APPROXIMATE HYBRID EQUIVALENT CIRCUIT

The analysis using the approximate hybrid equivalent circuit of Fig.1 for the common-emitter configuration is very similar to that just performed using the *re* model. Although time and priorities do not permit a detailed analysis of all the configurations discussed thus far, a brief overview of some of the most important will be included in this section to demonstrate the similarities in approach and the resulting equations.

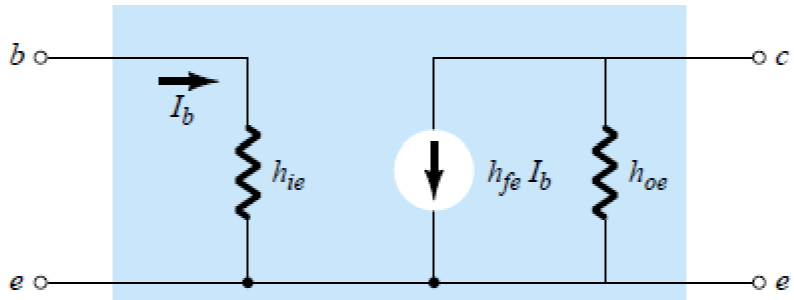


Fig.1: Approximate common-emitter hybrid equivalent circuit.

$h_{ie} = \beta r_e$, $h_{fe} = \beta$, $h_{oe} = 1/r_o$, $h_{fb} = -\alpha$, and $h_{ib} = r_e$

Fixed-Bias Configuration

the small-signal ac equivalent network will appear as shown in Fig. 8.38 using the approximate common-emitter hybrid equivalent model.

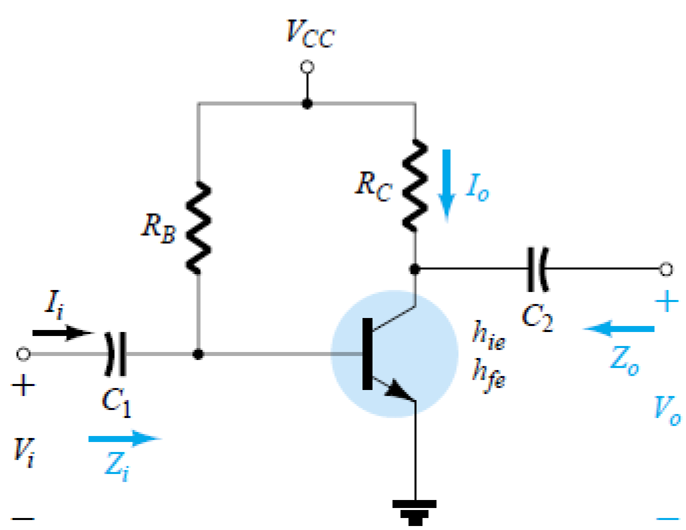


Fig.2: Fixed-bias configuration.

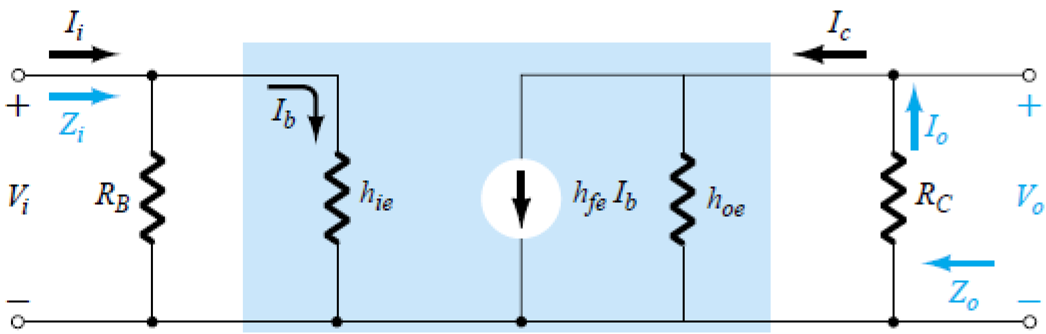


Fig.3: Substituting the approximate hybrid equivalent circuit into the ac equivalent network of Fig.2

$$\boxed{Z_i = R_B \| h_{ie}}$$

From Fig.3

$$\boxed{Z_o = R_C \| 1/h_{oe}}$$

From Fig.3

A_v: Using $R' = 1/h_{oe} \| R_C$,

$$V_o = -I_o\, R' = -I_C R'$$
$$= -h_{fe}\, I_b\, R'$$

and
$$I_b = \frac{V_i}{h_{ie}}$$

with
$$V_o = -h_{fe} \frac{V_i}{h_{ie}}\, R'$$

so that
$$\boxed{A_v = \frac{V_o}{V_i} = -\frac{h_{fe}(R_C \| 1/h_{oe})}{h_{ie}}}$$

