Lecture ( )

Magnetically Coupled Circuits

1) Introduction

The circuits we have considered so far may be regarded as conductively coupled, because one loop affects the neighboring loop through current conduction. When two loops with or without contacts between them affect each other through the magnetic field generated by one of them, they are said to be ***magnetically coupled.***

The ***transformer*** is an electrical device designed on the basis of the concept of magnetic coupling. It uses magnetically coupled coils to transfer energy from one circuit to another. Transformers are key circuit elements. They are used in power systems for stepping up or stepping down ac voltages or currents. They are used in electronic circuits such as radio and television receivers for such purposes as impedance matching, isolating one part of a circuit from another, and again for stepping up or down ac voltages and currents.

2) Mutual Inductance

When two inductors (or coils) are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter. This phenomenon is known as ***mutual inductance.***

2.1) Single inductor (a coil with N turns) : When current $\boldsymbol{i}$ flows through the coil, a magnetic flux $\emptyset$ is produced around it (Fig.2.1). According to Faraday's law, the voltage $\boldsymbol{v}$ induced in the coil is proportional to the number of turns $\boldsymbol{N}$ and the time rate of change of the magnetic flux ; that is,

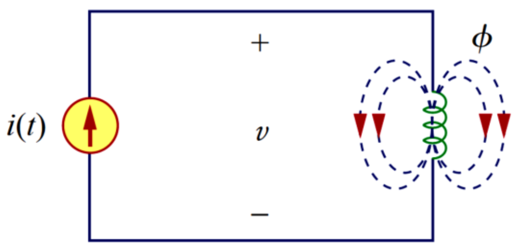


Fig.2.1

$$v = N\,\frac{d\emptyset}{dt} \qquad ...(2.1)$$

or Eq. (2.1) can be written as,

$$v = N\,\frac{d\emptyset}{di}\,\frac{di}{dt} = L\,\frac{di}{dt} \qquad ...(2.2)$$

where

$$L = N\,\frac{d\emptyset}{di} \qquad ...(2.3)$$

This inductance is commonly called ***self-inductance***, because it relates the voltage induced in a coil by a time-varying current in the same coil.

2.2) Two coils with self-inductances L_1 and L_2 :

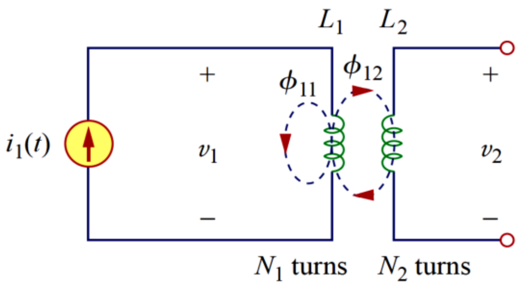


Fig.2.2

Consider Fig.2.2, For simplicity, assume that the second inductor carries no current. The magnetic flux $\emptyset_1$ emanating from coil 1 has two components:

1) component $\emptyset_{11}$ links only coil 1
2) component $\emptyset_{12}$ links both coils.

$\therefore\ \emptyset_1 = \emptyset_{11} + \emptyset_{12}$ ...(2.4)

Since the entire flux $\emptyset_1$ links coil 1, so

$$v_1 = N_1 \frac{d\emptyset_1}{dt} = N_1 \frac{d\emptyset_1}{di_1}\frac{di_1}{dt} = L_1 \frac{di_1}{dt} \qquad ...(2.5)$$

Where $L_1 = N_1 \frac{d\emptyset_1}{di_1}$ is the self inductance of coil 1.

Only flux $\emptyset_{12}$ links coil 2, so

$$v_2 = N_2 \frac{d\emptyset_{12}}{dt} = N_2 \frac{d\emptyset_{12}}{di_1}\frac{di_1}{dt} = M_{21} \frac{di_1}{dt} \qquad ...(2.6)$$

Where $M_{21} = N_2 \frac{d\emptyset_{12}}{di_1}$ ...(2.7)

M_{21} is known as the ***mutual inductance*** of coil 2 with respect to coil 1. Subscript 21 indicates that the inductance M_{12} relates the voltage induced in coil 2 to the current in coil 1. Thus, the open-circuit ***mutual voltage*** (or induced voltage) across coil 2 is

$$v_2 = M_{21} \frac{di_1}{dt} \qquad ...(2.8)$$

2.3) Suppose we now let current i_2 flow in coil 2, while coil 1 carries no current:

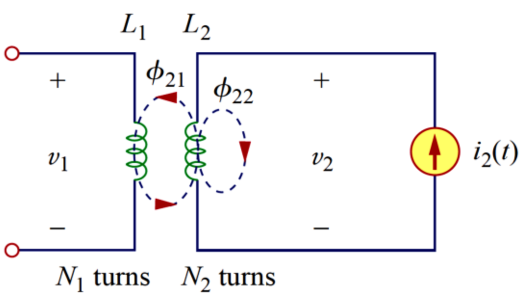


Fig.2.3

Consider Fig. 3, the magnetic flux $\emptyset_2$ emanating from coil 2 comprises flux $\emptyset_{22}$ that links only coil 2 and flux $\emptyset_{21}$ that links both coils.

$\therefore\ \emptyset_2 = \emptyset_{21} + \emptyset_{22}$ ...(2.9)

The entire flux $\emptyset_2$ links coil 2, so

$$v_2 = N_2 \frac{d\emptyset_2}{dt} = N_2 \frac{d\emptyset_2}{di_2}\frac{di_2}{dt} = L_2 \frac{di_2}{dt} \qquad ...(2.10)$$

Where $L_2 = N_2 \frac{d\emptyset_2}{di_2}$ is the self inductance of coil 2.

Only flux $\emptyset_{21}$ links coil 1. so

$$v_1 = N_1 \frac{d\emptyset_{21}}{dt} = N_1 \frac{d\emptyset_{21}}{di_2}\frac{di_2}{dt} = M_{12} \frac{di_2}{dt} \qquad ...(2.11)$$

Where $M_{12} = N_1 \frac{d\emptyset_{21}}{di_2}$ ...(2.12)

which is the ***mutual inductance*** of coil 1 with respect to coil 2. Thus, the open-circuit ***mutual voltage*** across coil 1 is

$$v_1 = M_{12} \frac{di_2}{dt} \qquad ...(2.13)$$

$$M_{12} = M_{21} = M \qquad ...(2.14)$$

Mutual inductance is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H). Like self-inductance L, mutual inductance M is measured in henrys (H).

3) ANALYSIS OF COUPLED CIRCUITS

Although mutual inductance M is always a positive quantity, the mutual voltage ($v = M\frac{di}{\mathrm{dt}}$) may be negative or positive, just like the self induced voltage ($v = L\frac{di}{\mathrm{dt}}$).

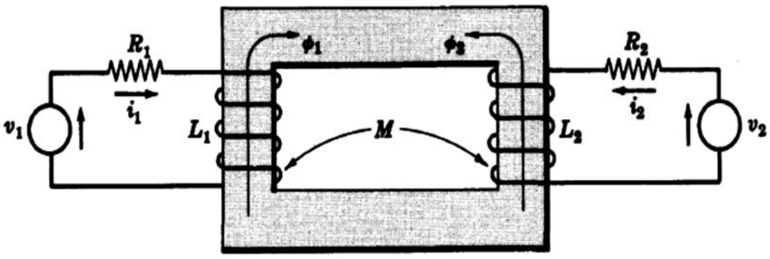


Fig.3.1

For Fig.3.1 . apply Kirchhoff's voltage law (KVL),

$$R_1 i_1 + L_1\frac{di_1}{dt} \mp M\frac{di_2}{dt} = v_1 \qquad \ldots(3.1)$$

$$R_2 i_2 + L_2\frac{di_2}{dt} \mp M\frac{di_1}{dt} = v_2 \qquad \ldots(3.2)$$

To determine the correct signs in Eqs(3.1) and (3.2) apply the right hand rule to each coil, allowing the fingers to wrap around in the direction of the assumed current. Then the right thumb points in the direction of the flux. Thus the positive directions of $\emptyset_1$, and $\emptyset_2$ are as shown in the figure. ***If fluxes $\emptyset_1$ and $\emptyset_2$ due to the assumed positive current directions aid one another, then the signs on the voltages of mutual inductance are the same as the signs on the voltages of self-inductance.***

So Eqs(3.1) and (3.2) becomes

$$R_1 i_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = v_1 \qquad \ldots(3.3)$$

$$R_2 i_2 + L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = v_2 \qquad \ldots(3.4)$$

In frequency domain (replacing each $\boldsymbol{\frac{d}{dt}}$ *by* $\boldsymbol{j\omega}$)Eqs(3.3) and (3.4) becomes,

$$(R_1 + j\omega L_1)I_1 - j\omega M I_2 = \mathrm{V}_1 \quad \rightarrow \mathrm{Z}_{11}\mathrm{I}_1 + \mathrm{Z}_{12}\mathrm{I}_2 = \mathrm{V}_1 \qquad \ldots(3.5)$$

$$-j\omega M I_1 + (R_2 + j\omega L_2)I_2 = \mathrm{V}_2 \quad \rightarrow \mathrm{Z}_{21}\mathrm{I}_1 + \mathrm{Z}_{22}\mathrm{I}_2 = \mathrm{V}_2 \qquad \ldots(3.6)$$

Where $Z_{11} = R_1 + j\omega L_1$, $Z_{22} = R_2 + j\omega L_2$, and $Z_{12} = Z_{21} = -j\omega M$ which is common to the two mesh currents I_1 and I_2.

4) NATURAL CURRENT

In Fig.4.1 . select current I_1 in agreement with the source V_1 and apply the right hand rule to determine the direction of the flux $\emptyset_{12}$. Now Lenz's law states that ***the polarity of the induced voltage is such that if the circuit is completed, a current will pass through the coil in a direction which creates a flux opposing the main flux set up by current I_1.*** Therefore when the switch is closed in the circuit of Fig.5, the direction of flux $\emptyset_{21}$ according to Lenz's law is as shown. Now apply the right hand rule with the thumb pointing in the direction of the fingers will wrap around coil 2 in the direction of the natural current. Then the mesh current equations are

$$(R_1 + j\omega L_1)I_1 - j\omega M I_2 = V_1 \quad \ldots(4.1)$$

$$-j\omega M I_1 + (R_2 + j\omega L_2)I_2 = 0 \quad \ldots(4.2)$$

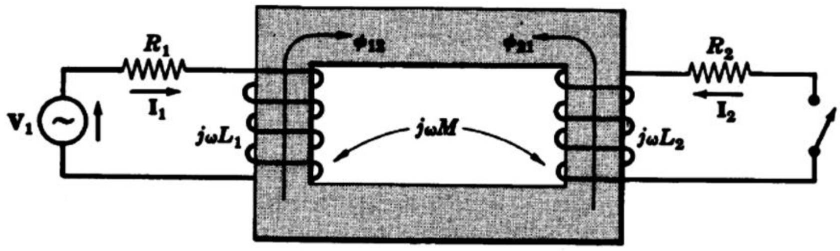


Fig.4.1

5) DOT RULE FOR COUPLED COILS

While the relative polarity for voltages of mutual inductance can be determined from sketches of the core which show the winding, the method is not practical. To simplify the diagrammatic representation of coupled circuits, the coils are marked with ***dots*** as shown in Fig.5.1 (c). ***On each coil, a dot is placed at the terminals which are instantaneously of the same polarity on the basis of the mutual inductance alone.***

To assign the dots on a pair of coupled coils,

1) select a current direction in one coil of the pair and place a dot at the terminal where this current enters the winding. The dotted terminal is instantaneously positive with respect to the other terminal of the coil.
2) Apply the right hand rule to find the corresponding flux in the second coil as shown in Fig.6 (a). Now in the second coil the flux must oppose the original flux, according to ***Lenz's law***. See Fig.6 (b).
3) Use the right hand rule to find the direction of the natural current, and since the voltage of mutual inductance is positive at the terminal where this natural current leaves the winding, place a dot at this terminal as shown in Fig.5.1 (b). With the instantaneous polarity of the coils given by the dots, the core is no longer needed in the diagram and the coupled coils may be illustrated as in Fig.5.1 (c).

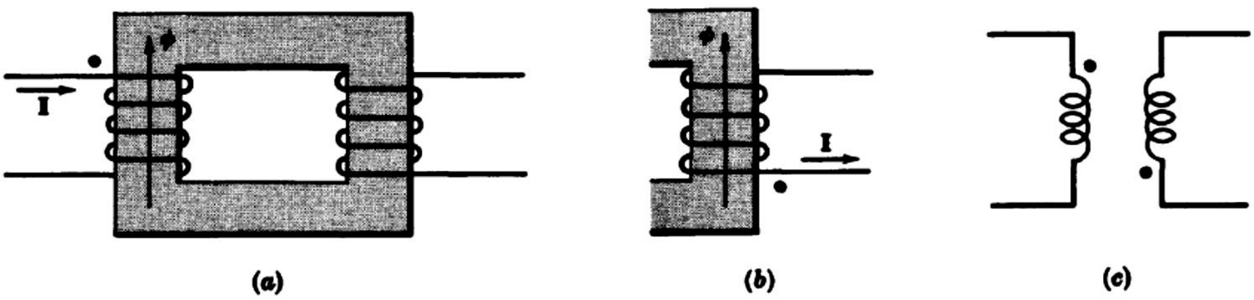


Fig.5.1

To determine the sign of the voltage of mutual inductance in the mesh current equations, we use the dot rule which states:

1) ***When both assumed currents enter or leave a pair of coupled coils at the dotted terminals, the signs of the M terms will be the same as the signs of the L terms;***
2) ***If one current enters at a dotted terminal and one leaves by a dotted terminal, the signs of the M terms are opposite to the signs of the L terms.***

or

3) ***If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.***
4) ***If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil.***

Thus, the reference polarity of the mutual voltage depends on the reference direction of the inducing current and the dots on the coupled coils.

Fig.5.2 shows when the signs of the M and L terms are opposite. Fig.5.3 shows two cases in which the signs of M and L are the same.

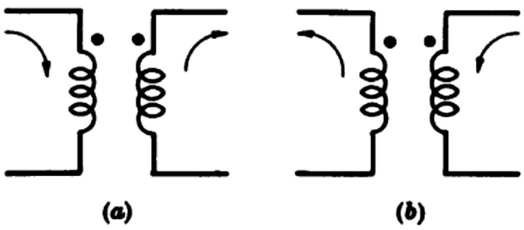


Fig.5.2

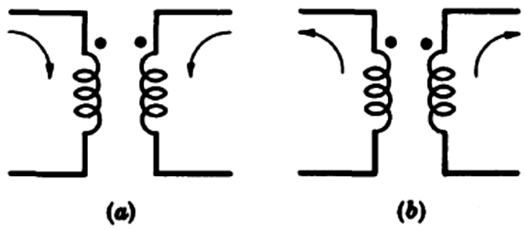


Fig.5.3

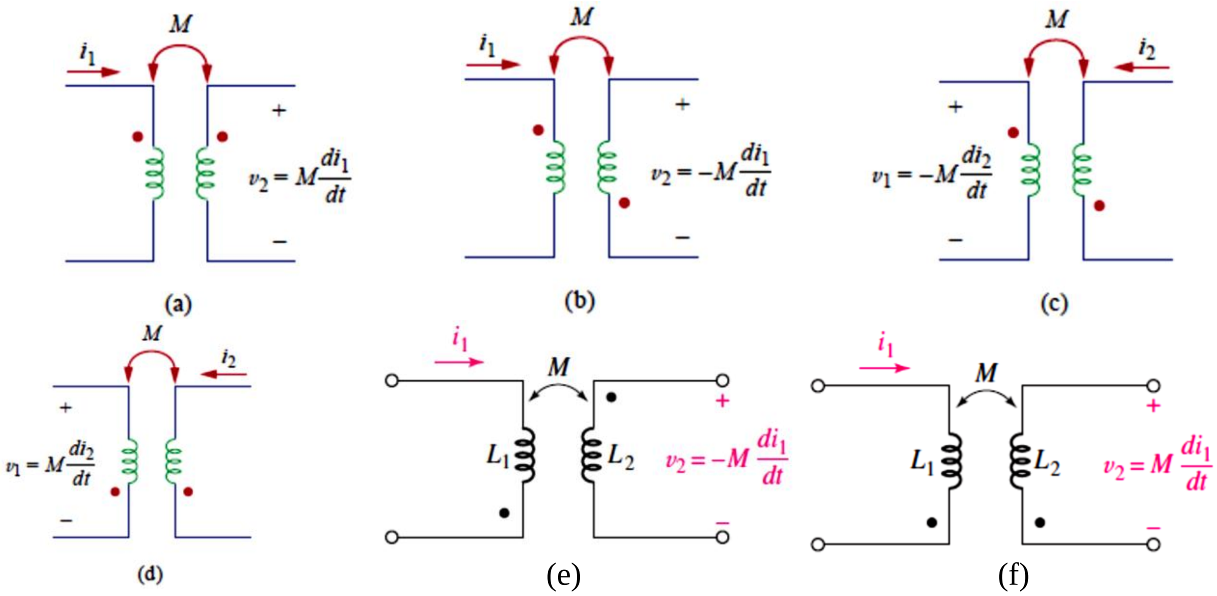


Fig.5.4 Examples illustrating how to apply the dot convention

Fig.5.5 shows the dot convention for coupled coils in series. For the coils in Fig.5.5 (a), the total inductance is

$L = L_1 + L_2 + 2M$ (Series-aiding connection)

For the coils in Fig.5.5 (b),

$L = L_1 + L_2 - 2M$ (Series-opposing connection)

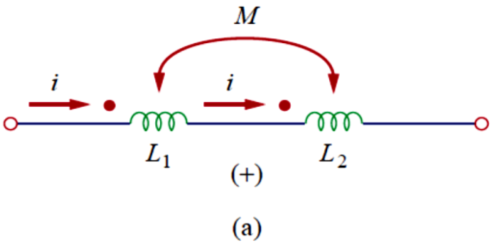


Fig.5.5 Dot convention for coils in series; (a) series-aiding connection, (b) series-opposing connection.

