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8- Frequency Response:-

There are two methods of analysis of systems to determine centain properties. If a time signal like step and ramp are used to excite the system and its "time response" or "time domain analysis." If sinusoidal signal of variable frequency is used to excite the system and the magnitude and phase of the steady state output from the system is measured, we call it "frequency response analysis" or "frequency domain analysis".

8-1 Bode plot:-

One of the important representations of the sinusoidal transfer function is a Bode plot. In this type of representation the magnitude of $G(j\omega)$ in db, i.e., ~~20 log K~~ $20 \log |G(j\omega)|$ is plotted against '$\log \omega$'. Similarly phase angle of $G(j\omega)$ is plotted against $\log \omega$.

The transfer function $G(j\omega)$ can be written as

$$G(j\omega) = |G(j\omega)| \angle \phi(\omega)$$

where $\phi(\omega)$ is the angle of $G(\omega)$, $|G(j\omega)|$ is the magnitude the open loop transfer function be given in time Constant formas,

$$G(s) = \frac{K(1+sT_a)(1+sT_b)}{s^r(1+sT_1)(1+sT_2)\cdots\left[1+2\delta\frac{s}{w_n}+\frac{s^2}{w_n^2}\right]}$$

Replacing $s \rightarrow jw$

$$G(jw) = \frac{K(1+jwT_a)(1+jwT_b)}{(jw)^r(1+jwT_1)(1+jwT_2)\cdots\left[1+2\delta\frac{jw}{w_n}+\left(\frac{jw}{w_n}\right)^2\right]}$$

$$|G(jw)| = \frac{K|1+jwT_a||1+jwT_b|\cdots}{|jw|^r|1+jwT_1||1+jwT_2|\cdots\left|\left[1+2\delta\frac{jw}{w_n}+\left(\frac{jw}{w_n}\right)^2\right]\right|\cdots}$$

Taking 20 log of $|G(jw)|$

$$20\log|G(jw)| = 20\log K + 20\log|1+jwT_a| + 20\log|1+jwT_b|$$
$$+\cdots - 20\log w^r - 20\log|1+jwT_1| - 20\log|1+jwT_2|$$
$$\cdots - 20\log\left|1+2\delta\frac{jw}{w_n}+\left(\frac{jw}{w_n}\right)^2\right|$$

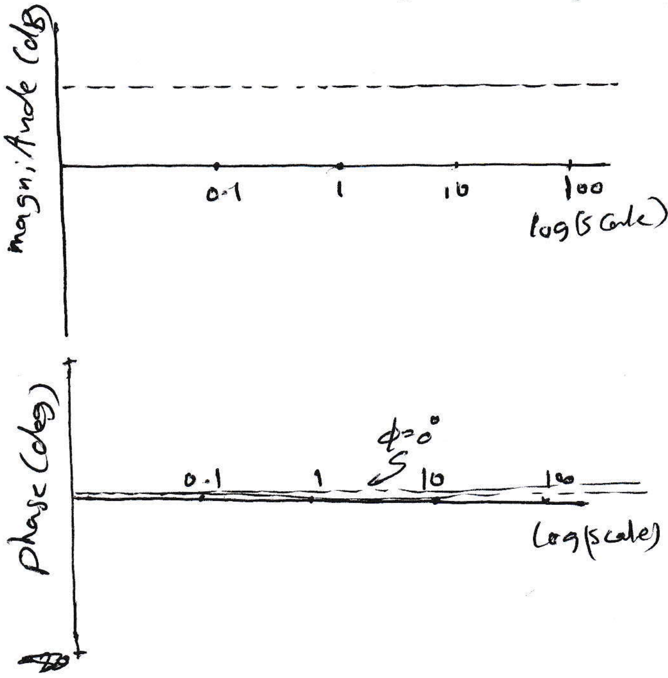
phase angle of $G(jw)$ is given by,

$$\phi(w) = \tan^{-1} wT_a + \tan^{-1} wT_b + \cdots$$
$$-r(90) - \tan^{-1} wT_1 - \tan^{-1} wT_2 - \tan^{-1}\frac{2\delta w w_n}{w_n^2-w^2}\cdots$$

let us draw Bode plot for each term in the transfer function $G(s)\,H(s)$.