A_i: Assuming that $R_B \gg h_{ie}$ and $1/h_{oe} \geq 10R_C$, then $I_b \cong I_i$ and $I_o = I_c = h_{fe} I_b = h_{fe}\, I_i$ with

$$\boxed{A_i = \frac{I_o}{I_i} \cong h_{fe}}$$

For the network of Fig. 8.39, determine:

EXAMPLE 8.11

(a) Z_i.
(b) Z_o.
(c) A_v.
(d) A_i.

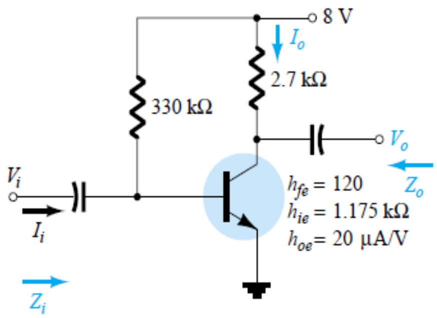


Figure 8.39 Example 8.11.

Solution

(a) $Z_i = R_B \| h_{ie} = 330\text{ k}\Omega \| 1.175\text{ k}\Omega$

$\cong h_{ie} = \mathbf{1.171\text{ k}\Omega}$

(b) $r_o = \dfrac{1}{h_{oe}} = \dfrac{1}{20\ \mu\text{A/V}} = 50\text{ k}\Omega$

$Z_o = \dfrac{1}{h_{oe}} \| R_C = 50\text{ k}\Omega \| 2.7\text{ k}\Omega = \mathbf{2.56\text{ k}\Omega} \cong R_C$

(c) $Av = -\dfrac{h_{fe}(R_C \| 1/h_{oe})}{h_{ie}} = -\dfrac{(120)(2.7\text{ k}\Omega \| 50\text{ k}\Omega)}{1.171\text{ k}\Omega} = \mathbf{-262.34}$

(d) $A_i \cong h_{fe} = 120$

Voltage-Divider Configuration

$$R' = R_1 \| R_2.$$

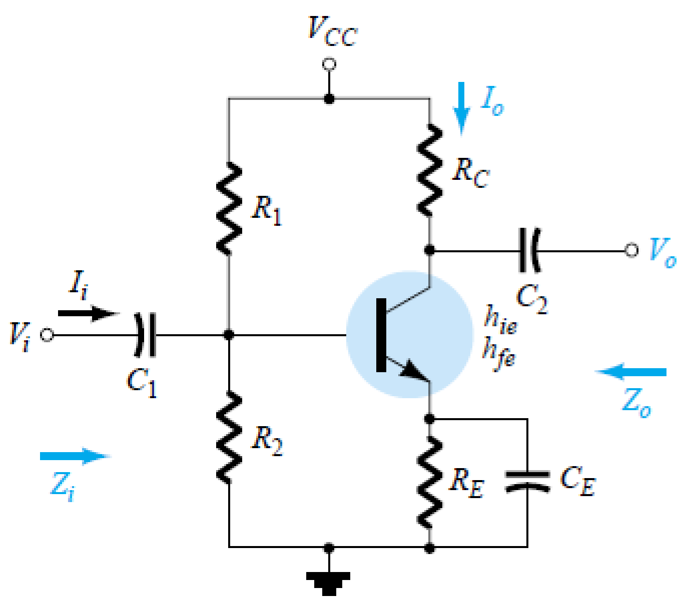


Fig.4: Voltage-divider bias configuration.

$$Z_i = R' \| h_{ie}$$

$$Z_o \cong R_C$$

A_v:

$$A_v = -\frac{h_{fe}(R_C \| 1/h_{oe})}{h_{ie}}$$

A_i:

$$A_i = -\frac{h_{fe}R'}{R' + h_{ie}}$$

Unbypassed Emitter-Bias Configuration

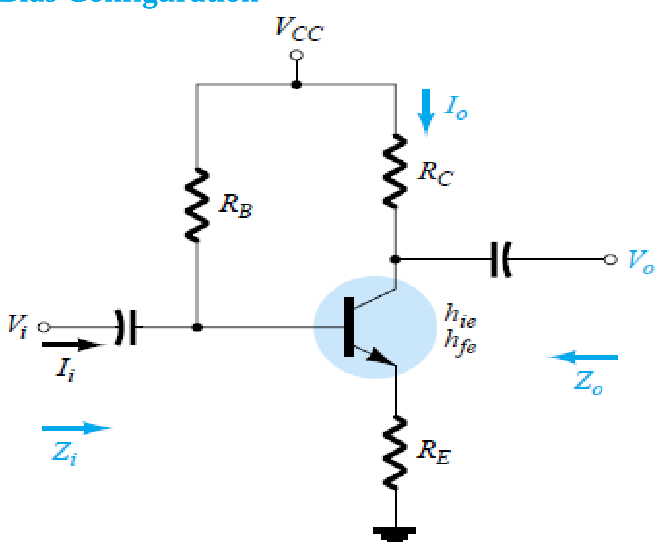


Fig. 5:CE unbypassed emitter-bias configuration.

Z_i:

$$Z_b \cong h_{fe} R_E$$

and

$$Z_i = R_B \| Z_b$$

Z_o:

$$Z_o = R_C$$

A_v:

$$A_v = -\frac{h_{fe}\, R_C}{Z_b} \cong -\frac{h_{fe} R_C}{h_{fe}\, R_E}$$

and

$$\boxed{A_v \cong -\frac{R_C}{R_E}}$$

A_i:

$$\boxed{A_i = \frac{h_{fe} R_B}{R_B + Z_b}}$$

Or

$$\boxed{A_i = -A_v \frac{Z_i}{R_C}}$$

COMPLETE HYBRID EQUIVALENT MODEL

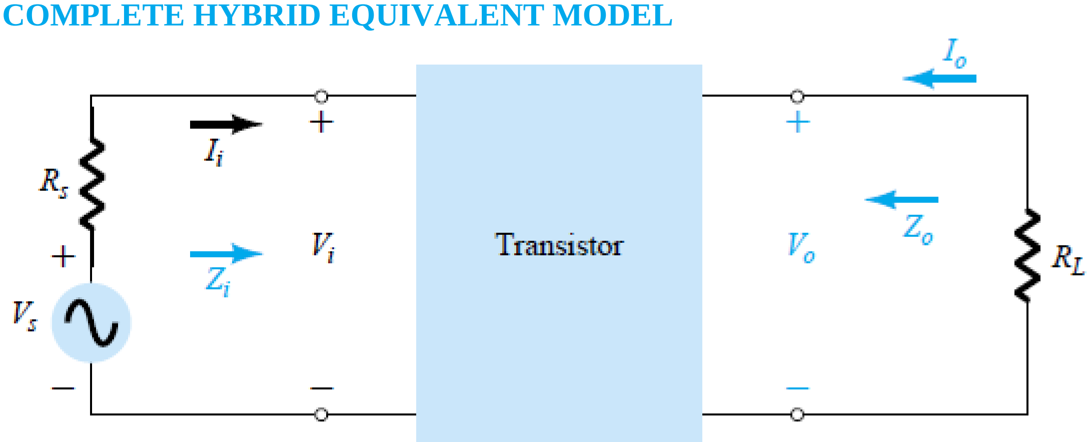


Fig.6: Two-port system.

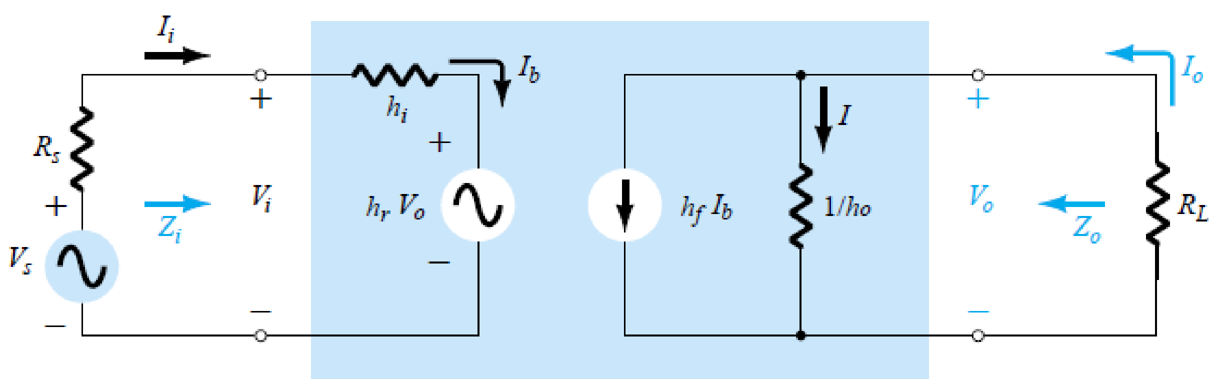


Fig.7: Substituting the complete hybrid equivalent circuit into the two-port system of Fig.6

