H.W:- Use G-S and Jacobi to solve

$$\left.\begin{array}{l} 2X_1 - X_2 + X_3 = -1 \\ 3X_1 + 3X_2 + 9X_3 = 0 \\ 3X_1 + 3X_2 + 5X_3 = 4 \end{array}\right\} \quad X^{(0)} = (0.5, 1.5, -0.5)$$

Ans: (1, 2, -1)

C) Bisection method:-

تابع الى الموضوع الاول

It is simple but slowly convergent method for determin a zero of $f(x)$ when $f(x)$ is contineous. It is based on the intermidiet theory:- Suppose the continous fun. $f(x)$ defined on the interval $[a,b]$ is given with $f(a)<0$ & $f(b)>0$, then there exist c, $a<c<b$.

Steps there solution:-

Given a contineous fun. $f(x)$ with an interval $[a_0, b_0]$ and satisfying $f(a_0)\,f(b_0) < 0$

1) compute $C_n = \frac{1}{2}(a_n + b_n)$, $n = 0, 1, 2, \ldots$

2) if $C_n \neq 0$, and $f(a_n)\,f(C_n) < 0$, then
   Set $a_{n+1} = a_n$, $b_{n+1} = C_n$
   Else $a_{n+1} = C_n$, $b_{n+1} = b_n$

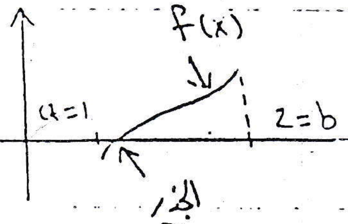
3) The iteration is stopped:-

$$\frac{|C_{n-1} - C_n|}{|C_n|} < \text{tolerance } (\epsilon)$$

Ex: $f(x) = x^3 + 4x^2 - 10$ has a root in [1,2] use bisection method to find that root.

$f(a) = f(1) = 1 + 4 - 10 = -5$

$f(b) = f(2) = 2^3 + 4 \times 4 - 10 = 14$



n	a_n	b_n	c_n	$f(a_n)$	$f(c_n)$
0	1	2	1.5	-Ve	+Ve
1	1	1.5	1.25	-Ve	-Ve
2	1.25	1.5	1.375	-Ve	+Ve
3	1.25	1.375	1.3125	-Ve	-Ve
11	1.364746	1.365234	1.364990	-Ve	-Ve
12	1.364990	1.365234	1.365112	-Ve	-Ve

$$\left| \frac{C_{11} - C_{12}}{C_{12}} \right| = 8.9 \times 10^{-5}$$

H.W: $f(x) = x^3 - x - 1$ has a root in [1,2] use bisection method to find that root assume $\epsilon = 10^{-2}$

Ans: 1.3203125

Z.T

1) (a,b) , 2 (الفردية فقط) , 3 , 4 , 6 , 7

مواعيد كوزات الفصل الثاني

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Sheet Z-transform

Q-6 : C)

$$x(n+2) - x(n+1) + 0.5\, x(n) = 1 \,,\; x(0) = 2 \,,\; x(1) = 3$$

$$Z\{x(n+p)\} = Z^{p}\left\{X(Z) - \sum_{k=0}^{p-1} x(k)\, Z^{-k}\right\}$$

$$\{Z^2 X(Z) - Z^2 x(0) - Z\, x(1)\} - \{Z\, X(Z) - Z\, x(0)\} +$$

$$0.5\, X(Z) = \frac{Z}{Z-1}$$

نعوض I.C ونشوف الحدود

$$\frac{X(Z)}{Z} = \frac{2Z^2 - Z}{(Z-1)(Z^2 - Z + 0.5)} = \frac{2Z^2 - Z}{(Z-1)\{(Z-\frac{1}{2})^2 + \frac{1}{4}\}}$$

$$= \frac{2Z^2 - Z}{(Z-1)\{(Z-\frac{1}{2}) + j\frac{1}{2}\}\{(Z-\frac{1}{2}) - j\frac{1}{2}\}}$$

$$\frac{X(Z)}{Z} = \frac{2Z^2 - Z}{(Z-1)\{Z - (\frac{1}{2} - j\frac{1}{2})\}\{Z - (\frac{1}{2} + j\frac{1}{2})\}}$$

$$= \frac{A}{Z-1} + \frac{B}{Z - (\frac{1}{2} - j\frac{1}{2})} + \frac{C}{Z - (\frac{1}{2} - j\frac{1}{2})}$$

اكمل الحل

Numerical method for 1-st order D.E :-

A D.E of the 1-st order is of the form

$y' = f(x, y)$ —— (1)