Example 1: Apply KVL for circuit shown in time domain and in frequency domain

Solution:

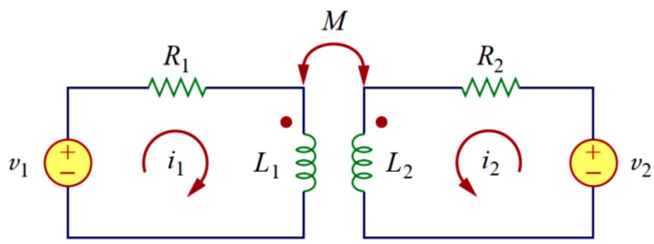
$$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M\frac{di_1}{dt}$$

or in frequency domain,

$$V_1 = (R_1 + j\omega L_1) I_1 + j\omega M I_2$$

$$V_2 = j\omega M I_1 + (R_2 + j\omega L_2) I_2$$

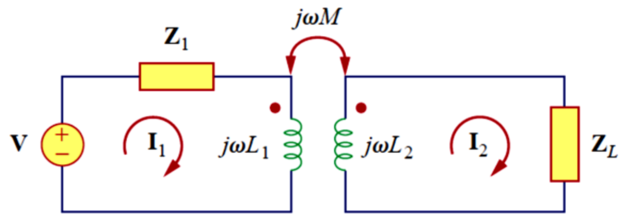


Example 2: Apply KVL for circuit shown in frequency domain

Solution:

$$V = (Z_1 + j\omega L_1) I_1 - j\omega M I_2$$

$$0 = -j\omega M I_1 + (Z_L + j\omega L_2) I_2$$



Example 3: Calculate the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit shown

Solution:

Apply KVL for coil 1,

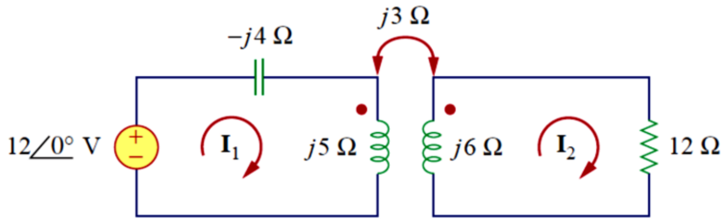
$$-12 + (-j4 + j5)I_1 - j3I_2 = 0$$

$$\therefore jI_1 - j3I_2 = 12 \qquad \ldots(1)$$

Apply KVL for coil 2,

$$-j3I_1 + (12 + j6)I_2 = 0$$

$$\therefore I_1 = \frac{(12+j6)}{j3} I_2 = (2 - j4)I_2 \qquad \ldots(2)$$



Substitute Equ.s (2) in (1), we get,

$$j(2-j4)I_2 - j3I_2 = 12 \qquad \rightarrow \qquad \therefore I_2 = \frac{12}{4-j} = 2.91\angle 14.04^o\ A$$

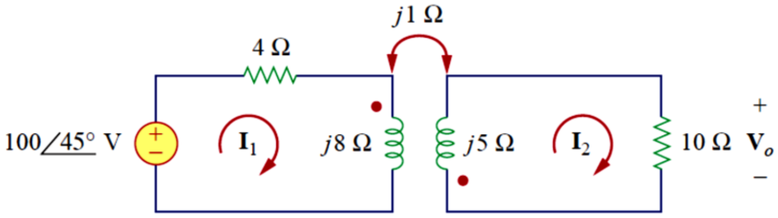
Now subs the value of I_2 in Equ. (2) we get,

$$I_1 = (2-j4)\times \frac{12}{4-j} = 13.01\angle{-49.39^o}\ A$$

H.W.1: Determine the voltage **V***o* in the circuit shown.

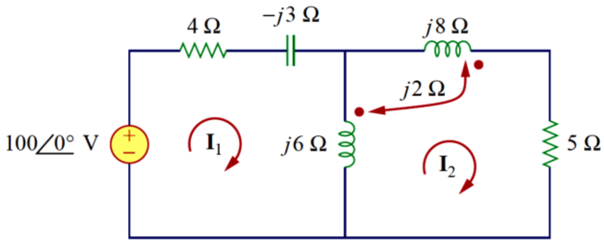
Answer:

$$V_o = 10\angle{-135^o}\ V$$



Example 4: Calculate the mesh currents in the circuit shown

Solution:



Apply KVL for loop 1,

$$-100 + I_1(4 - j3 + j6) - j6I_2 - j2I_2 = 0$$

$$\therefore\ (4+j3)I_1 - j8I_2 = 100 \qquad \ldots(1)$$

Apply KVL for loop 2,

$$-j2I_1 - 6jI_1 + (j6 + j8 + j2\times 2 + 5)I_2 = 0$$

$$\therefore\ -j8I_1 + (5+j18)I_2 = 0 \qquad \ldots(2)$$

Solve Equ.s (1) & (2) using matrix,

$$\begin{bmatrix} 4+j3 & -j8 \\ -j8 & 5+j18 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix} \ , \quad \Delta = \begin{vmatrix} 4+j3 & -j8 \\ -j8 & 5+j18 \end{vmatrix} = 30 + j87\ ,$$

$$\Delta_1 = \begin{vmatrix} 100 & -j8 \\ 0 & 5+j18 \end{vmatrix} = 100(5+j18)\ , \quad \Delta_2 = \begin{vmatrix} 4+j3 & 100 \\ -j8 & 0 \end{vmatrix} = j800$$

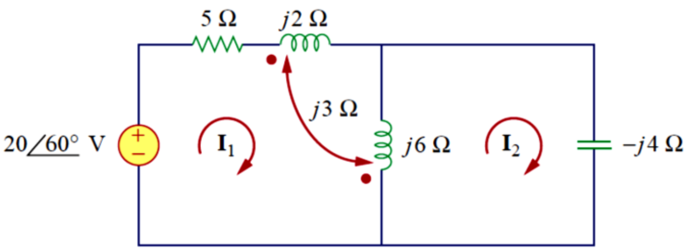
$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100(5+j18)}{30+j87} = \frac{1868.2\angle 74.5^o}{92.03\angle 71^o} = 20.3\angle 3.5^o\ \text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{j800}{30+j87} = \frac{800\angle 90^o}{92.03\angle 71^o} = 8.693\angle 19^o\ \text{A}$$

H.W.2: Determine the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit shown.

Answer:

$$3.583\angle 86.56^o\ \text{A}, = 5.383\angle 86^o\ \text{A}$$



6) Energy in a Coupled Circuit

Consider circuit shown in Fig.6.1 ,

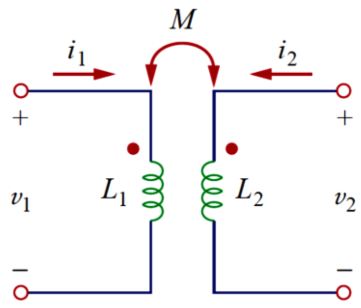


Fig.6.1

- If i_1 and i_2 are zero initially, the power and energy stored in the coils is zero.

  $p_1(t) = p_2(t) = 0 \quad \& \quad w_1 = w_2 = 0$
- If we let i_1 increase from zero to I_1 while maintaining $i_2 = 0$, so

$$p_1(t) = v_1 i_1 = i_1 L_1 \frac{di_1}{dt} \qquad \ldots(6.1)$$

$$w_1 = \int p_1 \, . \mathrm{dt} = L_1 \int_0^{I_1} i_1 \, . di_1 = \frac{1}{2} L_1 I_1^2 \qquad \ldots(6.2)$$

- If we now maintain $i_1 = I_1$ and increase i_2 from zero to I_2, the mutual voltage induced in coil 1 is $M_{12}\frac{di_2}{dt}$ while the mutual voltage induced in coil 2 is zero, since i_1 does not change. So

$$p_2(t) = i_1 M_{12} \frac{di_2}{dt} + i_2 v_2 = I_1 M_{12} \frac{di_2}{dt} + i_2 L_2 \frac{di_2}{dt} \qquad \ldots(6.3)$$

$$w_2 = \int p_2 \, dt = M_{12} I_1 \int_0^{I_1} di_2 + L_2 \int_0^{I_2} i_2 di_2 = M_{12} I_1 I_2 + \frac{1}{2} L_2 I_2^2 \qquad \ldots(6.4)$$

- The total energy stored in the coils when both i_1 and i_2 have reached constant values is,

$$w = w_1 + w_2 = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2 \qquad \ldots(6.5)$$

- If we first increase i_2 from zero to I_2 and later increase i_1 from zero to I_1, the total energy stored in the coils is

$$w = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{21} I_1 I_2 \qquad \ldots(6.6)$$

  Compare Eq.s (6.5) with (6.6) we get,

$$M_{12} = M_{21} = M \qquad \ldots(6.7)$$

$$w = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2 \qquad \ldots(6.8)$$

This equation was derived based on the assumption that the coil currents both entered the dotted terminals. If one current enters one dotted terminal while the other current leaves the other dotted terminal, the mutual voltage is negative, so that the mutual energy $\boldsymbol{M I_1 I_2}$ *is also negative. In that case,*

$$w = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 - M I_1 I_2 \qquad \ldots(6.9)$$

The instantaneous energy stored in the circuit the general expression

$$\boxed{w = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \mp M I_1 I_2} \qquad \ldots(6.10)$$

7) Coupling coefficient (k)

We will now establish an upper limit for the inductance M. mutual The energy stored in the circuit cannot be negative because the circuit is passive. This means that the quantity $(\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2)$ must be greater than or equal to zero:

$$\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2 \geq 0 \qquad \ldots(7.1)$$

To complete the square, we both add and subtract the term $i_1 i_2 \sqrt{L_1 L_2}$ on the right-hand side of Eq. (6.11) and obtain

$$\frac{1}{2}\left(i_1\sqrt{L_1} - i_2\sqrt{L_2}\right)^2 + i_1 i_2\left(\sqrt{L_1 L_2} - M\right) \geq 0 \qquad \ldots(7.2)$$

The squared term is never negative; at its least it is zero. Therefore, the second term on the right-hand side of Eq. (6.12) must be greater than zero; that is,

$$\sqrt{L_1 L_2} - M \geq 0$$

or

$$M \leq \sqrt{L_1 L_2} \qquad \ldots(7.3)$$

Thus, the mutual inductance cannot be greater than the geometric mean of the self-inductances of the coils. The extent to which the mutual inductance M approaches the upper limit is specified by the ***coefficient of coupling k***, given by

$$k = \frac{M}{\sqrt{L_1 L_2}} \qquad \ldots(7.4)$$

$$\boxed{M = k\sqrt{L_1 L_2}} \qquad \ldots(7.5)$$

Where $0 \leq k \leq 1$ or equivalently $0 \leq M \leq \sqrt{L_1 L_2}$.

The coupling coefficient is the fraction of the total flux emanating from one coil that links the other coil. For example, in Fig.6.2(a),

$$k = \frac{\emptyset_{12}}{\emptyset_1} = \frac{\emptyset_{12}}{\emptyset_{11} + \emptyset_{12}} \qquad \ldots(6.6)$$

and in Fig.6.2(b),

$$k = \frac{\emptyset_{21}}{\emptyset_2} = \frac{\emptyset_{21}}{\emptyset_{21} + \emptyset_{22}} \qquad \ldots(6.7)$$

1) For k = 1 (the entire flux produced by one coil links another coil, 100% coupling) the coils are said to be ***perfectly coupled.***
2) For k < 0.5 coils are said to be ***loosely coupled;***
3) For k > 0.5, coils are said to be ***tightly coupled.***

The coupling coefficient k is a measure of the magnetic coupling between two coils; $0 \leq k \leq 1$

coupling coefficient k depend on the closeness of the two coils, their core, their orientation, and their windings.

The air-core transformers used in radio frequency circuits are loosely coupled, whereas iron-core transformers used in power systems are tightly coupled. The linear transformers are mostly air-core; the ideal transformers are principally iron-core.

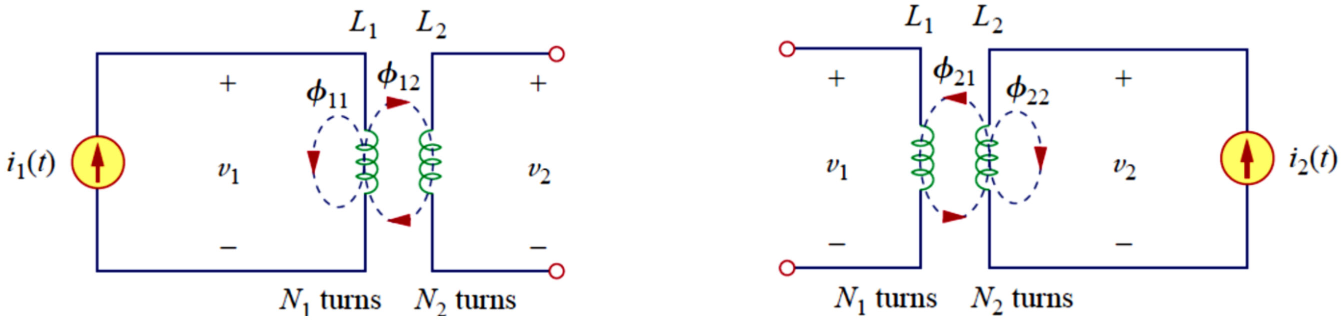
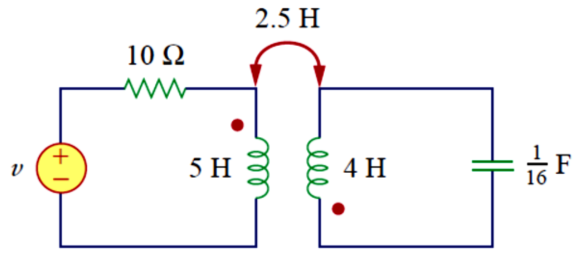


Fig.6.2

Example 5: Determine the coupling coefficient and calculate the energy stored in the coupled inductors at time t=1 s if v = 60cos(4t +30°) V.

Solution:



$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{5\times 4}} = 0.56 \quad \text{(tightly coupled)}$$

To find the energy stored, we need to calculate the current. To find the current, we need to obtain the frequency-domain equivalent of the circuit.

$$60\cos(4t+30^\circ) \Rightarrow 60\angle 30^\circ, \quad \omega = 4 \text{ rad/s}$$
$$5\text{ H} \Rightarrow j\omega L_1 = j20\ \Omega$$
$$2.5\text{ H} \Rightarrow j\omega M = j10\ \Omega$$
$$4\text{ H} \Rightarrow j\omega L_2 = j16\ \Omega$$
$$\frac{1}{16}\text{ F} \Rightarrow \frac{1}{j\omega C} = -j4\ \Omega$$

The frequency-domain equivalent is shown in Fig.. Now apply mesh analysis.

For mesh 1,

$$(10+j20)I_1 + j10I_2 = 60\angle 30^0 \qquad \dots(1)$$

For mesh 1,

$$j10I_1 + (j16 - j4)I_2 = 0 \qquad \dots(2)$$

Solve Eq.s (1) and (2) we get,

$I_1 = 3.905\angle{-19.4^o}$ A & $I_2 = 3.254\angle 160.6^o$ A

In time domain.

$i_1 = 3.905\cos(4t - 19.4^o)$ & $i_2 = 3.254\cos(4t + 160.6^o)$

At t=1s, 4t = 4 rad = 229.2°

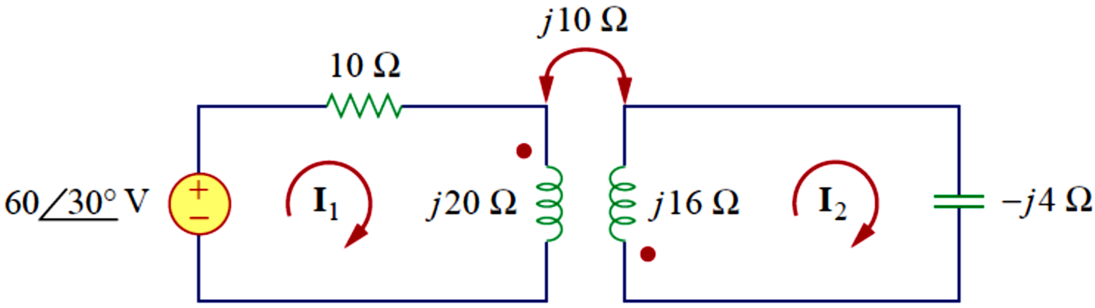
$i_1 = 3.905\cos(229.2^o - 19.4^o) = -3.389$ A

$i_2 = \ 3.254\cos(229.2^o + 160.6^o) = 2.824 \text{ A}$

The total energy stored in the coupled inductors is,

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + \ Mi_1 i_2$$

$$= \tfrac{1}{2}(5)(-3.389)^2 + \tfrac{1}{2}(4)(2.824)^2 + \ 2.5(-3.389)(2.824) = 20.73\,J$$

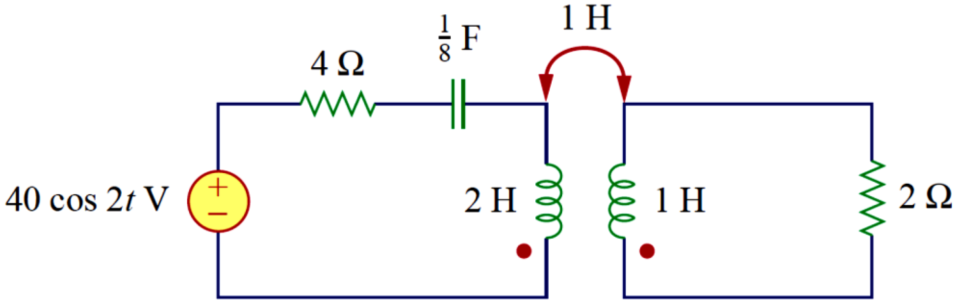


Frequency-domain equivalent circuit

H.W.3: Determine the coupling coefficient and calculate the energy stored in the coupled inductors at time t=1.5 .

Answer:

0.7071, 39.4 J.



8) Linear Transformers

A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils.

As shown in Fig.8.1, the coil that is directly connected to the voltage source is called ***the primary winding.*** The coil connected to the load is called ***the secondary winding.*** The resistances R_1 and R_2 are included to account for the losses (power dissipation) in the coils.

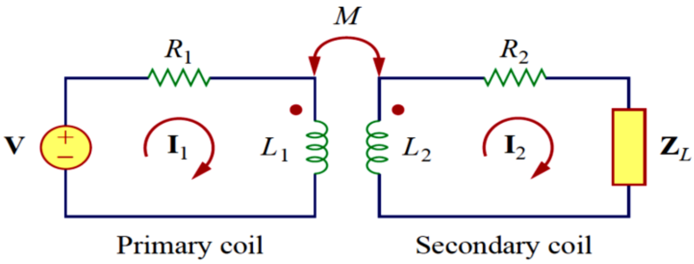


Fig.8.1 A linear transformer

The transformer is said to be ***linear if the coils are wound on a magnetically linear material(a material for which the magnetic permeability is constant).*** Such materials include air, plastic, Bakelite, and wood. In fact, most materials are magnetically linear. Linear transformers are sometimes called ***air-core transformers,*** although not all of them are necessarily air-core. They are used in radio and TV sets.