a) for $K \Rightarrow 20\log(K)$

$\phi = 0^\circ$

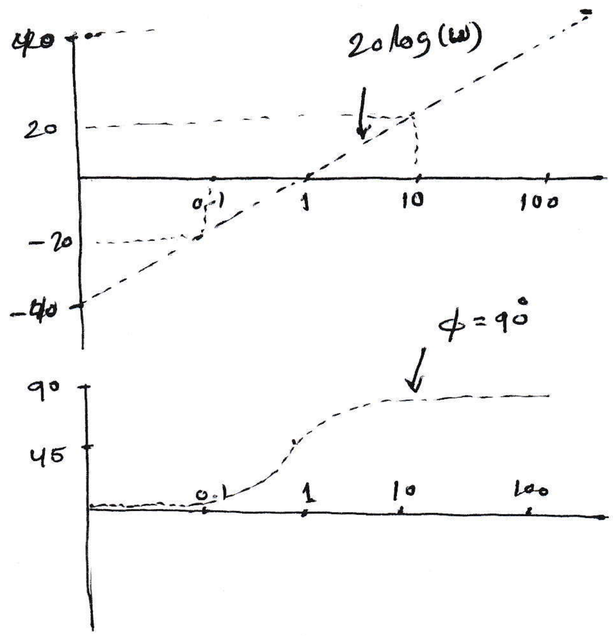


b) Zero at the frequency $j\omega$ with origin

s

$|j\omega| = 20\log(\omega)$

$\phi = 90^\circ$

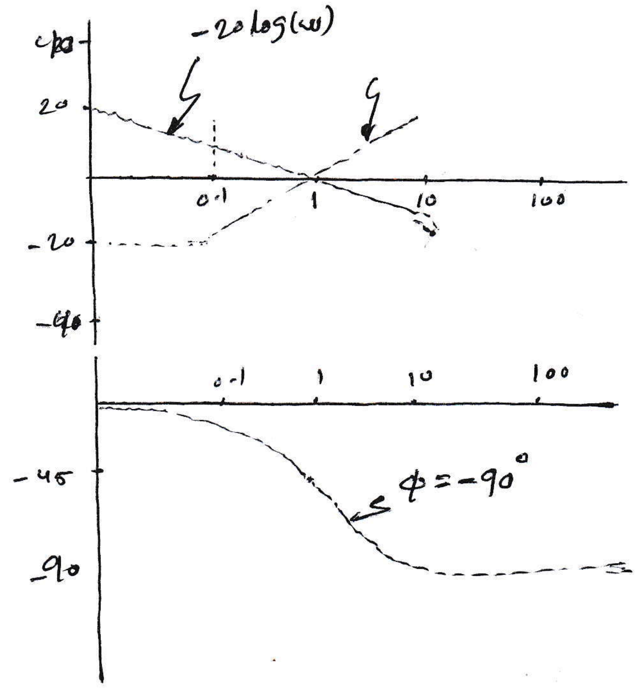


c) pole at the frequency $j\omega$ with origin

s

$\frac{1}{|j\omega|} = -20\log(\omega)$

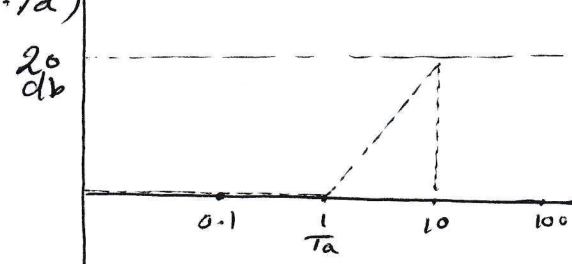
$\phi = -90^\circ$

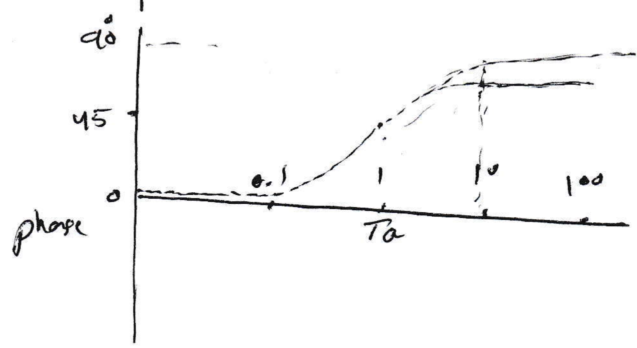


d) Zero at factor of the form $(s+T_a)$

$$|j\omega+T_a| = 20\log\sqrt{\omega^2+T_a^2}$$

$$\phi = \tan^{-1}$$

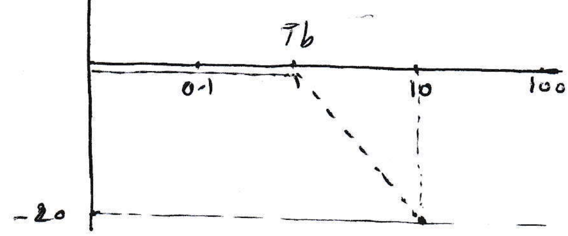


