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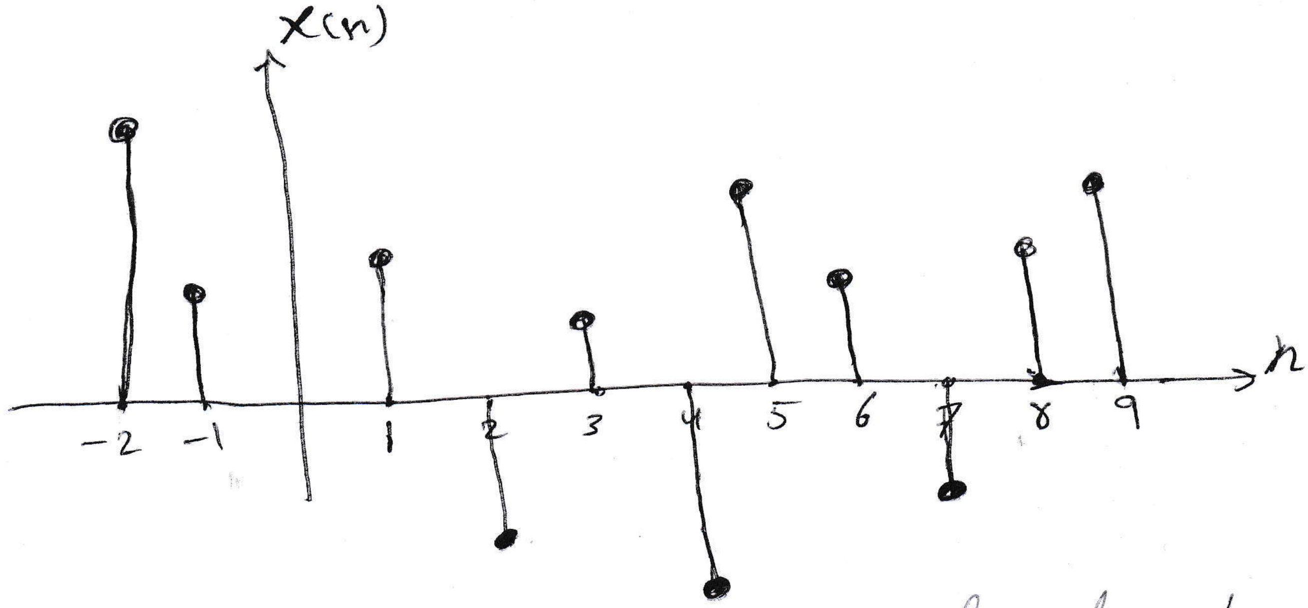
Background to DSP :-

A discrete-time signal is an indexed sequence of real or complex numbers. Thus, a discrete-time signal is a function of an integer-valued variable, n, that is denoted by $x(n)$. A discrete-time signal is undefined for noninteger values of n. Therefore, a real-valued signal $x(n)$ will be represented graphically in the form of a "Lollipop" plot as shown in figure



the graphical representation of a discrete-time signal $x(n)$

$$x = [x(0), x(1), \ldots, x(N-1)]^T$$

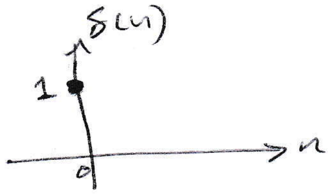
Discrete-time signals are often derived by sampling a continous-time signal, such as speed, with an analog-to-digital (A/D) converter. For example, a continous-time-signal $x_a(t)$ that is sampled at a rate of $f_s = \frac{1}{T_s}$ samples per second produces the sampled signal $x(n)$, which is related to $x_a(t)$ as follows:

$$x(n) = x_a(nT_s)$$

- Some Fundamental Sequences

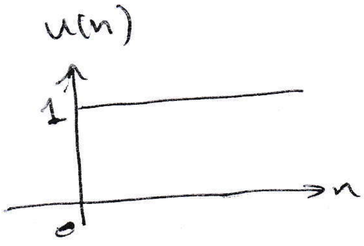
1) Unit Sample, $\delta(n)$

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$



2) Unit step, $u(n)$

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



$$u(n) = \sum_{k=-\infty}^{n} \delta(k)$$

A unit sample may be written as a difference of two steps

$$\delta(n) = u(n) - u(n-1)$$

3) the exponential sequence

$$x(n) = a^n$$

a: is a real or Complex

$a = e^{jw_o}$, where w_o is a real number

$$e^{jnw_o} = \cos(nw_o) + j\sin(nw_o)$$

- Signal Duration

Discrete-time signals may be conveniently classified in terms of their duration or extent.

a) A Finite-length sequence
it is equal to Zero for all values of n.

b) Infinite-length sequence
Signals that are not finite in length such as the unit step and the Complex exponential.

Infinite-length can be classified as either being right-sided, left-sided, or two sided.

1- A right-sided sequence
It is any infinite-length sequence that is equal to zero for all values of $n < n_o$ for some integer n_o,
Such as an unit step.

2- left-sided sequence.
Such as $x(n) = u(n-n_o) = \begin{cases} 1 & n \le n_o \\ 0 & n > n_o \end{cases}$

3) two-sided sequence.