The input impedance ($\boldsymbol{Z_{in}} = \frac{V}{\mathbf{I_1}}$) as seen from the source governs the behavior of the primary circuit.

Applying KVL to the two meshes in Fig.8.1 gives

$$V = (R_1 + j\omega L_1)I_1 - j\omega M I_2 \qquad \ldots(8.1)$$

$$0 = -j\omega M I_1 + (R_2 + j\omega L_2 + Z_L)I_2 \qquad \ldots(8.2)$$

From Eq.(8.2)

$$I_2 = \frac{j\omega M}{(R_2 + j\omega L_2 + Z_L)} I_1 \qquad \ldots(8.3)$$

Substitute Eq. (8.3) in (8.1) we get,

$$V = \left((R_1 + j\omega L_1) + \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)} \right) I_1 \qquad \ldots(8.4)$$

From Eq. (8.4) we get,

$$\boxed{\boldsymbol{Z_{in}} = \frac{V}{\mathbf{I_1}} = (R_1 + j\omega L_1) + \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)}} \qquad \ldots(8.5)$$

Notice that the input impedance comprises two terms. The first term, $(R_1 + j\omega L_1)$, is the primary impedance. The second term is due to the coupling between the primary and secondary windings. It is as though this impedance is reflected to the primary. Thus, it is known as the ***reflected impedance*** $\boldsymbol{Z_R}$ and

$$\boxed{Z_R = \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)}} \qquad \ldots(8.6)$$

It should be noted that the Eq. (8.5) or (8.6) is not affected by the location of the dots on the transformer, because the same result is produced when M is replaced by $-M$.

9) Conductively Coupled Equivalent Circuits

It is convenient to replace a magnetically coupled circuit by an equivalent circuit with no magnetic coupling. Now to replace the linear transformer in Fig.8.2 by an equivalent **T** or **Π** circuit, a circuit that would have no mutual inductance.

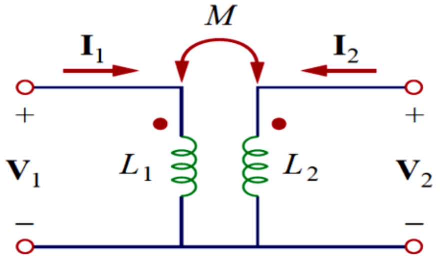


Fig.8.2

The voltage-current relationships for the primary and secondary coils give the matrix equation

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \ldots(9.1)$$

By matrix inversion, this can be written as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{L_2}{j\omega\left(L_1L_2 - M^2\right)} & \dfrac{-M}{j\omega\left(L_1L_2 - M^2\right)} \\ \dfrac{-M}{j\omega\left(L_1L_2 - M^2\right)} & \dfrac{L_1}{j\omega\left(L_1L_2 - M^2\right)} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \ldots(9.2)$$

1) For the T (or Y) network of Fig.8.3, mesh analysis provides the terminal equations as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & j\omega L_c \\ j\omega L_c & j\omega(L_b + L_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \ldots(9.3)$$

If the circuits in Fig.8.2 and Fig.8.3 are equivalents, Eqs. (9.1) and (9.3) must be identical. Equating terms in the impedance matrices of Eqs. (9.1) and (9.3) leads to

$$\boxed{L_a = L_1 - M, \qquad L_b = L_2 - M, \quad L_c = M} \qquad \ldots(9.4)$$

2) For the **Π** (or Δ) network in Fig.8.4, nodal analysis gives the terminal equations as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{j\omega L_A} + \frac{1}{j\omega L_C} & -\frac{1}{j\omega L_C} \\ -\frac{1}{j\omega L_C} & \frac{1}{j\omega L_B} + \frac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \ldots(9.5)$$

Equating terms in admittance matrices of Eqs. (9.2) and (9.5), we obtain

$$\boxed{L_A = \frac{L_1L_2 - M^2}{L_2 - M}, \qquad L_B = \frac{L_1L_2 - M^2}{L_1 - M}, \quad L_C = \frac{L_1L_2 - M^2}{M}} \qquad \ldots(9.6)$$

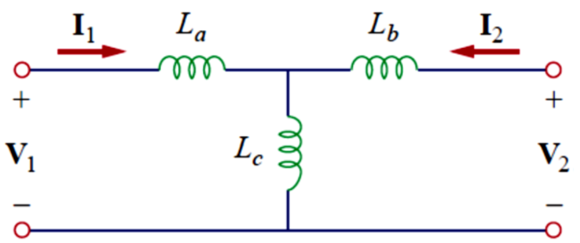


Fig.8.3 An equvalient **T** circuit

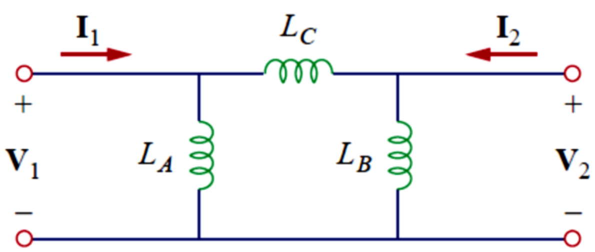
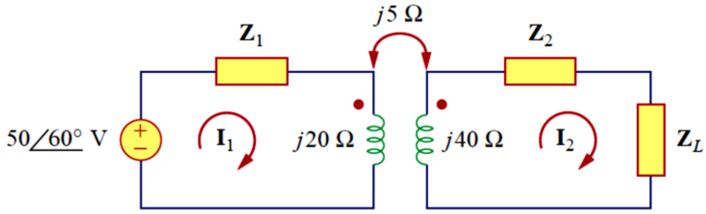


Fig.8.4 An equvalient **Π** circuit

Example 6: In the circuit shown, calculate the input impedance and current $\mathbf{I}_1$. Take $Z_1 = 60 - j100\ \Omega, Z_2 = 30 + j40\Omega$ and $Z_L = 80 + j60\Omega$

Solution: From Eq.(8.22)



$$Z_{in} = (Z_1 + j20) + \frac{5^2}{(j40 + Z_2 + Z_L)}$$

$$= 60 - j100 + j20 + \frac{25}{110 + j140}$$

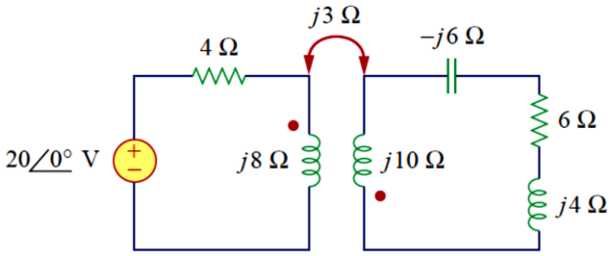
$$= 60.09 - j80.11 = 100.14\angle -53.1^o\Omega$$

$$I_1 = \frac{V}{Z_{in}} = \frac{50\angle 60^o}{100.14\angle -53.1^o} = 0.5\angle 113.1^o \text{ A}$$

H.W.4: Find the input impedance of the circuit and the current from the voltage source.

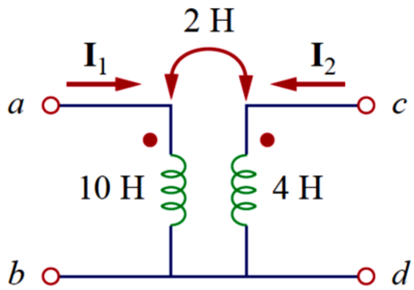
Answer:

$8.58\angle 58.05^o\Omega, 2.331\angle{-58.05^o}$ A



Example 7: Determine the T-equivalent circuit of the linear transformer shown.

Solution:

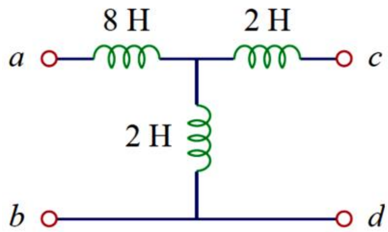


$L_1 = 10, L_2 = 4,$ and $M = 2$

$L_a = L_1 - M = 10 - 2 = 8$ H,

$L_b = L_2 - M = 4 - 2 = 2$ H,

$L_c = M = 2$ H

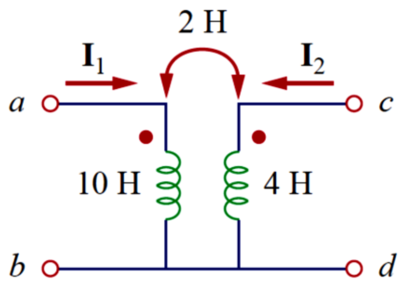


We have assumed that reference directions for currents and voltage polarities in the primary and secondary windings conform to those in Fig.15 . Otherwise, we may need to replace M with $-M$, as **Example 8** illustrates. The T-equivalent circuit is shown in Fig.

H.W.5: For the linear transformer , find the equivalent network.

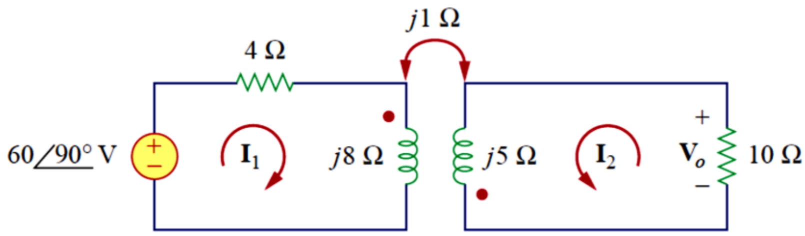
Answer:

$L_A = 18$ H, $L_B = 4.5$ H, $L_C = 18$ H



Example 8: Solve for I_1, I_2 and V_O using the T-equivalent circuit for the linear transformer.

Solution:



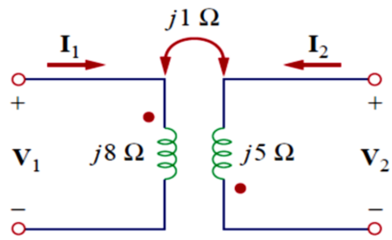
$L_1 = 8, L_2 = 5, \text{and } M = -1$

$L_a = L_1 - M = 8 - (-1) = 9 \text{ H},$

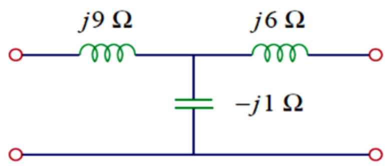
$L_b = L_2 - M = 5 - (-1) = 6 \text{ H},$

$L_c = M = -1 \text{ H}$

To represent the circuit in frequency domain as ω is not specified , we assume ω =1 rad, and the equavalent T circuit in frequency domain is shown in Fig.



(a)



(b)

Apply mesh analysis,

For mesh 1,

$$j6 = (4 + j9 - j1)I_1 + (-j1)I_2 \qquad \ldots(1)$$

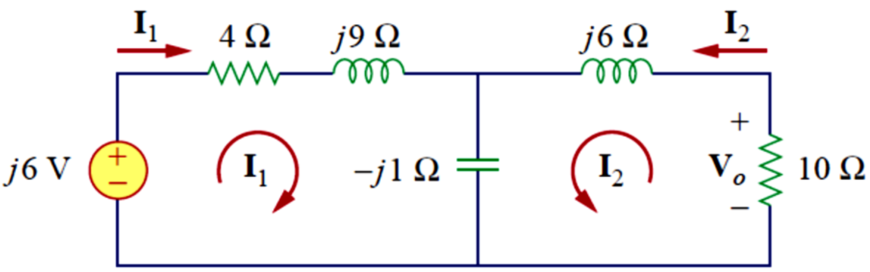
For mesh 2,

$$0 = (-j1)I_1 + (10 + j6 - j1)I_2 \qquad \ldots(2)$$

Solve Eq.s (1) & (2) we get,

$I_1 = 0.6 + j0.3 \text{ A}, \qquad I_2 = j0.06 \text{ A} \;\&\; V_O = -10I_2 = j0.6 \text{ V}$

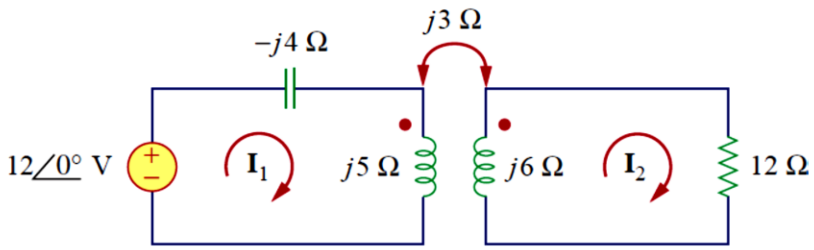
The complete equivalent T circuit in frequency domain is shown in Fig below.



H.W.6: Calculate the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit shown (**in example3**) using the T-equivalent model for the magnetically coupled coils.

Answer:

$13\angle{-49.4^o}\text{A}, \qquad 2.91\angle 14.04^o \text{ A}$



10) Ideal Transformers

An ideal transformer is one with perfect coupling (k =1). It consists of two (or more) coils with a large number of turns wound on a common core of high permeability. Because of this high permeability of the core, the flux links all the turns of both coils, thereby resulting in a perfect coupling. Consider the circuit in Fig.10.1.

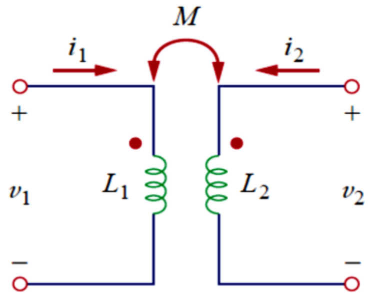


Fig.10.1

In the frequency domain,

$$V_1 = j\omega L_1 I_1 + j\omega M I_2 \rightarrow I_1 = \frac{V_1 - j\omega M I_2}{j\omega L_1} \quad \ldots(10.1)$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2 \quad \ldots(10.2)$$

Substituting Eq. (10.1) in (10.2) gives

$$V_2 = j\omega M\left(\frac{V_1 - j\omega M I_2}{j\omega L_1}\right) + j\omega L_2 I_2 = j\omega L_2 I_2 + \frac{M V_1}{L_1} - \frac{j\omega M^2 I_2}{L_1} \quad \ldots(10.3)$$

But $M = \sqrt{L_1 L_2}$ for perfect coupling (k =1). Hence,

$$V_2 = j\omega L_2 I_2 + \frac{\sqrt{L_1 L_2}\, V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1} = \sqrt{\frac{L_2}{L_1}}\, V_1 = nV_1 \quad \ldots(10.4)$$

where $n = \sqrt{\frac{L_2}{L_1}}$ and is called the ***turns ratio.***

As $L_1, L_2, M \rightarrow \infty$ such that n remains the same, the coupled coils become an ideal transformer.

A transformer is said to be ideal if it has the following properties:

1. Coils have very large reactances ($L_1, L_2, M \rightarrow \infty$).
2. Coupling coefficient is equal to unity (k =1).
3. Primary and secondary coils are lossless ($R_1 = 0, R_2 = 0$)

An ideal transformer is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances.

Iron-core transformers are close approximations to ideal transformers. These are used in power systems and electronics.

When a sinusoidal voltage is applied to the primary winding of ideal transformer as shown in Fig.10.2. and according to Faraday's law, the voltage across its windings are,

$$v_1 = N_1 \frac{d\emptyset}{dt} \quad \ldots(10.5)$$

$$v_2 = N_2 \frac{d\emptyset}{dt} \quad \ldots(10.6)$$

Divide Eq.(10.6) by (10.5), we get

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n \quad \ldots(10.7)$$

where n is, again, the ***turns ratio or transformation ratio***. We can use the phasor voltages and rather than the instantaneous values and Thus, Eq. (10.7) may be written as

$$\boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1} = n} \qquad \text{...(10.8)}$$

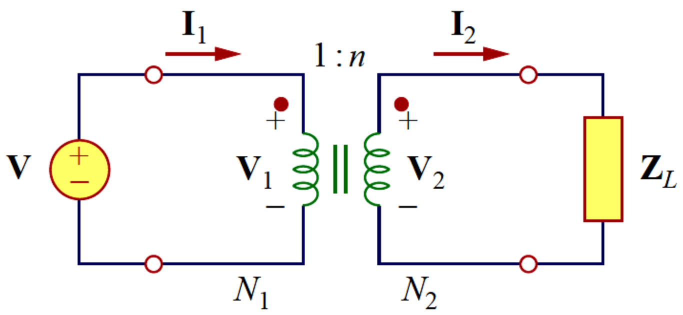


Fig.10.2

For the reason of power conservation, the energy supplied to the primary must equal the energy absorbed by the secondary, since there are no losses in an ideal transformer. This implies that

$$v_1 i_1 = v_2 i_2 \qquad \text{...(10.9)}$$

or

$$\frac{i_1}{i_2} = \frac{v_2}{v_1} = n \qquad \text{...(10.10)}$$

In phasor form,

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = n \qquad \text{...(10.11)}$$

$$\boxed{\therefore \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n}} \qquad \text{...(10.12)}$$

1) If n > 1 (i.e. $N_2 > N_1$& $V_2 > V_1$), then the transformer is called *'step-up transformer'*.
2) If n < 1 (i.e. $N_2 < N_1$& $V_2 < V_1$), then the transformer is called *'step-down transformer'*.
3) If n = 1 (i.e. $N_2 = N_1$& $V_2 = V_1$),then the transformer is called *'isolation transformer'* or *1:1 transformer*.

It is important that we know how to get the proper polarity of the voltages and the direction of the currents for the transformer in Fig.10.2. If the polarity of V_1 or V_2 or the direction of I_1 or I_2 is changed, n in Eqs. (10.7) to (10.12) may need to be replaced by $(-n)$. The two simple rules to follow are:

1) If V_1 and V_2 are *both positive* or *both negative* at the dotted terminals, use $(+n)$ in Eq.(10.8). Otherwise, use $(-n)$

2) If I_1 and I_2 *both enter into* or *both leave the* dotted terminals, use $(-n)$ in Eq.(10.12). Otherwise, use $(+n)$

The rules are demonstrated with the four circuits in Fig.10.3.

