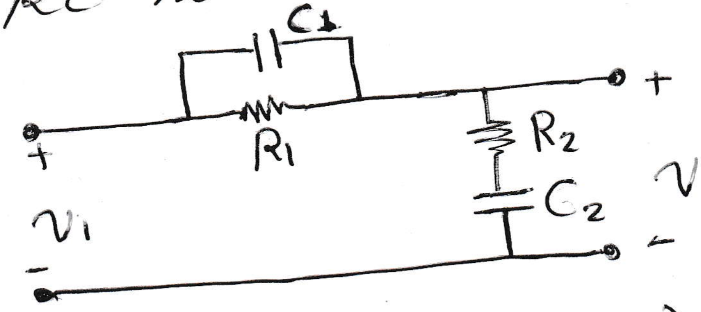
Lag-Lead Compensator

The general form of the transfer function of lag lead compensator is,

$$G_c(s) = \frac{s+\frac{1}{T_1}}{s+\frac{1}{\beta T_1}} \; \frac{s+\frac{1}{T_2}}{s+\frac{1}{\alpha T_2}}$$

$\beta > 1, \alpha < 1, T_1 T_2 > 0.$

The RC network as shown in fig below.



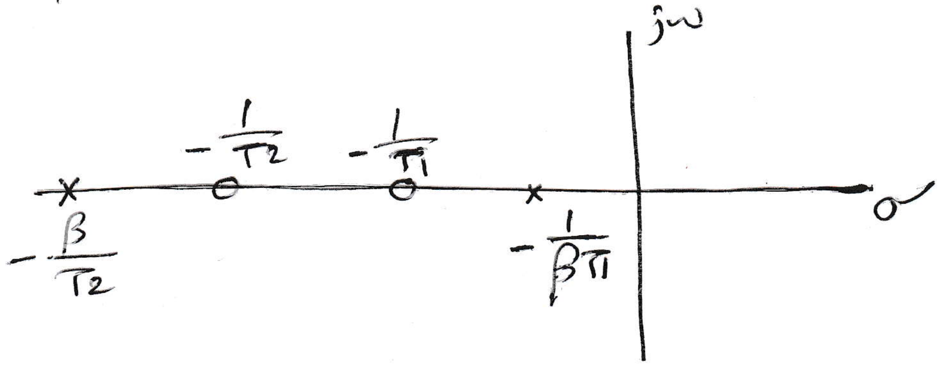
$$G_c(s) = \frac{V_2(s)}{V_c(s)} = \frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$$

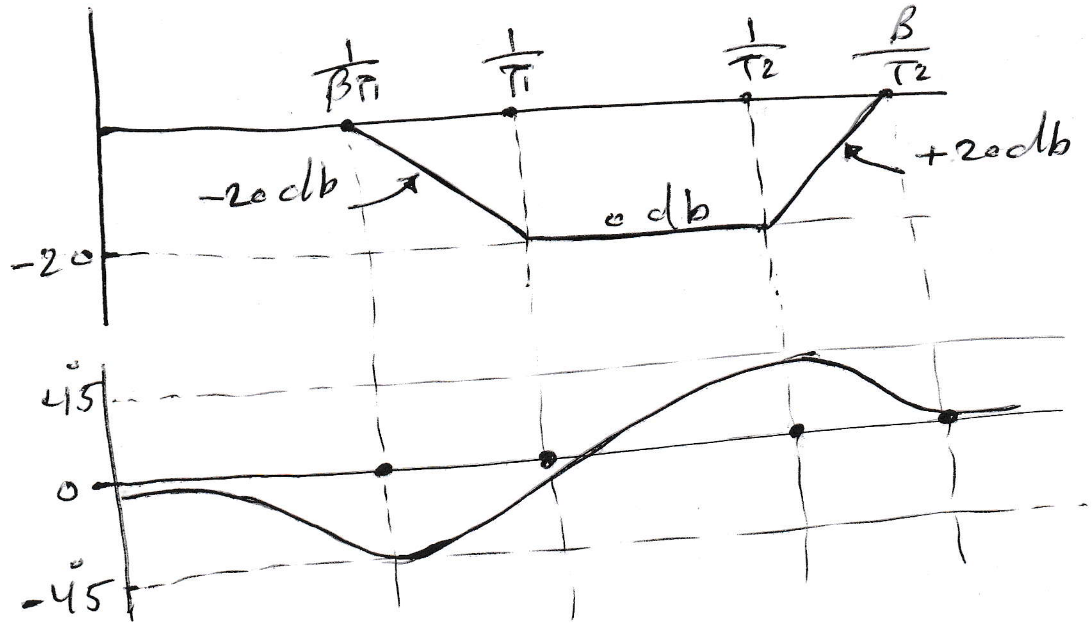
$$T_1 = \frac{1}{R_1C_1}, \quad T_2 = \frac{1}{R_2C_2}$$