Current Gain, $A_i = I_o/I_i$

Applying Kirchhoff's current law to the output circuit yields

$$I_o = h_f I_b + I = h_f I_i + \frac{V_o}{1/h_o} = h_f I_i + h_o V_o$$

Substituting $V_o = -I_o R_L$ gives us

$$I_o = h_f I_i - h_o R_L I_o$$

Rewriting the equation above, we have

$$I_o + h_o R_L I_o = h_f I_i$$

and

$$I_o(1 + h_o R_L) = h_f I_i$$

so that

$$\boxed{A_i = \frac{I_o}{I_i} = \frac{h_f}{1 + h_o R_L}}$$

Note that the current gain will reduce to the familiar result of $A_i = h_f$ if the factor $h_o R_L$ is sufficiently small compared to 1.

Voltage Gain, $Av = Vo/Vi$

Applying Kirchhoff's voltage law to the input circuit results in

$Vi = Iihi + hrVo$

Substituting $Ii = (1 + hoR_L)Io/h_f$ and $Io = Vo/R_L$ from above results in

$$V_i = \frac{-(1 + h_o R_L)h_i}{h_f R_L} V_o + h_r V_o$$

Solving for the ratio V_o/V_i yields

$$\boxed{A_v = \frac{V_o}{V_i} = \frac{-h_f R_L}{h_i + (h_i h_o - h_f h_r)R_L}}$$

In this case, the familiar form of $Av = -hfRL/hi$ will return if the factor $(hiho - hf\ hr)RL$ is sufficiently small compared to hi.

Input Impedance, $Z_i = V_i/I_i$

For the input circuit,

$$V_i = h_i I_i + h_r V_o$$

Substituting $V_o = -I_o R_L$

we have $V_i = h_i I_i - h_r R_L I_o$

Since $A_i = \frac{I_o}{I_i}$

$$I_o = A_i I_i$$

so that the equation above becomes

$$V_i = h_i I_i - h_r R_L A_i I_i$$

Solving for the ratio V_i/I_i, we obtain

$$Z_i = \frac{V_i}{I_i} = h_i - h_r R_L A_i$$

and substituting

$$A_i = \frac{h_f}{1 + h_o R_L}$$

yields

$$\boxed{Z_i = \frac{V_i}{I_i} = h_i \frac{- h_f h_r R_L}{1 + h_o R_L}}$$

The familiar form of *Zi* = *hi* will be obtained if the second factor is sufficiently smaller than the first.

Output Impedance, $Z_o = V_o/I_o$

The output impedance of an amplifier is defined to be the ratio of the output voltage to the output current with the signal V_s set to zero. For the input circuit with $V_s = 0$,

$$I_i = \frac{-h_r V_o}{R_s + h_i}$$

Substituting this relationship into the following equation obtained from the output circuit yields

$$I_o = h_f I_i + h_o V_o$$

$$= \frac{-h_f h_r V_o}{R_s + h_i} + h_o V_o$$

$$\boxed{Z_o = \frac{V_o}{I_o} = \frac{1}{h_o - [h_f h_r/(h_i + R_s)]}}$$

In this case, the output impedance will reduce to the familiar form *Zo* = 1/*ho* for the transistor when the second factor in the denominator is sufficiently smaller than the first

EXAMPLE 8.13 For the network of Fig. 8.49, determine the following parameters using the complete hybrid equivalent model and compare to the results obtained using the approximate model.

(a) Z_i and Z'_i.
(b) A_v.
(c) $A_i = I_o/I_i$ and $A'_i = I_o/I'_i$.
(d) Z_o (within R_C) and Z'_o (including R_C).