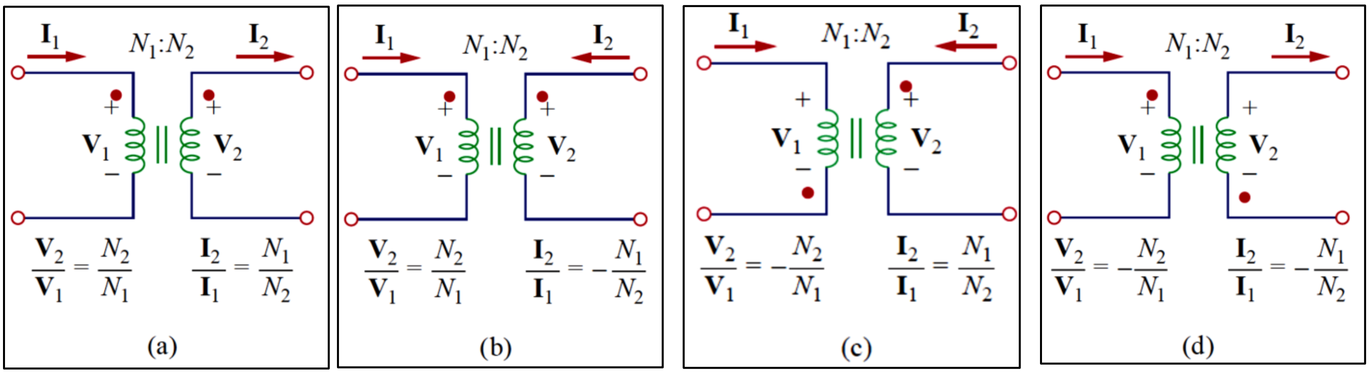


Fig.10.3

$$V_1 = \frac{V_2}{n} \quad or \quad V_2 = nV_1 \qquad \ldots(10.13)$$

$$I_1 = nI_2 \quad or \quad I_2 = \frac{I_1}{n} \qquad \ldots(10.14)$$

The complex power in the primary winding is

$$\boxed{S_1 = V_1 I_1^* = \frac{V_2}{n}(nI_2)^* = V_2 I_2^* = S_2} \qquad \ldots(10.15)$$

The ***input impedance*** is found as,

$$Z_{in} = \frac{V_1}{I_1} = \frac{1}{n^2}\frac{V_2}{I_2} \quad \& \quad Z_L = \frac{V_2}{I_2}$$

$$\boxed{\therefore\ Z_{in} = \frac{Z_L}{n^2}} \qquad \ldots(10.16)$$

The input impedance is also called the ***reflected impedance***, since it appears as if the load impedance is reflected to the primary side. This ability of the transformer to transform a given impedance into another impedance provides us a means of *impedance matching* to ensure maximum power transfer.
In analyzing a circuit containing an ideal transformer, it is common practice to eliminate the transformer by reflecting impedances and sources from one side of the transformer to the other. In the circuit of Fig.10.4, suppose we want to reflect the secondary side of the circuit to the primary side. We find the Thevenin equivalent of the circuit to the right of the terminals We obtain V_{TH} as the open-circuit voltage at terminals *a-b*, as shown in Fig.10.5(a).

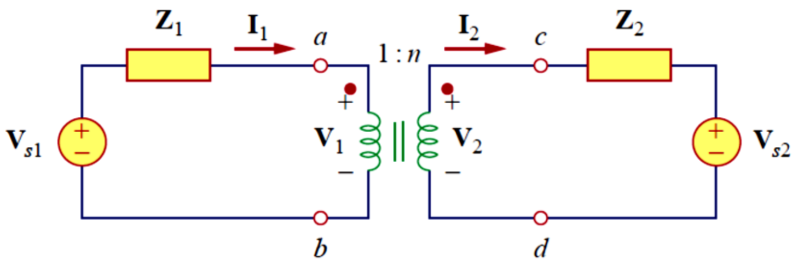


Fig.10.4

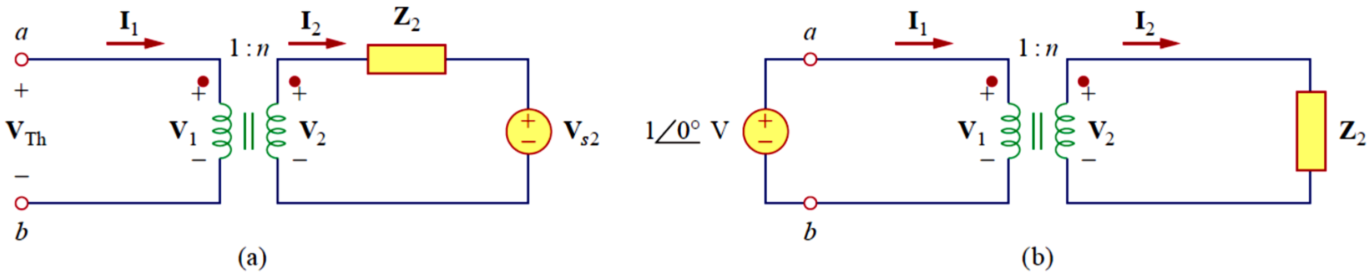


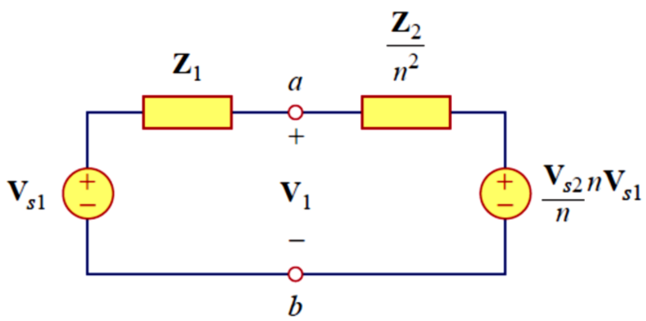
Fig.10.5(a) Obtaining $\mathbf{V}_{\mathrm{Th}}$ for Fig. 10.4, (b) obtaining $\mathbf{Z}_{\mathrm{Th}}$ for Fig. 10.4.

Since terminals *a-b* are open, $I_1 = 0 = I_2$ so that $V_1 = V_2$ Hence, From Eq. (10.13),

$$V_{TH} = V_1 = \frac{V_2}{n} = \frac{V_{s2}}{n} \qquad \ldots(10.17)$$

To get $\mathbf{Z}_{\mathrm{Th}}$ we remove the voltage source in the secondary winding and insert a unit source at terminals *a-b* as in Fig. 10.5(b). From Eqs. (10.13) and (10.13),

$$Z_{TH} = \frac{V_1}{I_1} = \frac{V_2}{n}\frac{1}{nI_2} = \frac{Z_2}{n^2}, \qquad V_2 = Z_2 I_2 \qquad \ldots(10.18)$$



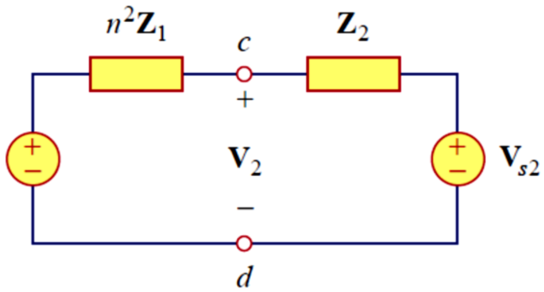


Fig. 10.6 Equivalent circuit for Fig.10.4 obtained by reflecting the secondary circuit to the primary side

Fig. 10.7 Equivalent circuit for Fig.10.4 obtained by reflecting the primary circuit to the secondary side

1) The general rule for eliminating the transformer and reflecting the secondary circuit to the primary side is: divide the secondary impedance by n^2, divide the secondary voltage by n, and multiply the secondary current by n.

2) The rule for eliminating the transformer and reflecting the primary circuit to the secondary side is: multiply the primary impedance by n^2, multiply the primary voltage by n, and divide the primary current by n.

Example 9: An ideal transformer is rated at 2400/120 V, 9.6 kVA, and has 50 turns on the secondary side. Calculate: (a) the turns ratio, (b) the number of turns on the primary side, and (c) the current ratings for the primary and secondary windings.

Solution:

(a) $n = \frac{v_2}{v_1} = \frac{120}{2400} = 0.05$

(b) $n = \frac{N_2}{N_1} \to 0.05 = \frac{50}{N_1} \to N_1 = 1000\ turns$

(c) $S_1 = V_1 I_1 = V_2 I_2 = 9.6$ kVA

$$I_1 = \frac{9600\text{ VA}}{V_1} = \frac{9600\text{ VA}}{2400} = 4\text{ A}$$

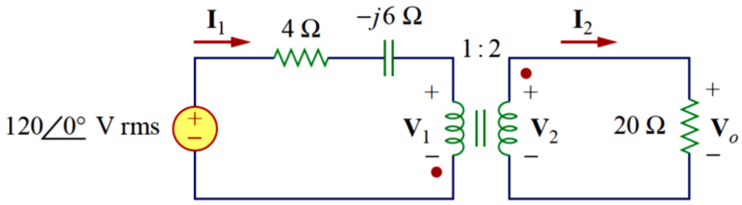
$$I_2 = \frac{9600\text{ VA}}{V_2} = \frac{9600\text{ VA}}{120} = 80\text{ A} \quad \text{or} \quad I_2 = \frac{I_1}{n} = \frac{4A}{0.05} = 80\text{ A}$$

H.W.7: The primary current to an ideal transformer rated at 3300/110 is 5 A. Calculate: (a) the turns ratio, (b) the kVA rating, (c) the secondary current.

Answer: (a) 1/30, (b) 16.5 kVA, (c) 150 A.

Example 10: For the ideal transformer circuit of Fig. shown, find: (a) the source current I_1 (b) the output voltage V_O and (c) the complex power supplied by the source.

Solution:



(a)The 20Ω impedance can be reflected to the primary side and we get

$$Z_R = \frac{20}{n^2} = \frac{20}{2^2} = 5\,\Omega$$

$\therefore Z_{in} = 4 - j6 + Z_R = 9 - j6 = 10.82\angle - 33.69^o\Omega$

$I_1 = \frac{120\angle-0^o}{Z_{in}} = \frac{120\angle-0^o}{10.82\angle-33.69^o} = 11.09\angle 33.69^o\ A$

(b) since both I_1 & I_2 leave the dotted terminals,

$I_2 = \left(-\frac{1}{n}\right) I_1 = -5.545\angle 33.69^o\ A$

$V_o = 20\, I_2 = 110.9\angle 213.69^o\ V$

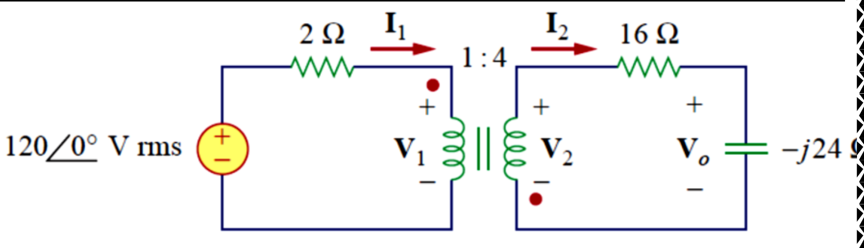
(c) $S_1 = V_s I_1^* = (120\angle 0^o)(11.09\angle{-33.69^o}) = 1330.8\angle{-33.69^o}$ VA

H.W.8: In the ideal transformer circuit of Fig. shown, find V_O and the complex power supplied by the source.

Answer:

$214.7\angle 116.56^o$ V,

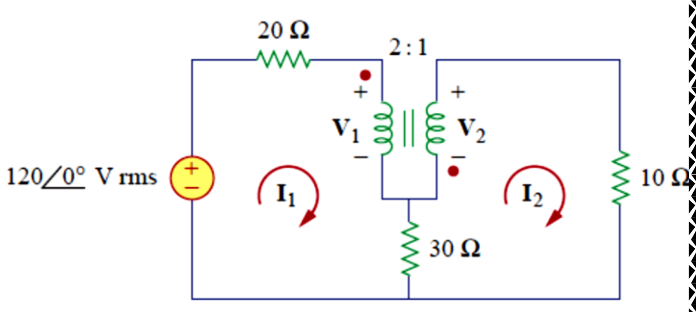
$4.293\angle{-26.56^o}$ kVA



Example 11: Calculate the power supplied to the 10- Ω resistor in the ideal transformer circuit of Fig. shown

Solution:

Reflection to the secondary or primary side cannot be done with this circuit: there is direct connection between the primary and secondary sides due to the 30- resistor. We apply mesh analysis.



For mesh 1,

$$50I_1 - 30I_2 + V_1 = 120 \qquad \ldots(1)$$

For mesh 2,

$$-30I_1 + 40I_2 - V_2 = 0 \qquad \ldots(2)$$

At the transformer terminals,

$$V_2 = -\frac{1}{2}V_1 \qquad \ldots(3)$$

$$I_2 = -2\,I_1 \qquad \ldots(4)$$

Solve Eq.s(1), (2), (3) and (4), we get

$I_2 = -0.7272$ A

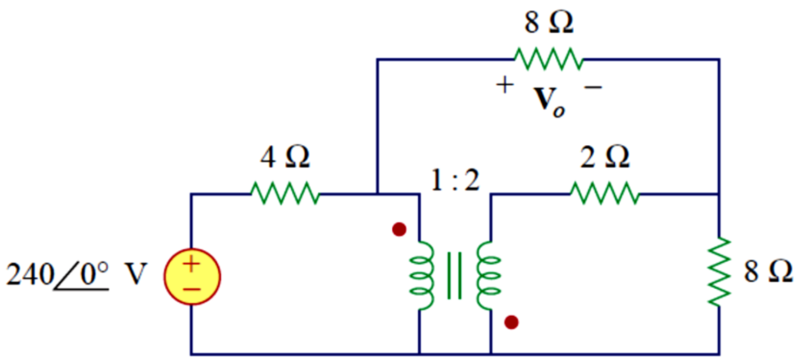
The power absorbed by 10- Ω resistor is,

$p = (-0.7272)^2(10) = 5.3$ W

H.W.9: Find $\mathbf{V}_o$ in the circuit of Fig. shown.

Answer:

96 V



11) Ideal Autotransformers

Unlike the conventional two-winding transformer we have considered so far, an ***autotransformer*** has a single continuous winding with a connection point called a ***tap*** between the primary and secondary sides. The tap is often adjustable so as to provide the desired turns ratio for stepping up or stepping down the voltage. This way, a variable voltage is provided to the load connected to the autotransformer.

An autotransformer is a transformer in which both the primary and the secondary are in a single winding.

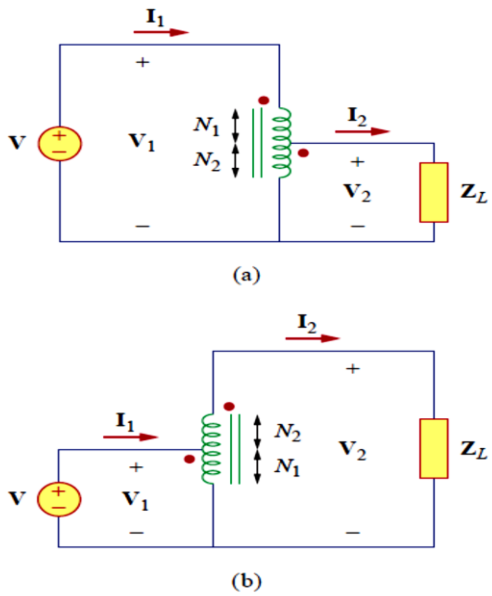


Fig.11.1

Fig.11.1, the autotransformer can operate in the ***step-down*** or ***stepup*** mode. The autotransformer is a type of power transformer. Its major advantage over the two-winding transformer is its ability to transfer larger apparent power. **Example 12** will demonstrate this. Another advantage is that an autotransformer is smaller and lighter than an equivalent two-winding transformer. However, since both the primary and secondary windings are one winding, ***electrical isolation*** (no direct electrical connection) is lost. The lack of electrical isolation between the primary and secondary windings is a major disadvantage of the autotransformer.

For the step-down autotransformer circuit of Fig.11.1(a),

$$\frac{V_2}{V_1} = \frac{N_1 + N_2}{N_2} = 1 + \frac{N_1}{N_2} \qquad \ldots(11.1)$$

$$S_1 = V_1 I_1^* = V_2 I_2^* = S_2 \qquad \ldots(11.2)$$

$$V_1 I_1 = V_2 I_2 \qquad \ldots(11.3)$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1 + N_2} \qquad \ldots(11.4)$$

For the step-up autotransformer circuit of Fig.11.1(a),

$$\frac{V_2}{V_1} = \frac{N_1 + N_2}{N_1} = 1 + \frac{N_2}{N_1}$$

$$S_1 = V_1 I_1^* = V_2 I_2^* = S_2 \qquad \ldots(11.5)$$

$$V_1 I_1 = V_2 I_2 \qquad \ldots(11.6)$$

$$\frac{I_1}{I_2} = \frac{N_1 + N_2}{N_1} = 1 + \frac{N_2}{N_1} \qquad \ldots(11.7)$$

A major difference between conventional transformers and autotransformers is that the primary and secondary sides of the autotransformer are not only coupled magnetically but also coupled conductively. The autotransformer can be used in place of a conventional transformer when electrical isolation is not required.

Example 12: Compare the power ratings of the two-winding transformer in Fig. (a) and the autotransformer in Fig. (b).

Solution:

For the two winding transformer, the power rating is,

$$S_1 = 0.2\times240 = 48 \text{ VA}$$

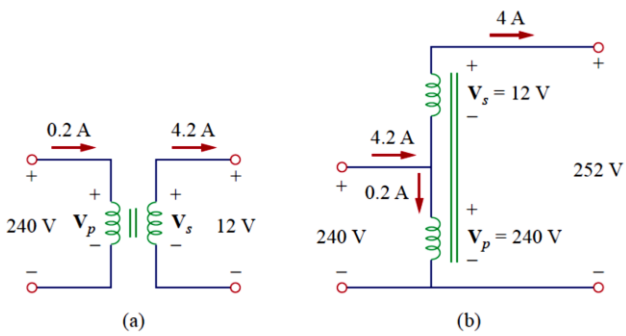
or $$S_2 = 4\times12 = 48 \text{ VA}$$

For the autotransformer, the power rating is,

$$S_1 = 4.2\times240 = 1008 \text{ VA}$$

or $$S_2 = 4\times252 = 1008 \text{ VA}$$

which is 21 times the power rating of the two-winding transformer.



H.W.10: Refer to **Example 12**. If the two-winding transformer is a 30-VA, 120/10V transformer, what is the power rating of the autotransformer?

