Approximate Analysis

The input section of the voltage-divider configuration can be represented by the network of Fig1. The resistance R_i is the equivalent resistance between base and ground for the transistor with an emitter resistor R_E. The reflected resistance between base and emitter is defined by $Ri = (\beta+1)R_E$. If Ri is much larger than the resistance R_2, the current $I_B$will be much smaller than I_2 (current always seeks the path of least resistance) and I_2 will be approximately equal to I_1. If we accept the approximation that $I_B$is essentially zero amperes compared to $I1$ or $I2$, then $I1 = I2$ and R_1 and R_2 can be considered series element

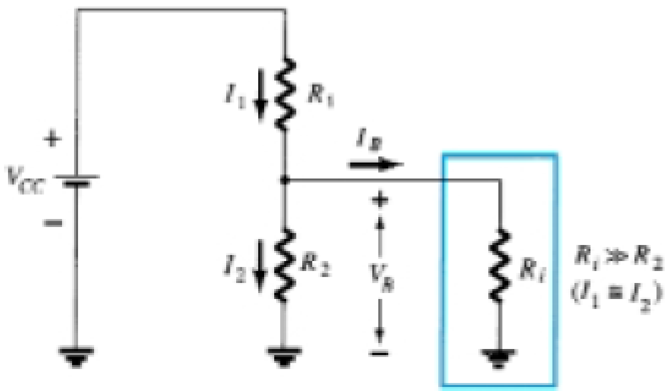

Fig.1: Partial-bias circuit for calculating the approximate base voltage V_B.

$$\boxed{V_B = \frac{R_2 V_{CC}}{R_1 + R_2}}$$

Since $Ri = (\beta+1)R_E$the condition that will define whether the approximate approach can be applied will be the following:

$$\boxed{\beta R_E \geq 10R_2}$$

In other words, if β times the value of $R_E$is at least 10 times the value of R_2, the approximate approach can be applied with a high degree of accuracy. Once $V_B$is determined, the level of $V_E$can be calculated from:

$$\boxed{V_E = V_B - V_{BE}}$$

and the emitter current can be determined from

$$\boxed{I_E = \frac{V_E}{R_E}}$$

and

$$\boxed{I_{C_Q} \cong I_E}$$

Assist. Lecturer: AMMAR ISSA ISMAEL

The collector-to-emitter voltage is determined by

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E$$

but since $I_E \cong I_C$,

$$\boxed{V_{CE_Q} = V_{CC} - I_C(R_C + R_E)}$$

Note in the sequence of calculations from above equations that β does not appear and $I_B$was not calculated. The *Q*-point (as determined by $I_{CQ}$and V_{CEQ}) is therefore independent of the value of β.

EAMPLE: 4.8:- Repeat the analysis of Fig.2 using the approximate technique, and compare solutions for $I_{CQ}$and V_{CEQ}.

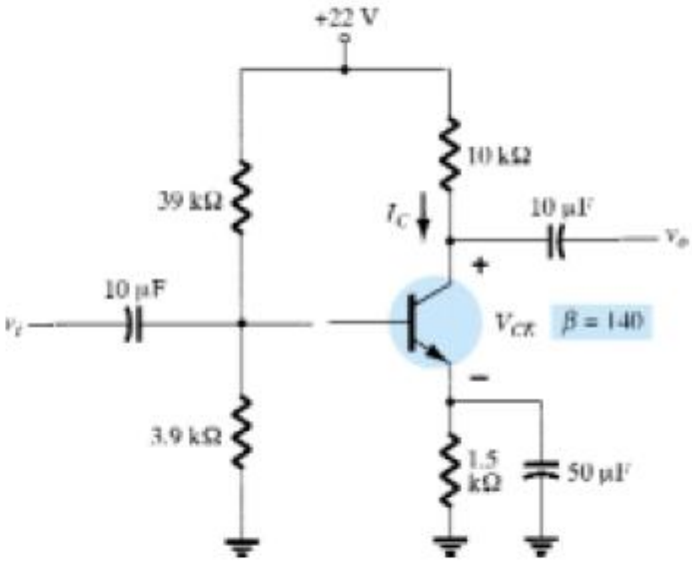

Fig.2: for example 4.7

Testing:

$$\beta R_E \geq 10R_2$$

$$(140)(1.5\ \text{k}\Omega) \geq 10(3.9\ \text{k}\Omega)$$

$$210\ \text{k}\Omega \geq 39\ \text{k}\Omega\ (\textit{satisfied})$$

$$V_B = \frac{R_2 V_{CC}}{R_1 + R_2} = \frac{(3.9\ \text{k}\Omega)(22\ \text{V})}{39\ \text{k}\Omega + 3.9\ \text{k}\Omega} = 2\ \text{V}$$

