Lecture ( )

Three-Phase Circuits

1) Introduction

A single-phase ac power system consists of a generator connected through a pair of wires (a transmission line) to a load. Fig.1.1 (a) depicts a single-phase twowire system, where V_p is the **rms** magnitude of the source voltage and ϕ is the phase. What is more common in practice is a single-phase threewire system, shown in Fig.1.1 (b). It contains two identical sources (equal magnitude and the same phase) that are connected to two loads by two outer wires and the neutral. For example, the normal household system is a single-phase three-wire system because the terminal voltages have the same magnitude and the same phase. Such a system allows the connection of both 120-V and 240-V appliances.

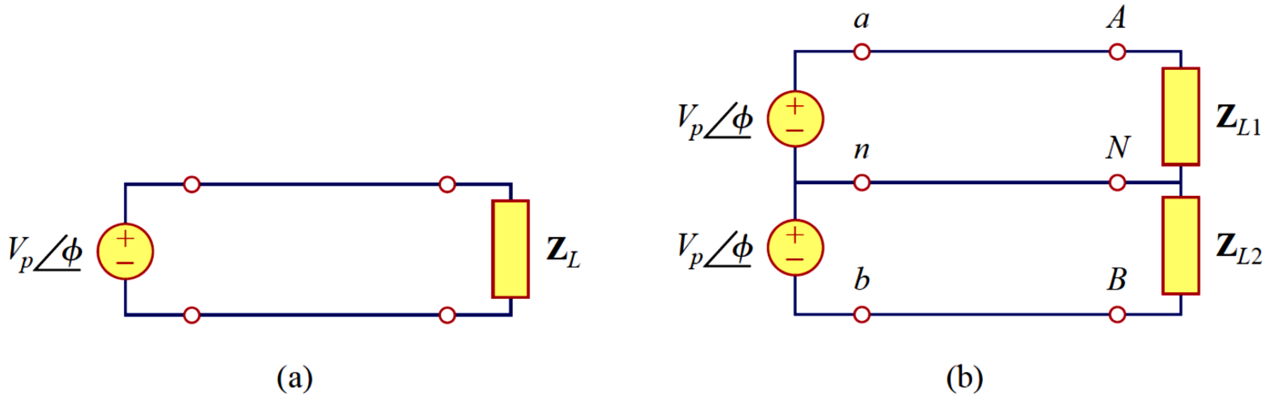


Fig.1.1 Single-phase systems: (a) two-wire type, (b) three-wire type.

Circuits or systems in which the ac sources operate at the same frequency but different phases are known as ***polyphase***.

Fig.1.2 shows a two-phase three-wire system, and Fig.1.3 shows a three-phase fourwire system.