Answer: 390 VA

Example 13: For the autotransformer circuit in Fig. shown. Calculate: (a)$\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_o$ if $\mathbf{Z}_L$=8+j6 Ω, and (b) the complex power supplied to the load.

Solution:

(a) This is a step-up autotransformer with,

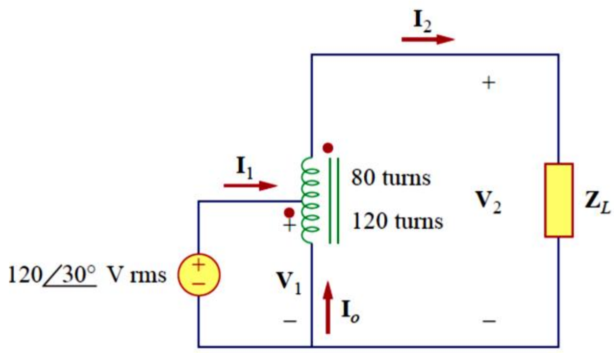
$$N_1 = 80, N_2 = 120, V_1 = 120\angle30^0$$

$$\frac{V_2}{V_1} = \frac{N_1 + N_2}{N_1} = \frac{200}{80}$$

$$\therefore V_2 = \frac{200}{80}V_1 = \frac{200}{80}(120\angle30^0) = 300\angle30^0 \text{ V}$$

$$I_2 = \frac{V_2}{Z_L} = \frac{300\angle30^0}{8+j6} = 30\angle-6.87^0 \text{ A}$$

$$\frac{I_1}{I_2} = \frac{N_1+N_2}{N_1} = \frac{200}{80} \rightarrow I_1 = \frac{200}{80}I_2 = \frac{200}{80}(30\angle-6.87^0\,) = 75\angle-6.87^0 \text{ A}$$



At the tap KCL gives

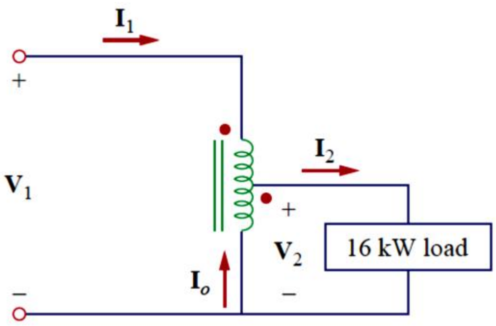
$$I_1 + I_o = I_2 \rightarrow I_o = I_2 - I_1 = 30\angle-6.87^0 - 75\angle-6.87^0 = 45\angle173.13^0 \text{ A}$$

(b) The complex power supplied to the load is

$$S_2 = V_2I_2^* = |I_2|^2Z_L = (30)^2\,(10\angle36.87^0) = 9\angle36.87^0 \text{ kVA}$$

H.W.11: In the autotransformer circuit of Fig. shown. Find currents $\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_o$. Take V_1= 1250 V, V_2= 500 V.

Answer: 12.8 A, 32 A, 19.2 A



12) Three-Phase Transformers

To meet the demand for three-phase power transmission, transformer connections compatible with three-phase operations are needed. We can achieve the transformer connections in two ways: by connecting three single-phase transformers, thereby forming a so-called ***transformer bank***, or by using a special three-phase transformer. For the same kVA rating, a three-phase transformer is always smaller and cheaper than three single-phase transformers. When single-phase transformers are used, one must ensure that they have the same turns ratio *n* to achieve a balanced three-phase system. There are four standard ways of connecting three single-phase transformers or a three-phase transformer for three-phase operations: Y-Y, Δ-Δ, Y-Δ, and Δ-Y.

For any of the four connections, the total apparent power S_T, real power P_T, and reactive power Q_T are obtained as

$$S_T = \sqrt{3}\, V_L I_L \quad \ldots(12.1)$$

$$P_T = S_T \cos\theta = \sqrt{3}\, V_L I_L \cos\theta \quad \ldots(12.2)$$

$$Q_T = S_T \sin\theta = \sqrt{3}\, V_L I_L \sin\theta \quad \ldots(12.3)$$

Where, V_L = line voltage , I_L = line current

(V_{Lp} = line voltage for the primary side, V_{Ls} = line voltage for the secondary side)

(I_{Lp} = line current for the primary side, I_{Ls} = line current for the secondary side)

For Y-Y & Δ-Δ transformer in Fig.12.1(a)(b),

$$V_{Ls} = nV_{Lp} \quad \ldots(12.4a)$$

$$I_{Ls} = \frac{I_{Lp}}{n} \quad \ldots(12.4b)$$

For Y-Δ, transformer in Fig.12.1(c),

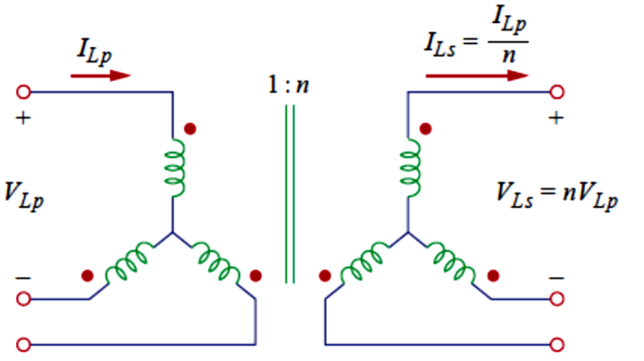
$$V_{Ls} = \frac{nV_{Lp}}{\sqrt{3}} \quad \ldots(12.5a)$$

$$I_{Ls} = \frac{\sqrt{3}\, I_{Lp}}{n} \quad \ldots(12.5b)$$

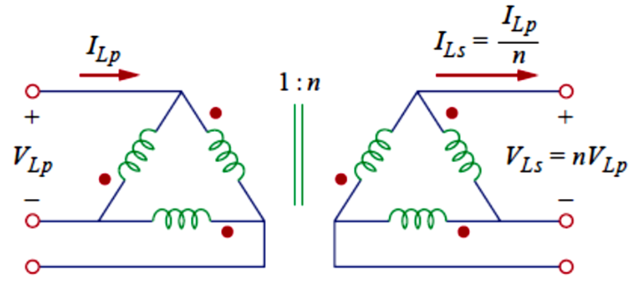
For Δ- Y, transformer in Fig.12.1(d),

$$V_{Ls} = n\sqrt{3}\, V_{Lp} \quad \ldots(12.6a)$$

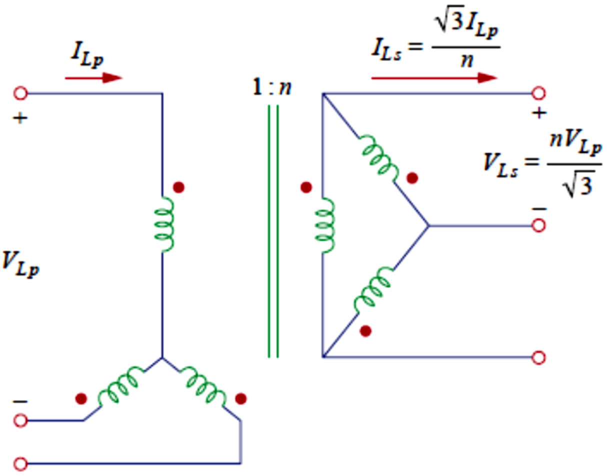
$$I_{Ls} = \frac{I_{Lp}}{n\sqrt{3}} \quad \ldots(12.6b)$$



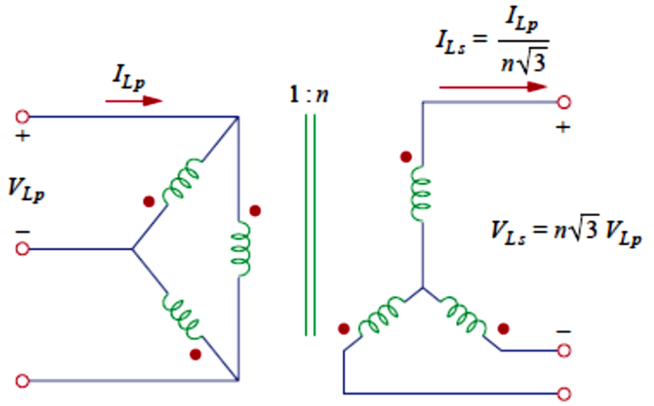
(a) Y-Y three-phase transformer connection



(b) Δ-Δ three-phase transformer connection



(c) Y-Δ three-phase transformer connection



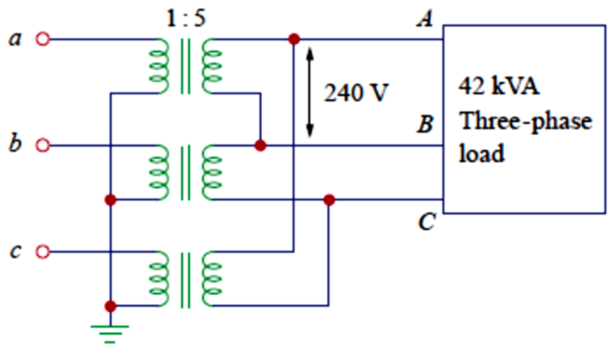
(d) Δ-Y three-phase transformer connection

Fig.12.1

Example 14: The 42-kVA balanced load depicted in Fig. is supplied by a threephase transformer. (a) Determine the type of transformer connections. (b) Find the line voltage and current on the primary side. (c) Determine the kVA rating of each transformer used in the transformer bank. Assume that the transformers are ideal.

Solution:

(a) The transformer is Y-Δ connected



(b)

$$I_{Ls} = \frac{S_T}{\sqrt{3}\, V_{Ls}} = \frac{42000}{\sqrt{3} \times 240} = 101\ A$$

$$I_{Ls} = \frac{\sqrt{3}\, I_{Lp}}{n} \rightarrow 101 = \frac{\sqrt{3}\, I_{Lp}}{5}$$

$$\therefore\ I_{Lp} = 292A$$

$$V_{Ls} = \frac{nV_{Lp}}{\sqrt{3}} \rightarrow 240 = \frac{5\,V_{Lp}}{\sqrt{3}}$$

$$\therefore\ V_{Lp} = 83.14\,V$$

(C) Because the load is balanced, each transformer equally shares the total load and since there are no losses (assuming ideal transformers), the kVA rating of each transformer is

$S = \frac{S_T}{3} = 14$ kVA

Or from primary side Y

$V_{Lp} = V_{Ls} = 240$

$I_{Ls} = \frac{I_{Lp}}{\sqrt{3}} = \frac{292}{\sqrt{3}} = 58.34\,A$

$S = 240 \times 58.34 = 14\,kVA$

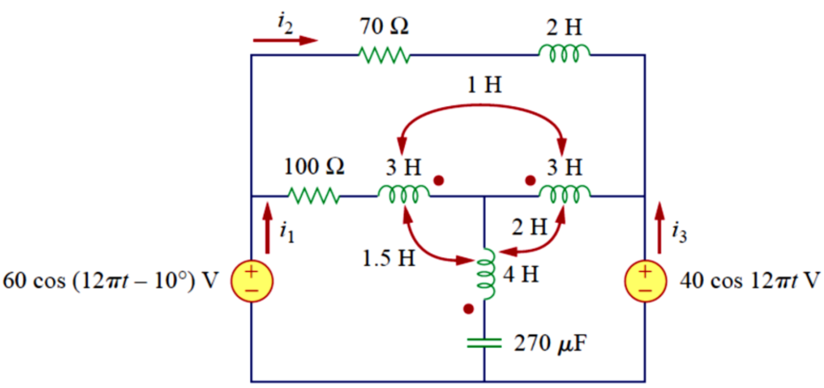
Or you can find it from secondary side Δ

H.W.12: A three-phase - transformer is used to step down a line voltage of 625 kV, to supply a plant operating at a line voltage of 12.5 kV. The plant draws 40 MW with a lagging power factor of 85 percent. Find: (a) the current drawn by the plant, (b) the turns ratio, (c) the current on the primary side of the transformer, and (d) the load carried by each transformer.

Answer: (a) 2.1736 kA, (b) 0.02, (c) 43.47 A, (d) 15.69 MVA.

H.W. : find i_1, i_2, and i_3 in Fig.

Answer:



$I_1 = 0.4654\angle{-70.25^o}$ A

$I_2 = 0.2114\angle{-75.75^o}$ A

$I_3 = 0.1095\angle{17.15^o}$ A

Or

$i_1 = 0.4654\cos(12\pi t - 70.25^o)$ A

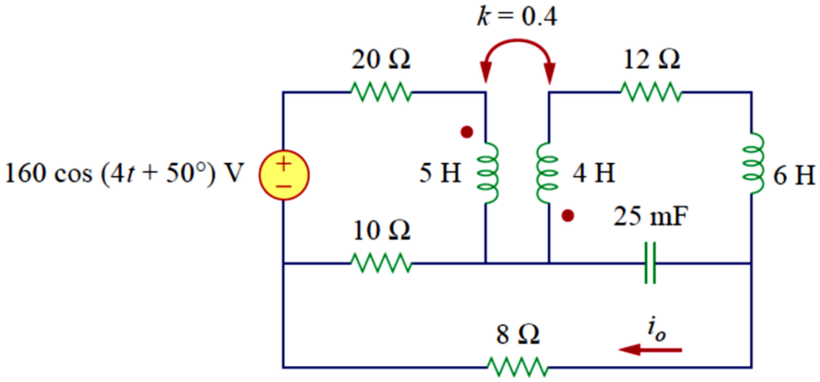
$i_2 = 0.2114\cos(12\pi t - 75.75^o)$ A

$i_3 = 0.1095\cos(12\pi t + 17.15^o)$ A

H.W. : find i_o in Fig.

Answer:

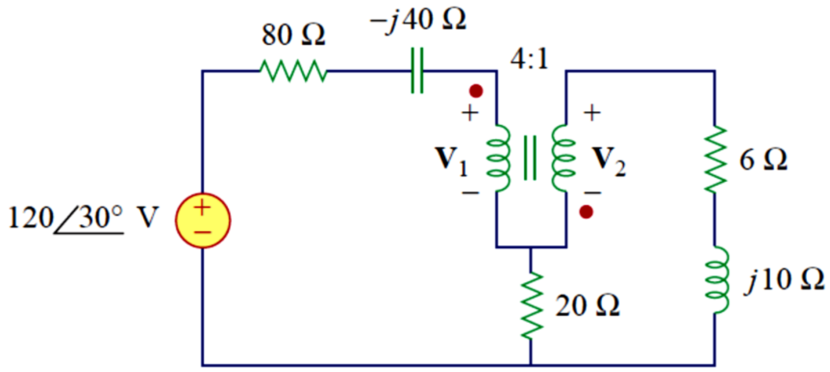
$2.012\cos(4t + 68.52^o)$ A



H.W. : find V_1 & V_2 in the ideal transformer circuit of Fig.

Answer:

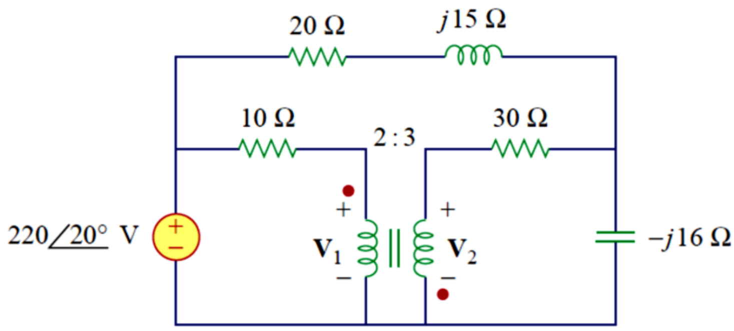
$91.12\angle 37.92^o$ V,
$22.78\angle -142.1^o$ V



H.W. : find V_1 & V_2 in the ideal transformer circuit of Fig.

Answer:

$138.82\angle 28.65^o$ V, $208.2\angle -151.4^o$ V



7) CONDUCTIVELY COUPLED EQUIVALENT CIRCUITS

It is possible in analysis to replace a mutually coupled circuit with a conductively coupled equivalent circuit. For Fig.9(a) the mesh current equations are,

$(R_1 + j\omega L_1)I_1 - j\omega M I_2 = V_1$ ...(27)

$-j\omega M I_1 + (R_2 + j\omega L_2)I_2 = V_2$ ...(28)

Equ.s (27) & (28) can be written in matrix form as:

$$\begin{bmatrix} R_1 + j\omega L_1 & -j\omega M \\ -j\omega M & R_2 + j\omega L_2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Or

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Note that:

1) the first loop impedance is the sum of first column impedances ($Z_{11}+Z_{21} = R_1 + j\omega L_1 - j\omega M$)
2) the second loop impedance is the sum of second column impedances ($Z_{12} + Z_{22} = R_2 + j\omega L_2 - j\omega M$),
3) the common impedance between the two loops is the diagonal impedance ($-Z_{11}$ or $-Z_{22} = j\omega M$)

the conductively copled circuit is shown in Fig.9(a).

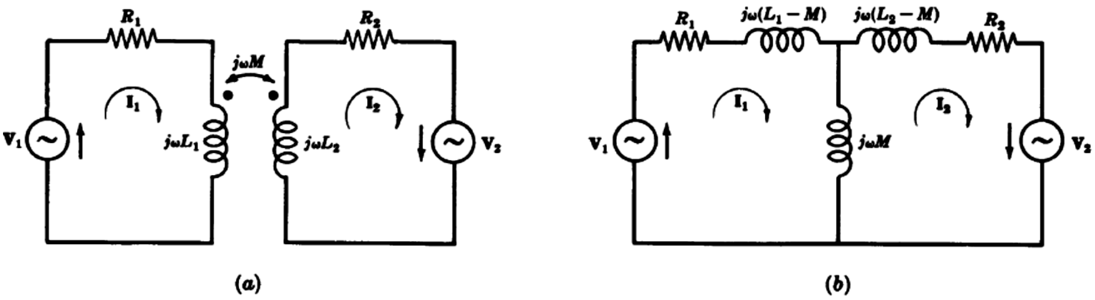


Fig.9

Note : The above method of analysis does not always lead to a physically realizable equivalent circuit. This is true when $M > L_1$ or $M > L_2$.

To replace the series connection of the mutually coupled coils shown in Fig.10 (a), proceed in the following manner. First apply the methods described above and obtain the dotted equivalent shown in Fig.10 (b). Then replace the dotted equivalent by the conductive equivalent shown in Fig.10 (c).