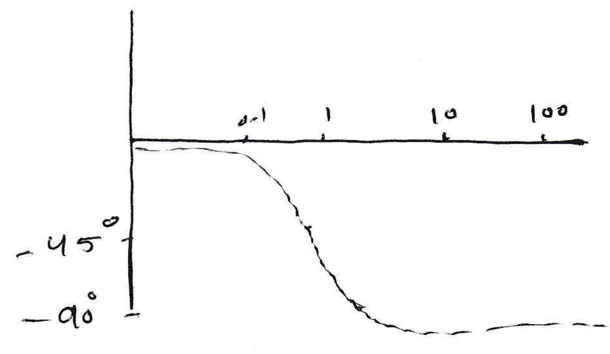


e) pole at factor of the form $\left(\frac{1}{s+T_b}\right)$

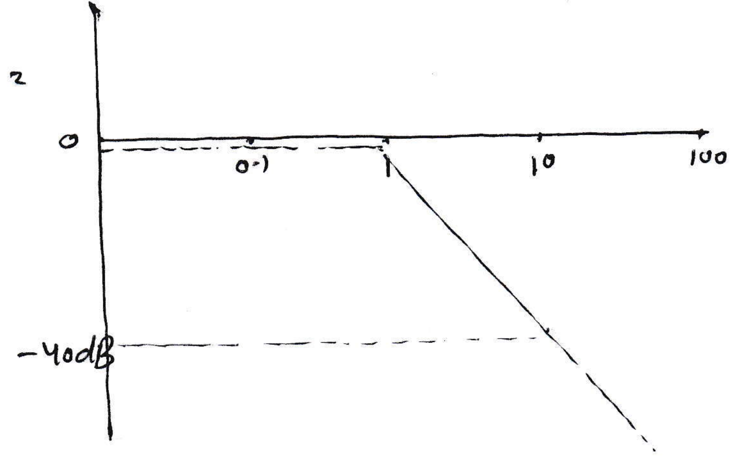
$$\frac{1}{|j\omega+T_b|} = -20\log\sqrt{\omega^2+T_b^2}$$

$$\phi = -\tan^{-1}$$



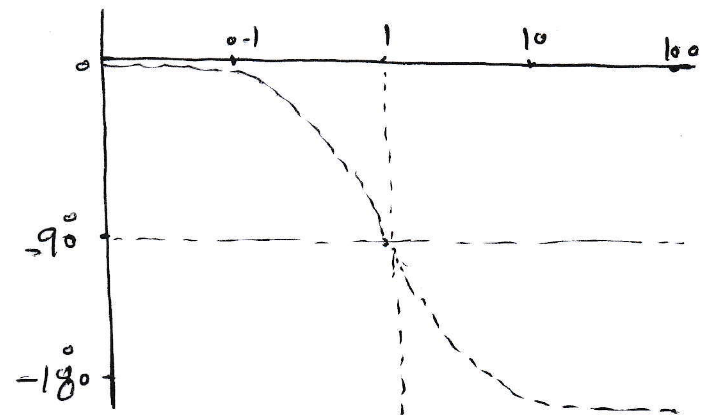


f) Double pole, $G(s) = \dfrac{1}{(s+a)^2}$



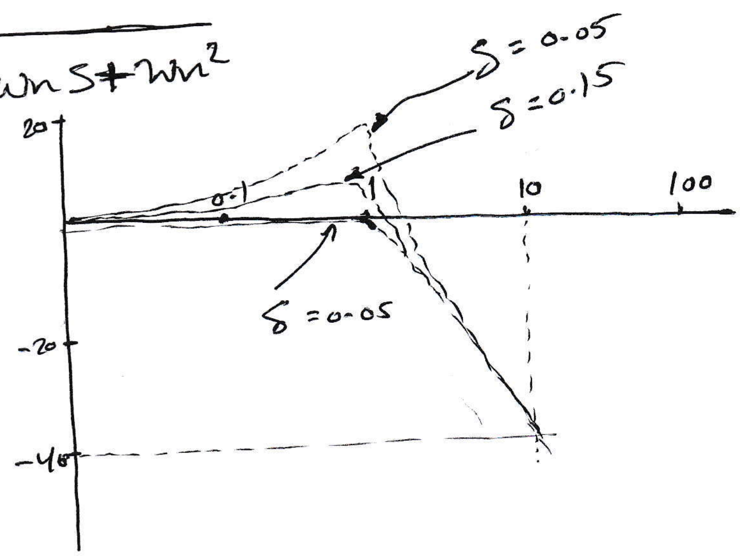
$\left|\dfrac{1}{j\omega+a}\right| = -40 \log \left|\sqrt{\omega^2+a^2}\right.$

$\phi = -2\tan^{-1}\dfrac{\omega}{a}$



g) Second order undamped response.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$