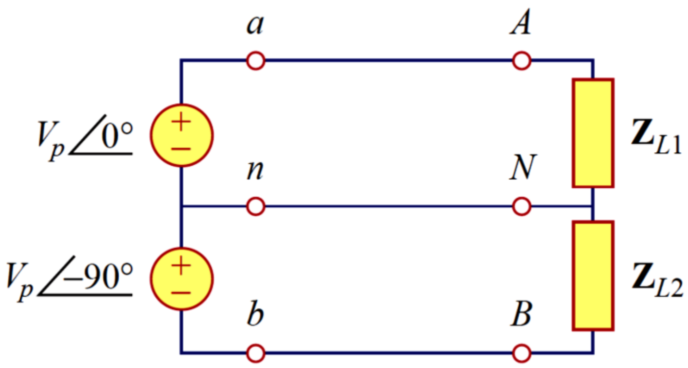


Fig.1.2 Two-phase three-wire system

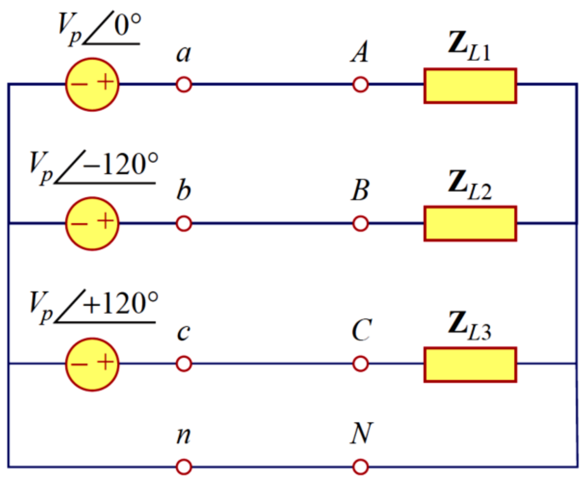


Fig.1.3 Three-phase four-wire system

A ***single-phase system***, a two-phase system is produced by a generator consisting of two coils placed ***perpendicular*** to each other so that the voltage generated by one lags the other by $\mathbf{90^0}$. A ***three-phase system*** is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by $\mathbf{120^0}$. Since the ***three-phase system*** is by far the most prevalent and most economical ***polyphase system***.

Three-phase systems are important for at least three reasons:

1) nearly all electric power is generated and distributed in three-phase, at the operating frequency of 60 Hz (or ω =377 rad/s) or 50 Hz (or ω =314 rad/s). When one-phase or two-phase inputs are required, they are taken from the three-phase system rather than generated independently. Even when more than three phases are needed—such as in the aluminum industry, where 48 phases are required for melting purposes—they can be provided by manipulating the three phases supplied.
2) the instantaneous power in a three-phase system can be constant (not pulsating. This results in uniform power transmission and less vibration of three-phase machines.
3) for the same amount of power, the three-phase system is more economical than the singlephase. The amount of wire required for a three-phase system is less than that required for an equivalent single-phase system.

2) *Balanced Three-Phase Voltages*

Three-phase voltages are often produced with a three-phase ac generator (or alternator) whose cross-sectional view is shown in Fig.2.1. The generator basically consists of a rotating magnet (called the ***rotor***) surrounded by a stationary winding (called the ***stator***). Three separate windings or coils with terminals *a*-*a*', *b*-*b*',and *c*-*c*' are physically placed $\mathbf{120^0}$ apart around the stator. As the rotor rotates, its magnetic field "cuts" the flux from the three coils and induces voltages in the coils.

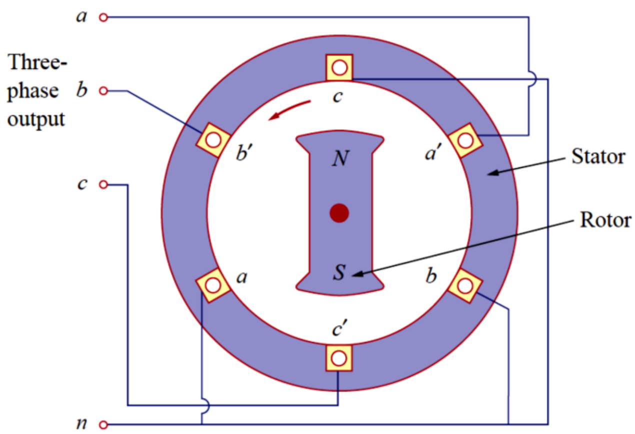


Fig.2.1 A three-phase generator

Because the coils are placed $\mathbf{120^0}$ apart, the induced voltages in the coils are equal in magnitude but out of phase by $\mathbf{120^0}$ (Fig.2.2). Since each coil can be regarded as a single-phase generator by itself, the three-phase generator can supply power to both single-phase and three-phase loads.

