KIRCHHOFF'S VOLTAGE LAW

the algebraic sum of the potential rises and drops around a closed path (or closed loop) is zero

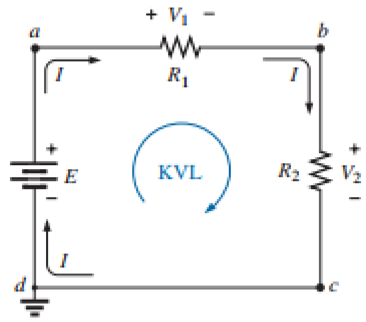


FIG. 26
Applying Kirchhoff's voltage law to a series dc circuit.

In symbolic form it can be written as

$$\boxed{\Sigma_{\circlearrowright} V = 0} \quad \text{(Kirchhoff's voltage law in symbolic form)}$$

$$+E - V_1 - V_2 = 0$$
$$E = V_1 + V_2$$

Kirchhoff's voltage law can also be written in the following form $\boxed{\Sigma_{\circlearrowright} V_{\text{rises}} = \Sigma_{\circlearrowright} V_{\text{drops}}}$

the sum of the voltage rises around a closed path will always equal the sum of the voltage drops.

EXAMPLE 8 Use Kirchhoff's voltage law to determine the unknown voltage for the circuit in Fig. 27.

Solution: When applying Kirchhoff's voltage law, be sure to concentrate on the polarities of the voltage rise or drop rather than on the type of element. In other words, do not treat a voltage drop across a resistive element differently from a voltage rise (or drop) across a source. If the polarity dictates that a drop has occurred, that is the important fact, not whether it is a resistive element or source.

Application of Kirchhoff's voltage law to the circuit in Fig. 27 in the clockwise direction results in

$$+E_1 - V_1 - V_2 - E_2 = 0$$

and
$$V_1 = E_1 - V_2 - E_2 = 16\text{ V} - 4.2\text{ V} - 9\text{ V}$$

so
$$V_1 = \mathbf{2.8\ V}$$

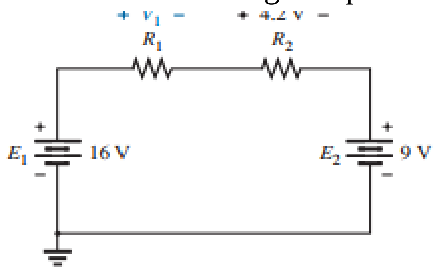


FIG. 27
Series circuit to be examined in Example 8.

EXAMPLE 9 Determine the unknown voltage for the circuit in Fig. 28.

Solution: In this case, the unknown voltage is not across a single resistive element but between two arbitrary points in the circuit. Simply apply Kirchhoff's voltage law around a path, including the source or resistor R_3. For the clockwise path, including the source, the resulting equation is the following:

$$+E - V_1 - V_x = 0$$

and
$$V_x = E - V_1 = 32\text{ V} - 12\text{ V} = \mathbf{20\ V}$$

For the clockwise path, including resistor R_3, the following results:

$$+V_x - V_2 - V_3 = 0$$

and
$$V_x = V_2 + V_3 = 6\text{ V} + 14\text{ V}$$

with
$$V_x = \mathbf{20\ V}$$

providing exactly the same solution.

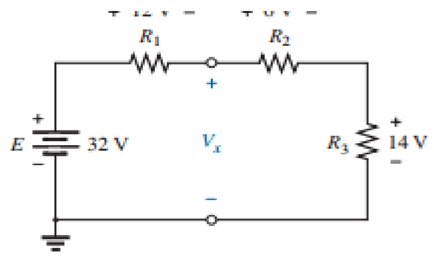


FIG. 28
Series dc circuit to be analyzed in Example 9.

EXAMPLE 10 Using Kirchhoff's voltage law, determine voltages V1 and V2 for the network in Fig. 29.