$$R_1R_2C_1C_2 = \alpha\beta T_1 T_2$$

and $$\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1} = \frac{1}{\beta T_1}+\frac{1}{\alpha T_2}$$

The pole zero configuration of a lag-lead network is given by





Design of Lag-Lead Compensator:

A lead Compensator increases the gain cross over frequency and hence the bandwidth is also increased.

For higher order Systems with large error Constants, the phase lead required may be large, which turn results larger bandwidth. this may be undesirable due to noise considerations. On the other hand a lag Compensator results in a lower bandwidth and generally a sluggish system. To satisfy the additional requirement on bandwidth, a lag or lead Compensator alone may not be satisfactory.

Hence a lag-lead Compensator is used. The procedure for the design of lag-lead Compensator is as follows:

1 - The required error Constant is satified by choosing a proper gain. The phase margin and the bandwidth are obtained from the Nichols chart. If the phase margin falls short and the bandwidth is smaller than the desired value, a lead Compensator is designed.

2 - If the bandwidth is larger than the desired value large compensator is attempted, if the phase margin desired is available at any frequency. If this results in a lower bandwidth then the desired bandwidth, a lag lead compensator is to be designed.

3 - To design a lag-lead compensator, we start with the design of a lag compensator. A lag compensator is designed to satisfy the phase margin requirement. It means that the gain cross over frequency, ω_{gc2}, is chosen to be higher than that to be used.

4 - the β and ω_{gc2} are known and the corner frequencies of the lag compensator can be obtained. Since $\alpha = \frac{1}{\beta}$, the frequency at which $20 \log \alpha$ is available on the magnitude plot is the new gain crossover frequency ω_{gc3}.

the maximum phase lead available is.

$$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$$

and $\omega_m = \omega_{gc3}$

with these parameters, a lead compensator is designed,

for the lead network,

$$\omega_1 = \frac{1}{\tau} = \omega_m\sqrt{\alpha}$$

$$\omega_2 = \frac{1}{\alpha\tau} = \frac{\omega_m}{\sqrt{\alpha}}$$

the log magnitude vs phase angle curve of the lag-lead compensator is drawn on the Nichols chart, from which the bandwidth can be obtained.

Ex

Consider the system,

$$G(s) = \frac{K}{s(s+5)(s+10)}$$

Design a compensator to satisfy the following specifications.

e_{ss} for a velocity input ≤ 0.045

$\phi_{pm} \geq 45^\circ$

Bandwidth $\omega_b = 11$ rad/sec.

Sol $\cancel{e}$ $e_{ss} = \frac{1}{K_V}$

$$K_V = \frac{1}{e_{ss}} = \frac{1}{0.04} = 25$$

$$\frac{K}{50} = 25 \implies K = 1250.$$

Bode plot and log magnitude Vs phase angle plot on Nichols chart are drawn below.

- From the graph, the phase cross over frequency is 10.5 rad/sec, the phase margin is -22°

From the graph of Nichols chart, the Bandwidth is 13.5 rad/sec.

Since the Bandwidth is already large, a lead Compensator will further increase it. A lag Compensator provides the required phase margin at a frequency of about 2.3 rad/sec, which makes the bandwidth much smaller than desirable. Hence a lag-lead Compensator only can satisfy all the specifications.