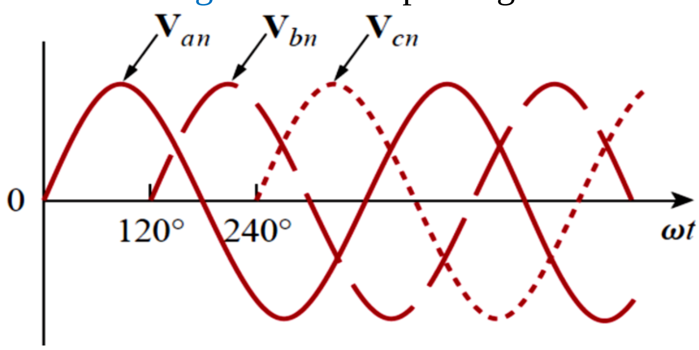


Fig.2.2 Generated voltages 120^0 apart from each other

A typical three-phase system consists of three voltage sources connected to loads by three or four wires (or transmission lines). (Threephase current sources are very scarce.) A three-phase system is equivalent to three single-phase circuits. The voltage sources can be either **wye-connected** as shown in Fig.2.3(a) or **delta-connected** as in Fig.2.3(b).

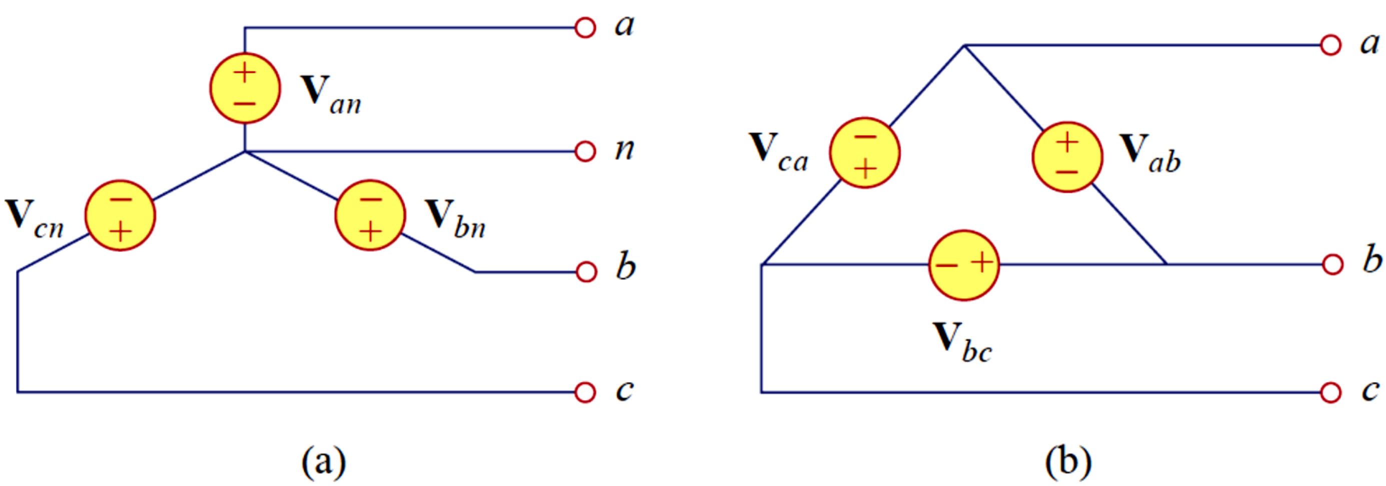


Fig.2.3 Three-phase voltage sources: (a) Y-connected source, (b) –connected source.

3) *Phase sequences*

Consider the wye-connected voltages in Fig. Fig.2.3 (a). The voltages $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$ and $\mathbf{V}_{cn}$ are called ***phase voltages.***

Where

$\mathbf{V}_{an}$: is the voltage between line ***a*** and the neutral line ***n***.

$\mathbf{V}_{bn}$: is the voltage between line ***b*** and the neutral line ***n***.

$\mathbf{V}_{cn}$: is the voltage between line ***c*** and the neutral line ***n***.