An infinite-length signal that is neither right-sided nor left-sided, such as the complex exponential.

- Periodic and Aperiodic Sequences :-

A signal $x(n)$ is said to be periodic if, for some positive real integer N,

$$x(n) = x(n+N) \quad \text{or} \quad y(n) = \sum_{k=-\infty}^{\infty} x(n-kN)$$

where N is fundamental period

- Signal Manipulations :-

Sequences are often altered and manipulated by modifying the index n as follows

$$y(n) = x(f(n))$$

a) Shifting

$$f(n) = n - n_0 .$$

If $y(n) = x(n-n_0)$

~~If~~ $x(n)$ is shifted to the right by "n_0" samples if "n_0" is positive as a delay.

$x(n)$ is shifted to the left by "n_0" samples if "n_0" is negative as an advance.

b) Reversal

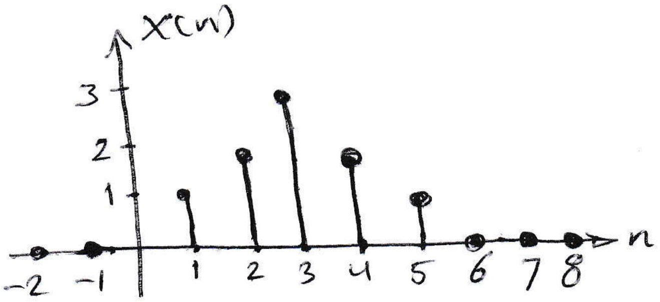
$f(n) = -n$ and simply involves "flipping" the signal $x(n)$ with respect to the index n.

c) Time scaling

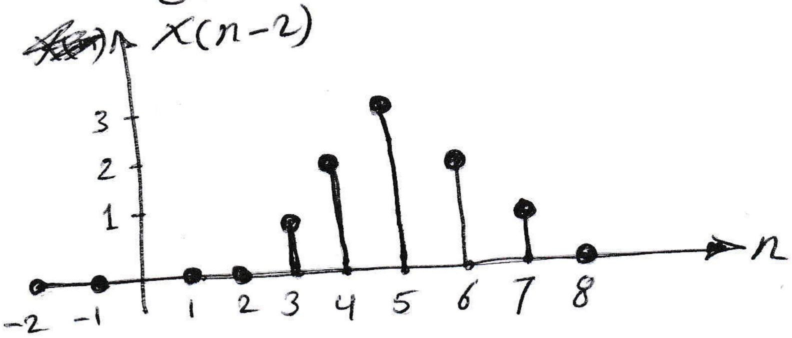
$f(n) = Mn$ or $f(n) = n/N$

where M and N are positive integers.

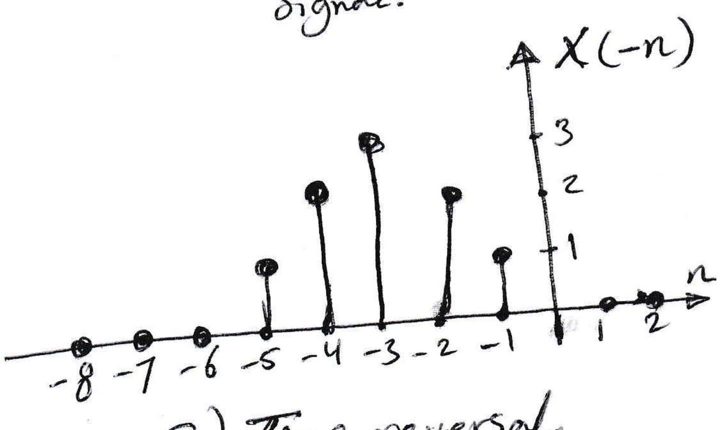
$$y(n) = \begin{cases} x\left(\frac{n}{N}\right) & n = 0, \pm N, \pm 2N, \ldots \\ 0 & \text{otherwise} \end{cases}$$



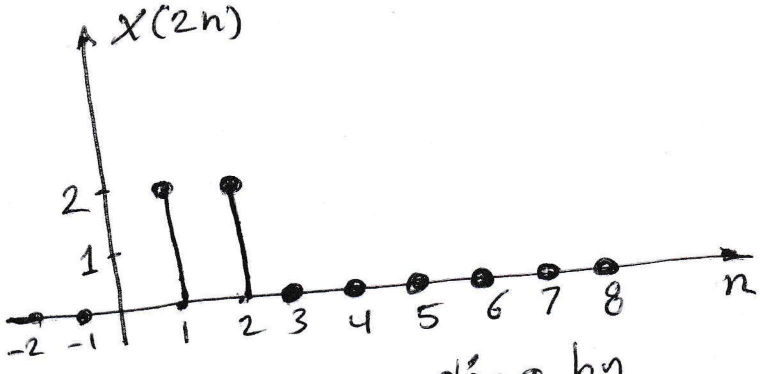
a) A discrete-time signal.