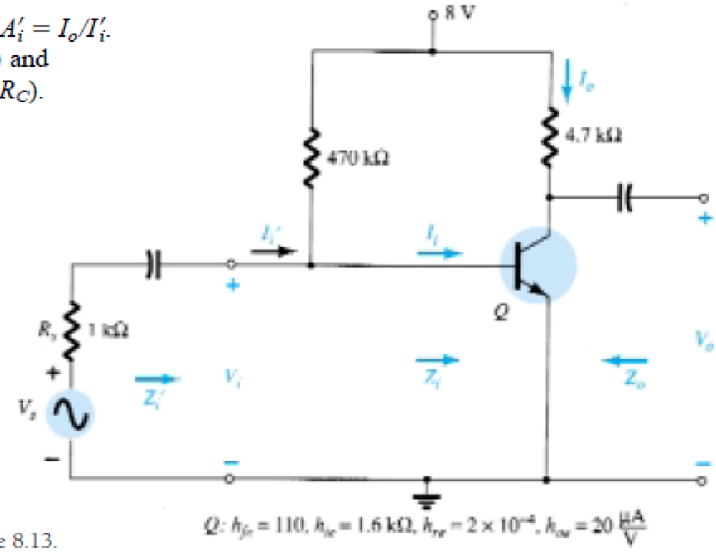


Figure 8.49 Example 8.13.

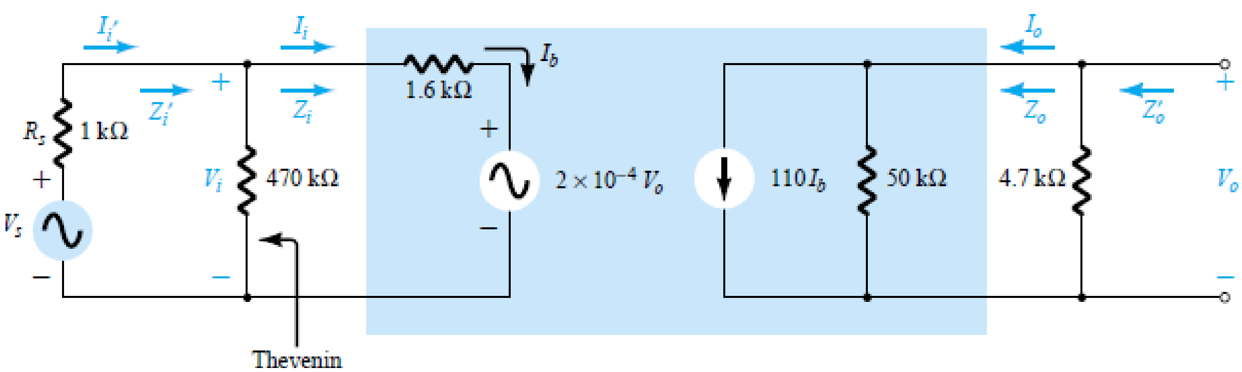


Fig.9: Substituting the complete hybrid equivalent circuit into the ac equivalent network

A Thévenin equivalent circuit for the input section of Fig. 8.50 will result in the input equivalent of Fig.10 since $E_{Th} = V_s$ and $R_{Th} = R_s$ = 1 k_ (a result of R_B = 470 kΩ being much greater than R_s =1 kΩ). In this example, $R_L = R_C$ and I_o is defined as the current through R_C

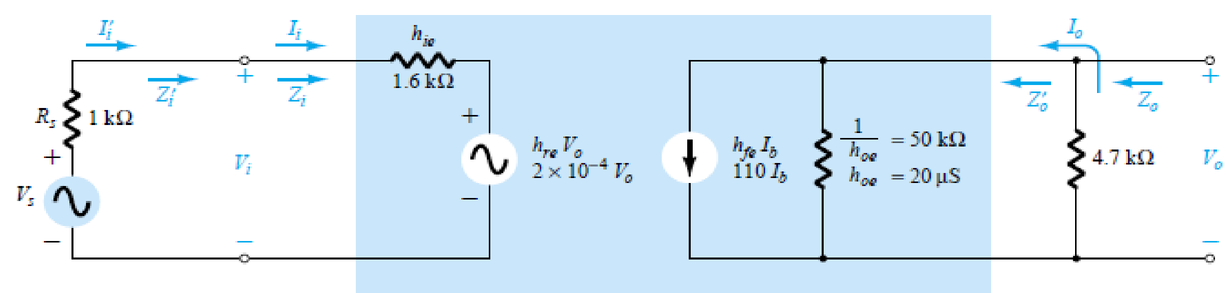


Fig.10: Replacing the input section of Fig.9 with a Thévenin equivalent circuit.

a)

$$Z_i = \frac{V_i}{I_i} = h_{ie} - \frac{h_{fe}h_{re}R_L}{1 + h_{oe}R_L}$$
$$= 1.6\ \text{k}\Omega - \frac{(110)(2 \times 10^{-4})(4.7\ \text{k}\Omega)}{1 + (20\ \mu\text{S})(4.7\ \text{k}\Omega)}$$
$$= 1.6\ \text{k}\Omega - 94.52\ \Omega$$
$$= \mathbf{1.51\ k\Omega}$$

versus 1.6 kΩ using simply h_{ie}.