Solution: For path 1, starting at point a in a clockwise direction, +25 V -V1 +15 V =0 V.so V1=25

For path 2, starting at point a in a clockwise direction, -V2- 20 V =0. so V2 = -20 V

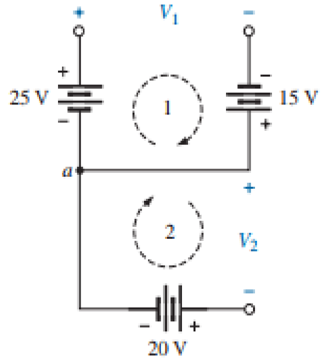


FIG. 29
Combination of voltage sources to be examined in Example 10.

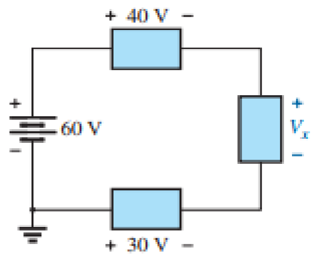


FIG. 30
Series configuration to be examined in Example 11.

Example 11 emphasizes the fact that when you are applying Kirchhoff's voltage law, the polarities of the voltage rise or drop are the important parameters, not the type of element involved.

EXAMPLE 11 Using Kirchhoff's voltage law, determine the unknown voltage for the circuit in Fig. 30.

Solution: Note that in this circuit, there are various polarities across the unknown elements since they can contain any mixture of components. Applying Kirchhoff's voltage law in the clockwise direction results in

$$+60\text{ V} - 40\text{ V} - V_x + 30\text{ V} = 0$$

and $\quad V_x = 60\text{ V} + 30\text{ V} - 40\text{ V} = 90\text{ V} - 40\text{ V}$

with $\quad V_x = \mathbf{50\text{ V}}$

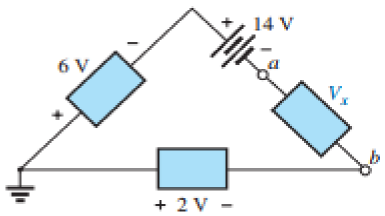


FIG. 31
Applying Kirchhoff's voltage law to a circuit in which the polarities have not been provided for one of the voltages (Example 12).

EXAMPLE 12 Determine the voltage V_x for the circuit in Fig. 31. Note that the polarity of V_x was not provided.

Solution: For cases where the polarity is not included, simply make an assumption about the polarity, and apply Kirchhoff's voltage law as before. If the result has a positive sign, the assumed polarity was correct. If the result has a minus sign, the **magnitude is correct,** but the assumed polarity must be reversed. In this case, if we assume point *a* to be positive and point *b* to be negative, an application of Kirchhoff's voltage law in the clockwise direction results in

$$-6\text{ V} - 14\text{ V} - V_x + 2\text{ V} = 0$$

and $\quad V_x = -20\text{ V} + 2\text{ V}$

so that $\quad V_x = \mathbf{-18\text{ V}}$

Since the result is negative, we know that point *a* should be negative and point *b* should be positive, but the magnitude of 18 V is correct.

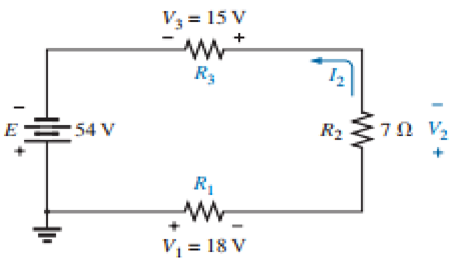


FIG. 32
Series configuration to be examined in Example 13.

EXAMPLE 13 For the series circuit in Fig. 32.

a. Determine V_2 using Kirchhoff's voltage law.
b. Determine current I_2.
c. Find R_1 and R_3.