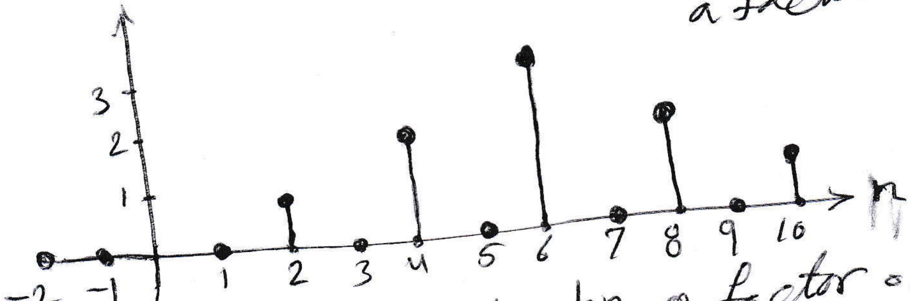
b) A delay by $n_0 = 2$.



c) Time reversal.



d) Down-sampling by a factor of 2.



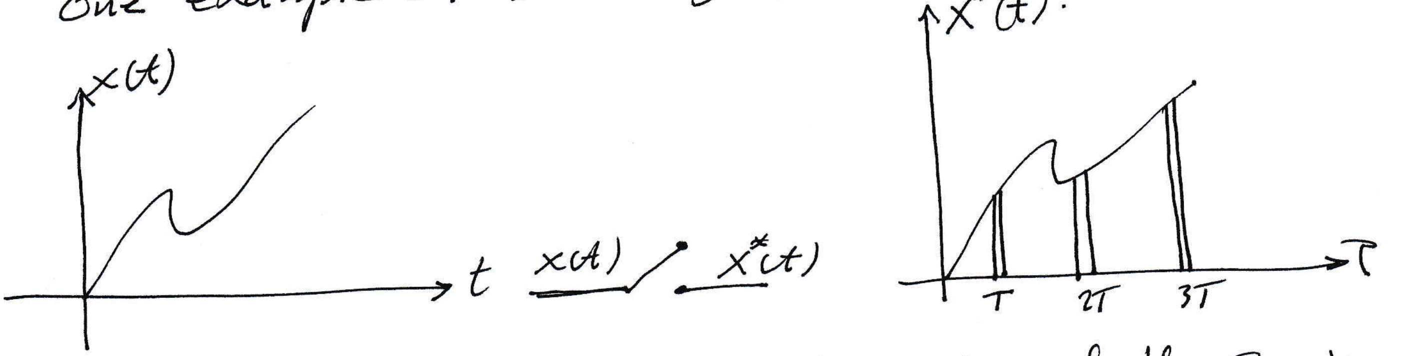
e) UP-sampling by a factor of 2.

the Z-transform

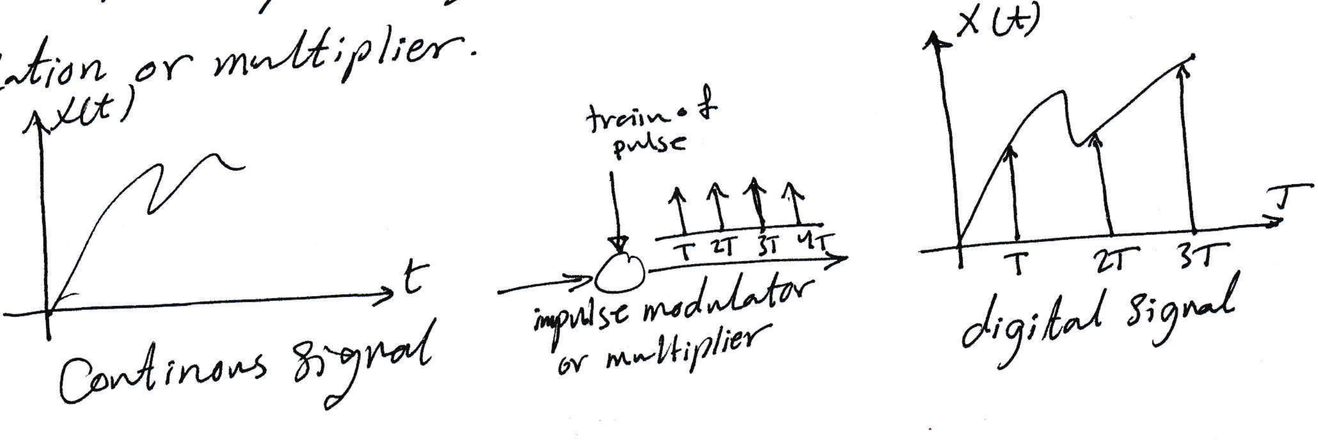
the Z-transform is a useful tool in the analysis of discrete-time signals and systems and is the discrete-time counterpart of the laplace transform for continous-time signals and systems the Z-transform may be used to solve constant coefficient difference equations, evaluate the response of a linear-time-invariant system to a given input, and design linear filters.

It deals with discrete signals signals are found is discrete time system, sampled data control system, computer control system, etc ---.