Note that the level of $V_B$is the same as E_{Th} determined in Example 4.7. Essentially, therefore, the primary difference between the exact and approximate techniques is the effect of R_{Th} in the exact analysis that separates E_{Th} and V_B.

$$V_E = V_B - V_{BE}$$
$$= 2\text{ V} - 0.7\text{ V}$$
$$= 1.3\text{ V}$$

compared to 0.85 mA with the exact analysis. Finally,

$$V_{CE_Q} = V_{CC} - I_C(R_C + R_E)$$

=22 V _ (0.867 mA)(10 kΩ +1.5 kΩ)

=22 V _ 9.97 V
= **12.03 V**

versus 12.22 V obtained in Example 4.7

The results for $I_{CQ}$and $V_{CEQ}$are certainly close, and considering the actual variation in parameter values one can certainly be considered as accurate as the other. The larger the level of $R_i$compared to R_2, the closer the approximate to the exact solution.

EXAMPLE:4.9:- Repeat the exact analysis of Example 4.7 if β is reduced to 70, and compare solutions for $I_{CQ}$and V_{CEQ}.

Solution

This example is not a comparison of exact versus approximate methods but a testing of how much the Q-point will move if the level of β is cut in half. R_{Th} and $E_{Th}$are the same:

$$R_{Th} = 3.55\text{ k}\Omega, \qquad E_{Th} = 2\text{ V}$$
$$I_B = \frac{E_{Th} - V_{BE}}{R_{Th} + (\beta + 1)R_E}$$
$$= \frac{2\text{ V} - 0.7\text{ V}}{3.55\text{ k}\Omega + (71)(1.5\text{ k}\Omega)} = \frac{1.3\text{ V}}{3.55\text{ k}\Omega + 106.5\text{ k}\Omega}$$
$$= 11.81\ \mu\text{A}$$
$$I_{C_Q} = \beta I_B$$
$$= (70)(11.81\ \mu\text{A})$$
$$= 0.83\text{ mA}$$

$$V_{CE_Q} = V_{CC} - I_C(R_C + R_E)$$
$$= 22\text{ V} - (0.83\text{ mA})(10\text{ k}\Omega + 1.5\text{ k}\Omega)$$
$$= \mathbf{12.46\text{ V}}$$

Tabulating the results, we have:

β	I_{C_Q} (mA)	V_{CE_Q} (V)
140	0.85	12.22
70	0.83	12.46

The results clearly show the relative insensitivity of the circuit to the change in β. Even though β is drastically cut in half, from 140 to 70, the levels of *ICQ* and *VCEQ*are essentially the same.

EXAMPLE:4.10:- Determine the levels of $I_{CQ}$and $V_{CEQ}$for the voltage-divider configuration of Fig.3using the exact and approximate techniques and compare solutions. In this case, the conditions of Eq. ($\beta R_E \geq 10R2$) will not be satisfied but the results will reveal the difference in solution if the criterion of Eq. ($\beta R_E \geq 10R2$) is ignored.

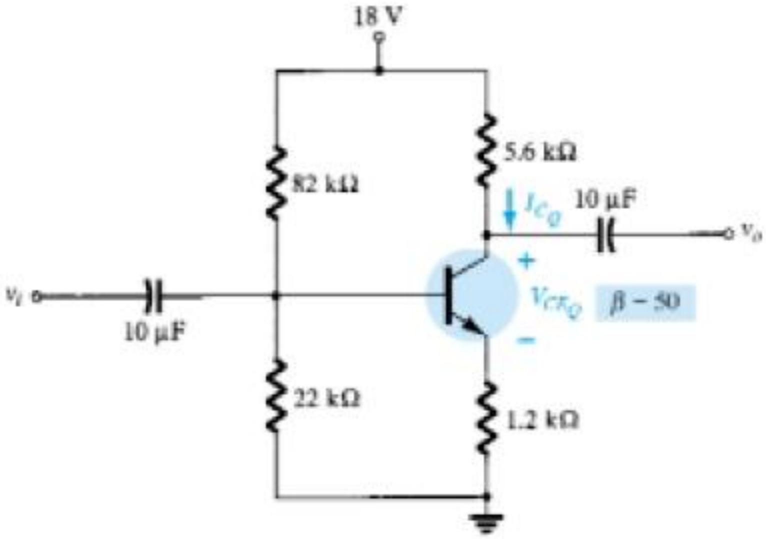

Fig.3: Voltage-divider configuration for Example 4.10.