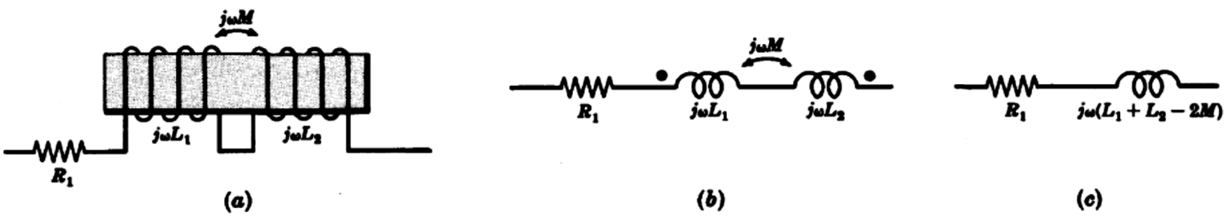


Fig.10

Lecture ()

Magnetically Coupled Circuits

Problems

H.W.(1): Two coils have a coupling coefficient $k = .85$ and coil 1 has 250 turns. With a current $i_1 = 2$ amp in coil 1, the total flux φ_1 is 3.0×10^{-4} weber. When i_1 is reduced linearly to zero in two milli- seconds the voltage induced in coil2 is 68.75 volts. Find L_1, L_2, M and N_2.

[Answer: 37.5 mH, 150 mH, 63.8 mH, 500]

H.W.(2): Two coupled coils, $L_1 = 0.8$ H and $L_2 = 0.2$ H, have a coefficient of coupling k = 0.90. Find the mutual inductance M and the turns ratio N_1=N_2.

[Answer: 0.36 H, 2]

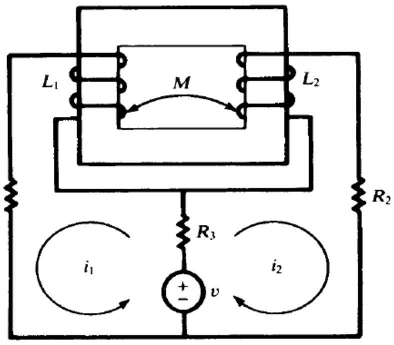
H.W.(3): Two coupled coils, N_1 =100 and N_2 = 800, have a coupling coefficient k =0.85. With coil 1 open and a current of 5 A in coil 2, the flux is $\emptyset_2$ = 0.35 mWb. Find L_1, L_2, and M.

[Answer: 0.875 mH, 56 mH, 5.95 mH]

H.W.(4): Two identical coupled coils have an equivalent inductance of 80 mH when connected series aiding, and 35 mH in series opposing. Find L_1, L_2, M, and k.

[Answer: 28.8 mH, 28.8 mH, 11.25 mH, 0.392]

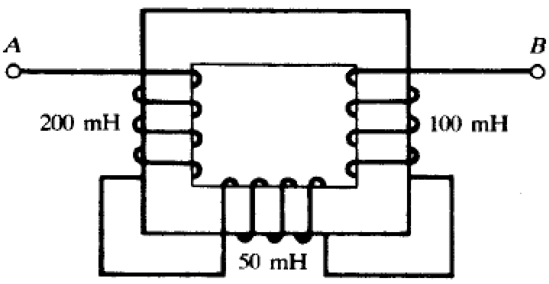
H.W.(5): Write the mesh current equations for the coupled circuit shown in Fig. shown. Obtain the dotted equivalent circuit and write the same equations.



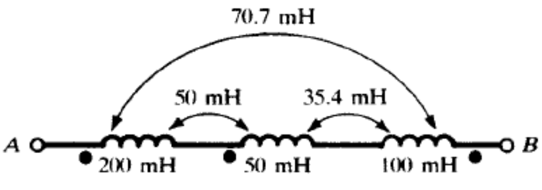
[Answer]

$$(R_2 + R_3)i_2 + L_2\frac{di_2}{dt} + R_3 i_1 + M\frac{di_1}{dt} = v$$

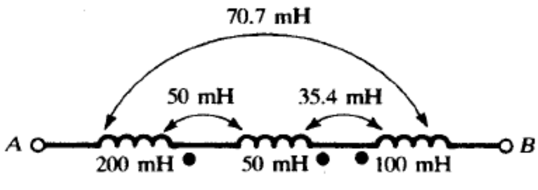
H.W.(6): Obtain two forms of the dotted equivalent circuit for the coupled coils shown in Fig.



[Answer]

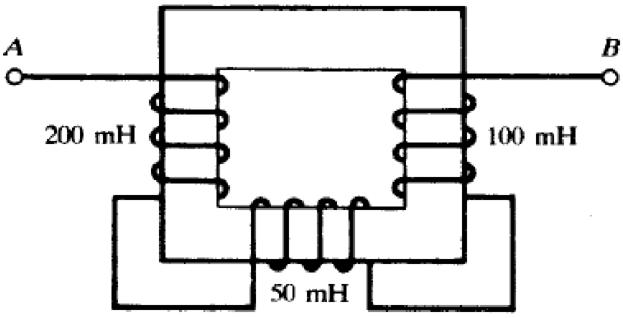


(a)



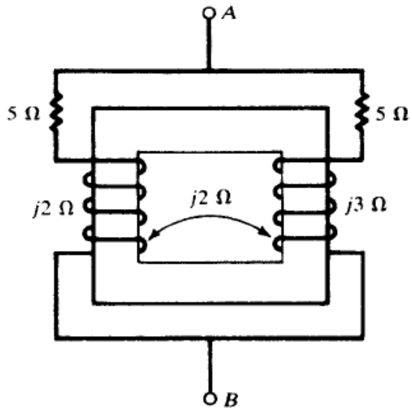
(b)

H.W.(7): The three coupled coils shown in Fig. have coupling coefficients of 0.5. Obtain the equivalent inductance between the terminals AB.



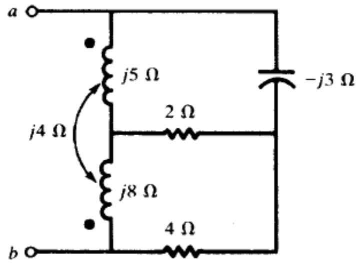
[Answer: 239 mH]

H.W.(8): (a) Obtain the equivalent impedance at terminals AB of the coupled circuit shown in Fig. shown (b) Reverse the winding sense of one coil and repeat.



[Answer: $(a)3.4\angle 41.66^0\Omega, (b)2.54\angle 5.37^0\Omega$**]**

H.W.(9): For the coupled circuit shown in Fig., find the input impedance at terminals ab.

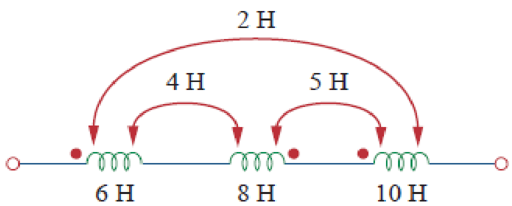


[Answer: 6.22 + j4.65 Ω]

H.W.(10): Two coils are mutually coupled, with L_1 = 25 mH, L_2 = 60 mH, and k = 0.5. Calculate the maximum possible equivalent inductance if: (a) the two coils are connected in series (b) the coils are connected in parallel

[Answer:123.7 mH, 24.31 mH]

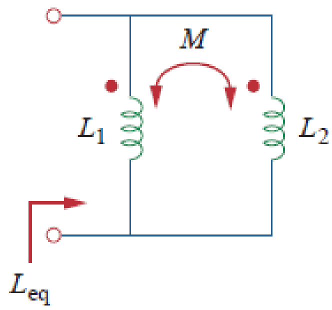
H.W.(11): For the three coupled coils in Fig. shown, calculate the total inductance.



[Answer:10H]

H.W.(12): For the coupled coils in Fig. shown, show that

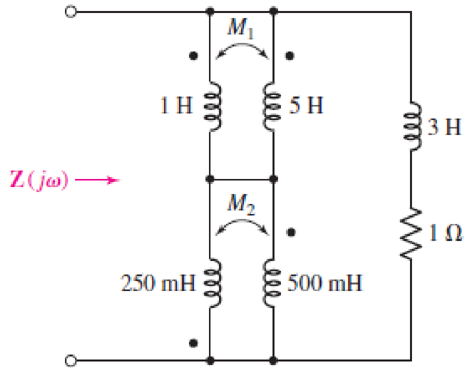
$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$



[Answer:]

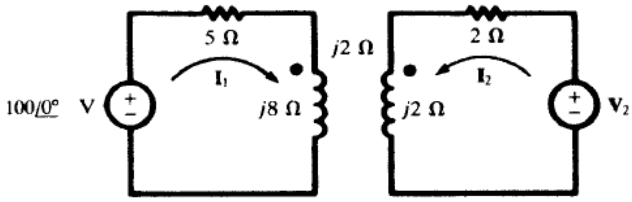
H.W.(13): With respect to the network shown in Fig. shown, derive an expression for $\mathbf{Z}(j\omega)$ if M_1 and M_2 are set to their respective maximum values.

[Answer



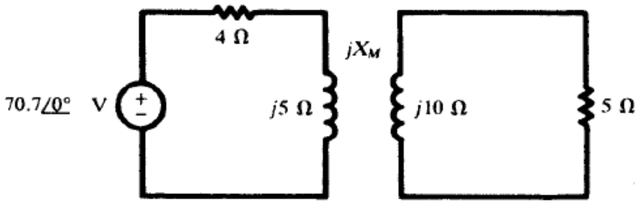
H.W.(14): In the coupled circuit shown in Fig. , find V_2 for which $I_1 = 0$. What voltage appears at the 8 Ω inductive reactance under this condition?

[Answer:
$141.4\angle{-45^0}$V, $100\angle{0^0}$V (+at dot)]



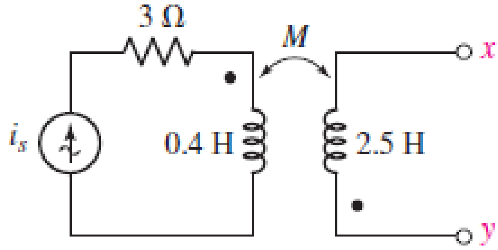
H.W.(15): Find the mutual reactance X_M for the coupled circuit of Fig, if the average power in the 5Ω resistor is 45.24 W.

[Answer:4Ω]



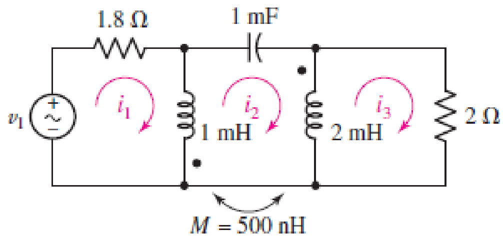
H.W.(16): Let $is = 2 \cos 10t$ A in the circuit of Fig. shown, and find the total energy stored in the passive network at $t = 0$ if $k = 0.6$ and terminals x and y are (a) left open-circuited; (b) short-circuited.

[Answer: 0.8 J; 0.512 J.]



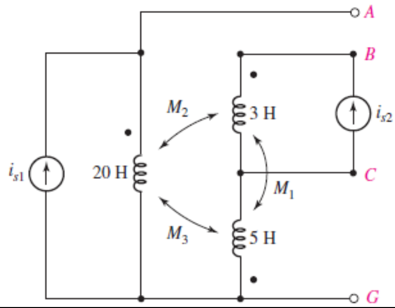
H.W.(17): For the circuit of Fig, (a) draw the phasor representation; (b) write a complete set of mesh equations; (c) calculate $i_2(t)$ if $v_1(t) = 8 \sin 720t$ V.

[Answer:]



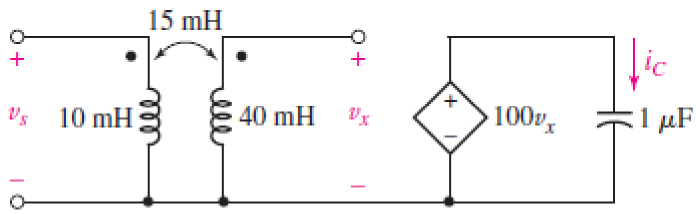
H.W.(18): For the circuit of Fig. shown, $M_1 = 1$ H, $M_2 = 1.5$ H, and $M_3 = 2$ H. If $i_{s1} = 8 \cos 2t$ A and $i_{s2} = 7 \sin 2t$ A, calculate (a) $\mathbf{V}_{AB}$; (b) $\mathbf{V}_{AG}$; (c) $\mathbf{V}_{CG}$.

[Answer:]



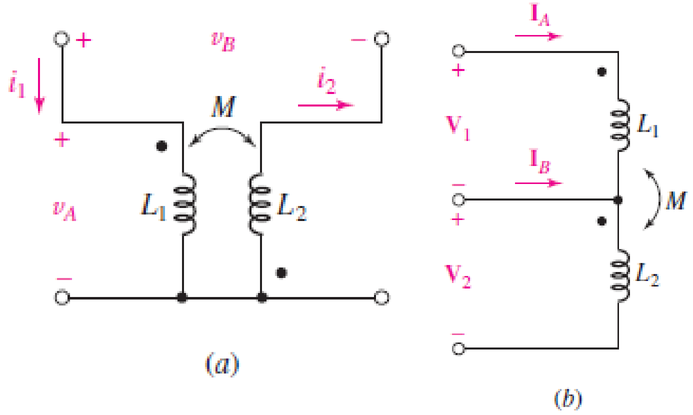
H.W.(19): Determine an expression for $i_C(t)$ valid for t >0 in the circuit of Fig. shown , if $v_s\ (t) = 10t^2 u(t)/(t^2 + 0.01)$ V.

[Answer:]



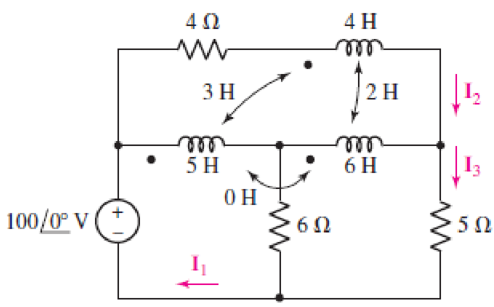
H.W.(20): For the coupled inductor network of Fig.a, set L_1= 20 mH, L_2 = 30 mH, M = 10 mH, and obtain equations for v_A and v_B if (a) i_1= 0 and i_2= 5 sin 10t; (b) i_1= 5 cos 20t and i_2= 2 cos (20t – 100∘) mA. (c) Express $\mathbf{V}_1$ and $\mathbf{V}_2$ as functions of $\mathbf{I}_A$ and $\mathbf{I}_B$ for the network shown in Fig. b.

[Answer:]



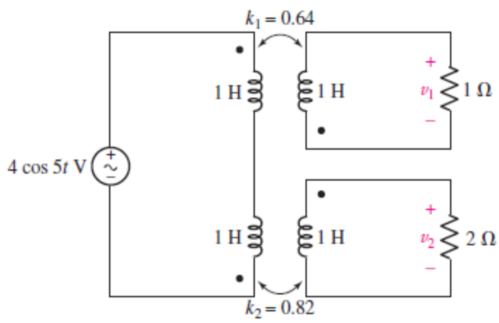
H.W.(21): Note that there is no mutual coupling between the 5 H and 6 H inductors in the circuit of Fig. shown. (a) Write a set of equations in terms of $\mathbf{I}_1(j\omega)$, $\mathbf{I}_2(j\omega)$, and $\mathbf{I}_3(j\omega)$. (b) Find $\mathbf{I}_3(j\omega)$ if ω = 2 rad/s.

[Answer:]



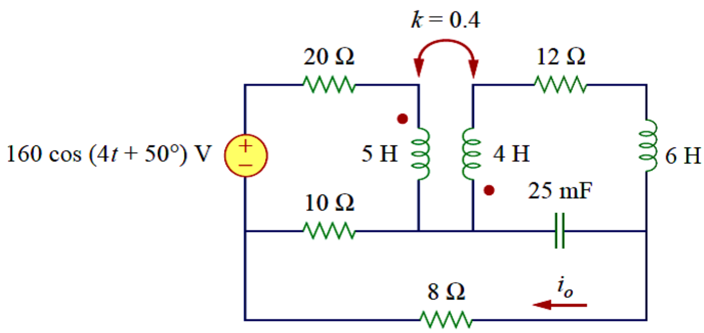
H.W.(22): Compute v_1, v_2, and the average power delivered to each resistor in the circuit of Fig. shown.

[Answer:]



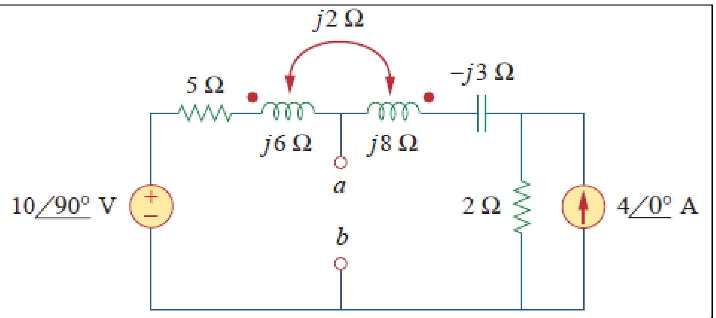
H.W.(23): find i_o in Fig.

[Answer:
$2.012\cos(4t + 68.52^o)$ A]



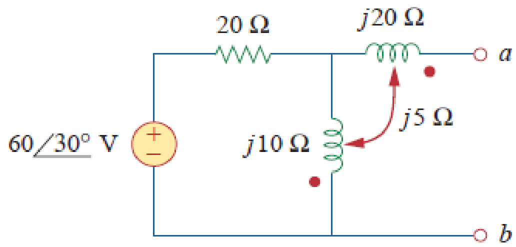
H.W.(24): Obtain the Thevenin equivalent circuit for the circuit in Fig. shown at terminals *a*-*b*.

[Answer:V_{Th} = 5.349∠34.11°, Z_{Th} = 2.332∠50°ohms]



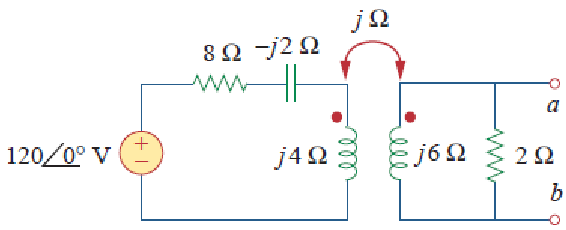
H.W.(25): Find the Norton equivalent for the circuit in Fig. shown at terminals *a*-*b*.