for magnitude $|G(j\omega)|$

for $\omega \equiv \omega_n$

~~20 log~~

$\Rightarrow 20\log\dfrac{1}{2\delta}$ dB

for $\omega \ll \omega_n \Rightarrow 0$ dB

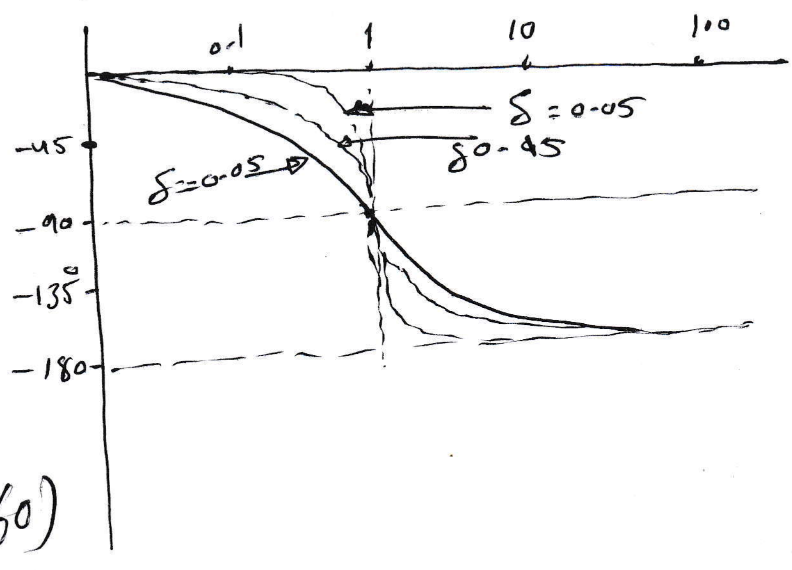
for $\omega \gg \omega_n \Rightarrow -40$ dB

for phase $\angle G(j\omega)$

$\omega = \omega_n = -90^\circ$

$\omega \ll \omega_n \Rightarrow \phi = 0^\circ$

$\omega \gg \omega_n \Rightarrow \phi = -180^\circ$



EX obtain the Bode plot of the system given by the transfer function $G(s) = \frac{1}{2s+1}$

by converting $s \to j\omega$

$G(j\omega) = \frac{1}{2j\omega+1}$, $\omega = \frac{1}{2}$, the break point or corner frequency

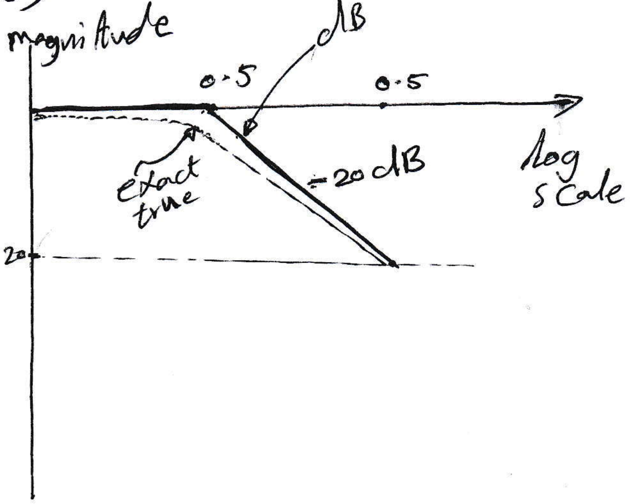
- for $\omega \ll \frac{1}{2} \Rightarrow G(j\omega) = 1 \Rightarrow 20\log|G(j\omega)| = 20\log(1) = 0$
- for $\omega \gg \frac{1}{2} \Rightarrow G(j\omega) = \frac{1}{2j\omega} \Rightarrow 20\log|G(j\omega)|$

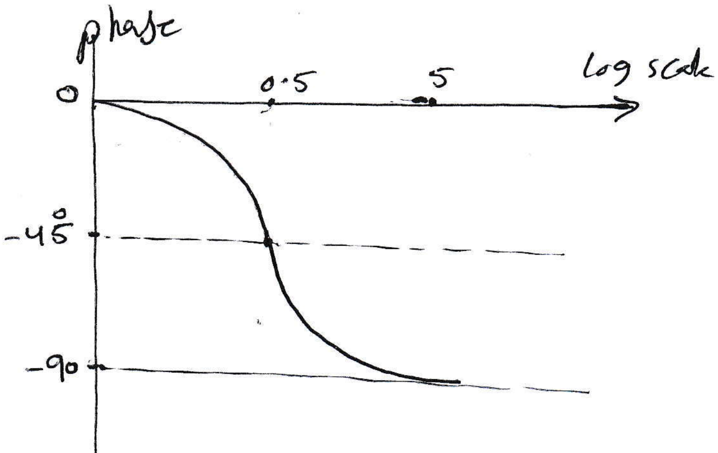
$20\log|G(j\omega)| = 20\log\left|\frac{1}{2j\omega}\right| = 20\log(1) - 20\log(2\omega) = -20\log(2\omega)$

$= 20\log\left(0.5/\sqrt{\omega^2+0.5^2}\right)$

$\phi = 0 - \tan^{-1}(2\omega) = -\tan^{-1}(2\omega)$

ω	magnitude measure	exact	phase
0.05	0	-0.043	0
0.5	0	-3.01	-45°
5	-20dB	-20.043	-90
10			





EX obtain the bode plot of the system given by the transfer function $G(s) = \dfrac{4}{s^2+s+4}$

Replacing $s = j\omega$ in the transfer function.