Exact Analysis

$$\text{Eq. (4.33):} \quad \beta R_E \geq 10R_2$$
$$(50)(1.2\ \text{k}\Omega) \geq 10(22\ \text{k}\Omega)$$
$$60\ \text{k}\Omega \ngeq 220\ \text{k}\Omega\ (\textit{not satisfied})$$

$$R_{\text{Th}} = R_1 \| R_2 = 82\ \text{k}\Omega \| 22\ \text{k}\Omega = 17.35\ \text{k}\Omega$$

$$E_{\text{Th}} = \frac{R_2 V_{CC}}{R_1 + R_2} = \frac{22\ \text{k}\Omega(18\ \text{V})}{82\ \text{k}\Omega + 22\ \text{k}\Omega} = 3.81\ \text{V}$$

$$I_B = \frac{E_{\text{Th}} - V_{BE}}{R_{\text{Th}} + (\beta + 1)R_E} = \frac{3.81\ \text{V} - 0.7\ \text{V}}{17.35\ \text{k}\Omega + (51)(1.2\ \text{k}\Omega)} = \frac{3.11\ \text{V}}{78.55\ \text{k}\Omega}$$
$$= 39.6\ \mu\text{A}$$

$$I_{C_Q} = \beta I_B = (50)(39.6\ \mu\text{A}) = \mathbf{1.98\ mA}$$

$$V_{CE_Q} = V_{CC} - I_C(R_C + R_E)$$
$$= 18\ \text{V} - (1.98\ \text{mA})(5.6\ \text{k}\Omega + 1.2\ \text{k}\Omega)$$
$$= \mathbf{4.54\ V}$$

Approximate Analysis

$$V_B = E_{\text{Th}} = 3.81\ \text{V}$$
$$V_E = V_B - V_{BE} = 3.81\ \text{V} - 0.7\ \text{V} = 3.11\ \text{V}$$
$$I_{C_Q} \cong I_E = \frac{V_E}{R_E} = \frac{3.11\ \text{V}}{1.2\ \text{k}\Omega} = \mathbf{2.59\ mA}$$
$$V_{CE_Q} = V_{CC} - I_C(R_C + R_E)$$
$$= 18\ \text{V} - (2.59\ \text{mA})(5.6\ \text{k}\Omega + 1.2\ \text{k}\Omega)$$
$$= \mathbf{3.88\ V}$$

Tabulating the results, we have:

	I_{C_Q} (mA)	V_{CE_Q} (V)
Exact	1.98	4.54
Approximate	2.59	3.88

The results reveal the difference between exact and approximate solutions. $I_{CQ}$is about30% greater with the approximate solution, while $V_{CEQ}$is about 10% less. The results are notably different in magnitude, but even though β$R_E$is only about three times larger than R_2, the results are still relatively close to each other. For the future, however, our analysis will be dictated by Eq.(βR_E≥10R2)to ensure a close similarity betweenexact and approximate solutions.

Assist. Lecturer: AMMAR ISSA ISMAEL

DC BIAS WITH VOLTAGE FEEDBACK

An improved level of stability can also be obtained by introducing a feedback path from collector to base. Although the *Q*-point is not totally independent of beta (even under approximate conditions),the sensitivity to changes inbeta or temperature variations is normally less than encountered for the fixed-bias or emitter-biased configurations.

$$V_{CC} - I'_C R_C - I_B R_B - V_{BE} - I_E R_E = 0$$

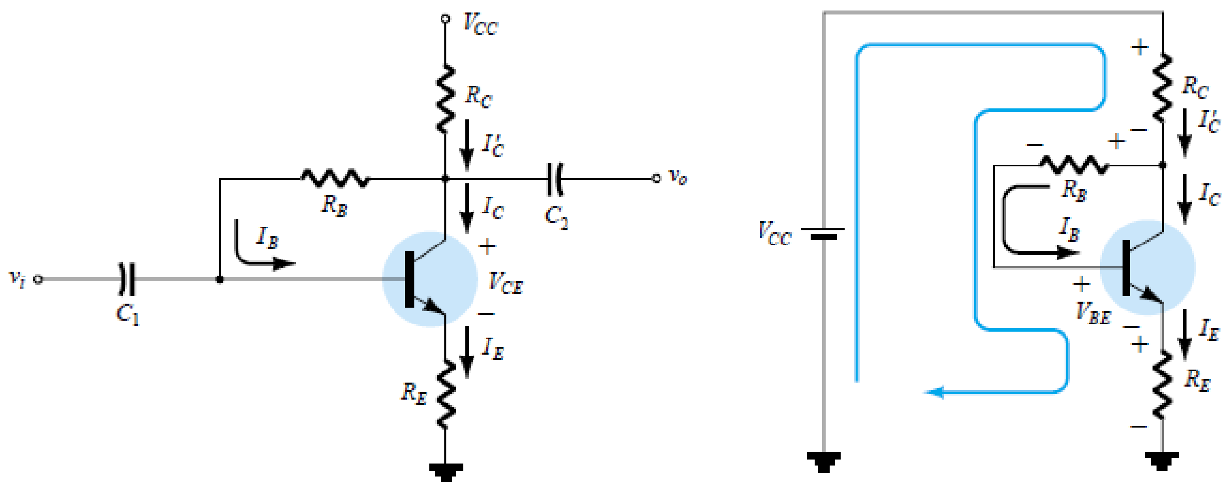

Fig.4: dc bias circuit with voltage feedback.