[Answer:I_N = 1.404∠9.44°A, Z_N = (1 + j19.5) ohms]



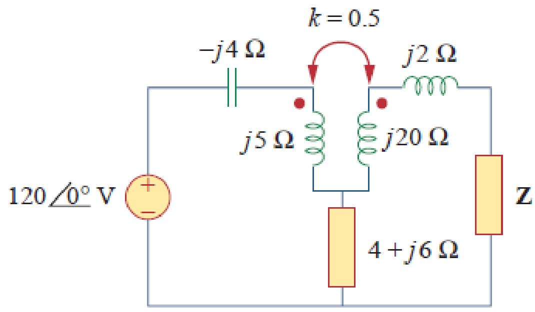
H.W.(26): Find the Norton equivalent for the circuit in Fig. shown at terminals *a*-*b*.

[Answer: I_N = 1.6246∠-12.91°A, Z_N = 1.894∠19.53°ohms]



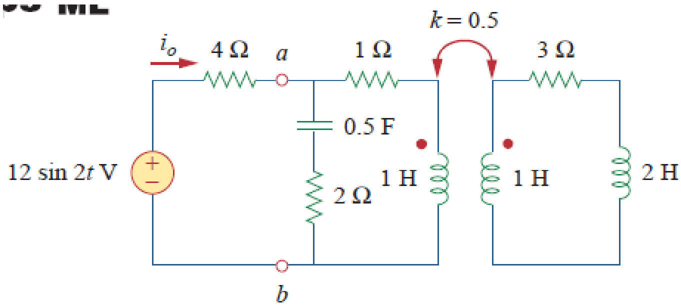
H.W.(27): Find the Thevenin equivalent to the left of the load **Z** in the circuit of Fig.

[Answer:V_{TH} = 61.37∠-46.22°A, Z_{TH} = (2.215 + j29.12) ohms]



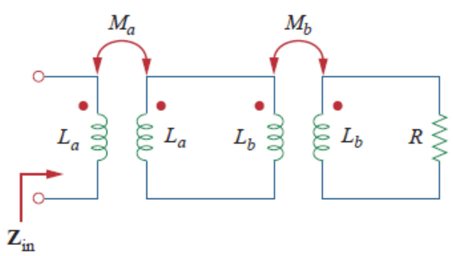
H.W.(28): For the network in Fig. , find $\mathbf{Z}_{ab}$ and $\mathbf{I}_o$.

[Answer: Z_{ab} = 1.5085∠17.9°ohms io = 2.2sin(2t – 4.88°) A]



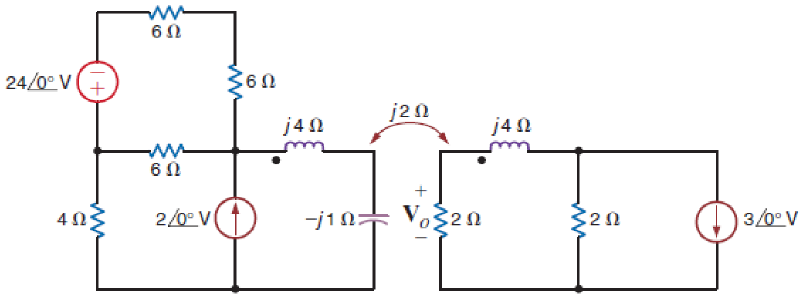
H.W.(29): Two linear transformers are cascaded as shown in Fig.. Show that

$$\mathbf{Z}_{in} = \frac{\omega^2 R(L_a^2 + L_aL_b - M_a^2) + j\omega^3(L_a^2L_b + L_aL_b^2 - L_aM_b^2 - L_bM_a^2)}{\omega^2(L_aL_b + L_b^2 - M_b^2) - j\omega R(L_a + L_b)}$$



[Answer:]

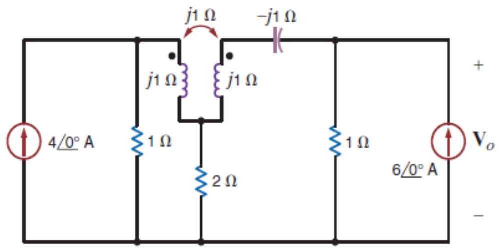
H.W.(30): Find $\mathbf{V}_o$ in the network in Fig. shown.



[Answer:]

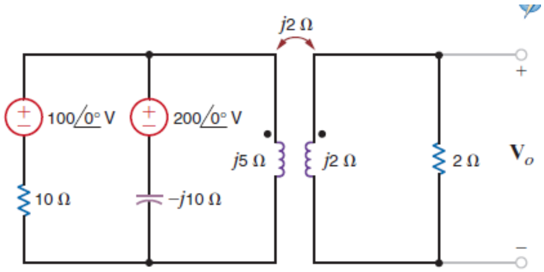
H.W.(31): Find $\mathbf{V}_o$ in the network in Fig. shown.

[Answer:]



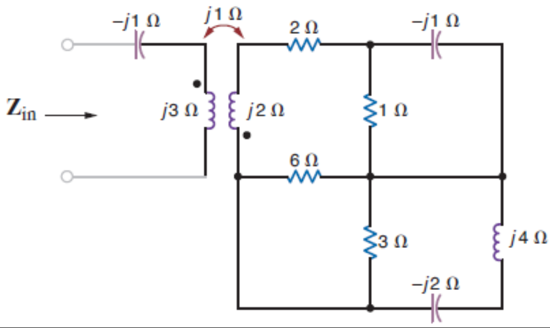
H.W.(32): Find $\mathbf{V}_o$ in the network in Fig. shown.

[Answer: $\boxed{V_o = 36.8 \angle 72.9^\circ \text{ V}}$ **]**



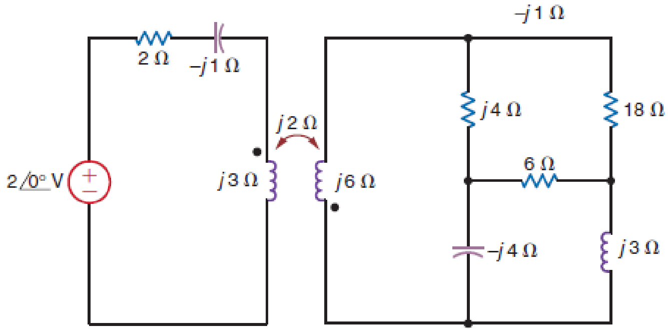
H.W.(33): Determine the impedance seen by the source in the network in Fig. shown.

[Answer:]

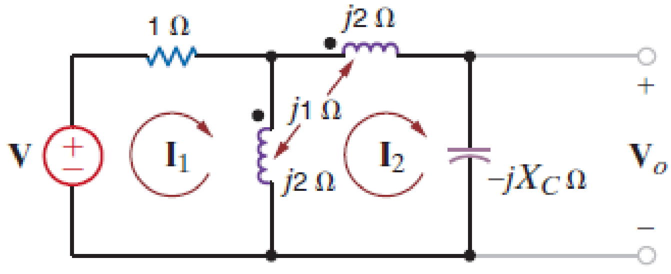


H.W.(34): Determine the impedance seen by the source in the network in Fig. shown.

[Answer:]

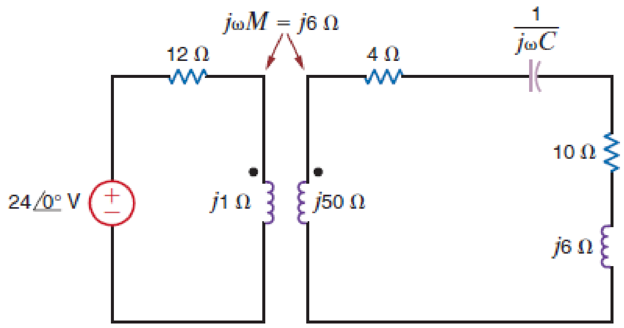


H.W.(35): determine the value of X_C such that the output voltage is equal to twice the input voltage.



[Answer: $\boxed{X_c = 2\Omega}$ **]**

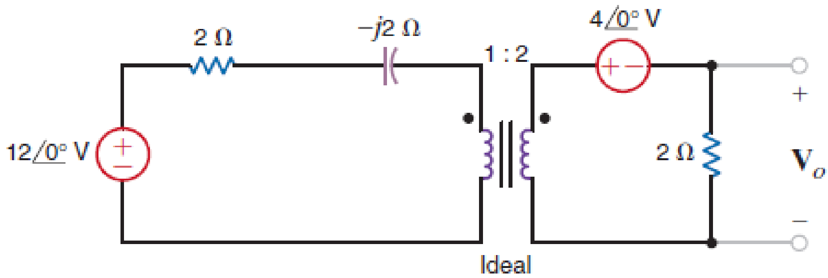
H.W.(36): determine the value of the capacitor *C* that will cause the impedance seen by the 24∠0° V voltage source to be purelyresistive, *f* =60 Hz.



[Answer]

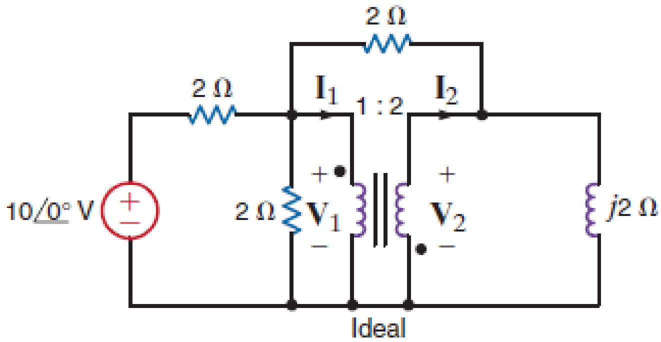
$\boxed{C = \left\{ \begin{matrix} 53.8\mu F \\ 99\mu F \end{matrix} \right\} \text{ either will work!}}$

H.W.(37): Given the network in Fig. shown, form an equivalent circuit for the transformer and primary, and use the resultant network to compute $\mathbf{V}_o$.



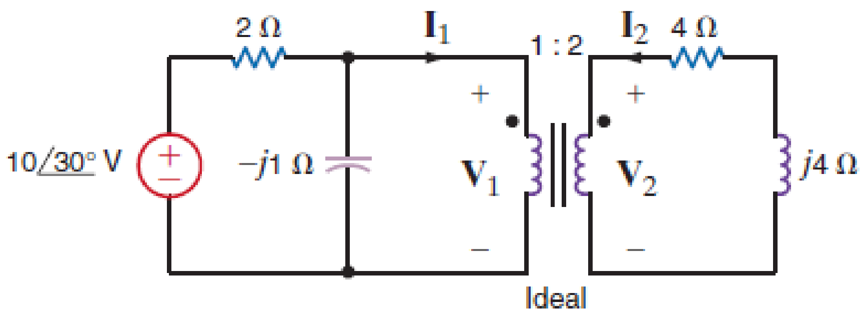
[Answer: V_o **= 3.12∠38.66° V.]**

H.W.(38): Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in the network in Fig. shown.



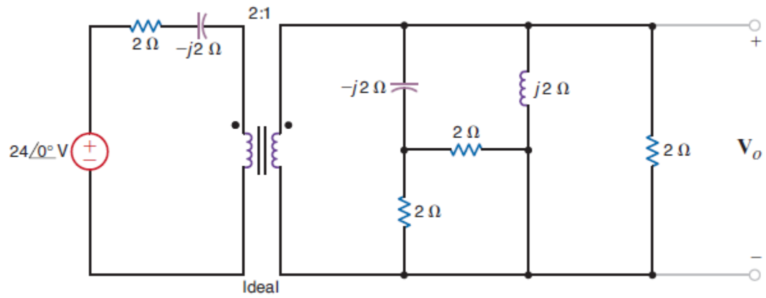
[Answer:
V_2 = 1.71∠-160° V.
V_1 = 0.85∠20° V;
I_2 = 1.54∠166.3° A;
I_1 = 3.08∠-13.7° A]

H.W.(39): Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$ and $\mathbf{V}_2$ in the network in Fig. shown.



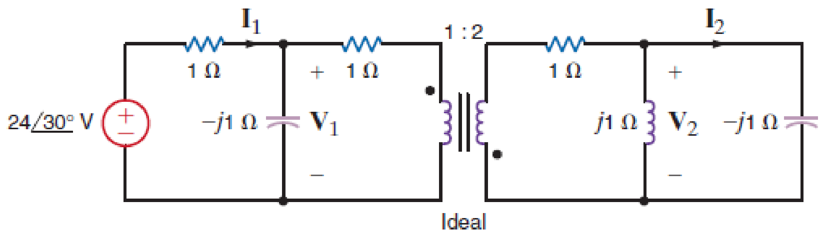
[Answer:]

H.W.(40): Find $\mathbf{V}_o$ in the network in Fig. shown.



[Answer:]

H.W.(41): Determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$ and $\mathbf{V}_2$ in the network in Fig. shown.



$I_2 = 34.0\angle -105°$ A

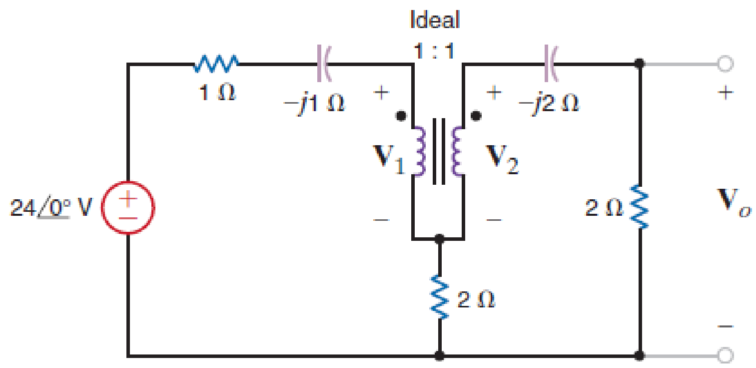
$V_1 = 17.0\angle -15°$ V

$V_2 = 34.0\angle 165°$ V

$I_1 = 17.0\angle 75°$ A

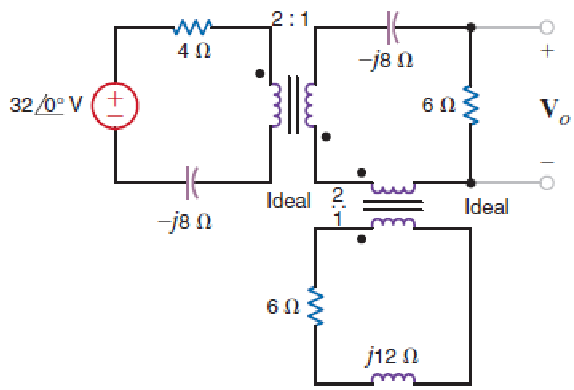
H.W.(42): Find $\mathbf{V}_o$ in the network in Fig. shown.

[Answer:11.32∠45°]



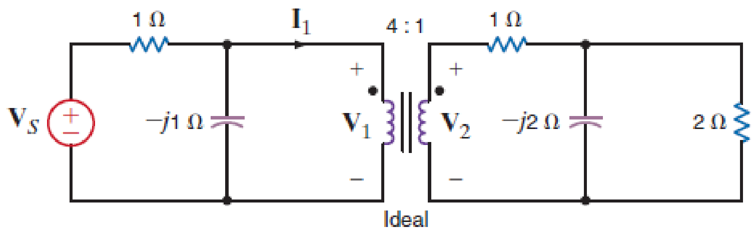
H.W.(43): Find $\mathbf{V}_o$ in the network in Fig. shown.

[Answer:1.96∠129°]



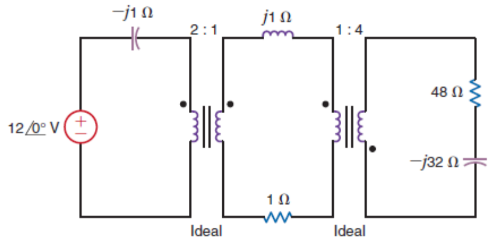
H.W.(44): Determine the input impedance seen by the source in Fig. shown.

[Answer:1.42∠- 43.9°]



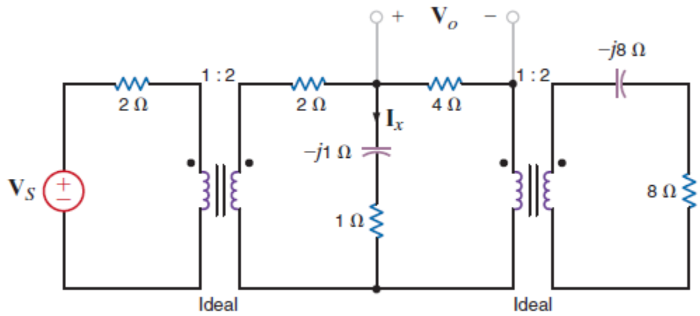
H.W.(45): Determine the input impedance seen by the source in Fig. shown.

[Answer:]

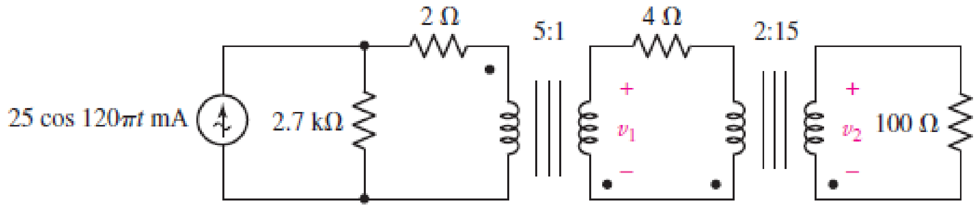


H.W.(46): In the network in Fig. shown, if $I_x = 4\angle 30°$ find V_0 .

[Answer: 3.58∠3.43°]

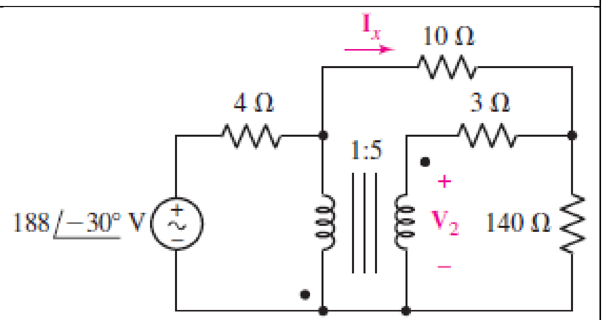


H.W.(47): With respect to the circuit depicted in Fig. shown, calculate (*a*) the voltages v_1 and v_2; (*b*) the average power delivered to each resistor.