If the voltage sources have the same amplitude and frequency **ω** and are out of phase with each other by **120⁰** the voltages are said to be ***balanced.*** Thus , Balanced phase voltages are equal in magnitude and are out of phase with each other by 120^0.
This implies that,

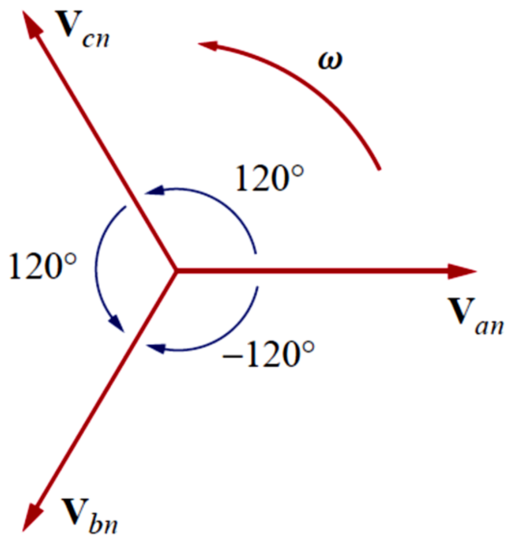
$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0 \qquad \textbf{(3.1)}$$

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}| \qquad \textbf{(3.2)}$$

Since the three-phase voltages are **120⁰** out of phase with each other, there are two possible combinations. One is shown in Fig.3.1(a) and known as the ***abc sequence or positive sequence.*** This sequence is produced when the rotor in Fig.2.1 rotates ***counterclockwise direction.***

$$\begin{aligned} \mathbf{V}_{an} &= V_p\angle 0^\circ \\ \mathbf{V}_{bn} &= V_p\angle -120^\circ \\ \mathbf{V}_{cn} &= V_p\angle -240^\circ = V_p\angle +120^\circ \end{aligned} \qquad \textbf{(3.3)}$$

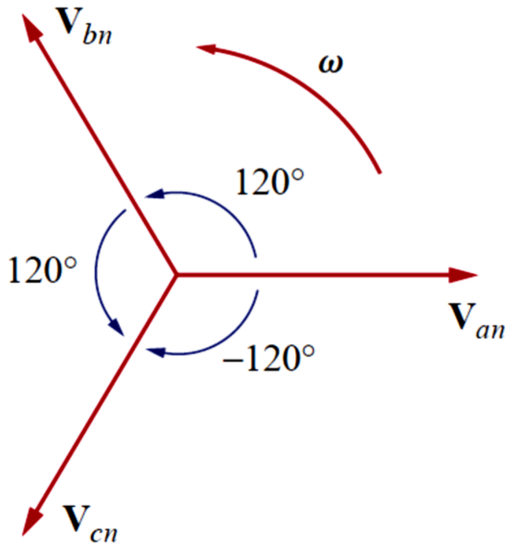
where $\mathbf{V}_p$ is the effective or rms value of the phase voltages.



(a)

The other is shown in Fig.3.1(b) and known as the ***acb sequence or negative sequence.*** This sequence is produced when the rotor in Fig.2.1 rotates ***clockwise direction.***

$$\begin{aligned} \mathbf{V}_{an} &= V_p\angle 0^\circ \\ \mathbf{V}_{cn} &= V_p\angle -120^\circ \\ \mathbf{V}_{bn} &= V_p\angle -240^\circ = V_p\angle +120^\circ \end{aligned} \qquad \textbf{(3.4)}$$



(b)

Fig.3.1Phase sequences: (a) *abc* or positive sequence, (b) *acb* or negative sequence.

$$\begin{aligned} \mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} &= V_p\angle 0^\circ + V_p\angle -120^\circ + V_p\angle +120^\circ \\ &= V_p(1.0 - 0.5 - j0.866 - 0.5 + j0.866) \qquad \textbf{(3.5)} \\ &= 0 \end{aligned}$$

The phase sequence is the time order in which the voltages pass through their respective maximum values.

Like the generator connections, a three-phase load can be either ***wye-connected*** or ***delta-connected***, depending on the end application. Fig.3.2(a) shows a wye-connected load, and Fig.3.2(b) shows a delta-connected load. The neutral line in Fig.3.2(a) may or may not be there, depending on whether the system is four- or three-wire. (And, of course, a neutral connection is topologically impossible for a delta connection.)

A wye- or delta-connected load is said to be ***unbalanced*** if the phase impedances are not equal in magnitude or phase. So A balanced load is one in which the phase impedances are equal in magnitude and in phase.