$y(x_0) = y_0$ = initial Value

which is also called an initial Value problem

eq. (1) Can be solved using

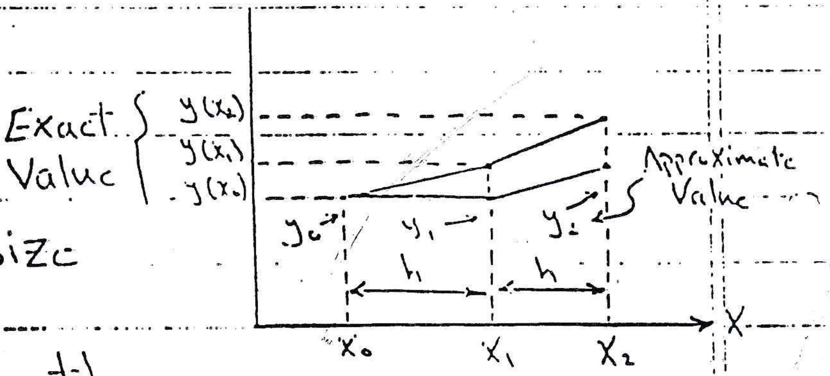
1- Euler method

2- Runge-kutta (R-K) method

1) Euler method :-

It is an approximation of the curve $y(x)$ by Polygon

$$y'_0 = \frac{y_1 - y_0}{h}$$



$h = x_{n+1} - x_n$ = step size

$\therefore y_1 = y_0 + h y'_0$

using Taylor series, then

$$\boxed{y_{n+1} = y_n + h y'_n = y_n + h f(x_n, y_n)}$$

is called Euler method (App. solution)

Note:- The omission of the further terms of the above equation causes of error (Trancation ...

$\int u \cdot dv = u \cdot v - \int v \cdot du$, $u = x$, $dv = e^{x}$

$= x(-e^{x}) - \int -e^{x} \, dx$ (122)

error).

To Find the exact solution:-

1) $y' + f(x) \cdot y = r(x)$ .... تشابه المعادلة

2) $y(x) = e^{-\ell} \left\{ \int e^{\ell} r(x) \, dx + c \right\}$ = Exact sol.

$\ell = \int f(x) \, dx$

Ex:- Find the exact solution of $y' = 2y$ if $y(0) = 1$

$y' - 2y = 0$

$\therefore f(x) = -2$, $r(x) = 0$

$\ell = \int f(x) \, dx = \int -2 \, dx = -2x$

$\therefore y(x) = e^{2x} \left\{ \int e^{-2x} (0) \, dx + c \right\}$

$y(x) = e^{2x} c$

لايجاد قيمة الثابت نعوض بـ I.C :-

$1 = e^{0} c$, $\therefore c = 1$

$y(x) = e^{2x}$ (دائماً الحل المضبوط يكون بدلالة X)

Ex:- Use Euler method to solve $y' = x + y$, $h = 0.2$, $n = 5$, $y(0) = 0$ Calculate the error?

$y_{n+1} = y_n + 0.2 (x_n + y_n)$ ← App. sol.

$y(x_n) = e^{x} - x - 1$ = Exact sol. (as H.W)

n	X_n	y_n	y_{n+1}	$y(X_n)$	$\lvert error \rvert = \lvert Exact - App. \rvert$
0	0	$y_0 = 0$	$y_1 = 0$	0	0
1	0.2	$y_1 = 0$	$y_2 = 0.04$	$0.0124 = y(x_1)$	0.0214
2	0.4	0.04	0.128	0.0918	0.0518
3	0.6	0.128	0.2736	0.222	0.094
4	0.8	0.2736	0.488	0.4255	0.152
5	1	0.488	0.7856	0.718	0.229

حساب الخطأ

H.W: Use Euler method to solve $y' = x^2 + y$, $h = 0.025$, $X_0 = 0$, $y(0) = 1$, $n = 8$, calculate the error. ($y(x) = 3e^x - x^2 - 2x - 2$) exact sol. (as H.W)

2) Runge - kutta (R-k) method:-

It is more accurate than Euler method because it has the advantage of eliminating the evaluation of the derivatives of $f(x, y)$ and hence reducing the compitation. it depends on four auxiliary contities A_n, B_n, C_n & D_n (Runge-kutta order four-method)

