Antennas & Wave Propagation

Electronic Dep.

3rd Stage

Lecture Two

Analysis of The Radiative Field

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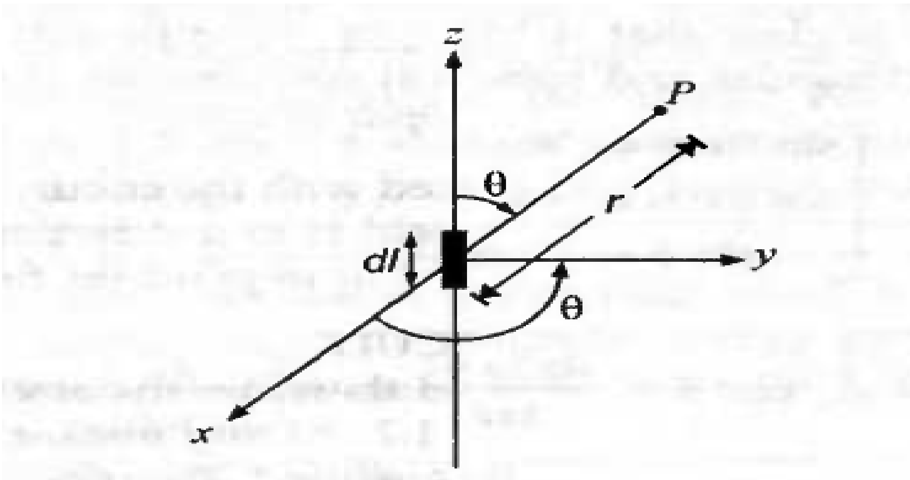
Analysis of The Radiative Field

- Assume an elementary doublet of length dl such that (approximately dl = 0.1λ.
- such antenna called Hertzian antenna



→ Magnetic potential at a distance r from this current element is

$$A = \mu_0 \int \frac{i\,dl}{4\pi r}$$

→ The current in the doublet oscillatory with frequency

$$i = I_0 \sin \omega t$$

→There will be a time lag *r/c* [c is the velocity of electromagnetic propagation] at the point P

$$i = I_0 \sin \omega \left(t - \frac{r}{c} \right)$$

$$A = \mu_0 \int \frac{I_0 \sin \omega \left(t - \frac{r}{c} \right) dl}{4\pi r}$$

- Referring to figure (1), current element *dl* is symmetrically placed about the origin with its axis along the z axis, the current on the element has only a z-component, *A* also has only the A_z component. Thus, the magnetic potential at *P is*

$$A_z = \frac{\mu_0}{4\pi} \int \frac{I_0 \sin \omega \left(t - \frac{r}{c} \right) dl}{r}$$

$$\int_0^{dl} I dl = I dl$$

$$A_z = \frac{\mu_0}{4\pi} \frac{I_0 \, dl \sin \omega \left(t - \frac{r}{c} \right)}{r}$$

$$H = \frac{1}{\mu_0} (\nabla \times A)$$

Thus, using curl equation from the Appendix-IV

$$H_r a_r + H_\theta a_\theta + H_\phi a_\phi = \frac{1}{\mu_0} \begin{vmatrix} \frac{1}{r^2 \sin\theta} a_r & \frac{1}{r \sin\theta} a_\theta & \frac{1}{r} a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin\theta A_\phi \end{vmatrix}$$

Equating the coefficients of the a_r, a_θ and a_ϕ, gets

$$H_r = \frac{1}{\mu_0 r \sin\theta} \left[\frac{\partial}{\partial \theta} (A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi} \right]$$

$$\boxed{H_\phi = \frac{I_0 dl \sin\theta}{4\pi}\left[\frac{\sin\omega\left(t-\frac{r}{c}\right)}{r^2} + \frac{\omega}{rc}\cos\omega\left(t-\frac{r}{c}\right)\right]}$$

Thus, two fields will be

$$\text{Induction field (Near field)} = \frac{I_0\, dl \sin\theta}{4\pi}\left(\frac{1}{r^2}\sin\omega\left(t-\frac{r}{c}\right)\right)$$

$$\text{Radiation field (Far field)} = \frac{I_0\, dl \sin\theta}{4\pi}\left(\frac{\omega\cos\omega\left(t-\frac{r}{c}\right)}{cr}\right)$$

$$\nabla \times H = \in \frac{\partial E}{\partial t} \qquad \text{(as } J = 0 \text{ for free space)}$$

The above equation in its components form can be written as follows :

$$\begin{vmatrix} \frac{1}{r^2 \sin\theta} & \frac{1}{r\sin\theta} a_\theta & \frac{1}{r} a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & 0 & r\sin\theta\, H_\phi \end{vmatrix} = \in \left(\frac{\partial E_r}{\partial t} + \frac{\partial E_\theta}{\partial t} + \frac{\partial E_\phi}{\partial t} \right)$$

$$\boxed{E_r = \frac{2 I_0\, dl \cos\theta}{4\pi\varepsilon} \left[\frac{\sin \omega t'}{c r^2} - \frac{\cos \omega t'}{\omega r^3} \right]}$$

$$\boxed{E_\theta \text{ (Radiation)} = \left(\frac{\omega I_0\, dl \sin\theta}{4\pi\varepsilon\, c^2 r} \right) \cos\omega \left(t - \frac{r}{c} \right)}$$

- The field is maximum at the equator and equal to zero at poles.
- The field intensities increase with frequency.
- Both the fields are in time phase indicating transfer of energy.
- They vary with the sine of the angle 0, and thus being maximum at 0 = n/2

Thanks 4 Listening

Any Question Please...