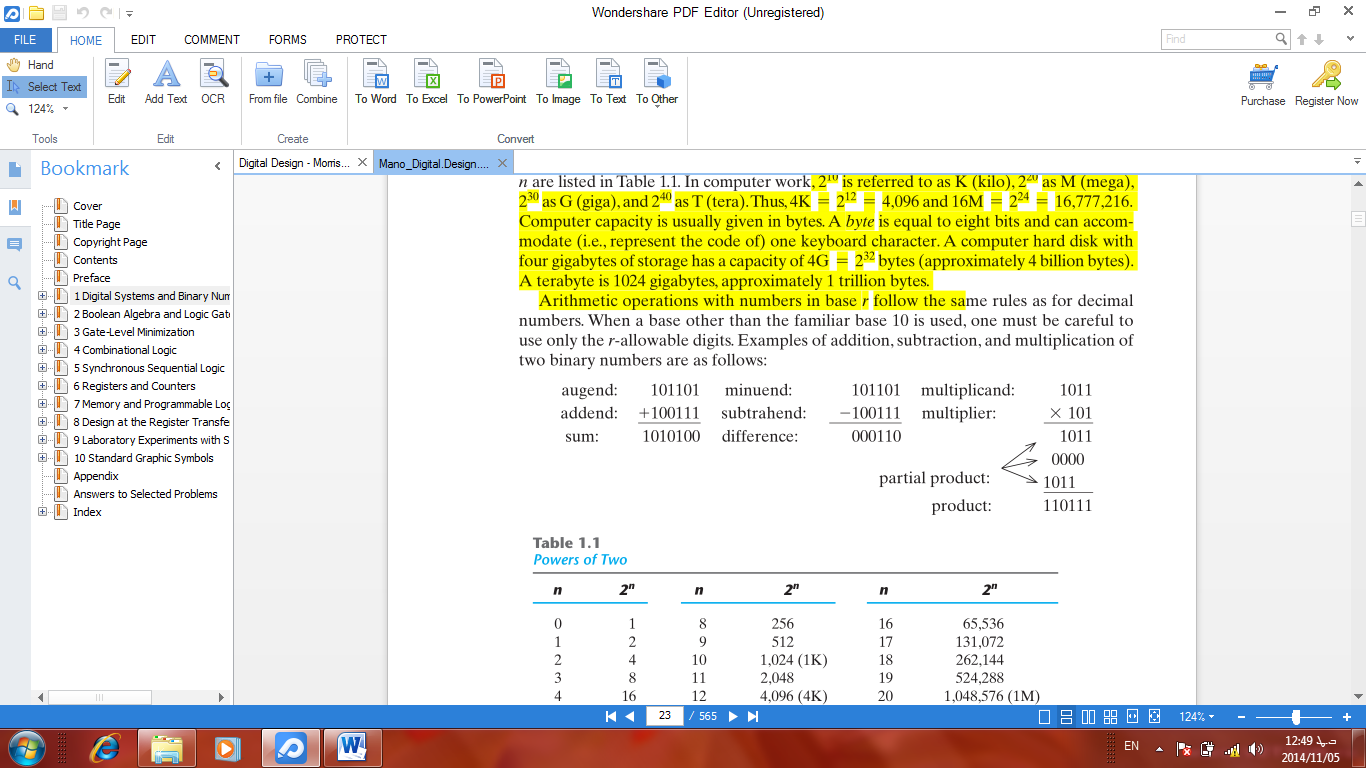
(B65F)16=11\*163+6\*162+5\*161+15\*160=(46,687)10

As noted before, the digits in a binary number are called bits. When a bit is equal to 0, it does not contribute to the sum during the conversion. Therefore, the conversion from binary to decimal can be obtained by adding only the numbers with powers of two corresponding to the bits that are equal to 1. For example,

(110101)2=32+16+4+1=(53)10

There are four 1’s in the binary number. The corresponding decimal number is the sum of the four powers of two. Zero and the first 24 numbers obtained from 2 to the power of n are listed in Table 1.1 . In computer work, 210is referred to as K (kilo), 220 as M (mega), 230 as G (giga), and 240 as T (tera). Thus, 4K=212=4,096 and 16M=224=16,777,216. Computer capacity is usually given in bytes. A byte is equal to eight bits and can accommodate (i.e., represent the code of) one keyboard character. A computer hard disk with four gigabytes of storage has a capacity of 4G=232 bytes (approximately 4 billion bytes). A terabyte is 1024 gigabytes, approximately 1 trillion bytes. 

Arithmetic operations with numbers in base r follow the same rules as for decimal numbers. When a base other than the familiar base 10 is used, one must be careful to use only the r‐allowable digits. Examples of addition, subtraction, and multiplication of two binary numbers are as follows:



The sum of two binary numbers is calculated by the same rules as in decimal, except that the digits of the sum in any significant position can be only 0 or 1. Any carry obtained in a given significant position is used by the pair of digits one significant position higher. Subtraction is slightly more complicated. The rules are still the same as in decimal, except that the borrow in a given significant position adds 2 to a minuend digit. (A borrow in the decimal system adds 10 to a minuend digit.) Multiplication is simple: The multiplier digits are always 1 or 0; therefore, the partial products are equal either to a shifted (left) copy of the multiplicand or to 0.

The conversion of a number base r to decimal is done by expanding the number in a power series and adding all the terms as shown previously. We now present a general procedure for the reverse operation of convening a decimal number to a number in base ,: If the number includes a radix point, it is necessary to separate the number into an integer pm and a fraction part, since eachpartmust be converteddifferently. The conversion of decimalinteger to a number in base r is done by dividing the number and all successive quotients by r and accumulating the remainders. This procedure is best illustrated by example.

EXAMPLE 1-1

EXAMPLE 1.1

Convert decimal 41 to binary. First, 41 is divided by 2 to give an integer quotient of 20 and a remainder of 12. Then the quotient is again divided by 2 to give a new quotient and remainder. The process is continued until the integer quotient becomes 0. The coefficients of the desired binary number are obtained from the remainders as follows:

Integer

Quotient Remainder Coefficient

41/2= 20 + a0=1

20/2= 10 + 0 a1=0

10/2= 5 + 0 a2=0

5/2= 2 + a3=1

2/2= 1 + 0 a4=0

1/2= a5=1

Therefore, the answer is (41)10=(a5a4a3a2a1a0)2=(101001)2.