- for lag compensator, choose the gain crossover frequency to be 4 rad/sec. At this frequency the gain of uncompensated system is 16 db.

$$\therefore\ 20 \log \beta = 16$$

$$\beta = 6.3 \simeq 8$$

Choose $\omega_1 = 1$ rad/sec, then $\omega_2 = \frac{1}{8} = 0.125$ rad/sec

the transfer function of the lag Compensator is $G_{c1}(s) = \frac{s+1}{8s+1}$

- let us now design a lead compensator. Since $\beta = 8$, $\alpha = \frac{1}{\beta} = \frac{1}{8} = 0.125$.

the maximum phase lead by the lead network is,

$$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$$

$$= \sin^{-1}\frac{7}{9}$$

$$= 51$$

The gain to be provided by the lead network is

$$10 \log \alpha = 10 \log \frac{1}{8} = 9 \text{ db.}$$

this gain occurs at frequency, $\omega_m = 6.8$ rad/sec on the lag compensated gain plot.

$$\omega_m = 6.8 \text{ rad/sec.}$$

the corner frequencies of the lead network are,

$$\omega_1 = \omega_m\sqrt{\alpha} = \frac{6.8}{\sqrt{8}} = 2.4 \text{ rad/sec}$$

$$\tau_1 = 0.416 \text{ sec.}$$

$$\omega_2 = \frac{\omega_m}{\sqrt{\alpha}} = 6.8\sqrt{8} = 19.23 \text{ rad/sec}$$

$$\tau_2 = 0.052 \text{ sec.}$$

The transfer function of the lead compensator is

$$G_{c2}(s) = \frac{0.416S+1}{0.052S+1}$$

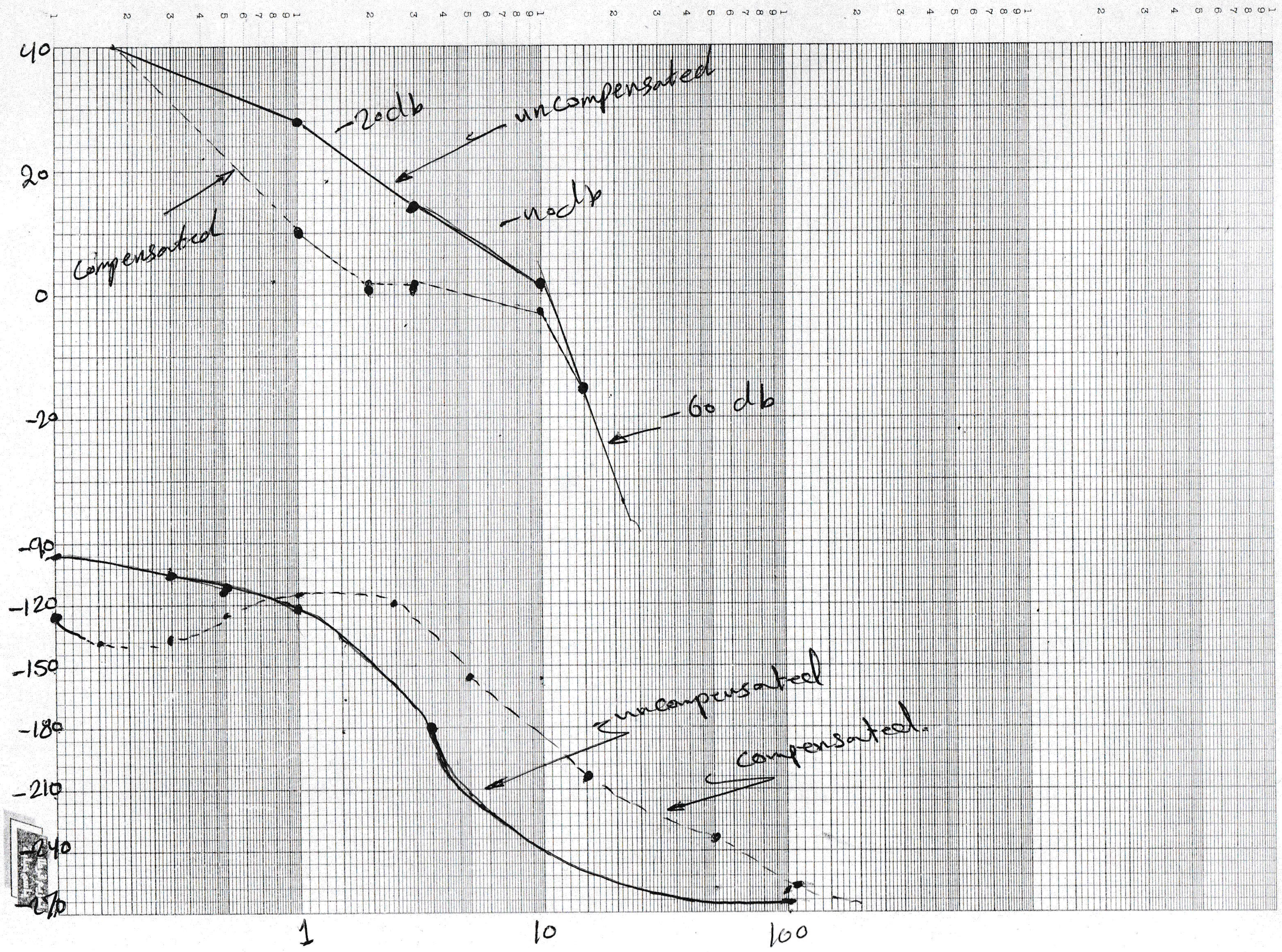
The open loop transfer function of the compensated system becomes.