Solutions:

a. Applying Kirchhoff's voltage law in the clockwise direction starting at the negative terminal of the supply results in

$$-E + V_3 + V_2 + V_1 = 0$$

and $$E = V_1 + V_2 + V_3 \text{ (as expected)}$$

so that $$V_2 = E - V_1 - V_3 = 54\text{ V} - 18\text{ V} - 15\text{ V}$$

and $$V_2 = \mathbf{21\ V}$$

b. $I_2 = \dfrac{V_2}{R_2} = \dfrac{21\text{ V}}{7\ \Omega}$

$I_2 = \mathbf{3A}$

c. $R_1 = \dfrac{V_1}{I_1} = \dfrac{18\text{ V}}{3\text{ A}} = \mathbf{6\ \Omega}$

with $R_3 = \dfrac{V_3}{I_3} = \dfrac{15\text{ V}}{3\text{ A}} = \mathbf{5\ \Omega}$

EXAMPLE 14 Using Kirchhoff's voltage law and Fig. 12, verify Eq. (1).

Solution: Applying Kirchhoff's voltage law around the closed path:

$$E = V_1 + V_2 + V_3$$

Substituting Ohm's law:

$$I_sR_T = I_1R_1 + I_2R_2 + I_3R_3$$

but $$I_s = I_1 = I_2 = I_3$$

so that $$I_sR_T = I_s(R_1 + R_2 + R_3)$$

and $$R_T = \boldsymbol{R_1 + R_2 + R_3}$$

which is Eq. (1).

Solutions:

a. Applying Kirchhoff's voltage law in the clockwise direction starting at the negative terminal of the supply results in

$$-E + V_3 + V_2 + V_1 = 0$$

and $E = V_1 + V_2 + V_3$ (as expected)

so that $V_2 = E - V_1 - V_3 = 54\text{ V} - 18\text{ V} - 15\text{ V}$

and $\mathbf{V_2 = 21\text{ V}}$

b. $I_2 = \dfrac{V_2}{R_2} = \dfrac{21\text{ V}}{7\ \Omega}$

$\mathbf{I_2 = 3A}$

c. $R_1 = \dfrac{V_1}{I_1} = \dfrac{18\text{ V}}{3\text{ A}} = \mathbf{6\ \Omega}$

with $R_3 = \dfrac{V_3}{I_3} = \dfrac{15\text{ V}}{3\text{ A}} = \mathbf{5\ \Omega}$

EXAMPLE 14 Using Kirchhoff's voltage law and Fig. 12, verify Eq. (1).

Solution: Applying Kirchhoff's voltage law around the closed path:

$$E = V_1 + V_2 + V_3$$

Substituting Ohm's law:

$$I_sR_T = I_1R_1 + I_2R_2 + I_3R_3$$

but $I_s = I_1 = I_2 = I_3$

so that $I_sR_T = I_s(R_1 + R_2 + R_3)$

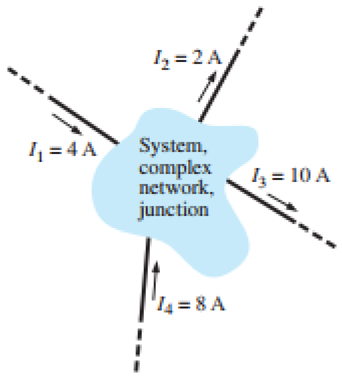
and $R_T = \mathbf{R_1 + R_2 + R_3}$

which is Eq. (1).

KIRCHHOFF'S CURRENT LAW

Kirchhoff's current law (KCL): The algebraic sum of the currents entering and leaving a junction (or region) of a network is zero.

The law can also be stated in the following way: The sum of the currents entering a junction (or region) of a network must equal the sum of the currents leaving the same junction (or region).



$\boxed{\Sigma I_i = \Sigma I_o}$

FIG. 30
Introducing Kirchhoff's current law.

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_4 = I_2 + I_3$$
$$4\text{ A} + 8\text{ A} = 2\text{ A} + 10\text{ A}$$
$$\mathbf{12\text{ A} = 12\text{ A}} \quad \text{(checks)}$$

In technology, the term **node** is commonly used to refer to a junction of two or more branches. Therefore, this term is used frequently in the analyses to follow.

EXAMPLE 16 Determine currents I_3 and I_4 in Fig. 32 using Kirchhoff's current law.