$$G(j\omega) = \frac{4}{(j\omega)^2 + j\omega + 4} = \frac{\omega_n^2}{(j\omega)^2 + 2\delta\omega_n j + \omega_n^2}$$

$$\omega_n^2 = 4 \Rightarrow \omega_n = 2$$

$$2\delta\omega_n = 1 \Rightarrow \delta = \frac{1}{2*2}$$

$$G(j\omega) = \frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\delta\left(\frac{j\omega}{\omega_n}\right) + 1} = \frac{1}{\left(\frac{j\omega}{2}\right)^2 + 0.5\left(\frac{j\omega}{2}\right) + 1}$$

the breakpoint or corner frequency is $\omega = \omega_n$,

$\omega \ll \omega_n$, for small values of ω,

$G(j\omega) \simeq 1. \Rightarrow 20 \log |G(j\omega)| = 20 \log (1) = 0$

- for larger values of ω, $\omega \gg \omega_n$, we get

$$G(j\omega) \simeq \frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2} = \frac{1}{\left(\frac{j\omega}{2}\right)^2}$$

$$20 \log |G(j\omega)| = 20 \log \left(\frac{1}{\left(\frac{\omega}{\omega_n}\right)^2}\right) = 20 \log (1) - 20 \log \left(\frac{\omega}{\omega_n}\right)^2$$

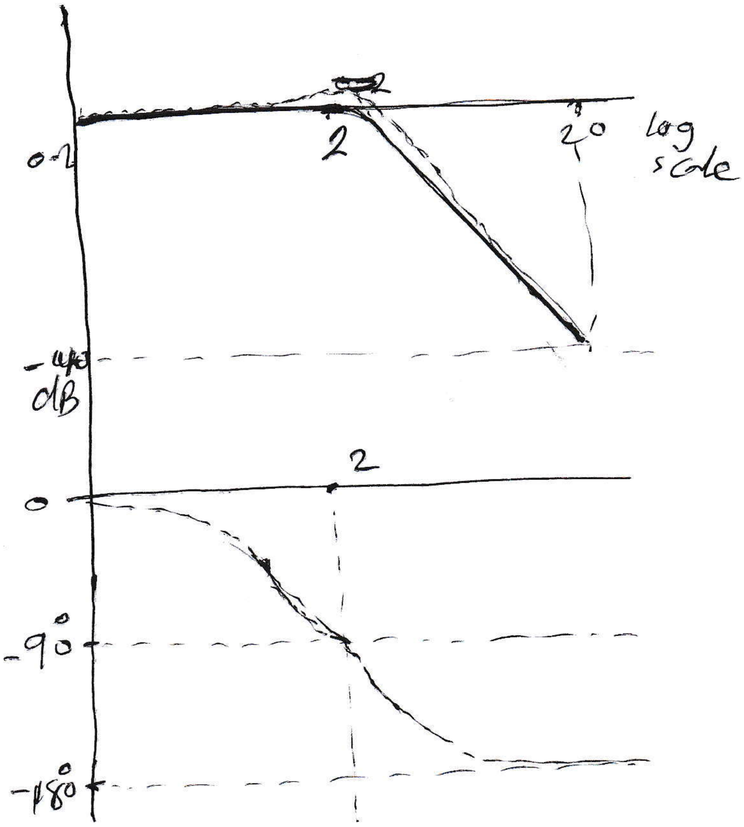
$$= -40 \log \frac{\omega}{\omega_n} = -40 \log \frac{\omega}{2}$$

$\phi = 0 - \tan^{-1}$

$\omega = 0.2 \Rightarrow \phi = 0^\circ$

$\omega = 2 \Rightarrow \phi = -90^\circ$

$\omega = 20 \Rightarrow \phi = -180^\circ$



EX Plot the Bode magnitude and phase for the system with transfer function.