$I'_C \cong I_C = \beta I_B$ and $I_E \cong I_C$

$$V_{CC} - \beta I_B R_C - I_B R_B - V_{BE} - \beta I_B R_E = 0$$

Gathering terms, we have

$$V_{CC} - V_{BE} - \beta I_B (R_C + R_E) - I_B R_B = 0$$

and solving for I_B yields

$$\boxed{I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta(R_C + R_E)}}$$

In general, the equation for $I_B$has had the following format:

$$I_B = \frac{V'}{R_B + \beta R'}$$

Since $I_C = \beta I_B$,

$$I_{C_Q} = \frac{\beta V'}{R_B + \beta R'}$$

In general, the larger $\beta R'$ is compared to R_B, the less the sensitivity of $I_{CQ}$to variationsin beta.

Obviously, if $\beta R' \gg R_B$and $R_B+\beta R' \cong \beta R'$, then

$$I_{C_Q} = \frac{\beta V'}{R_B + \beta R'} \cong \frac{\beta V'}{\beta R'} = \frac{V'}{R'}$$

$$I_E R_E + V_{CE} + I'_C R_C - V_{CC} = 0$$

Since $I'_C \cong I_C$ and $I_E \cong I_C$, we have

$$I_C(R_C + R_E) + V_{CE} - V_{CC} = 0$$

and $$\boxed{V_{CE} = V_{CC} - I_C(R_C + R_E)}$$

Example:4.11: Determine the quiescent levels of *ICQ* and *VCEQ* for the network of Fig.5

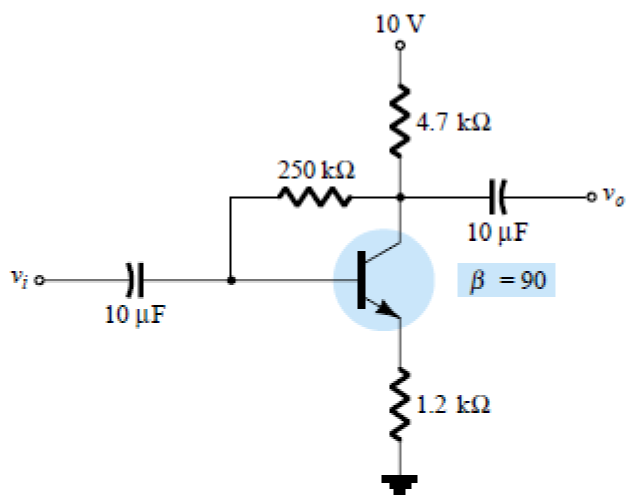

Fig.5: example 4.11

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta(R_C + R_E)}$$
$$= \frac{10\text{ V} - 0.7\text{ V}}{250\text{ k}\Omega + (90)(4.7\text{ k}\Omega + 1.2\text{ k}\Omega)}$$
$$= \frac{9.3\text{ V}}{250\text{ k}\Omega + 531\text{ k}\Omega} = \frac{9.3\text{ V}}{781\text{ k}\Omega}$$
$$= 11.91\ \mu\text{A}$$
$$I_{C_Q} = \beta I_B = (90)(11.91\ \mu\text{A})$$
$$= \mathbf{1.07\ mA}$$
$$V_{CE_Q} = V_{CC} - I_C(R_C + R_E)$$
$$= 10\text{ V} - (1.07\text{ mA})(4.7\text{ k}\Omega + 1.2\text{ k}\Omega)$$
$$= 10\text{ V} - 6.31\text{ V}$$
$$= \mathbf{3.69\ V}$$

EXAMPLE 4.12: Repeat Example 4.11 using a beta of 135 (50% more than Example 4.11).

I_B=8.89 µA

I_{CQ}= 1.2 mA, V_{CEQ}= 2.92 V

Even though the level of β increased 50%, the level of I_{CQ}only increased 12.1%while the level of V_{CEQ}decreased about 20.9%.

EXAMPLE 4.13: homework

EXAMPLE.4.15: Determine V_C and V_B for the network of Fig.6.