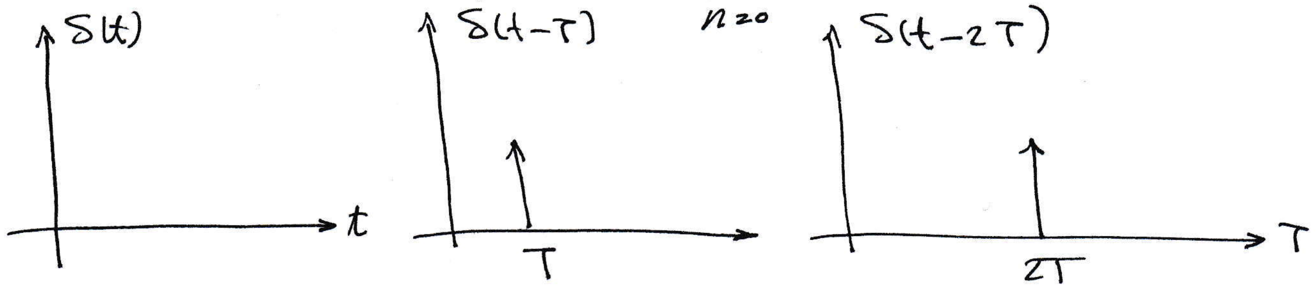
One example of such signal is the output of sampler.



To simplify the mathematical description of the sampling process; the sampler may be approximated by an impulse modulation or multiplier.



$\delta_T(t)$ = Train of unit impulses = $\sum_{n=0}^{\infty} \delta(t-nT)$



$$x^*(t) = x(t)\,\delta(t) = x(t)\sum_{n=0}^{\infty}\delta(t-nT)$$

$$= x(t)\,\delta(t) + x(t)\,\delta(t-T) + x(t)\,\delta(t-2T) + \cdots\cdots$$

$$= x(0)\,\delta(t) + x(T)\,\delta(t-T) + x(2T)\,\delta(t-2T) + \cdots$$

$$\therefore\ x^*(t) = \sum_{n=0}^{\infty} x(nT)\,\delta(t-nT)$$

take Laplace transform of both sides.

$$\mathcal{L}x^*(t) = \mathcal{L}\,x(0)\,\delta(t) + \mathcal{L}\,x(T)\,\delta(t-T) + \mathcal{L}\,x(2T)\,\delta(t-2T)$$

$$= x(0)\,\delta(t) + x(T)\,\delta(t-T) + x(2T)\,\delta(t-2T) + \cdots$$

$$= x(0)\cdot 1 + x(T)\cdot e^{-sT} + x(2T)\cdot e^{-2sT} + \cdots\cdots$$

or $$\mathcal{L}\,x^*(t) = \sum_{n=0}^{\infty} x(nT)\,e^{-snT}$$

let $Z = e^{sT}$ or $s = \frac{1}{T}\ln Z$

$$\mathcal{L}\,x^*(t) = x^*(s) = \sum_{n=0}^{\infty} x(nT)\,e^{-snT}$$

$$\therefore\ x^*(s)\Big|_{s=\frac{1}{T}\ln Z} = x(z) = Zx(t) = \sum_{n=0}^{\infty} x(nT)\,Z^{-n}$$

Z-transform of some elemantry function:-

1) Unit Step function:-

$$Z\,u(t) = \sum_{n=0}^{\infty} u(nT)\, Z^{-n} = u(0) + u(T)\, z^{-1} + u(2T)\, z^{-2}$$

$$= 1 + z^{-1} + z^{-2} + \cdots$$

$$\therefore \boxed{Z\,u(t) = \frac{1}{1-z^{-1}} = \frac{Z}{Z-1}}$$

2) Exponential function

$$Z\,e^{-at} = \sum_{n=0}^{\infty} e^{-anT}\, Z^{-n} = \sum_{n=0}^{\infty} (e^{aT} Z)^{-n}$$

$$= 1 + (e^{aT} Z)^{-1} + (e^{aT} Z)^{-2} + (e^{aT} Z)^{-3} + \cdots$$

The sum of the upper geometric series

$$\text{Sum} = \frac{1}{1-(e^{aT} Z)^{-1}} = \frac{Z}{Z-e^{-aT}}$$

$$\therefore \boxed{Z\,e^{-at} = \frac{Z}{Z-e^{-aT}}}$$

3) Z-transform of the function t:- (ramp)

$$Z(t) = \sum_{n=0}^{\infty} (nT)\, Z^{-n} = T z^{-1} + 2T z^{-2} + 3 z^{-3} + \cdots$$

$$= T z^{-1}[1 + 2 z^{-2} + 3 z^{-3} + \cdots]$$

$$\therefore Z(t) = T z^{-1} * \frac{1}{(1-z^{-1})^2}$$

$$= \frac{T Z^{-1}}{[Z^{-1}(Z-1)]^2} \Rightarrow \therefore \quad Z(t) = \frac{T Z^{-1}}{Z^{-2}(Z-1)^2} = \frac{T Z}{(Z-1)^2}$$

(4) impulse function $\delta(t)$:-

$$Z\,\delta(t) = \sum_{n=-\infty}^{\infty} \delta(nT)\, Z^{-1} = 1$$

Theorem of Z-transform :