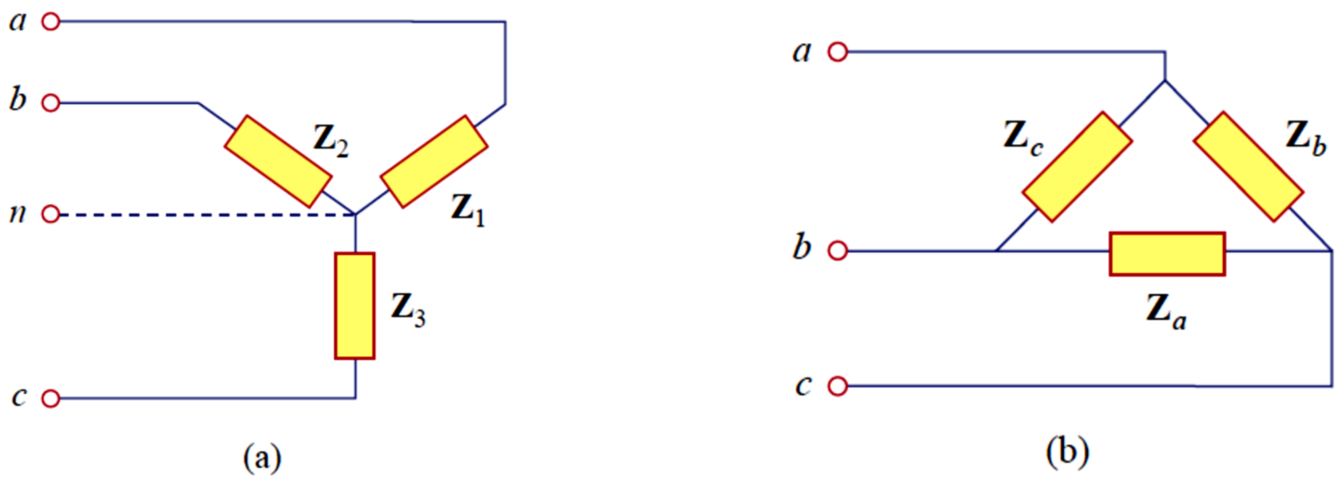


Fig.3.2 Two possible three-phase load configurations:
(a) a Y-connected load, (b) a Δ-connected load.

For ***a balanced wye-connected load,***

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y \qquad (3.6)$$

where $\mathbf{Z}_Y$ is the load impedance per phase.

For ***a balanced delta-connected load,***

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta \qquad (3.7)$$

where Z_Δ is the load impedance per phase.

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \quad \text{or} \quad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta \qquad (3.8)$$

so we know that a wye-connected load can be transformed into a deltaconnected load, or vice versa, using Eq. (3.8). so we know that a wye-connected load can be transformed into a deltaconnected load, or vice versa, using Eq. (3.8).

Since both the three-phase source and the three-phase load can be either wye- or delta-connected, we have four possible connections:

- Y-Y connection (i.e., Y-connected source with a Y-connected load).
- Y-Δ connection.
- Δ-Δ connection.
- Δ-Y connection.

It is appropriate to mention here that a balanced delta-connected load is more common than a balanced wye-connected load. This is due to the ease with which loads may be added or removed from each phase of a delta-connected load. This is very difficult with a wye-connected load because the neutral may not be accessible. On the other hand, delta-connected sources are not common in practice because of the circulating current that will result in the delta-mesh if the three-phase voltages are slightly unbalanced.

Example 1: Determine the phase sequence of the set of voltages
$v_{an} = 200\cos(\omega t + 10^0)\,, v_{bn} = 200\cos(\omega t - 230^0)\,, v_{cn} = 200\cos(\omega t - 110^0)$

Solution:

The voltages can be expressed in phasor form as

$$\mathbf{V}_{an} = 200\underline{/10^\circ}\text{ V},\qquad \mathbf{V}_{bn} = 200\underline{/-230^\circ}\text{ V},\qquad \mathbf{V}_{cn} = 200\underline{/-110^\circ}\text{ V}$$

We notice that $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ by 120° and $\mathbf{V}_{cn}$ in turn leads $\mathbf{V}_{bn}$ by 120°. Hence, we have an *acb* sequence.