$$G(s)\,G_{c1}(s)\,G_{c2}(s) = \frac{1250}{s(s+5)(s+10)} \times \frac{s+1}{8s+1} \times \frac{0.416s+1}{0.052s+1}.$$

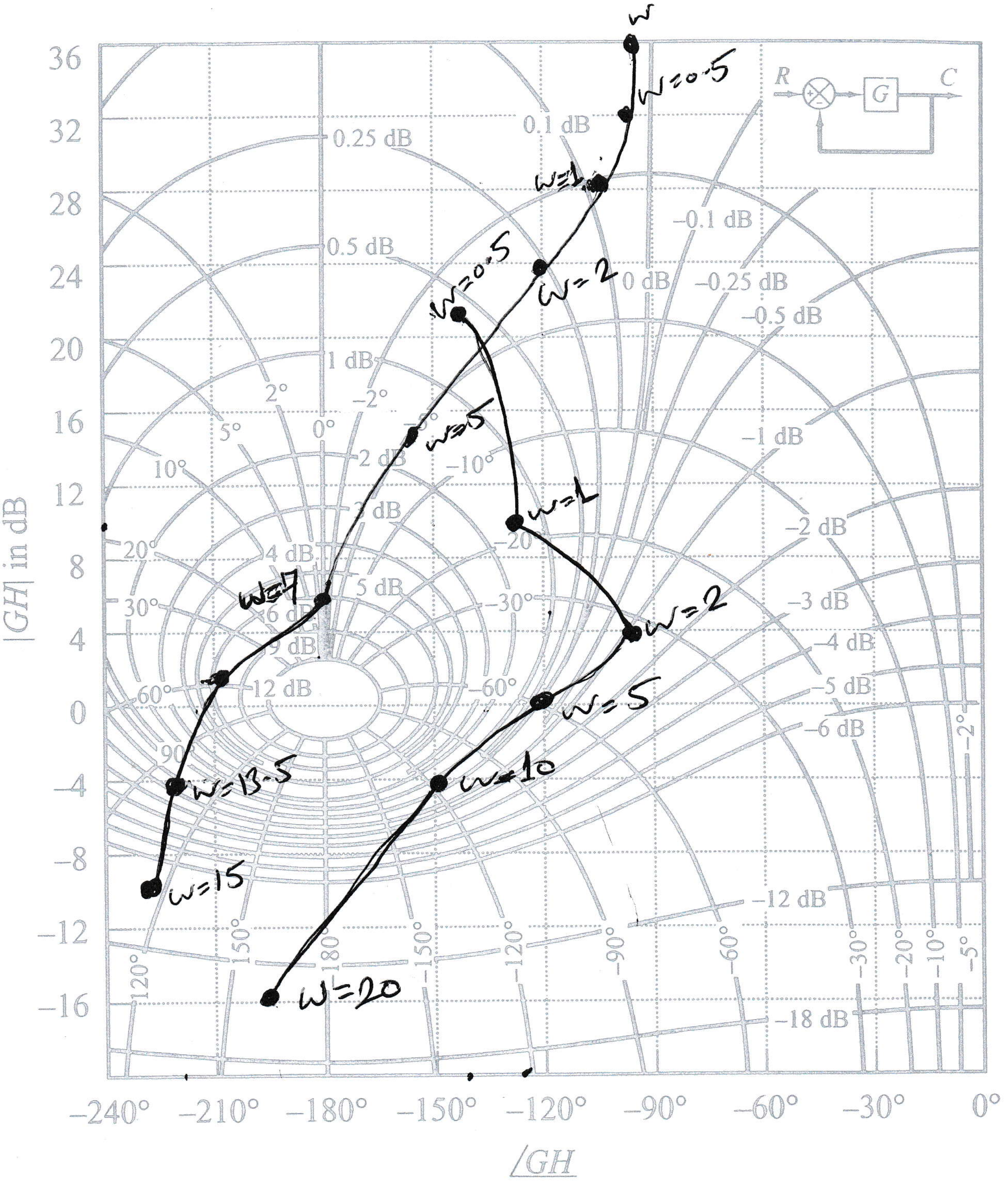
The phase margin of the lag-lead compensated network is 46°, which is acceptable. The values on Nichol's chart, the intersection of -3 db line with the curve is obtained at a frequency of 11 rad/sec where $|G| = 7$ db at frequency of 11 rad/sec