1) Linearity :- If $Z x_1(t) = X_1(Z)$, $Z X_2(t) = X_2(Z)$

& if a, b are Constant then,

$$Z(a x_1(t) + b X_2(t)) = \sum_{n=0}^{\infty} [a\, X_1(nT) + b X_2(nT)]\, Z^{-n}$$

$$= \sum_{n=0}^{\infty} a X_1(nT)\, Z^{-n} + \sum_{n=0}^{\infty} b X_2(nT)\, Z^{-n}$$

$$\boxed{Z(a\, X_1(t) + b\, X_2(t)) = a X_1(Z) + b\, X_2(Z)}$$

2) Time shifting :- if $Z X(t) = X(Z)$ then,

a) $$Z[X(t-mT)] = \sum_{n=0}^{\infty} x(nT - mT)\, Z^{-n}$$

$$= \sum_{n=0}^{\infty} X[(n-m)T]\, Z^{-n} * Z^{m} * Z^{m}$$

$$= Z^{-m} \sum_{n=0}^{\infty} X[(n-m)T]\, Z^{-(n-m)}$$

$$Z\{x(t-mT)\} = z^{-m}\sum_{n=m}^{\infty} x[(n-m)T]\, z^{-(n-m)}$$

let $n-m=l$

$$\therefore \; = z^{-m}\sum_{l=0}^{\infty} x(lT)\, z^{-l}$$

$$\therefore \boxed{Z\{x(t-mT)\} = z^{-m}X(z)}$$

Ex find $Zu(t-2T)$

$$Zu(t) = \frac{z}{z-1} \quad \therefore Zu(t-2T) = z^{-2}\frac{z}{z-1} = \frac{z^{-1}}{z-1}$$

B) $Z\,x(t+mT) = Zx(nT+mT)$

$$= Zx\{(n+m)T\} = Z\,x_{n+m}$$

$$Z\,x_{n+m} = \sum_{n=0}^{\infty} x_{n+m} \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} x_{n+m}\, z^{-(n+m)} * z^{m}$$

$$= z^{m} \cdot \sum_{n=0}^{\infty} x_{n+m}\, z^{-(n+m)}$$ let $n+m=K$ & $K=m$

$$= z^{m}\sum_{K=0}^{\infty} x_K \cdot z^{-K}$$

We have $\sum_{K=0}^{\infty} = \sum_{K=0}^{m-1} + \sum_{K=m}^{\infty} \Rightarrow \sum_{K=m}^{\infty} = \sum_{K=0}^{\infty} - \sum_{K=0}^{m-1}$

$$\therefore \ Z x_{n+m} = Z^{m}\left[\sum_{K=0}^{\infty} x_K \cdot Z^{-K} - \sum_{K=0}^{m-1} x_K \cdot Z^{-K}\right.$$

$$\therefore \ \boxed{Z x_{n+m} = Z^{m}\left[X(Z) - \sum_{K=0}^{m-1} x_K * Z^{-K}\right.}$$

$\underline{Ex}$ find $Zu(t+3T)$?

$$Zu(t+3T) = Z^{3}\left[u(Z) - \sum_{K=0}^{2} u_K Z^{-K}\right]$$

$$= Z^{3}\left[\frac{Z}{Z-1} - (u(0) + u_1 Z^{-1} + u_2 Z^{-2})\right]$$

$$Zu(t+3T) = Z^{3}\left[\frac{Z}{Z-1} - (1 + Z^{-1} + Z^{-2})\right]$$

③ If $X(Z) = \sum_{n=0}^{\infty} x(nT) \cdot Z^{-n}$

then $$\frac{dX(Z)}{dZ} = \sum_{n=0}^{\infty} x(nT) * -n * Z^{-n-1} = -Z^{-1} \sum_{n=0}^{\infty} n x(nT) Z^{-n}$$

$$= -Z^{-1} \sum_{n=0}^{\infty} n\, x(nT) Z^{-n}$$

$$= -\frac{Z^{-1}}{T} \sum_{n=0}^{\infty} (nT)\, x(nT) \cdot Z^{-n}$$

$$\therefore \ \boxed{\frac{dX(Z)}{dZ} = \frac{-Z^{-1}}{T} Z\{t\, x(t)\}}$$

or $$\boxed{-TZ \frac{dX(Z)}{dZ} = Z\{t\, x(t)\}}$$

Ex if $Z(t) = \frac{TZ}{(Z-1)^2}$ then find t^2

$$Z(t^2) = Z\,t \cdot t = -T \cdot Z\frac{d}{dZ} * \frac{TZ}{(Z-1)^2} = -TZ\,\frac{(-TZ-T)}{(Z-1)^3}$$

$$\therefore Z\,t^2 = \frac{T^2 Z(Z+1)}{(Z-1)^3}$$

4) $Z \sin wt$

Knowing that $Z\,e^{\pm at} = \frac{Z}{Z - e^{\pm at}}$

$$Z \sin wt = \frac{1}{2j}\left\{Z e^{jwt} - Z e^{-jwt}\right\}$$

$$= \frac{1}{2j}\left\{\frac{Z}{Z - e^{jwT}} - \frac{Z}{Z - e^{-jwT}}\right\}$$

$$= \frac{Z\left\{e^{jwT} - e^{-jwT}\right\}/2j}{Z^2 - Ze^{-jwT} - Ze^{jwT} + 1}$$