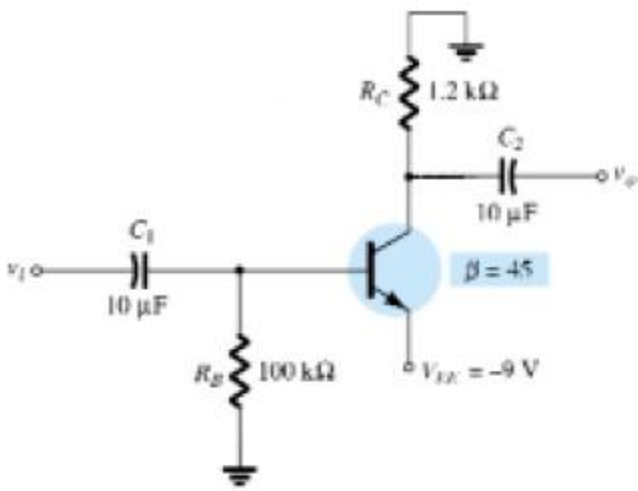

Fig.6: EXAMPLE.4.15

$$-I_B R_B - V_{BE} + V_{EE} = 0$$

and
$$I_B = \frac{V_{EE} - V_{BE}}{R_B}$$

Substitution yields

$$I_B = \frac{9\text{ V} - 0.7\text{ V}}{100\text{ k}\Omega}$$
$$= \frac{8.3\text{ V}}{100\text{ k}\Omega}$$
$$= 83\ \mu\text{A}$$
$$I_C = \beta I_B$$
$$= (45)(83\ \mu\text{A})$$
$$= 3.735\text{ mA}$$

$$
\begin{aligned}
V_C &= -I_C R_C \\
&= -(3.735 \text{ mA})(1.2 \text{ k}\Omega) \\
&= \mathbf{-4.48 \text{ V}} \\
V_B &= -I_B R_B \\
&= -(83 \ \mu\text{A})(100 \text{ k}\Omega) \\
&= \mathbf{-8.3 \text{ V}}
\end{aligned}
$$

EXAMPLE.4.16: Determine *VCEQ* and *IE* for the network of Fig.7.

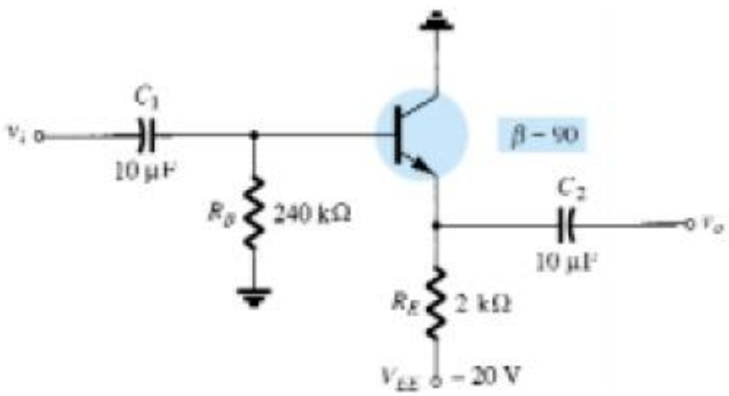

Fig.7: example 4.16

Applying Kirchhoff's voltage law to the input circuit will result in

$$-I_B R_B - V_{BE} - I_E R_E + V_{EE} = 0$$

but
$$I_E = (\beta + 1) I_B$$

and
$$V_{EE} - V_{BE} - (\beta + 1) I_B R_E - I_B R_B = 0$$

with
$$I_B = \frac{V_{EE} - V_{BE}}{R_B + (\beta + 1) R_E}$$

Substituting values yields

$$
\begin{aligned}
I_B &= \frac{20 \text{ V} - 0.7 \text{ V}}{240 \text{ k}\Omega + (91)(2 \text{ k}\Omega)} \\
&= \frac{19.3 \text{ V}}{240 \text{ k}\Omega + 182 \text{ k}\Omega} = \frac{19.3 \text{ V}}{422 \text{ k}\Omega} \\
&= 45.73 \ \mu\text{A} \\
I_C &= \beta I_B \\
&= (90)(45.73 \ \mu\text{A}) \\
&= 4.12 \text{ mA}
\end{aligned}
$$

Applying Kirchhoff's voltage law to the output circuit, we have

$$-V_{EE} + I_E R_E + V_{CE} = 0$$

but $$I_E = (\beta + 1)I_B$$

and $$V_{CE_Q} = V_{EE} - (\beta + 1)I_B R_E$$
$$= 20\text{ V} - (91)(45.73\ \mu\text{A})(2\text{ k}\Omega)$$
$$= \mathbf{11.68\ V}$$
$$I_E = \mathbf{4.16\ mA}$$

EXAMPLE.4.17: Determine the voltage *VCB* and the current *IB* for the common-base configuration of Fig.8.