$\omega_b = 11$ rad/sec

which is satisfactory.



40
20
0
-20
-90
-120
-150
-180
-210
-240
-270
1
10
100
-20db
uncompensated
-40db
compensated
-60 db
uncompensated
compensated.



36
32
28
24
20
16
12
8
4
0
−4
−8
−12
−16
|GH| in dB
−240°
−210°
−180°
−150°
−120°
−90°
−60°
−30°
0°
/GH
R
G
C
0.25 dB
0.1 dB
−0.1 dB
0.5 dB
0 dB
−0.25 dB
−0.5 dB
1 dB
2 dB
3 dB
4 dB
5 dB
6 dB
9 dB
12 dB
−1 dB
−2 dB
−3 dB
−4 dB
−5 dB
−6 dB
−12 dB
−18 dB
w
w=0.5
w=1
w=2
w=5
w=7
w=13.5
w=15
w=0.5
w=1
w=2
w=5
w=10
w=20

H.W (10)

Q1:- A unity feedback System has a open loop transfer function,

$$G(s) = \frac{2}{s(s+0.5)}$$

What is the steady state error for a unit velocity input of the system. Design a lag Compensator so that the steady State error remains the same but a phase margin of 45° is achieved. Also obtain the RC network to realise this Compensator.

Q2:- For a unity feedback system with,

$$G(s) = \frac{K}{s(s+2)}$$

design a lead Compensator so that the steady state error for a unit velocity input is 0.05 and the phase margin is 50°. What is the gain margin for these compensated system.

Q3 Design a lag-lead Compensator for the unity feedback system with,

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

and satisfying the specifications,

$K_v = 10 \ sec^{-1}$

$\phi_{pm} = 50^\circ$

$B.W > 2$ rad/sec.

Q4: Design a lag Compensator for

$$G(s) = \frac{1}{s(s+2)}$$

so that the dominant poles of the closed-loop system are located at $s = -1 \pm j$ and the steady-state error to a unit-ramp input is less than 0.2.

Q5: Design a lead Compensator for

$$G(s) = \frac{1}{s^2}$$

if the dominant poles of the closed loop system are located at $s = -2 \pm 2j$.