$$G(s) = \frac{2000(s+0.5)}{s(s+10)(s+50)}$$

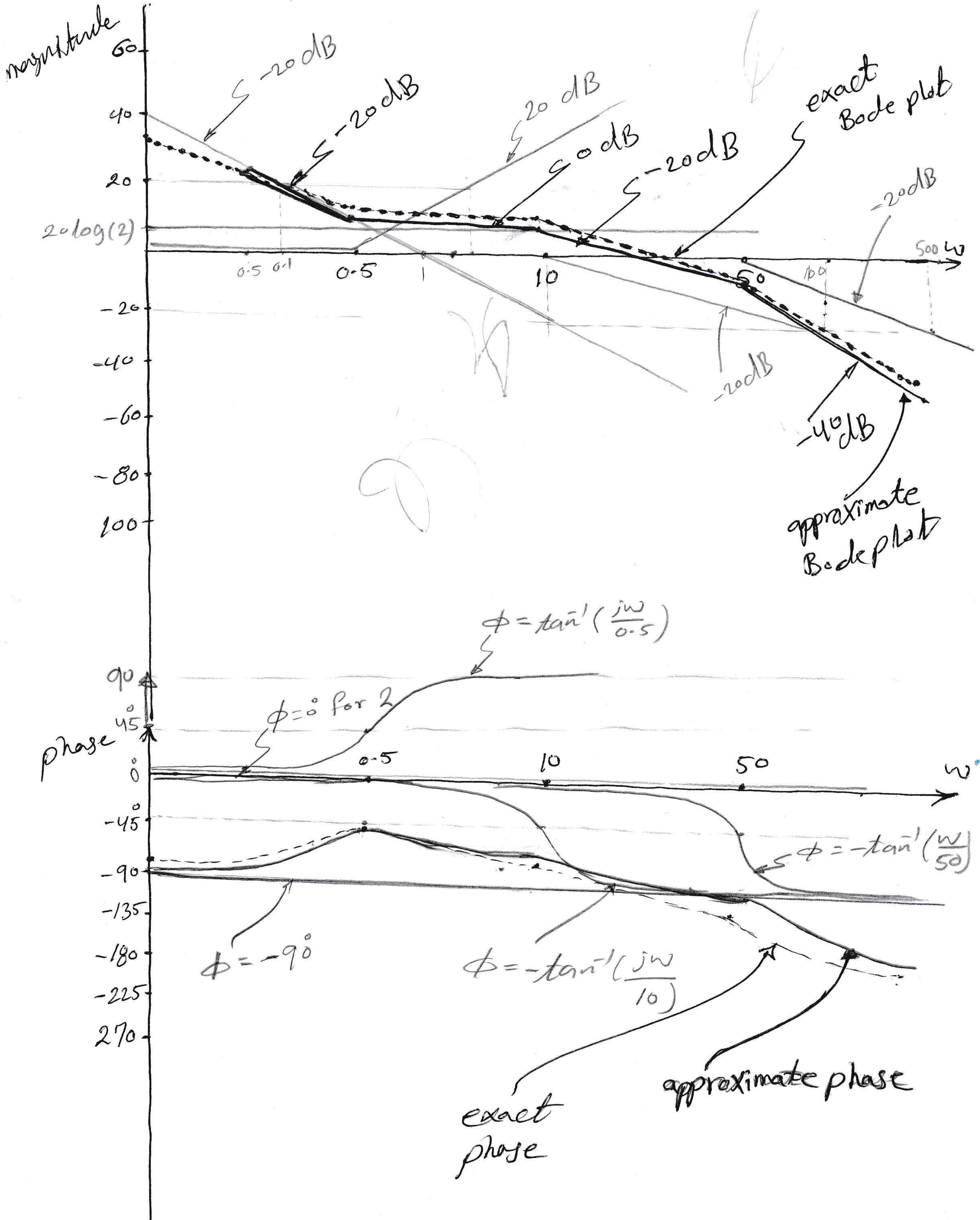
① by converting $s = j\omega$

$$G(j\omega) = \frac{2000(j\omega + 0.5)}{(j\omega)(j\omega+10)(j\omega+50)} = \frac{2\left(\frac{j\omega}{0.5}+1\right)}{j\omega\left(\frac{j\omega}{10}+1\right)\left(\frac{j\omega}{50}+1\right)}$$