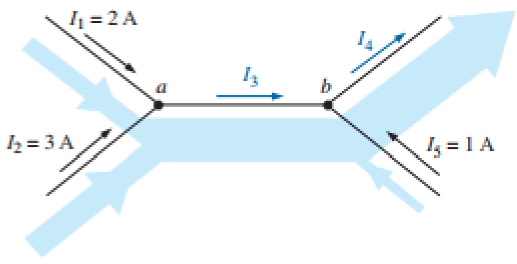


FIG. 32
Two-node configuration for Example 16.

Solution: There are two junctions or nodes in Fig. 32. Node a has only one unknown, while node b has two unknowns. Since a single equation can be used to solve for only one unknown, we must apply Kirchhoff's current law to node a first.

At node a

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 = I_3$$
$$2\text{ A} + 3\text{ A} = I_3 = \mathbf{5\text{ A}}$$

At node b, using the result just obtained,

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_5 = I_4$$
$$5\text{ A} + 1\text{ A} = I_4 = \mathbf{6\text{ A}}$$

Note that in Fig. 32, the width of the blue-shaded regions matches the magnitude of the current in that region.

EXAMPLE 17 Determine currents I_1, I_3, I_4, and I_5 for the network in Fig. 33.

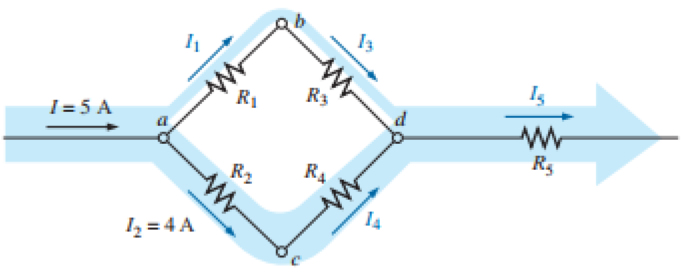


FIG. 33
Four-node configuration for Example 17.

Solution: In this configuration, four nodes are defined. Nodes a and c have only one unknown current at the junction, so Kirchhoff's current law can be applied at either junction.

At node a

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$
$$5\text{ A} = I_1 + 4\text{ A}$$

and
$$I_1 = 5\text{ A} - 4\text{ A} = \mathbf{1\text{ A}}$$

VOLTAGE DIVISION IN A SERIES CIRCUIT

the voltage across series resistive elements will divide as the magnitude of the resistance levels.

In other words,

in a series resistive circuit, the larger the resistance, the more of the applied voltage it will capture.

In addition,

the ratio of the voltages across series resistors will be the same as the ratio of their resistance levels.

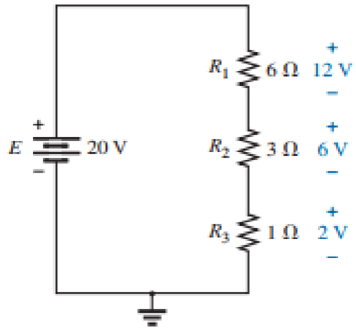


FIG. 33
Revealing how the voltage will divide across series resistive elements.

Voltage Divider Rule (VDR)

the voltage across a resistor in a series circuit is equal to the value of that resistor times the total applied voltage divided by the total resistance of the series configuration.

of the circuit. The rule itself can be derived by analyzing the simple series circuit in Fig. 36.

First, determine the total resistance as follows:

$$R_T = R_1 + R_2$$

Then
$$I_s = I_1 = I_2 = \frac{E}{R_T}$$

Apply Ohm's law to each resistor:

$$V_1 = I_1R_1 = \left(\frac{E}{R_T}\right)R_1 = R_1\frac{E}{R_T}$$

$$V_2 = I_2R_2 = \left(\frac{E}{R_T}\right)R_2 = R_2\frac{E}{R_T}$$

The resulting format for V_1 and V_2 is

$$\boxed{V_x = R_x\frac{E}{R_T}} \quad \text{(voltage divider rule)} \qquad \textbf{(10)}$$

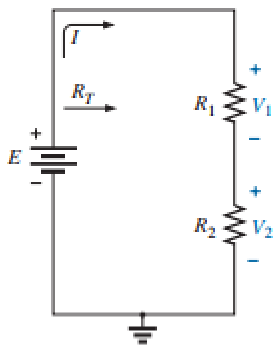


FIG. 36
Developing the voltage divider rule.

