Ex. Discuss the continuity of the function at $X=0$, $X=1$

$$F(x) = \begin{cases} x + \frac{1}{x} & , \text{For } x < 0 \\ -x^3 & , \text{For } 0 \le x < 1 \\ -1 & , \text{For } 1 \le x < 2 \end{cases}$$

at $X=0$

1) $F(x) = -x^3$

$F(0) = -(0)^3 = -0 = 0$

2) $\lim\limits_{x \to 0^+} (-x^3) = -(0)^3 = 0$

$\lim\limits_{x \to 0^-} \left(x + \frac{1}{x}\right) = 0 + \frac{1}{0} = \infty$

$\because \lim\limits_{x \to 0^+} F(x) \neq \lim\limits_{x \to 0^-} F(x) \neq F(x)$

The Function discontinuos at $X=0$

at $X=1$

1) $F(x) = -1$

$F(1) = -1$

2) $\lim_{x \to 1^+} F(x) = \lim_{x \to 1^+} (-1) = -1$

$\lim_{x \to 1^-} (-x^3) = -(1)^3 = -1$

$\therefore \lim_{x \to 1^-} F(x) = \lim_{x \to 1^+} F(x) = F(x)$

the function is continous at $x = 1$

the inverse trigonometric Functions

الدوال العكسية

1) $y = \sin^{-1} x$, $x = \sin y$

2) $y = \cos^{-1} x$, $x = \cos y$

3) $y = \tan^{-1} x$, $x = \tan y$

4) $y = \sec^{-1} x$, $x = \sec y$

5) $y = \cot^{-1} x$, $x = \cot y$

6) $y = \csc^{-1} x$, $x = \csc y$

المتطابقات الخاصة بالدوال العكسية

1) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

2) $\sin^{-1}(\sin \theta) = \theta$, $\sin(\sin^{-1} \theta) = \theta$

3) $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

4) $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$

5) $\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$

6) $\csc^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$

7) $\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right)$

* مشتقات الدوال العكسية

1) $$\frac{d}{dx}\sin^{-1}u = \frac{\frac{du}{dx}}{\sqrt{1-u^2}}$$

2) $$\frac{d}{dx}\cos^{-1}u = \frac{-\frac{du}{dx}}{\sqrt{1-u^2}}$$

3) $$\frac{d}{dx}\tan^{-1}u = \frac{\frac{du}{dx}}{1+u^2}$$

4) $$\frac{d}{dx}\cot^{-1}u = \frac{-\frac{du}{dx}}{1+u^2}$$

5) $$\frac{d}{dx}(\sec^{-1}u) = \frac{\frac{du}{dx}}{|u|\sqrt{u^2-1}}$$

6) $$\frac{d}{dx}\csc^{-1}u = \frac{-\frac{du}{dx}}{|u|\sqrt{u^2-1}}$$

Ex. Find $\frac{dy}{dx}$ For $y = 3^{\sin^{-1} 2x}$

$$\frac{dy}{dx} = 3^{\sin^{-1} 2x} * \frac{2}{\sqrt{1-4x^2}} * \ln 3$$

Ex. $y = \sec^{-1} 5x + \pi^{\cos x}$

$$\frac{dy}{dx} = \frac{5}{|5x|\sqrt{25x^2-1}} + \pi^{\cos x} * (-\sin x) . \ln \pi$$

$$= \frac{1}{|x|\sqrt{25x^2-1}} - \pi^{\cos x} * \sin x . \ln \pi$$

Ex. $y = \tan^{-1}(\sqrt{x+1})$

$$\frac{dy}{dx} = \frac{1}{1+(\sqrt{x+1})^2} * \frac{1}{2\sqrt{x+1}}$$

$$= \frac{1}{1+x+1} * \frac{1}{2\sqrt{x+1}}$$

$$= \frac{1}{2+x} * \frac{1}{2\sqrt{x+1}}$$

Ex. $y = e^{\sin^{-1}(\cos x)}$

$$\frac{dy}{dx} = e^{\sin^{-1}(\cos x)} * \frac{-\sin x}{\sqrt{1-\cos^2 x}}$$

Ex. $y = x \cdot \ln(\sec^{-1} x)$

$$\frac{dy}{dx} = x * \frac{1}{\sec^{-1} x} * \frac{1}{|x| \sqrt{x^2-1}} + \ln(\sec^{-1} x) * 1$$