H.W.1: Given that $v_{bn} = 110\angle 30^0\ V$, find v_{an} and v_{cn} assuming a possitive (abc) sequence.

Answer:

$$110\underline{/150^\circ}\text{ V},\ 110\underline{/-90^\circ}\text{ V}$$

3.1) Balanced Wye-Wye Connection

We begin with the Y-Y system, because any balanced three-phase system can be reduced to an equivalent Y-Y system. Therefore, analysis of this system should be regarded as the key to solving all balanced three-phase systems.

A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.

Consider the balanced four-wire Y-Y system of Fig.3.3, where a Y-connected load is connected to a Y-connected source. We assume a balanced load so that load impedances are equal.

Where

Z_Y: is the total load impedance per phase

Z_s: is the source impedance (or the internal impedance of the phase winding of the generator)

Z_ℓ: is the line impedance

Z_L: is the load impedance per phase

Z_n: is the impedance of the neutral line

since these impedances are in series. As illustrated in Fig.3.3, thus

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L \qquad (3.9)$$

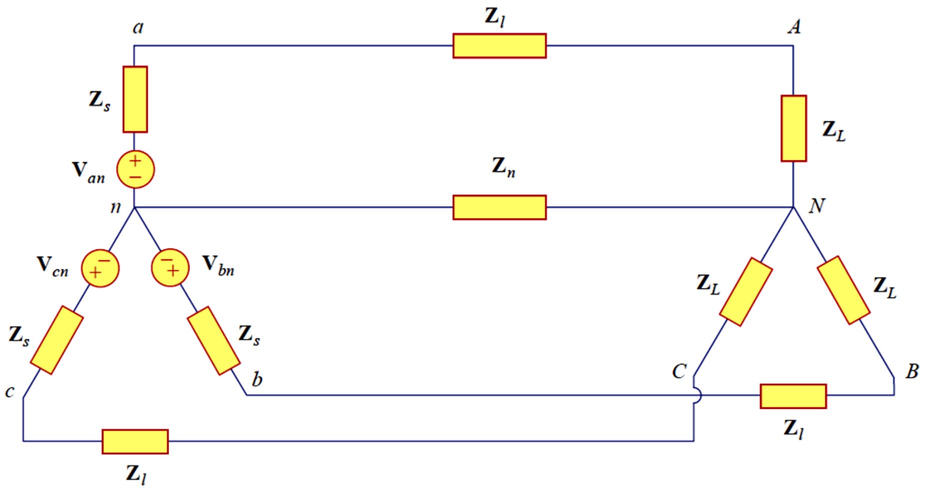


Fig.3.3 A balanced Y-Y system, showing the source, line, and load impedances.

NOTE: $\mathbf{Z}_s$ and $\mathbf{Z}_\ell$ are often very small compared with $\mathbf{Z}_L$, so one can assume that $\mathbf{Z}_Y = \mathbf{Z}_L$ (neglecting $\mathbf{Z}_s$ and $\mathbf{Z}_\ell$) if no source or line impedance is given. So Fig.3.3 can be simplified as in Fig.3.4 .