$$= \frac{Z \sin wT}{Z^2 - Z\left\{e^{-jwT} + e^{jwT}\right\} + 1}$$

$$\therefore \boxed{Z \sin wt = \frac{Z \sin wt}{Z^2 - 2Z\cos wT + 1}}$$

Initial value theorem :-

$$X(Z) = \mathcal{Z}\,X(t) = \mathcal{Z}\,x(nT) = \sum_{n\geq 0}^{\infty} x(nT)\, Z^{-n}$$

$$X(0) = \lim_{t\to 0} X(t) = \lim_{n\to 0} x(nT) = \lim_{Z\to\infty} X(Z)$$

EX Given $X(Z) = \dfrac{Z}{Z - e^{-at}}$. find the initial value $X(0)$

$$\frac{\cancel{Z}}{Z(1 - \frac{e^{-at}}{Z})} = \frac{1}{1-0} = 1$$

$$X(0) = \lim_{Z\to\infty} \frac{Z}{Z - e^{-at}} = 1$$

EX Final value theorem :-

$$\lim_{t\to\infty} X(t) = \lim_{n\to\infty} x(nT) = \lim_{Z\to 1} (1 - Z^{-1})\, X(Z)$$

EX find final value for $X(t) = u(t)$

$$\mathcal{Z}\,x(t) = \mathcal{Z}\,u(t) = \frac{Z}{Z-1}$$

$$\therefore \text{final Value} = \lim_{Z\to 1} (1 - Z^{-1}) \frac{Z}{Z-1}$$

$$= \lim_{Z\to 1} (1 - Z^{-1}) \frac{1}{(1 - Z^{-1})} = 1$$

The inverse Z-transform:-

$$Zx(t) = Z(x)(nT) = X(Z) = \sum_{n=0}^{\infty} x(nT)\, Z^{-n}$$

for example

$$Ze^{-at} = Ze^{-anT} = Ze^{(-aT)^n} = \frac{Z}{Z-e^{-aT}}$$

but $Z^{-1} \dfrac{Z}{Z-e^{-aT}} = e^{-aT} = (e^{-aT})^n$

EX

$$Z^{-1}\frac{Z}{Z-3} = (3)^n, \quad Z^{-1}\frac{Z}{Z-0.4} = (0.4)^n$$

$$Z^{-1}\frac{Z}{Z-1} = (1)^n, \quad Z^{-1}\frac{Z}{Z+1} = (-1)^n$$

Method of obtaining Inverse Z-transform:-

1- Power Series inversion:-

$$X(Z) = Zx(t) = Zx(nT) = \sum_{n=0}^{\infty} x(nT)\, Z^{-n} = x(0) + x(T)Z^{-1} + x(2T)Z^{-2} + \cdots$$

$$\therefore Z^{-1}X(Z) = x(nT) = x(0), x(T), x(2T), \cdots$$

or $x_0, x_1, x_2, x_3, \cdots$

the power Series of $X(Z)$ Contains the discrete value $x_0, x(T), x(2T), \cdots$, as a coeffrents of power Series.

② Inversion Integral :-

If $X(Z)$ has one or ~~multiple~~ poles we can use the general "residue form".

$$\boxed{Res\left[X(Z)\,Z^{n-1}\right]\Big|_{Z=Z_0} = \frac{1}{(r-1)!}\frac{d^{r-1}}{dZ^{r-1}}\left[X(Z)\,(Z-Z_0)^r\,Z^{n-1}\right]}$$

EX Find $Z^{-1}\dfrac{Z}{Z-1}$?

$$Res\left[\frac{Z}{Z-1}\,Z^{n-1}\right] = \frac{1}{0!}\left[\frac{Z}{Z-1}(Z-1)\,Z^{n-1}\right] = Z^{n}\Big|_{Z=1} = (1)^n$$

EX $Z^{-1}\dfrac{Z}{(Z-2)(Z-3)}$

$$Res_1\left[\frac{Z}{(Z-2)(Z-3)}\,Z^{n-1}\right]_{Z=2} = \left[\frac{Z}{(Z-2)(Z-3)} * (Z-2)\,Z^{n-1}\right]_{Z=2}$$

$$= \frac{Z^n}{Z-3} = \frac{(2)^n}{-1} = -(2)^n$$

$$Res_2\left[\frac{Z}{(Z-2)(Z-3)}\,Z^{n-1}\right]_{Z=3} = \left[\frac{Z}{(Z-2)(Z-3)}(Z-3)\,Z^{n-1}\right]_{Z=3}$$

$$= (3)^n$$

$$\therefore Z^{-1}\frac{Z}{(Z-2)(Z-3)} = -(2)^n + (3)^n$$

$\underline{\underline{EX}}$ find $Z^{-1} \dfrac{3Z}{(Z-1)^2(Z+2)}$

$$Res_1 \left[X(Z)\, Z^{n-1}\right] = \frac{1}{1} \frac{d}{dZ} \left[\frac{3Z}{(Z-1)^2(Z+2)} * (Z-1)^2 Z^{n-1}\right]\Bigg|_{Z=1}$$