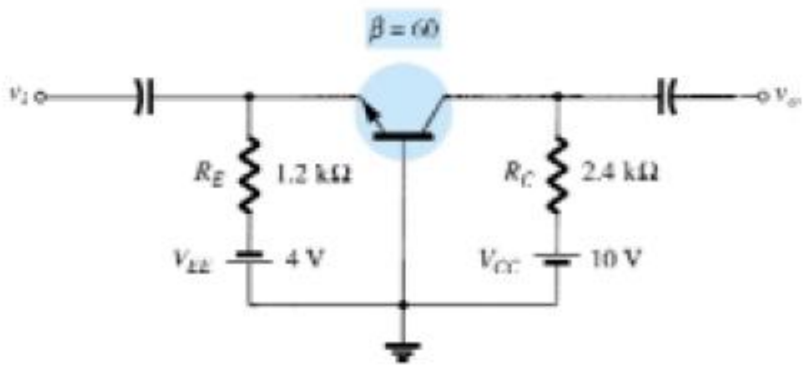

Fig.8: EXAMPLE.4.17

Solution

Applying Kirchhoff's voltage law to the input circuit yields

$$-V_{EE} + I_E R_E + V_{BE} = 0$$

and $$I_E = \frac{V_{EE} - V_{BE}}{R_E}$$

Substituting values, we obtain

$$I_E = \frac{4\text{ V} - 0.7\text{ V}}{1.2\text{ k}\Omega} = 2.75\text{ mA}$$

Applying Kirchhoff's voltage law to the output circuit gives

$$-V_{CB} + I_C R_C - V_{CC} = 0$$

and $$V_{CB} = V_{CC} - I_C R_C \text{ with } I_C \cong I_E$$
$$= 10\text{ V} - (2.75\text{ mA})(2.4\text{ k}\Omega)$$
$$= \mathbf{3.4\ V}$$
$$I_B = \frac{I_C}{\beta}$$
$$= \frac{2.75\text{ mA}}{60}$$
$$= \mathbf{45.8\ \mu A}$$

***EXAMPLE 4.18* :Homework**

DESIGN OPERATIONS

EXAMPLE.4.19: Given the device characteristics of Fig.9a, determine V_{CC}, R_B, and R_C for the fixed bias configuration of Fig.9b.

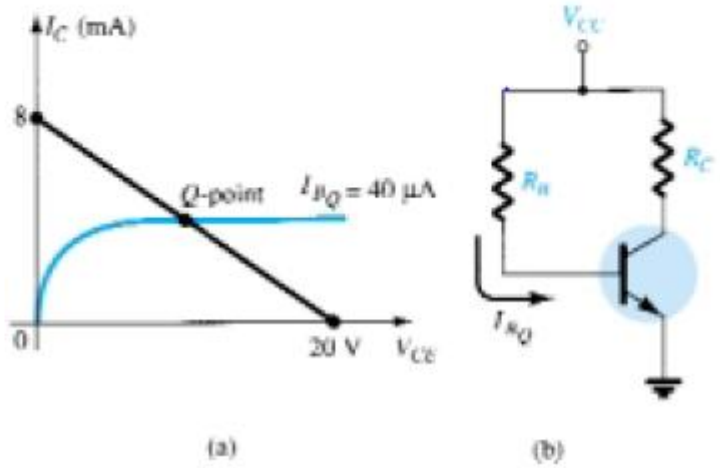

Fig.9: EXAMPLE.4.19

Solution

From the load line

$$V_{CC} = \mathbf{20\ V}$$

$$I_C = \left.\frac{V_{CC}}{R_C}\right|_{V_{CE} = 0\text{ V}}$$

and

$$R_C = \frac{V_{CC}}{I_C} = \frac{20\text{ V}}{8\text{ mA}} = \mathbf{2.5\ k\Omega}$$

$$I_B = \frac{V_{CC} - V_{BE}}{R_B}$$

with

$$R_B = \frac{V_{CC} - V_{BE}}{I_B}$$

$$= \frac{20\text{ V} - 0.7\text{ V}}{40\ \mu\text{A}} = \frac{19.3\text{ V}}{40\ \mu\text{A}}$$

$$= \mathbf{482.5\ k\Omega}$$

Standard resistor values:
R_C =2.4 k
R_B = 470 k
Using standard resistor values gives
I_B = 41.1 µA
which is well within 5% of the value specified.

EXAMPLE 4.20 homework

EXAMPLE. 4.21: The emitter-bias configuration of Fig.10 has the following specifications: I_{CQ} =(1/2) I_{Csat}, I_{Csa}t 8 mA, V_C = 18 V, and β = 110. Determine R_C, R_E, and R_B.