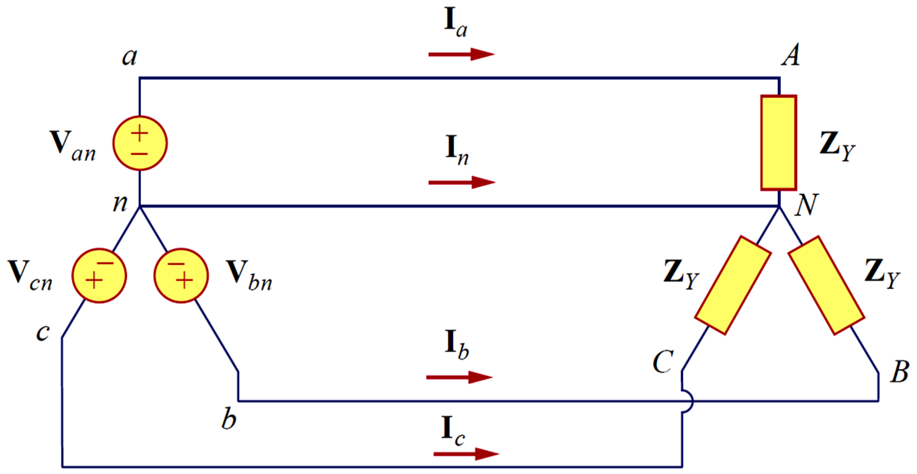


Fig.3.4 A balanced Y-Y connection.

For positive sequence, the *phase* voltages (or line-toneutral voltages) are

$$\left.\begin{aligned} V_{an} &= V_p\angle 0^0 \\ V_{bn} &= V_p\angle -120^0 \\ V_{cn} &= V_p\angle +120^0 \end{aligned}\right\} \qquad (\mathbf{3.10})$$

The *line-to-line* voltages or simply *line* voltages $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$ are related to the phase voltages. For example

$$V_{ab} = V_{an} + V_{nb} = V_{an} - V_{bn} = V_p\angle 0^0 - V_p\angle -120^0$$

$$= V_p\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}\,V_p\angle 30^0 \qquad \mathbf{(3.11a)}$$

Similarly,

$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}\,V_p\angle -90^0 \qquad \mathbf{(3.11b)}$$

$$V_{ca} = V_{cn} - V_{an} = \sqrt{3}\,V_p\angle -210^0 \qquad \mathbf{(3.11c)}$$

Thus, the magnitude of the line voltages V_L is $\sqrt{3}$ times the magnitude of the phase voltages V_p or

$$V_L = \sqrt{3}\,V_p \qquad \mathbf{(3.12)}$$

Where

$$V_p = |V_{an}| = |V_{bn}| = |V_{cn}| \qquad \mathbf{(3.13)}$$

$$V_L = |V_{ab}| = |V_{bc}| = |V_{ca}| \qquad \mathbf{(3.14)}$$

Also the line voltages lead their corresponding phase voltages by $\mathbf{30^0}$ as illustrated in Fig.3.5 (a) which also shows how to determine V_{ab} from the phase voltages, while Fig.3.5 (b) shows the same for the three line voltages. Notice that V_{ab} leads V_{bc} by $\mathbf{120^0}$ and leads V_{ca} by $\mathbf{120^0}$ so that the line voltages sum up to zero as do the phase voltages.

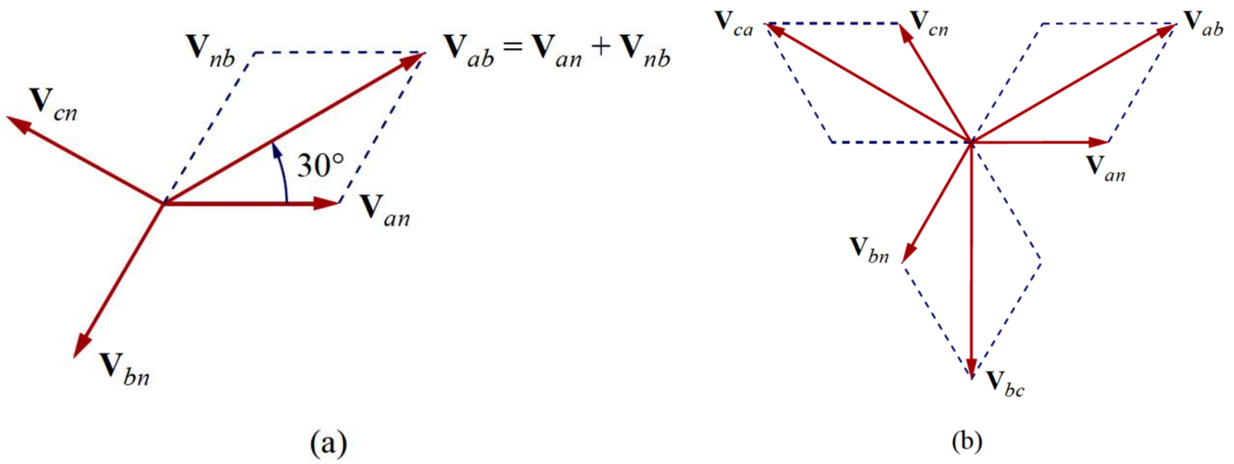


Fig.3.5 Phasor diagrams showing the relationship between line voltages & phase voltages.