Voltage Sources and Ground

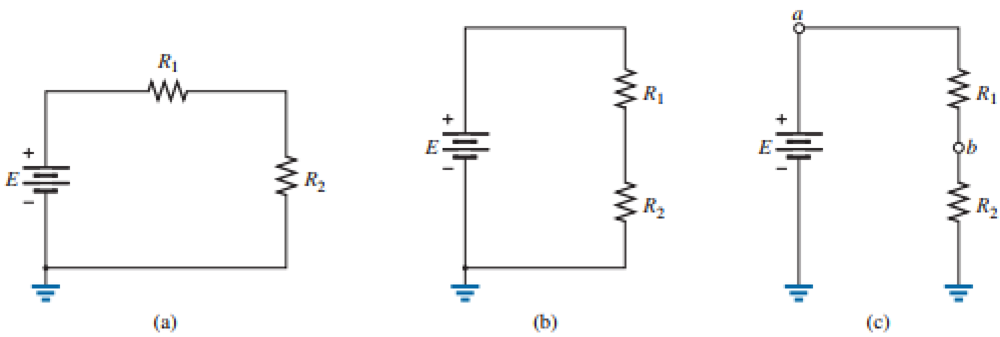


FIG. 46
Three ways to sketch the same series dc circuit.

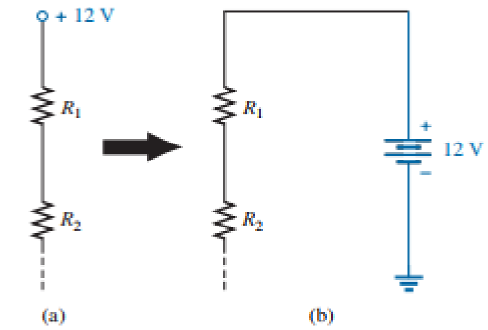


FIG. 47
Replacing the special notation for a dc voltage source with the standard symbol.

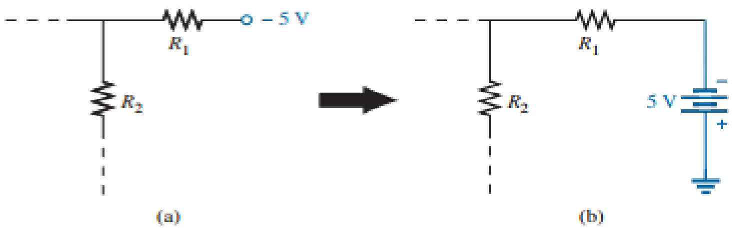


FIG. 48
Replacing the notation for a negative dc supply with the standard notation.

Double-Subscript Notation

FIG. 50
Defining the sign for double-subscript notation.

The double-subscript notation Vab specifies point a as the higher potential. If this is not the case, a negative sign must be associated with the magnitude of Vab.

Single-Subscript Notation

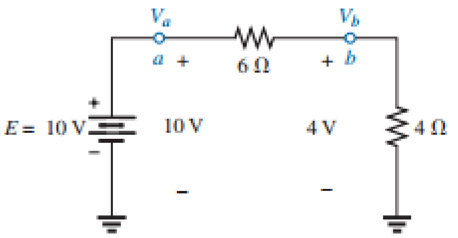
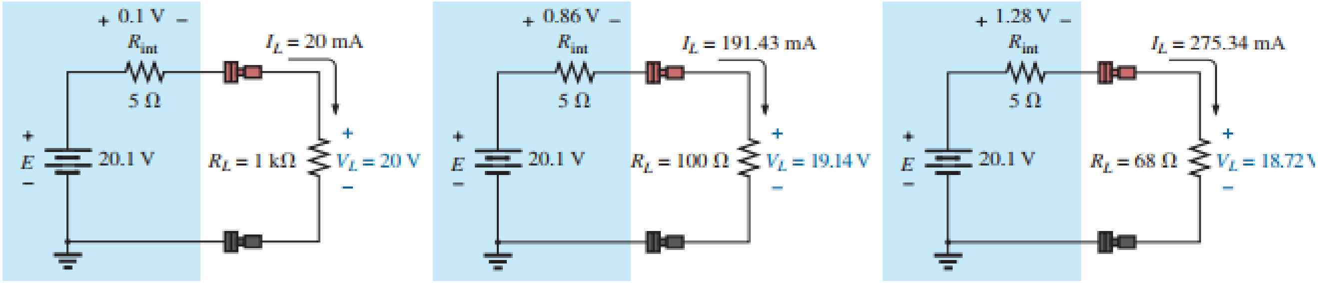