It is ~~dre~~ dbrector - corrector method
تصحيح

steps of solution :-

given an initial Value problem

$y' = f(x, y)$, $y(x_0) = y_0$ & $x_{n+1} = x_0 + (n+1)h$

$A_n = h f(x_n, y_n)$

$B_n = h f\{x_n + \frac{h}{2}, y_n + \frac{A_n}{2}\}$

$C_n = h f\{x_n + \frac{h}{2}, y_n + \frac{B_n}{2}\}$

$D_n = h f\{x_n + h, y_n + C_n\}$

$y_{n+1} = y_n + \frac{1}{6}(A_n + 2B_n + 2C_n + D_n)$

Ex : Solve using (R-K) method $y' = x + y$, $h = 0.2$, $n = 5$, $y(0) = 0$, Calculate the error.

solution :- $y' = f(x, y) = x_n \boxed{+} y_n$

$A_n = h f(x_n, y_n) = 0.2(x_n \boxed{+} y_n)$ ——— ①

$B_n = 0.2 \{x_n + 0.1 \boxed{+} y_n + \frac{A_n}{2}\}$ ——— ②

نعوض (1) ب (2)

$B_n = 0.22(x_n + y_n) + 0.02$ ——— ③

$C_n = 0.2 \{x_n + 0.1 \boxed{+} y_n + \frac{B_n}{2}\}$ ——— ④

نعوضها (3) ب (4)

$C_n = 0.222(x_n + y_n) + 0.022$ ——— ⑤

$D_n = 0.2 \{x_n + 0.2 \boxed{+} y_n + C_n\}$ ——— ⑥

نعوض (5) ب (6)

$$D_n = 0.2444\,(X_n + y_n) + 0.0444 \quad \text{--- (7)}$$

$$\therefore\; y_{n+1} = y_n + \frac{1}{6}(A_n + 2B_n + 2C_n + D_n) \quad \text{--- (8)}$$

نعوض المعادلات 1,3,5,7 في المعادلة اعلاه (8)

$$y_{n+1} = y_n + 0.2214\,(X_n + y_n) + 0.0214 = \text{App. sol.}$$

$$y(X_n) = e^X - X - 1$$
$$= \text{Exact sol. (as H.w)}$$

n	X_n	y_n	y_{n+1}	$y(X_n)$	Error
0	0	0	$y_1 = 0.0214$	0	0
1	0.2	$y_1 = 0.0214$	$y_2 = 0.91818$	$0.0214 = y(X_1)$	0
2	0.4	0.091818	0.222107	0.0918	$1.8 * 10^{-5}$
3	0.6	0.222107	0.425521	0.222	$1.7 * 10^{-4}$
4	0.8	0.425521	0.718251	0.4255	$2.1 * 10^{-5}$
5	1	0.718251	1.120071	0.718	$2.5 * 10^{-4}$

لحساب نسبة الخطأ

H.W: Solve using R-K method, $\acute{y} = X^2 + y$, $X_0 = 0$, $y_0 = 1$, $n = 8$, $h = 0.025$. Calculate the error.

$$y(x) = \text{Exact sol.} = 3e^X - X^2 - 2X - 2 \quad \text{(as H.w)}$$

umerical Differentiation :-

1) Difference formula.
2) Three points formula.

1) Difference formula :-

$$f'(x_0) = \frac{f(x_0+h)-f(x_0)}{h} - \underbrace{\frac{h}{2} f''(x)}_{error} , \quad h \neq 0$$

If $h>0$, it is called forward difference formula.
If $h<0$, " " " backward " "

Ex: Find $f'(1.8)$ for $f(x)=\ln x$ using $h=0.1, 0.01, 0.001$

$$f'(1.8) = \frac{f(1.8+h)-f(1.8)}{h} = \frac{\ln(1.8+h)-\ln(1.8)}{h}$$

h	$f'(1.8)$	$\lvert Error \rvert = \frac{h}{2} f''(x) = \frac{h}{2} \frac{1}{(1.8)^2}$
0.1	0.540672	0.0154321
0.01	0.551018	0.001543
0.001	0.555401	0.000154

$f'(x) = 1/x \quad f''(x) = -1/x^2$

Exact sol. $= f'(x) = 1/1.8 = 0.5555$

$\therefore$ Error = Exact sol. − App. sol.