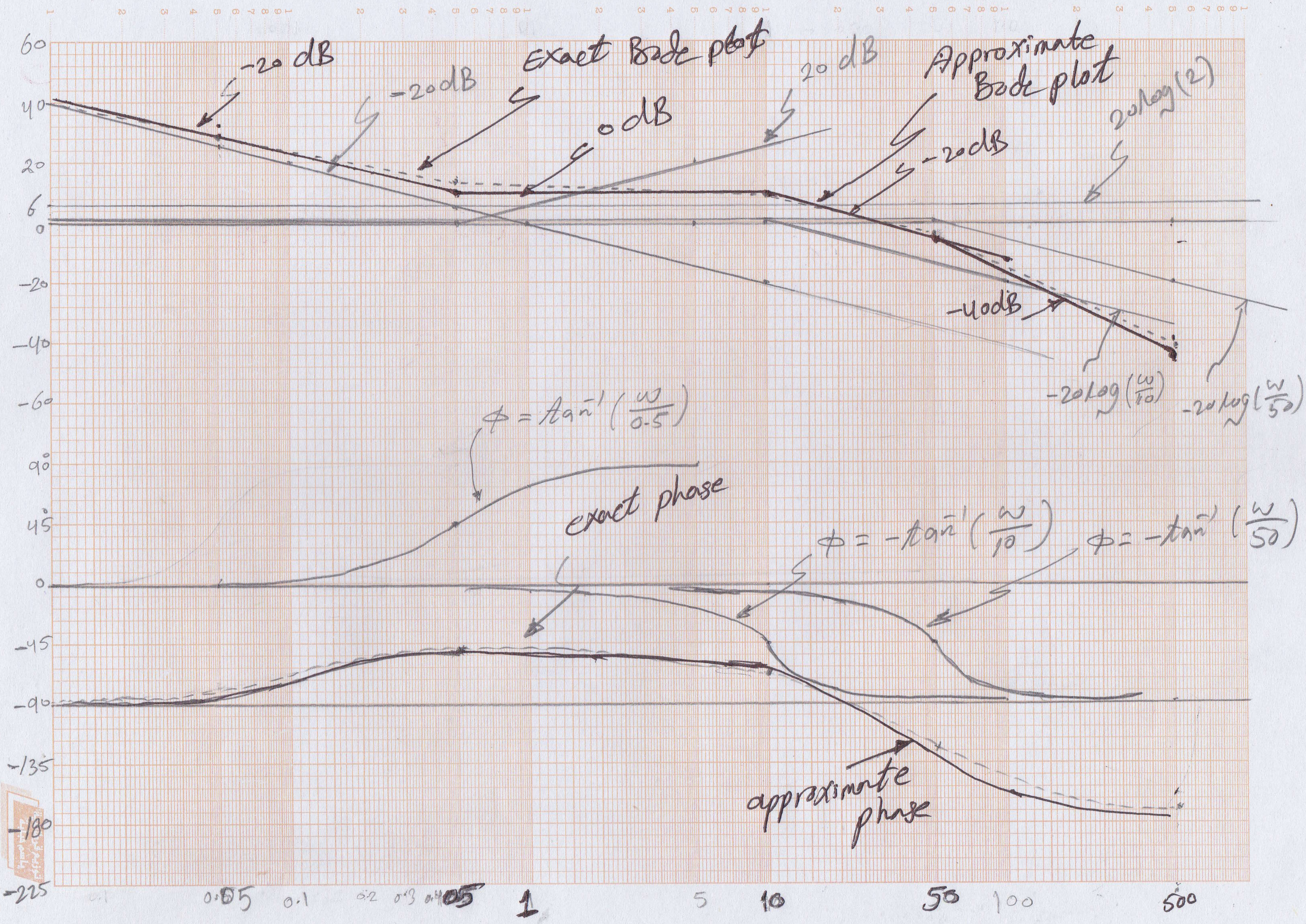
$$|G(jw)| = 20\log\left[\frac{2000*(\sqrt{\omega^2+0.5^2}}{\omega*(\sqrt{\omega^2+100})*(\sqrt{\omega^2+50^2}}\right]$$

ω	Magnitude Exact	Magnitude approximat	phase
0.05	32.0841	32	-84.63°
0.5	15		-48.43°
10	8.8714		-59.1723°
50	-5.1184		-124.26°
500	-41.983		-173.2009°

$$\phi = 0 + \tan^{-1}\frac{\omega}{0.5} - 90 - \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{\omega}{50}$$



magnitude
60
40
20
$20\log(2)$
-20
-40
-60
-80
100
-20dB
-20dB
20 dB
0dB
-20dB
exact Bode plot
-20dB
0.5 0.1
0.5
1
10
50
100
500 w
-20dB
-40dB
approximate Bode plot
$\phi = \tan^{-1}(\frac{jw}{0.5})$
90
45
phase
0
$\phi = 0°$ for 2
0.5
10
50
w
-45
-90
-135
-180
-225
270
$\phi = -\tan^{-1}(\frac{w}{50})$
$\phi = -90°$
$\phi = -\tan^{-1}(\frac{jw}{10})$
exact phase
approximate phase



Exact Bode plot
Approximate Bode plot
-20 dB
-20dB
0 dB
20 dB
-20dB
-40dB
20log(2)
-20log(ω/10)
-20log(ω/50)
ϕ = tan⁻¹(ω/0.5)
exact phase
ϕ = -tan⁻¹(ω/10)
ϕ = -tan⁻¹(ω/50)
approximate phase
60
40
20
6
0
-20
-40
-60
90°
45°
0
-45
-90
-135
-180
-225
0.05
0.1
0.2
0.3
0.4
0.5
1
5
10
50
100
500

EX Draw the frequency response of the system given by the transfer function.

$$G(s) = \frac{10}{s(s^2 + 0.4s + 4)}$$

$$G(s) = \frac{\frac{10}{4}}{s\left(\frac{s^2}{4} + 0.2\,\frac{s}{2} + 1\right)}$$