FIG. 51
Defining the use of single-subscript notation for voltage levels.

$$\boxed{V_{ab} = V_a - V_b}$$

The single-subscript notation Va specifies the voltage at point a with respect to ground (zero volts). If the voltage is less than zero volts, a negative sign must be associated with the magnitude of Va

VOLTAGE REGULATION AND THE INTERNAL RESISTANCE OF VOLTAGE SOURCES
every practical (real-world) supply has an internal resistance in series with idealized voltage source



(a) (b) (c)

FIG. 66
Demonstrating the effect of changing a load on the terminal voltage of a supply.

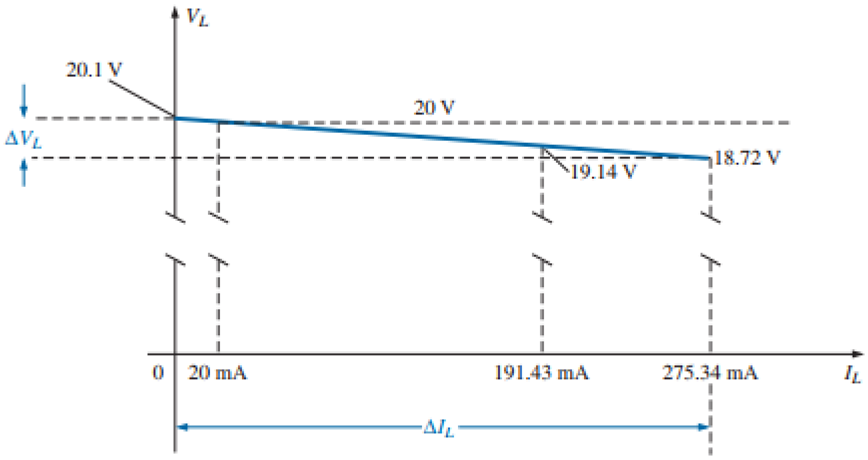
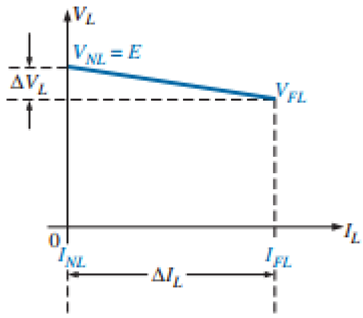


FIG. 67
Plotting V_L versus I_L for the supply in Fig. 66.

$$\boxed{R_{int} = \frac{\Delta V_L}{\Delta I_L}} \qquad \text{(ohms, } \Omega\text{)}$$

which for the plot in Fig. 67 results in

$$R_{int} = \frac{\Delta V_L}{\Delta I_L} = \frac{20.1\ \text{V} - 18.72\ \text{V}}{275.34\ \text{mA} - 0\ \text{mA}} = \frac{1.38\ \text{V}}{275.34\ \text{mA}} = \mathbf{5\ \Omega}$$



$$\boxed{VR = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\%}$$

voltage regulation (abbreviated VR; often called load regulation on specification sheets)

CURRENT DIVIDER RULE

For two parallel elements of equal value, the current will divide equally.

For parallel elements with different values, the smaller the resistance, the greater is the share of input current.

For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistance values.

EXAMPLE 21

a. Determine currents I_1 and I_3 for the network in Fig. 39.
b. Find the source current I_s.