$$Z_i' = 470\ \text{k}\Omega \| Z_i \cong Z_i = \mathbf{1.51\ k\Omega}$$

b)

$$A_v = \frac{V_o}{V_i} = \frac{-h_{fe}R_L}{h_{ie} + (h_{ie}h_{oe} - h_{fe}h_{re})R_L}$$
$$= \frac{-(110)(4.7\ \text{k}\Omega)}{1.6\ \text{k}\Omega + [(1.6\ \text{k}\Omega)(20\ \mu\text{S}) - (110)(2 \times 10^{-4})]4.7\ \text{k}\Omega}$$
$$= \frac{-517 \times 10^3\ \Omega}{1.6\ \text{k}\Omega + (0.032 - 0.022)4.7\ \text{k}\Omega}$$
$$= \frac{-517 \times 10^3\ \Omega}{1.6\ \text{k}\Omega + 47\ \Omega}$$
$$= \mathbf{-313.9}$$

versus −323.125 using $A_v \cong -h_{fe}R_L/h_{ie}$.

c)

$$A_i = \frac{I_o}{I_i} = \frac{h_{fe}}{1 + h_{oe}R_L} = \frac{110}{1 + (20\ \mu\text{S})(4.7\ \text{k}\Omega)}$$
$$= \frac{110}{1 + 0.094} = \mathbf{100.55}$$

versus 110 using simply h_{fe}. Since 470 kΩ >> Z_i, $I_i' \cong I_i$ and $A_i' \cong$ **100.55** also.

d)

$$Z_o = \frac{V_o}{I_o} = \frac{1}{h_{oe} - [h_{fe}h_{re}/(h_{ie} + R_s)]}$$
$$= \frac{1}{20\ \mu\text{S} - [(110)(2 \times 10^{-4})/(1.6\ \text{k}\Omega + 1\ \text{k}\Omega)]}$$
$$= \frac{1}{20\ \mu\text{S} - 8.46\ \mu\text{S}}$$
$$= \frac{1}{11.54\ \mu\text{S}}$$
$$= \mathbf{86.66\ k\Omega}$$

which is greater than the value determined from $1/h_{oe} = 50\text{ k}\Omega$.

$$Z'_o = R_C \| Z_o = 4.7\text{ k}\Omega \| 86.66\text{ k}\Omega = \mathbf{4.46\text{ k}\Omega}$$

versus $4.7\text{ k}\Omega$ using only R_C.

Exercises

1. For the network of Fig. 8.64:
   (a) Determine Z_i and Z_o.
   (b) Find A_v and A_i.
   (c) Repeat part (a) with $r_o = 20\text{ k}\Omega$.
   (d) Repeat part (b) with $r_o = 20\text{ k}\Omega$.

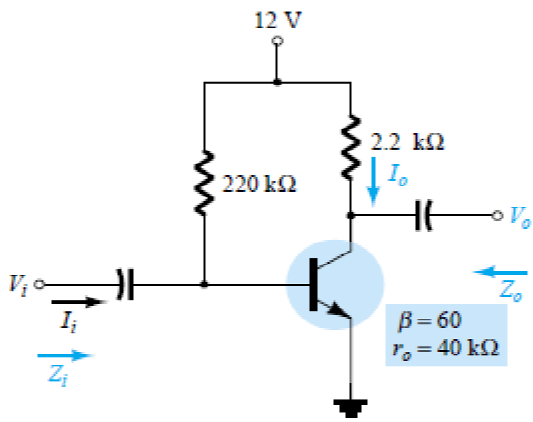


Figure 8.64 Problems 1 and 21

* **3.** For the network of Fig. 8.66:
   (a) Calculate I_B, I_C, and r_e.
   (b) Determine Z_i and Z_o.
   (c) Calculate A_v and A_i.
   (d) Determine the effect of $r_o = 30\text{ k}\Omega$ on A_v and A_i.