by Replacing $s = j\omega$ in the transfer function

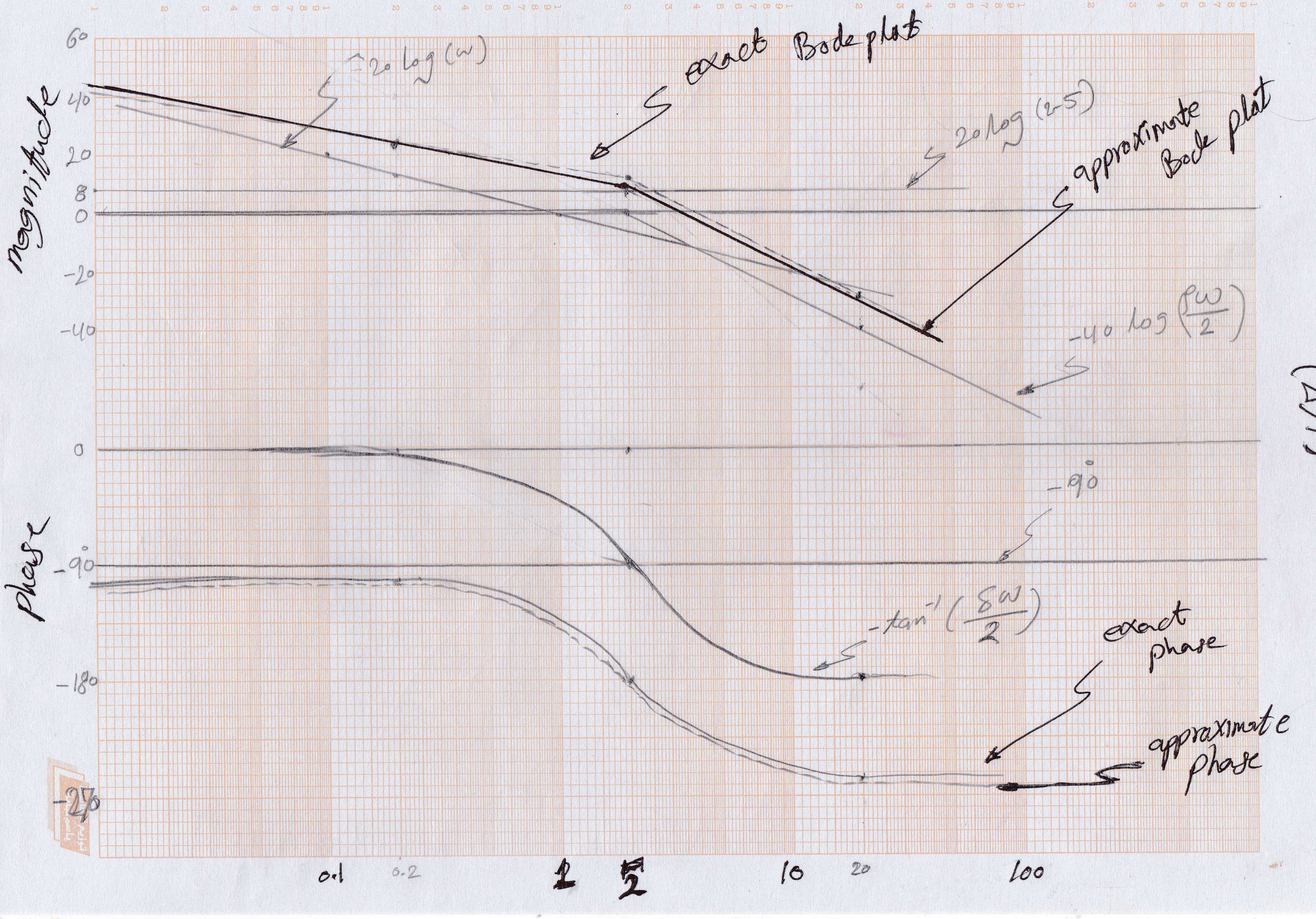
$$G(j\omega) = \frac{10}{4}\;\frac{1}{j\omega\left(\left(\frac{j\omega}{2}\right)^2 + 0.2\left(\frac{j\omega}{2}\right) + 1\right)}$$

the corner frequency is $\omega = 2$

$$|G(j\omega)| = 20\log\left[\frac{10/4}{\omega\sqrt{\left(\frac{\omega}{2}\right)^2 + \left(\left(\frac{\omega}{2}\right)^2 + 1\right)^2}}\right]$$

$$\text{phase} = 0 - 90^\circ - 2\tan^{-1}\left(\frac{\omega}{2}\right)$$

ω	Magnitude	phase
0.2	22.02	-101.4212°
2	15.9175	-180°
20	-57.9759 (166)	-258.5788°


magnitude
phase
60
40
20
8
0
-20
-40
0
-90
-180
-270
$20\log(\omega)$
exact Bode plot
$20\log(2.5)$
approximate Bode plot
$-40\log\left(\frac{8\omega}{2}\right)$
$-90°$
$-\tan^{-1}\left(\frac{8\omega}{2}\right)$
exact phase
approximate phase
0.1
0.2
1
2
10
20
100

EX Sketch the bode plot of the transfer function

$$H(s) = \frac{100(s+10)}{s^2 + 1000s}$$

$$H(jw) = \frac{100(jw+10)}{(jw)^2 + 1000jw}$$

$$= \frac{100(jw+10)}{jw(jw+1000)}$$

$$= \frac{100 * 10\left(\frac{jw}{10}+1\right)}{1000\, jw\left(\frac{jw}{1000}+1\right)}$$

$$= \frac{\left(\frac{jw}{10}+1\right)}{jw\left(\frac{jw}{1000}+1\right)}$$


phase
magnitude
-180°
-90°
0
90°
-40
-20
0
20
40
60
1
10
100
1000
10 000
-20 dB
0dB
-20 db