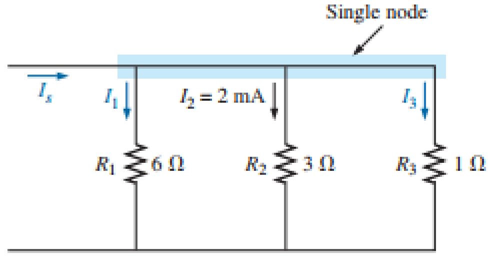


FIG. 39
Parallel network for Example 21.

Solutions:

a. Since R_1 is twice R_2, the current I_1 must be one-half I_2, and

$$I_1 = \frac{I_2}{2} = \frac{2\text{ mA}}{2} = \mathbf{1\ mA}$$

Since R_2 is three times R_3, the current I_3 must be three times I_2, and

$$I_3 = 3I_2 = 3(2\text{ mA}) = \mathbf{6\ mA}$$

b. Applying Kirchhoff's current law gives

$$\Sigma I_i = \Sigma I_o$$
$$I_s = I_1 + I_2 + I_3$$
$$I_s = 1\text{ mA} + 2\text{ mA} + 6\text{ mA} = \mathbf{9\ mA}$$

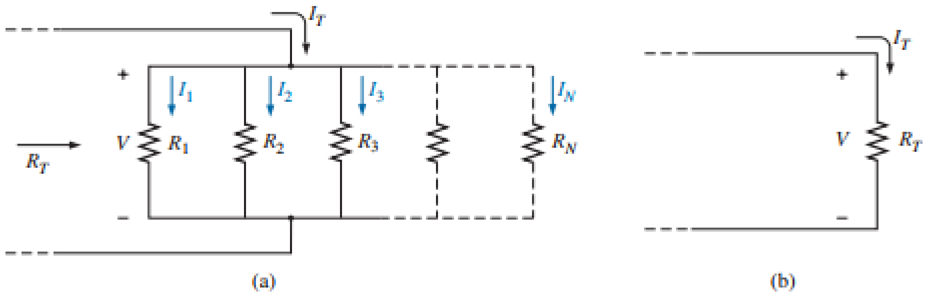


FIG. 40
Deriving the current divider rule: (a) parallel network of N parallel resistors; (b) reduced equivalent of part (a).

Since the voltage V is the same across parallel elements, the following is true:

$$V = I_1R_1 = I_2R_2 = I_3R_3 = \cdots = I_xR_x$$

where the product I_xR_x refers to any combination in the series.

Substituting for V in the above equation for I_T, we have

$$I_T = \frac{I_xR_x}{R_T}$$

Solving for I_x, the final result is the **current divider rule:**

$$\boxed{I_x = \frac{R_T}{R_x}I_T} \qquad \textbf{(14)}$$

the current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistance of the resistor of interest and multiplied by the total current entering the parallel configuration.

Special Case: Two Parallel Resistors

For the case of two parallel resistors as shown in Fig. 42, the total resistance is determined by

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Substituting R_T into Eq. (14) for current I_1 results in

$$I_1 = \frac{R_T}{R_1} I_T = \frac{\left(\dfrac{R_1 R_2}{R_1 + R_2}\right)}{R_1} I_T$$

and

$$\boxed{I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_T} \qquad \textbf{(15a)}$$

Similarly, for I_2,

$$\boxed{I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T} \qquad \textbf{(15b)}$$

Eq. (15) states that

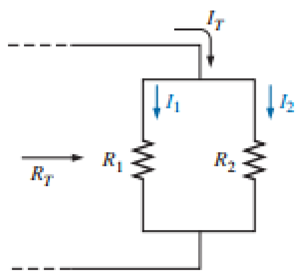


FIG. 42
Deriving the current divider rule for the special case of only two parallel resistors.