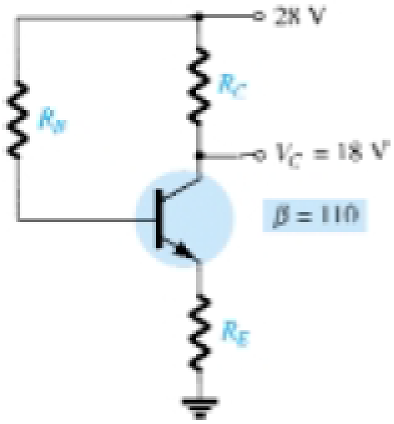

Fig.10: EXAMPLE.4.21

Solution

$$I_{C_Q} = \tfrac{1}{2} I_{C_{sat}} = 4 \text{ mA}$$

$$R_C = \frac{V_{R_C}}{I_{C_Q}} = \frac{V_{CC} - V_C}{I_{C_Q}}$$

$$= \frac{28 \text{ V} - 18 \text{ V}}{4 \text{ mA}} = \mathbf{2.5\ k\Omega}$$

$$I_{C_{sat}} = \frac{V_{CC}}{R_C + R_E}$$

and

$$R_C + R_E = \frac{V_{CC}}{I_{C_{sat}}} = \frac{28 \text{ V}}{8 \text{ mA}} = 3.5 \text{ k}\Omega$$

$$R_E = 3.5 \text{ k}\Omega - R_C$$

$$= 3.5 \text{ k}\Omega - 2.5 \text{ k}\Omega$$

$$= \mathbf{1\ k\Omega}$$

$$I_{B_Q} = \frac{I_{C_Q}}{\beta} = \frac{4 \text{ mA}}{110} = 36.36\ \mu\text{A}$$

$$I_{B_Q} = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E}$$

and

$$R_B + (\beta + 1)R_E = \frac{V_{CC} - V_{BE}}{I_{B_Q}}$$

with
$$R_B = \frac{V_{CC} - V_{BE}}{I_{B_Q}} - (\beta + 1)R_E$$
$$= \frac{28\text{ V} - 0.7\text{ V}}{36.36\ \mu\text{A}} - (111)(1\text{ k}\Omega)$$
$$= \frac{27.3\text{ V}}{36.36\ \mu\text{A}} - 111\text{ k}\Omega$$
$$= \mathbf{639.8\ k\Omega}$$

For standard values:

$R_C = 2.4\text{ k}\Omega,\ R_E = 1\text{ k}\Omega,\ R_B = 620\text{ k}\Omega$

TRANSISTOR SWITCHING NETWORKS

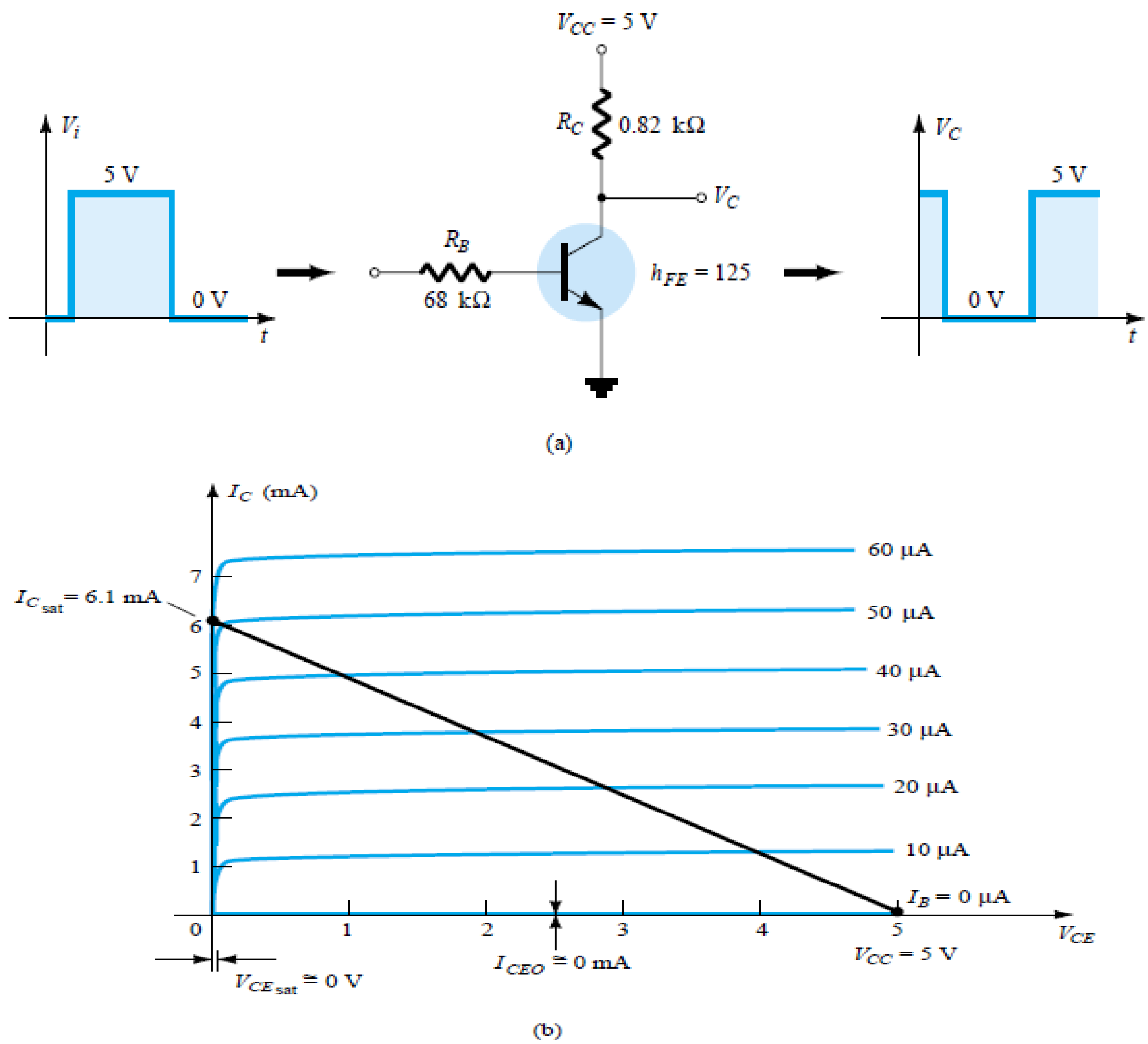

Fig.11: Transistor inverter.

Proper design for the inversion process requires that the operating point switch from cutoff to saturation along the load line depicted in Fig11. In addition, we will assume that $V_{CE} = V_{CEsat} = 0$ V rather than the typical 0.1- to 0.3-V level.

When $Vi = 5$ V, the transistor will be "on" and the design must ensure that the network is heavily saturated by a level of I_B greater than that associated with the I_B curve appearing near the saturation level. In Fig. 11b, this requires that $IB = 50$ µA.

The saturation level for the collector current for the circuit of Fig. 11a is defined by

$$\boxed{I_{C_{\text{sat}}} = \frac{V_{CC}}{R_C}}$$

The level of I_B in the active region just before saturation results can be approximated by the following equation:

$$I_{B_{\max}} \cong \frac{I_{C_{\text{sat}}}}{\beta_{\text{dc}}}$$

For the saturation level we must therefore ensure that the following condition is satisfied:

$$\boxed{I_B > \frac{I_{C_{\text{sat}}}}{\beta_{\text{dc}}}}$$

For the network of Fig. 11b, when $Vi = 5$ V, the resulting level of IB is the following:

$$I_B = \frac{V_i - 0.7\text{ V}}{R_B} = \frac{5\text{ V} - 0.7\text{ V}}{68\text{ k}\Omega} = 63\ \mu\text{A}$$

and $$I_{C_{\text{sat}}} = \frac{V_{CC}}{R_C} = \frac{5\text{ V}}{0.82\text{ k}\Omega} \cong 6.1\text{ mA}$$

Testing

$$I_B = 63\ \mu\text{A} > \frac{I_{C_{\text{sat}}}}{\beta_{\text{dc}}} = \frac{6.1\text{ mA}}{125} = 48.8\ \mu\text{A}$$

which is satisfied. Certainly, any level of I_B greater than 60 µA will pass through a Q-point on the load line that is very close to the vertical axis. For $Vi = 0$ V, $I_B = 0$ µA, and since we are assuming that $I_C = I_{CEQ} = 0$mA, the voltage drop across R_C as determined by $V_{RC} = I_C R_C = 0$ V, resulting in $V_C = 5$ V for the response indicated in Fig. 11a. In addition to its contribution to computer logic, the transistor can also be employed as a switch using the same extremities of the load line. At saturation, the current I_C is quite high and the voltage V_{CE} very low. The result is a resistance level between the two terminals determined by:

$$R_{sat} = \frac{V_{CE_{sat}}}{I_{C_{sat}}}$$

Using a typical average value of $V_{CE_{sat}}$ such as 0.15 V gives

$$R_{sat} = \frac{V_{CE_{sat}}}{I_{C_{sat}}} = \frac{0.15\text{ V}}{6.1\text{ mA}} = 24.6\ \Omega$$

which is a relatively low value and $\cong 0\ \Omega$ when placed in series with resistors in the kilohm range.

For Vi = 0 V, the cutoff condition will result in a resistance level of the following magnitude:

$$R_{cutoff} = \frac{V_{CC}}{I_{CEO}} = \frac{5\text{ V}}{0\text{ mA}} = \infty\ \Omega$$

resulting in the open-circuit equivalence. For a typical value of I_{CEQ} = 10 μA, the magnitude of the cutoff resistance is

$$R_{cutoff} = \frac{V_{CC}}{I_{CEO}} = \frac{5\text{ V}}{10\ \mu\text{A}} = 500\text{ k}\Omega$$

which certainly approaches an open-circuit equivalence for many situations.