$$= \frac{d}{dZ}\left[\frac{3Z}{(Z+2)}\,(Z^{n-1})\right]_{Z=1} = \frac{d}{dZ}\left[\frac{3Z^n}{Z+2}\right]_{Z=1}$$

$$= \frac{(Z+2) * 3n * Z^{n-1} - 3Z^n}{(Z+2)^2}\Bigg|_{Z=1} = \frac{9n}{9} - \frac{3}{9} = n - \frac{1}{3}$$

$$Res_2\left[X(Z)\, Z^{n-1}\right] = \left[\frac{3Z}{(Z-1)^2(Z+2)}\,(Z+2)\, Z^{n-1}\right]_{Z=-2}$$

$$= \frac{3Z^n}{(Z-1)^2}\Bigg|_{Z=-2} = \frac{3(-2)^n}{9} = \frac{1}{3}(-2)^n$$

$$\therefore Z^{-1} \frac{3Z}{(Z-1)^2(Z+2)} = n - \frac{1}{3} + \frac{1}{3}(-2)^n$$

3) Partial Fraction expansion method :-

First divide $X(Z)$ by Z, then obtain Coefficient of Partial fraction as before, then obtain the inverse term by term after multiply by Z again.

EX $Z^{-1}\dfrac{2Z^2-1.5Z}{Z^2-1.5Z+0.5}$

$$\frac{X(Z)}{Z}=\frac{2Z-1.5}{Z^2-1.5Z+0.5}=\frac{2Z-1.5}{(Z-0.5)(Z-1)}$$

$$=\frac{A}{Z-0.5}+\frac{B}{Z-1}$$

$$A=\lim_{Z\to 0.5}(Z-0.5)\frac{2Z-1.5}{(Z-0.5)(Z-1)}=1$$

$$B=\lim_{Z\to 1}(Z-1)\frac{2Z-1.5}{(Z-0.5)(Z-1)}=1$$

$$\therefore \frac{X(Z)}{Z}=\frac{1}{Z-0.5}+\frac{1}{Z-1}\Rightarrow X(Z)=\frac{Z}{Z-0.5}+\frac{Z}{Z-1}$$

$$\therefore X(nT)=(0.5)^n+(1)^n$$

EX find $Z^{-1}\dfrac{1}{(Z-1)(Z+3)}$ by partial fraction :

$$\frac{X(Z)}{Z}\Rightarrow\frac{X(Z)}{Z}=\frac{1}{(Z-1)(Z+3)Z}$$

$$\frac{X(Z)}{Z}=\frac{A}{Z-1}+\frac{B}{Z+3}+\frac{C}{Z}$$

$$A=\lim_{Z\to 1}\frac{Z}{(Z-1)(Z+3)Z}(Z-1)=\frac{1}{(1+3)\times 1}=\frac{1}{4}$$

$$B = \lim_{Z \to -3} \frac{1}{(Z+3)(Z-1)Z}(Z+3) = \frac{+1}{+12} = \frac{1}{12}$$

$$C = \lim_{Z \to 0} \frac{1}{(Z-1)(Z+3)\cancel{Z}}\cancel{Z} = -\frac{1}{3}$$

$$\therefore \frac{X(Z)}{Z} = \frac{1/4}{(Z-1)} + \frac{1/12}{Z+3} + \frac{-1/3}{Z}$$

$$\therefore X(nT) = \frac{1}{4}(1)^n + \frac{1}{12}(-3)^n - \frac{1}{3}\delta(n)$$

Ex find $Z^{-1} \dfrac{Z}{Z^2 - Z + 0.5}$

We know that $Z \sin wt = \dfrac{Z \sin WT}{Z^2 - 2Z\cos WT + 1}$

$$Z e^{-at} \sin wt = \frac{Z e^{-aT} \sin WT}{Z^2 - 2Z e^{-aT}\cos WT + e^{-2aT}}$$

By Comparing $0.5 = e^{-2aT} \Rightarrow (e^{-aT})^2 = \frac{1}{2}$

$$\therefore e^{-aT} = \frac{1}{\sqrt{2}}$$

$$2e^{-aT}\cos WT = 1 \Rightarrow 2\frac{1}{\sqrt{2}}\cos WT = 1$$

$$\therefore \cos WT = \frac{1}{\sqrt{2}} \Rightarrow WT = \frac{\pi}{4} = 45^\circ$$

$$\sin WT = \frac{1}{\sqrt{2}}$$

$$\therefore e^{-aT}\sin WT = \frac{1}{\sqrt{2}} * \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$Z e^{-aT}\sin WT = \frac{1/2\, Z}{Z^2 - Z + 0.5}$$

$$= Z^{-1} \frac{1/2\, Z}{Z^2 - Z + 0.5} = e^{-anT} \sin n\omega T$$

$$= (e^{-aT})^n \sin \omega n T = \left(\frac{1}{\sqrt{2}}\right)^n \sin n\frac{\pi}{4}$$

$$\therefore Z^{-1} \frac{Z}{Z^2 - Z + 0.5} = 2 * \left(\frac{1}{\sqrt{2}}\right)^n \sin n\frac{\pi}{4}$$