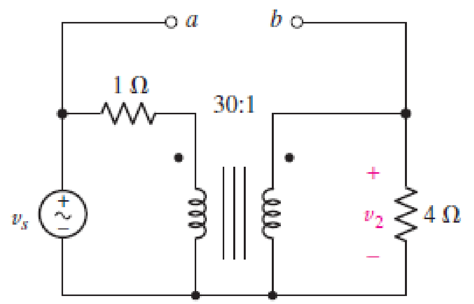
H.W.(48): Calculate $\mathbf{I}_x$ and $\mathbf{V}_2$ as labeled in Fig. shown.

[Answer:]

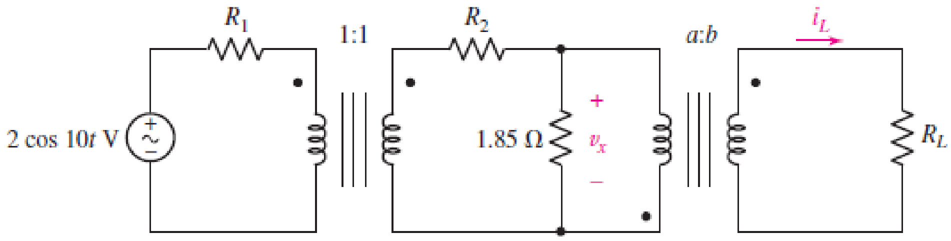


H.W.(49): For the circuit of Fig. shown, v_s= 117 sin 500t V. Calculate v_2 if the terminals marked *a* and *b* are (*a*) left open-circuited; (*b*) short-circuited; (*c*) bridged by a 2 Ωresistor.

[Answer:]

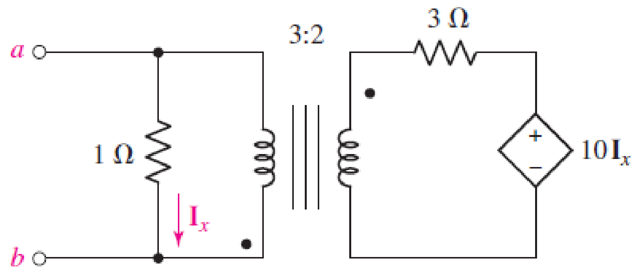


H.W.(50): For the circuit of Fig. shown, $R_1 = 1\ \Omega$, $R_2 = 4\ \Omega$, and $R_L = 1\ \Omega$. Select a and b to achieve a peak voltage of 200 V magnitude across R_L.



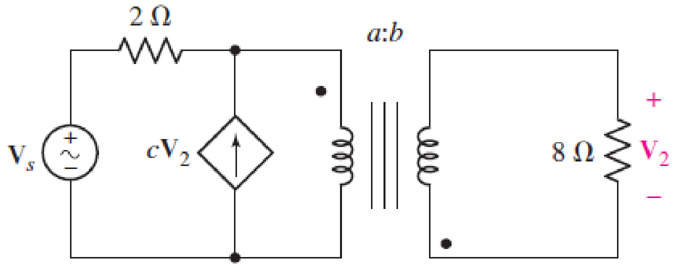
[Answer:]

H.W.(51): Determine the Thévenin equivalent of the network in Fig. shown as seen looking into terminals a and b.



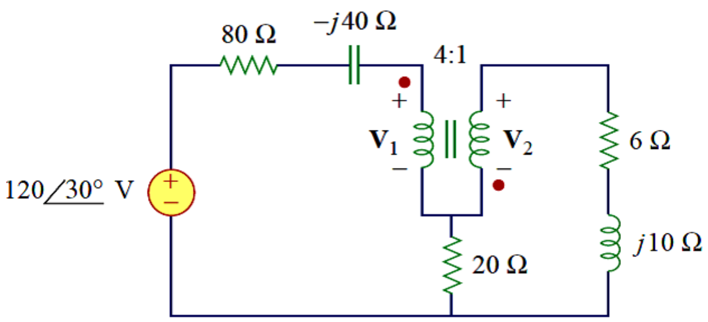
[Answer:]

H.W.(52): For the circuit of Fig. , take $c = -2.5$ mS and select values of a and b such that 100W average power is delivered to the 8Ω load when $\mathbf{V}_s = 5\angle{-35^\circ}$ V.



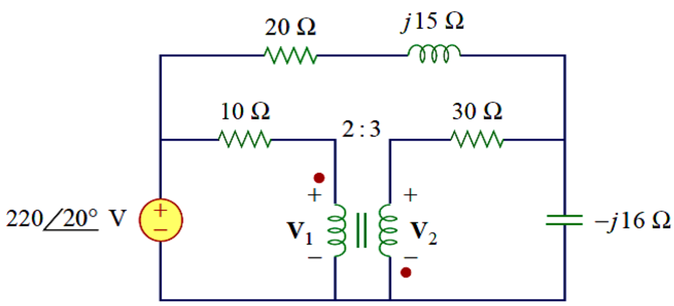
[Answer:]

H.W.(53): find V_1&V_2 in the ideal transformer circuit of Fig.



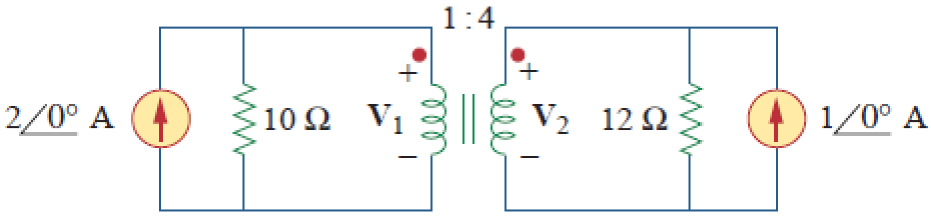
[Answer:
$91.12\angle 37.92^{o}$ V, $22.78\angle{-142.1^{o}}$ V]

H.W.(54): find V_1&V_2 in the ideal transformer circuit of Fig.



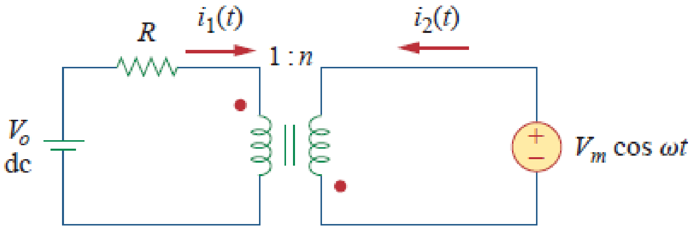
[Answer:$138.82\angle 28.65^{o}$ V, $208.2\angle{-151.4^{o}}$ V]

H.W.(55): Obtain $\mathbf{V}_1$ and $\mathbf{V}_2$ in the ideal transformer circuit of Fig.



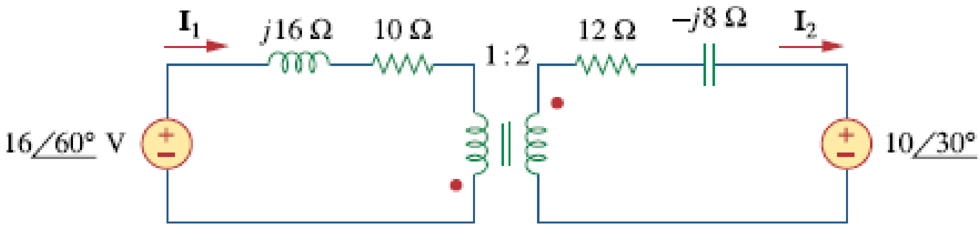
[Answer:v_1 = 4.186 V and v_2 = 4v = 16.744 V]

H.W.(56): In the ideal transformer circuit of Fig., find $i_1(t)$ and $i_2(t)$.



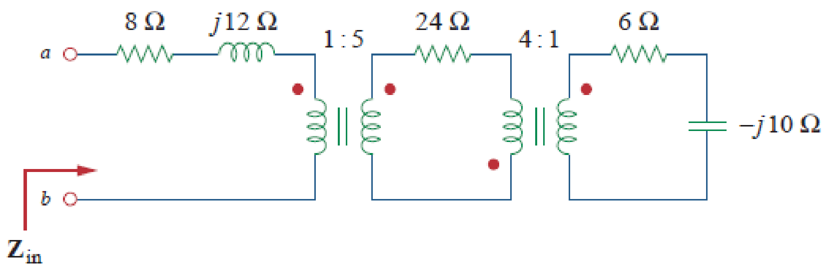
[Answer:$i_1(t) = (v_m/Rn)\cos\omega t$ A, and $i_2(t) = (-v_m/(n^2R))\cos\omega t$ A]

H.W.(57): Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. below. (b) Switch the dot on one of the windings. Find $\mathbf{I}_1$ and $\mathbf{I}_2$ again.



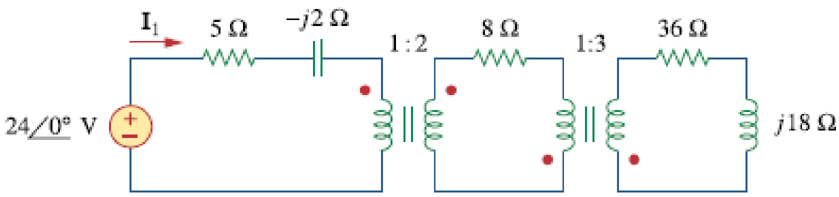
[Answer (a) $I_1 = 1.072\angle 5.88°$A, and $I_2 = 0.536\angle 185.88°$A (b) $I_1 = 0.625\angle 25$ A, and $I_2 = 0.3125\angle 25°$A]

H.W.(58): Calculate the input impedance for the network in Fig.



[Answer:Z_{in} = (12.8 + j5.6) ohms]

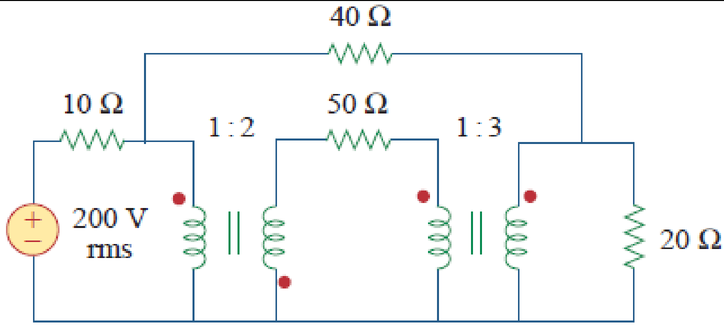
H.W.(59): Use the concept of reflected impedance to find the input impedance and current $\mathbf{I}_1$ in Fig.



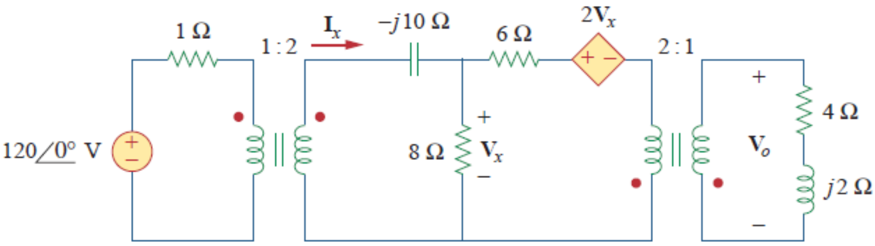
[Answer:$I_1 = 8.95\angle 10.62°$A]

H.W.(60): Calculate the average power dissipated by the 20- Ωresistor in Fig.

[Answer: 11.05 W]



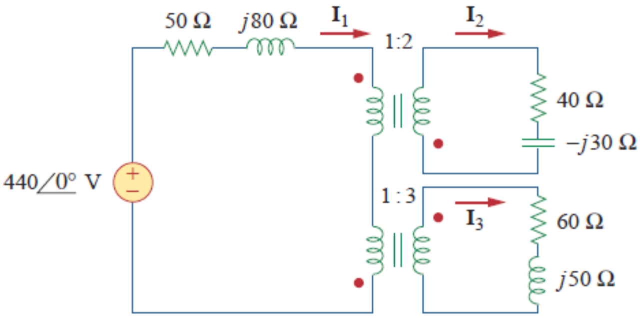
H.W.(61): Find $\mathbf{I}_x$ and $\mathbf{V}_x$ in the circuit of Fig.



[Answer: iX = 1.08∠33.91° A, Vx = 15.14∠–34.21° V.]

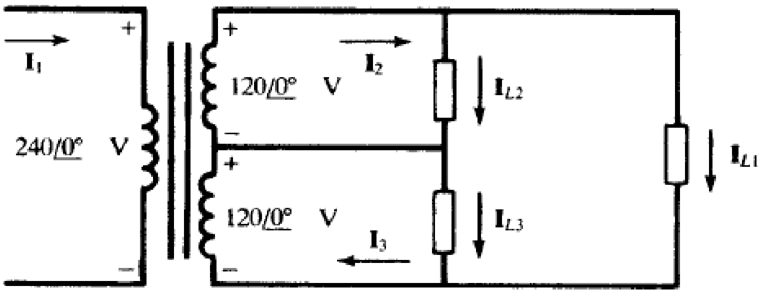
H.W.(62): Determine $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the ideal transformer circuit of Fig.

[Answer: I_1 = 4.028∠–52.38° A, I_2 = 2.019∠–52.11° A, I_3 = 1.338∠–52.2° A.]



H.W.(63): For the ideal transformer shown in Fig, find I_1, given

$\mathbf{I}_{L1} = 10.0\angle 0°$ A $\quad \mathbf{I}_{L2} = 10.0\angle -36.87°$ A $\quad \mathbf{I}_{L3} = 4.47\angle -26.57°$ A

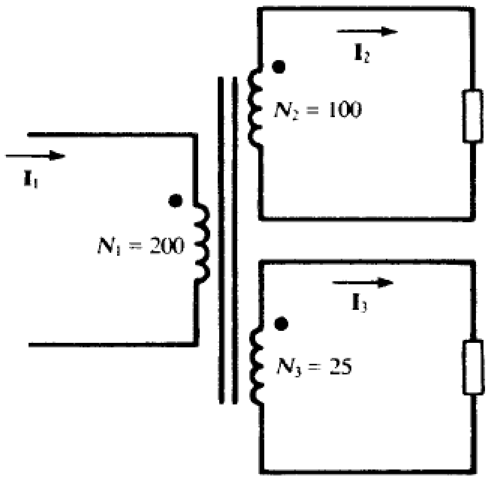


[Answer: $16.5\angle -14.04°$ A **]**

H.W.(64): For the ideal transformer shown in Fig., find I_1, given

$\mathbf{I}_2 = 50\underline{/-36.87^\circ}$ A and $\mathbf{I}_3 = 16\underline{/0^\circ}$ A.

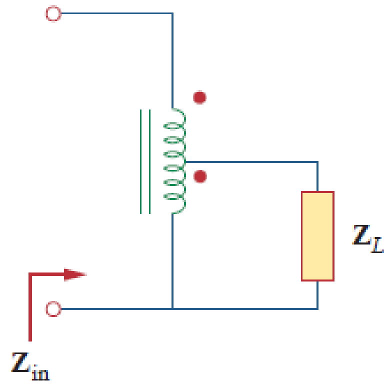
[Answer: $26.6\underline{/-34.29^\circ}$ A **]**



H.W.(65): In the autotransformer circuit in Fig., show that

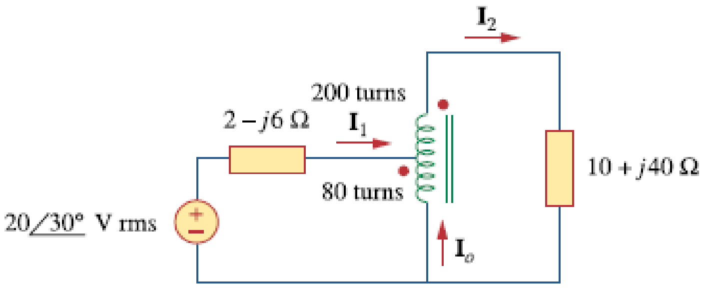
$$\mathbf{Z}_{in} = \left(1 + \frac{N_1}{N_2}\right)^2 \mathbf{Z}_L$$

[Answer:]



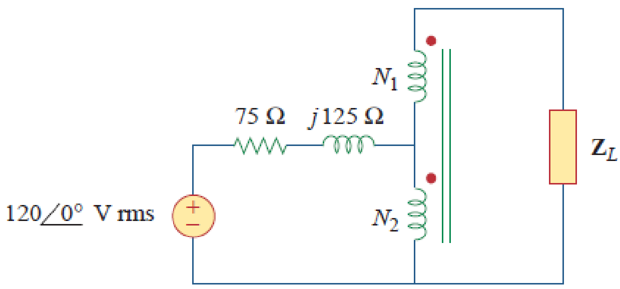
H.W.(66): In the ideal autotransformer of Fig., calculate $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_o$ Find the average power delivered to the load.

[Answer:
I_1 = 1.245∠–33.76°A,
I_2 = 0.8893∠–33.76°A,
I_o = 0.3557∠146.2°A,
p = 7.51 watts]



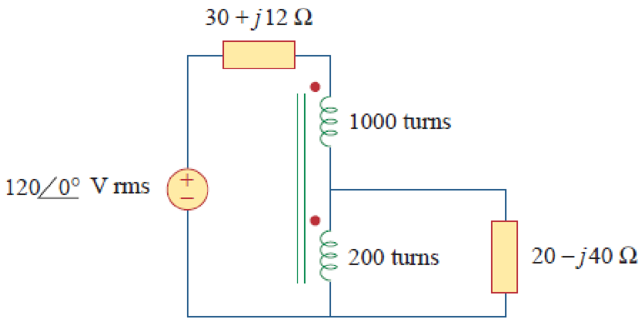
H.W.(67): In the circuit of Fig., $\mathbf{Z}_L$ is adjusted until maximum average power is delivered to $\mathbf{Z}_L$. Find $\mathbf{Z}_L$ and the maximum average power transferred to it. Take N_1 = 600 turns and N_2 = 200 turns.

[Answer: Z_L = (1.2 – j2) kΩ, p= 5.333 w]



H.W.(68): In the ideal transformer circuit shown in Fig., determine the average power delivered to the load.

[Answer: 74.9 watts.]



H.W.(69): Considering the autotransformer shown in Fig. ideal, obtain the currents I_1, I_{cb}, and I_{dc}.

[Answer:]
$3.70\angle 22.5^\circ$ A, $2.12\angle 86.71^\circ$ A, $10.34\angle 11.83^\circ$ A