VOLTAGE SOURCES IN PARALLEL

voltage sources can be placed in parallel only if they have the same voltage

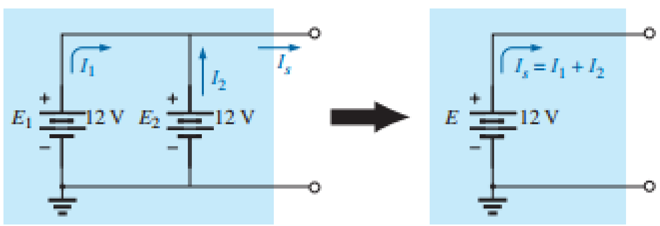


FIG. 46
Demonstrating the effect of placing two ideal supplies of the same voltage in parallel.

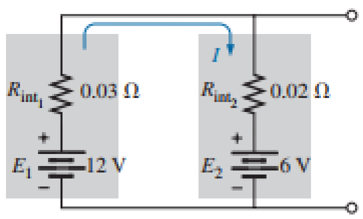


FIG. 47
Examining the impact of placing two lead-acid batteries of different terminal voltages in parallel.

$$I = \frac{E_1 - E_2}{R_{int_1} + R_{int_2}} = \frac{12\text{ V} - 6\text{ V}}{0.03\ \Omega + 0.02\ \Omega} = \frac{6\text{ V}}{0.05\ \Omega} = 120\text{ A}$$

it is always recommended that when you are replacing batteries in series or parallel, replace all the batteries.

OPEN AND SHORT CIRCUITS

an open circuit can have a potential difference (voltage) across its terminals, but the current is always zero amperes.

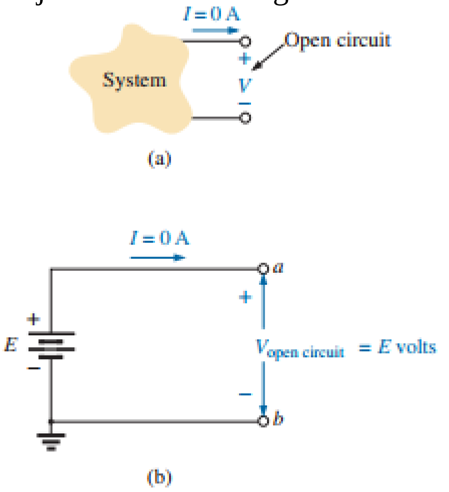


FIG. 48
Defining an open circuit.

a short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.

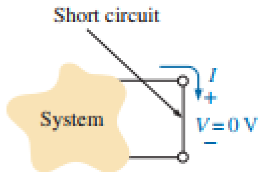


FIG. 50
Defining a short circuit.

EXAMPLE 27 Determine the unknown voltage and current for each network in Fig. 56.

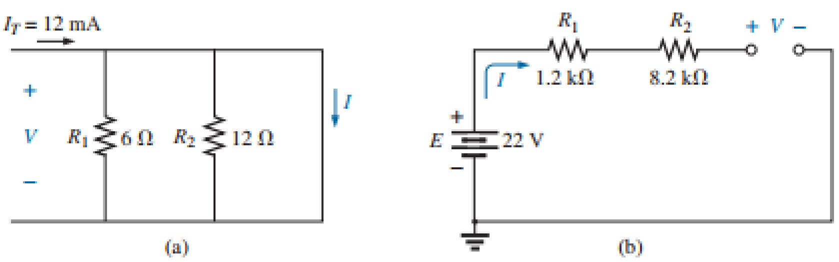


FIG. 56
Networks for Example 27.

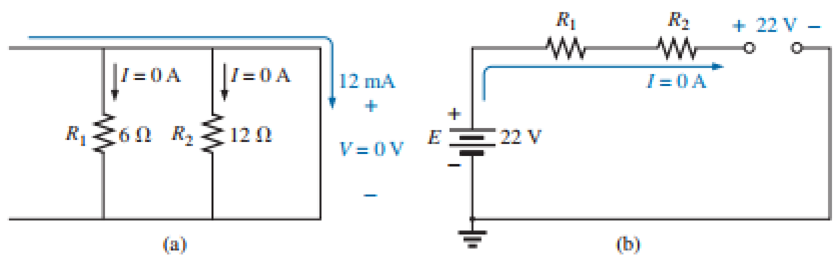


FIG. 57
Solutions to Example 27.