Ex find $Z^{-1} \dfrac{Z(Z+0.5)}{Z^2 - 0.5Z + 0.25}$

$$Z \cos \omega T = \frac{Z(Z - \cos \omega T)}{Z^2 - 2Z\cos \omega T + 1}$$

$$\therefore Z\, e^{-aT} \cos \omega T = \frac{Z e^{-aT}(Z - e^{-aT}\cos \omega T)}{Z^2 - 2Z e^{-aT}\cos \omega T + e^{-2aT}}$$

By Comparing

$$\frac{1}{4} = e^{-2aT} \Rightarrow \frac{1}{4} = (e^{-aT})^2 \Rightarrow e^{-aT} = \frac{1}{2}$$

$$2 * \frac{1}{2} \cos \omega T = 0.5 \Rightarrow \cos \omega T = 0.5 \Rightarrow \omega T = \frac{\pi}{3}$$

$$Z\, e^{-anT} \cos \omega T = \frac{Z(Z - 0.5 * 0.5)}{Z^2 - Z * \frac{1}{2} * 0.5 Z + 0.25}$$

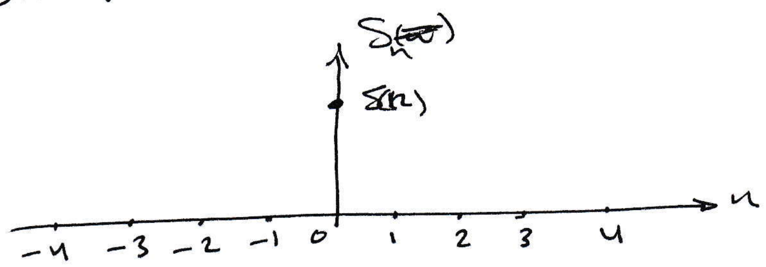
$$\therefore Z^{-1} \frac{Z(Z - 0.25)}{Z^2 - 0.5Z + 0.25} = (e^{-aT})^n \cos \omega n T$$

$$= (e^{-aT})^n \cos n\frac{\pi}{3}$$

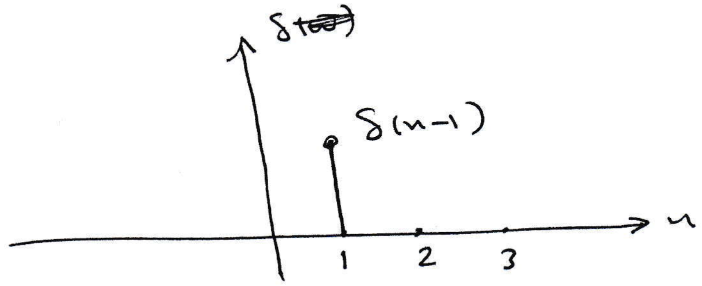
$$= \left(\frac{1}{2}\right)^n \cos \frac{n\pi}{3}$$

Unit Step Sequences "δ_n" :-

$$\delta_n = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

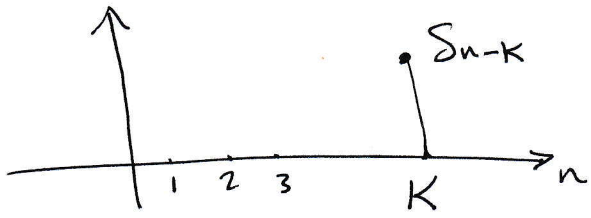


$$\delta_{n-1} = \begin{cases} 1 & n = 1 \\ 0 & n \neq 1 \end{cases}$$



general

$$\boxed{\delta_{n-K} = \begin{cases} 1 & n = K \\ 0 & n \neq K \end{cases}}$$

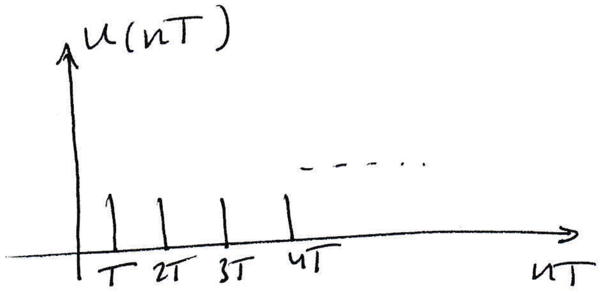


$$Z\,\delta_n = 1$$

$$Z\,\delta_{n-1} = Z^{-1}$$

In general $\Rightarrow$ $\boxed{Z\,\delta_{n-K} = Z^{-K}}$

Unit step sequences :-



$$Z\,u(n) = \frac{Z}{Z-1}$$

$$Z(u_n - \delta_n) = Z\,u_n - Z\,\delta_n$$

$$= \frac{Z}{Z-1} - 1$$

$$\boxed{Z(u_n - \delta_n) = \frac{1}{Z-1}}$$