وعليه كلما قلت قيمة h زادت الدقة

2) Three points formula :-

In this method the points are equally spaced

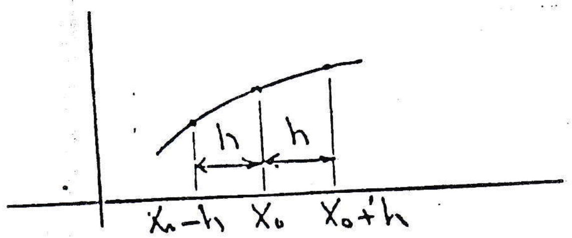
$x_0, \; x_1 = x_0 + h, \; x_2 = x_0 + 2h, \; h \neq 0$

Formula (1) :-

$$f'(x_0) = \frac{1}{2h}\left\{-3f(x_0) + 4f(\overbrace{x_0+h}^{x_1}) - f(\overbrace{x_0+2h}^{x_2})\right\} + \underbrace{\frac{h^2}{3} f'''(x)}_{\text{error}}$$

Formula (2) :-

$$f'(x_0) = \frac{1}{2h}\left\{f(x_0+h) - f(x_0-h)\right\} - \underbrace{\frac{h^2}{6} f'''(x)}_{\text{error}}$$



Error = Exact sol. – App. sol.

Formula (2) has the advantage over formula (1)

1) half the error.
2) evaluated only at two points.

Ex: Find $f'(2)$ for $f(x) = xe^x$, $h=0.1$ using three points formula(s).

x	$f(x) = xe^x$
1.9	12.703199
2	14.778112
2.1	17.148957
2.2	19.85503

القيم من الجدول

formula (1) :-

$$f'(2) = \frac{1}{0.2}\left\{-3f(2) + 4f(2.1) - f(2.2)\right\} = 22.03231$$

formula (2) :-

$$f'(2) = \frac{1}{0.2}\left\{f(2.1) - f(1.9)\right\} = 22.22879$$

$$\text{Exact sol.} = xe^x + e^x = 2e^2 + e^2 = 22.167168$$

$$\therefore \text{Error} = 22.167168 - \begin{cases} 22.03231 \\ 22.22879 \end{cases} = \begin{cases} 0.1126 \\ -0.0616 \end{cases}$$

Exact − App.

Numerical solution of non linear system:-

Non linear system can be solved by using:-

1) Fixed point method (Jacobi & G-S methods).
2) Newton method.

1) Fixed point method:-

∴ ~~The nonlinear sys~~tem is in the form:-

$$f_1(x_1, x_2, \dots, x_n) = f_1(X) = 0$$
$$f_2(x_1, x_2, \dots, x_n) = f_2(X) = 0$$
$$\vdots$$
$$f_n(x_1, x_2, \dots, x_n) = f_n(X) = 0$$
$$X = (x_1, x_2, \dots, x_n)$$
$$F(x) = \{f_1(X), f_2(X), \dots, f_n(X)\}$$

مواعيد الامتحانات اليومية ٨/٣ ، ٢٩/٣ ، ١٩/٤ شعبة (د)

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Ex: solved the nonlinear sys.

$$f_1(x_1, x_2, x_3) = 3x_1 - \cos(x_2 x_3) - 1/2 = 0$$
$$f_2(x_1, x_2, x_3) = x_1^2 - 81(x_2 + 0.1)^2 + \sin x_3 + 1.06 = 0$$
$$f_3(x_1, x_2, x_3) = e^{-x_1 x_2} + 20x_3 + \frac{10\pi - 3}{3} = 0$$

$$X^{(0)} = \underset{x_1^{(0)}}{0.1}, \underset{x_2^{(0)}}{0.1}, \underset{x_3^{(0)}}{-0.1}$$

a) Jacobi method :-

$$X_1^{(k)} = \frac{1}{3}\cos(X_2^{(k-1)}\, X_3^{(k-1)}) + 1/6$$

$$X_2^{(k)} = \frac{1}{9}\left\{\left[X_1^{(k-1)}\right]^2 + \sin X_3^{(k-1)} + 1.06\right\}^{1/2} - 0.1$$

$$X_3^{(k)} = -\frac{1}{20}\, e^{-X_1^{(k-1)} X_2^{(k-1)}} - \frac{10\pi - 3}{60}$$

k	$X_1^{(k)}$	$X_2^{(k)}$	$X_3^{(k)}$
0	0.1	0.1	−0.1
1	0.499983	0.0094411	−0.523101
2	0.499995	0.00002	−0.523633
3	0.5	0.00001	−0.523598
4	=	$3*10^{-8}$	=
5	=	$2*10^{-8}$	=