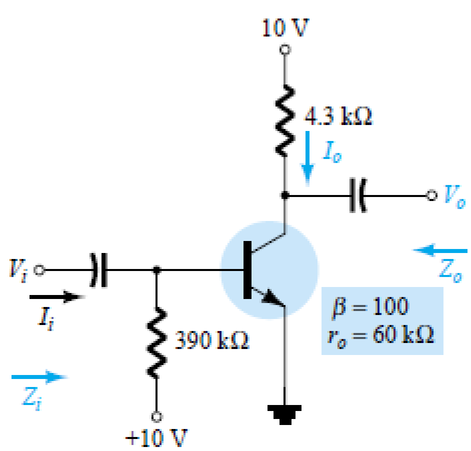


Figure 8.66 Problem 3

1. (a) r_e: $I_B = \dfrac{V_{CC} - V_{BE}}{R_B} = \dfrac{12V - 0.7V}{220k\Omega} = 51.36\mu A$

$I_E = (\beta+1) I_B = (60+1)(51.36\mu A)$

$= 3.13 mA$

$r_e = \dfrac{26mV}{I_E} = \dfrac{26mV}{3.13mA} = 8.31\Omega$

$Z_i = R_B \| \beta r_e = 220k\Omega \| (60)(8.31\Omega) = 220k\Omega \| 498.6\Omega$

$= \underline{497.47\Omega}$

$r_o \geq 10R_C \therefore Z_o = R_C = \underline{2.2k\Omega}$

(b) $A_v = -\dfrac{R_C}{r_e} = -\dfrac{2.2k\Omega}{8.31\Omega} = -\underline{264.74}$

$A_i \cong \beta = \underline{60}$

(c) $Z_i = 497.47\Omega$ (the same)

$Z_o = r_o \| R_C = 20k\Omega \| 2.2k\Omega = \underline{1.98k\Omega}$

(d) $A_v = -\dfrac{R_C \| r_o}{r_e} = \dfrac{-1.98k\Omega}{8.31\Omega} = -\underline{238.27}$

$A_i = -A_v Z_i / R_C = -(-238.27)(497.47\Omega)/2.2k\Omega$

$= \underline{53.88}$

3. (a) $I_B = \dfrac{V_{CC} - V_{BE}}{R_B} = \dfrac{10V - 0.7V}{390k\Omega} = \underline{23.85\mu A}$

$I_E = (\beta+1) I_B = (101)(23.85\mu A) = 2.41mA$

$r_e = \dfrac{26mV}{I_E} = \dfrac{26mV}{2.41mA} = \underline{10.79\Omega}$

$I_C = \beta I_B = (100)(23.85\mu A) = \underline{2.38mA}$

(b) $Z_i = R_B \| \beta r_e = 390k\Omega \| (100)(10.79\Omega) = 390k\Omega \| 1.08k\Omega$

$= \underline{1.08k\Omega}$

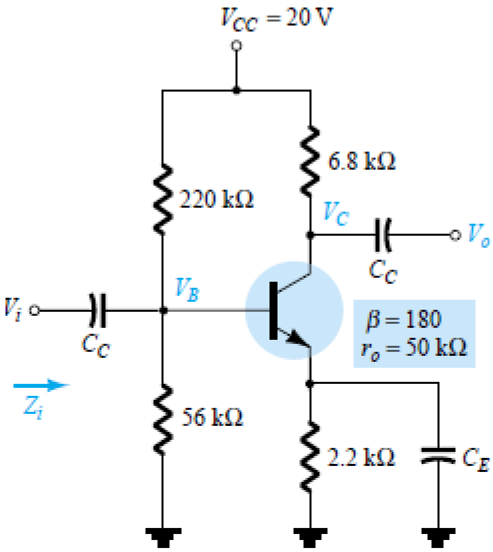
$r_o \geq 10R_C \therefore Z_o = R_C = \underline{4.3k\Omega}$

(c) $A_v = -\dfrac{R_C}{r_e} = -\dfrac{4.3k\Omega}{10.79\Omega} = -\underline{398.52}$

$A_i \cong \beta = \underline{100}$

6. For the network of Fig. 8.69:

(a) Determine r_e.
(b) Calculate V_B and V_C.
(c) Determine Z_i and $A_v = V_o/V_i$.



6. Test $\beta R_E \geq 10 R_2$

$(180)(2.2\,k\Omega) \overset{?}{\geq} 10(56\,k\Omega)$

$396\,k\Omega < 560\,k\Omega$ (not satisfied)

Use exact analysis:

(a) $R_{Th} = 56\,k\Omega \parallel 220\,k\Omega = 44.64\,k\Omega$

$$E_{Th} = \frac{56\,k\Omega(20V)}{220\,k\Omega + 56\,k\Omega} = 4.058V$$

$$I_B = \frac{E_{Th} - V_{BE}}{R_{Th} + (\beta+1)R_E} = \frac{4.058V - 0.7V}{44.64\,k\Omega + (181)(2.2\,k\Omega)}$$

$$= 7.58\,\mu A$$

$$I_E = (\beta+1)I_B = (181)(7.58\,\mu A)$$

$$= 1.372\,mA$$

$$r_e = \frac{26\,mV}{I_E} = \frac{26\,mV}{1.372\,mA} = \underline{18.95\,\Omega}$$