Ex. find $\frac{dy}{dx}$ for $\sin^{-1}(\cos y) = \ln \sec^{-1} x + e^{\csc^{-1} x}$

$$\frac{1}{\sqrt{1-\cos^2 y}} * (-\sin y) * \frac{dy}{dx} = \frac{1}{\sec^{-1} x} * \frac{1}{|x| \sqrt{x^2-1}} + e^{\csc^{-1} x} * \frac{-1}{|x| \sqrt{x^2-1}}$$

$$\frac{dy}{dx} = \frac{\frac{1}{\sec^{-1} x} * \frac{1}{|x| \sqrt{x^2-1}} - \frac{e^{\csc^{-1} x}}{|x| \sqrt{x^2-1}}}{\frac{-\sin y}{\sqrt{1-\cos^2 y}}}$$

تكامل الدوال العكسية

1) $$\int \frac{\text{مشتقة الزاوية}}{\sqrt{a^2 - (\text{الزاوية})^2}} = \sin^{-1} \frac{\text{الزاوية}}{a} + c$$

2) $$\int \frac{-\text{مشتقة الزاوية}}{\sqrt{a^2 - (\text{الزاوية})^2}} = \cos^{-1} \frac{\text{الزاوية}}{a} + c$$

3) $$\int \frac{\text{مشتقة الزاوية}}{a^2 + (\text{الزاوية})^2} = \frac{1}{a} \tan^{-1} \frac{\text{الزاوية}}{a} + c$$

4) $$\int \frac{-\text{مشتقة الزاوية}}{a^2 + (\text{الزاوية})^2} = \frac{1}{a} \cot^{-1} \frac{\text{الزاوية}}{a} + c$$

5) $$\int \frac{\text{مشتقة الزاوية}}{|\text{الزاوية}| \sqrt{(\text{الزاوية})^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{\text{الزاوية}}{a} + c$$

6) $$\int \frac{-(\text{مشتقة الزاوية})}{|\text{الزاوية}| \sqrt{(\text{الزاوية})^2 - a^2}} = \frac{1}{a} \csc^{-1} \frac{\text{الزاوية}}{a} + c$$

في جميع تكاملات الدوال العكسية نحتاج الى مشتقة الزاوية حتى نكامل

Ex. Find $\int \frac{dx}{\sqrt{9-x^2}}$

$a^2 = 9 \Rightarrow a = 3$

الزاوية$^2 = x^2 \Rightarrow$ الزاوية $= x$

$$\int \frac{dx}{\sqrt{9-x^2}} = \sin^{-1}\frac{x}{a} + c$$

$$= \sin^{-1}\frac{x}{3} + c$$

Ex. $\int \frac{x}{1+x^4}\,dx$

$$= \int \frac{x}{1+(x^2)^2}\,dx \times \frac{2}{2}$$

$$= \frac{1}{2}\int \frac{2x}{1+(x^2)^2}\,dx$$

$$= \frac{1}{2}\tan^{-1}\frac{x^2}{1} + c$$

$$= \frac{1}{2}\tan^{-1}x^2 + c$$

Ex. $\int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}\, dx$

$$\int e^{\sin^{-1}x} * \frac{1}{\sqrt{1-x^2}} = e^{\sin^{-1}x} + c$$

Ex. $\int \frac{\tan^{-1}x}{1+x^2}\, dx$

$$\int \tan^{-1}x * \frac{1}{1+x^2}\, dx$$

$$= \frac{(\tan^{-1}x)^2}{2} + c$$

Ex. $\int \frac{\sec^2 x\, dx}{\sqrt{5-9\tan^2 x}}$

$a = \sqrt{5}$
$x = 3\tan x$

$$\int \frac{\sec^2 x\, dx}{\sqrt{5-9\tan^2 x}} * \frac{3}{3}$$

$$= \frac{1}{3}\int \frac{3\sec^2\, dx}{\sqrt{5-9\tan^2 x}} = \frac{1}{3}\sin^{-1}\frac{3\tan x}{\sqrt{5}} + c$$

Ex. $\displaystyle\int \frac{dx}{(1+4x^2)(2+\tan^{-1}2x)}$

$$\int \frac{(2+\tan^{-1}2x)^{-1}\,dx}{(1+4x^2)}$$

$$= \int (2+\tan^{-1}2x)^{-1} * \frac{dx}{(1+4x^2)} * \frac{2}{2}$$

$$= \frac{1}{2}\ln|2+\tan^{-1}2x| + c$$

Ex $\displaystyle\int \frac{e^{\tan^{-1}2t}}{1+4t^2}\,dt$

$$\int e^{\tan^{-1}2t} * \frac{dt}{1+4t^2} * \frac{2}{2}$$

$$= \frac{1}{2}\int e^{\tan^{-1}2t} * \frac{2\,dt}{1+(2t)^2}$$

$$= \frac{1}{2}e^{\tan^{-1}2t} + c$$