$\therefore X \cong 0.5\ ,\ 0\ ,\ -0.523598$
$\qquad\ \ X_1 \quad X_2 \quad X_3$

للتأكد نعوض بمعادلات السؤال

b) G-S method :-

$$X_1^{(k)} = \frac{1}{3}\cos X_2^{(k-1)}\, X_3^{(k-1)} + 1/6$$

$$X_2^{(k)} = \frac{1}{9}\left\{\left[X_1^{(k)}\right]^2 + \sin X_3^{(k-1)} + 1.06\right\}^{1/2} - 0.1$$

$$X_3^{(k)} = -\frac{1}{20}\, e^{-X_1^{(k)} X_2^{(k)}} - \frac{10\pi - 3}{60}$$

k	$X_1^{(k)}$	$X_2^{(k)}$	$X_3^{(k)}$
0	0.1	0.1	-0.1
1	0.499983	0.02222	-0.523046
2	0.499977	0.00002	-0.523598
3	0.5	4×10^{-8}	-0.523598
4	=	0	=

$\therefore X \cong \underset{X_1}{0.5}, \underset{X_2}{0}, \underset{X_3}{-0.523598}$

2) Newton's method :-

$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}, \quad f' \neq 0$ for 1 - Variable

$$X^{(k)} = X^{(k-1)} - J[X^{(k-1)}]^{-1} \cdot F[X^{(k-1)}] \quad \text{—— (1)}$$

$$\begin{bmatrix} x_1^{(k)} \\ \vdots \\ x_n^{(k)} \end{bmatrix} = \begin{bmatrix} x_1^{(k-1)} \\ \vdots \\ x_n^{(k-1)} \end{bmatrix} - \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & & \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}^{-1} \begin{bmatrix} f_1[X^{(k-1)}] \\ \vdots \\ f_n[X^{(k-1)}] \end{bmatrix} \quad \text{—— (2)}$$

Ex: Solve the non-linear sys. using Newton's method

$$\left.\begin{aligned} f_1(X_1, X_2, X_3) &= 3X_1 - \cos(X_2 X_3) - 1/2 = 0 \\ f_2(X_1, X_2, X_3) &= X_1^2 - 81(X_2 + 0.1)^2 + \sin X_3 + 1.06 = 0 \\ f_3(X_1, X_2, X_3) &= e^{-X_1 X_2} + 20 X_3 + \frac{10\pi - 3}{3} = 0 \end{aligned}\right\} \text{—— (3)}$$

$X^{(0)} = \underset{X_1^{(0)}}{0.1}, \underset{X_2^{(0)}}{0.1}, \underset{X_3^{(0)}}{-0.1}$

$$J=\begin{bmatrix} 3 & x_3^{(k-1)}\sin x_2^{(k-1)}\, x_3^{(k-1)} & x_2^{(k-1)}\sin x_2^{(k-1)}\, x_3^{(k-1)} \\ 2x_1^{(k-1)} & -162\,[x_2^{(k-1)}+0.1] & \cos x_3^{(k-1)} \\ -x_2^{(k-1)}\, e^{-x_1^{(k-1)} x_2^{(k-1)}} & -x_1^{(k-1)}\, e^{-x_1^{(k-1)} x_2^{(k-1)}} & 20 \end{bmatrix} \quad \text{—④}$$

عندما k=1 نعوض I.C في المعادلة (4) ونأخذ لها inverse فينتج

$$J^{-1}=\begin{bmatrix} 0.333 & 1.03*10^{-5} & -1.7*10^{-5} \\ 2.1*10^{-3} & -0.03 & 1.5*10^{-3} \\ 1.6*10^{-3} & -1.5*10^{-4} & 0.05 \end{bmatrix} \quad \text{—⑤}$$

عندما k=1 نعوض I.C في المعادلة (3) نحصل ع

$$\left.\begin{aligned} f_1[x^{(0)}] &= -1.19995 \\ f_2[x^{(0)}] &= -2.26983 \\ f_3[x^{(0)}] &= 8.46202 \end{aligned}\right\} \text{—⑥}$$

نعوض (5) و (6) بـ (2) ع

$$\begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ x_3^{(1)} \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \\ -0.1 \end{bmatrix} - [J^{-1}] \begin{bmatrix} -1.19995 \\ -2.26983 \\ 8.46202 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 \\ 0.019466 \\ -0.52152 \end{bmatrix}$$

وهذه تعتبر ك I.C عندما تصبح k=2 ويعاد الحل من بدايته

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$
0	0.1	0.1	-0.1
1	0.5	0.019466	-0.52152
2	0.50004	-0.001588	-0.523557
3	0.5	-0.00001	-0.523598
4	0.5	0	=

$$\therefore X \cong \underset{x_1}{0.5}\ ,\ \underset{x_2}{0}\ ,\ \underset{x_3}{-0.523598}$$

للتأكد نعوض بمعادلات السؤال