(b) $V_E = I_E R_E = (1.372mA)(2.2k\Omega) = 3.02V$

$V_B = V_E + V_{BE} = 3.02V + 0.7V$

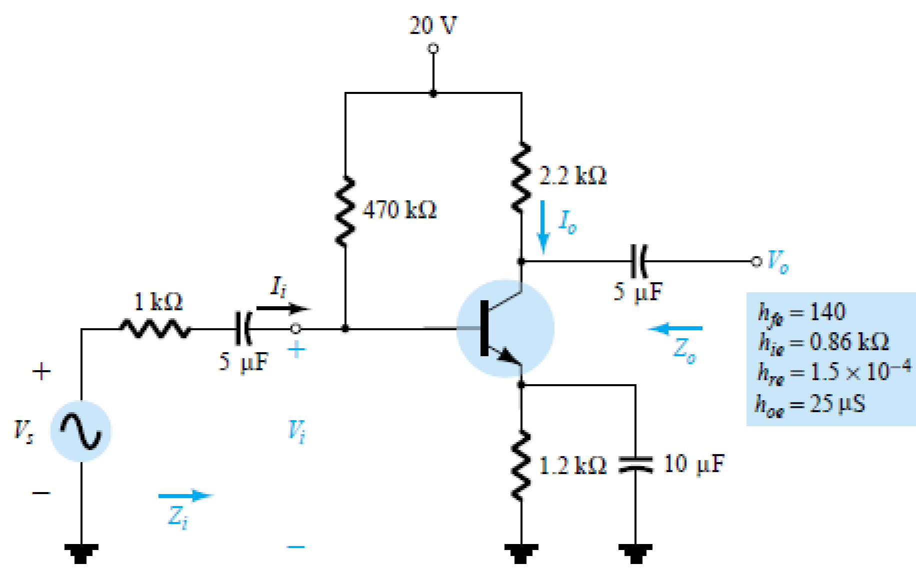
$= \underline{3.72V}$

$V_C = V_{CC} - I_C R_C$

$= 20V - \beta I_B R_C = 20V - (180)(7.58\mu A)(6.8k\Omega)$

$= \underline{10.72V}$

* **25.** For the network of Fig. 8.83, determine:
(a) Z_i.
(b) A_v.
(c) $A_i = I_o/I_i$.
(d) Z_o.



25. a. $Z_i = h_{ie} - \dfrac{h_{fe} h_{re} R_L}{1 + h_{oe} R_L}$

$= 0.86k\Omega - \dfrac{(140)(1.5\times10^{-4})(2.2k\Omega)}{1+(25\mu S)(2.2k\Omega)}$

$= 0.86k\Omega - 43.79\Omega$

$= 816.21\Omega$

$Z_i' = R_B \| Z_i = 470k\Omega \| 801$

$= 470k\Omega \| 816.21\Omega$

$= \underline{814.8\Omega}$

b. $A_v = \dfrac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{fe} h_{re}) R_L}$

$= \dfrac{-(140)(2.2k\Omega)}{0.86k\Omega + ((0.86k\Omega)(25\mu S) - (140)(1.5\times10^{-4}))2.2k\Omega}$

$= \underline{-357.68}$

c. $A_i = \dfrac{I_o}{I_i} = \dfrac{h_{fe}}{1 + h_{oe} R_L} = \dfrac{140}{1+(25\mu S)(2.2k\Omega)}$

$= 132.70$

$A_i' = \dfrac{I_o}{I_i'} = \left(\dfrac{I_o}{I_i}\right)\left(\dfrac{I_i}{I_i'}\right)$ $\qquad I_i = \dfrac{470k\Omega I_i'}{470k\Omega + 0.816k\Omega}$

$= (132.70)(0.998)$ $\qquad \dfrac{I_i}{I_i'} = 0.998$

$= \underline{132.43}$

d. $Z_o = \dfrac{1}{h_{oe} - (h_{fe} h_{re} / (h_{ie} + R_s))}$

$= \dfrac{1}{25\times10^{-6} - ((140)(1.5\times10^{-4})/(0.86k\Omega + 1k\Omega))}$

$= 72.94k\Omega$

$Z_o' = R_C \| Z_o = 2.2k\Omega \| 72.94k\Omega$

$= \underline{2.14k\Omega}$