Applying KVL to each phase in Fig.3.4, we obtain the line currents as

$$\left.\begin{aligned} I_a &= \frac{V_{an}}{Z_Y} \\ I_b &= \frac{V_{bn}}{Z_Y} = \frac{V_{an}\angle -120^0}{Z_Y} = I_a\angle -120^0 \\ I_c &= \frac{V_{cn}}{Z_Y} = \frac{V_{an}\angle -240^0}{Z_Y} = I_a\angle -240^0 \end{aligned}\right\} \qquad (\mathbf{3.15})$$

$$I_a + I_b + I_c = 0 \qquad (\mathbf{3.16})$$

$$I_n = -(I_a + I_b + I_c) = 0 \qquad (\mathbf{3.17a})$$

$$V_{nN} = Z_n I_n = 0 \qquad (\mathbf{3.17b})$$

that is, the voltage across the neutral wire is zero. The neutral line can thus be removed without affecting the system. In fact, in long distance power transmission, conductors in multiples of three are used with the earth itself acting as the neutral conductor. Power systems designed in this way are well grounded at all critical points to ensure safety.
While the ***line*** current is the current in each line, the ***phase*** current is the current in each phase of the source or load. ***In the Y-Y system, the line current is the same as the phase current.***

An alternative way of analyzing a balanced Y-Y system is to do so on a "per phase" basis. We look at one phase, say phase a, and analyze the single-phase equivalent circuit in Fig.3.6. The single-phase analysis yields the line current $\mathbf{I_a}$ as

$$I_a = \frac{V_{an}}{Z_Y} \qquad (\mathbf{3.18})$$

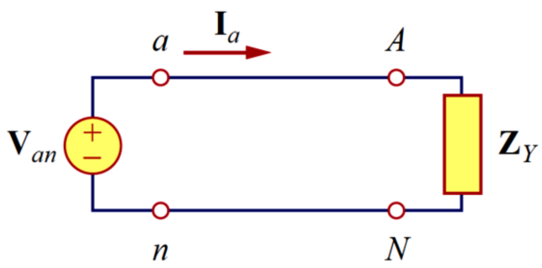


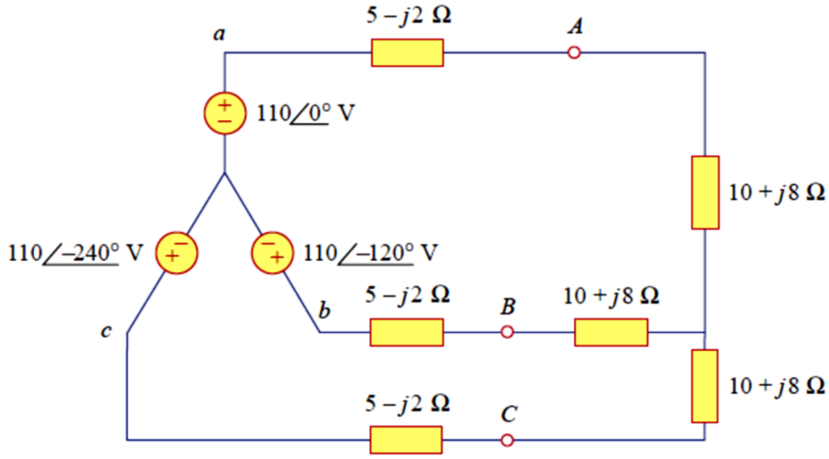
Fig.3.6 A single-phase equivalent circuit

From $\mathbf{I_a}$ we use the phase sequence to obtain other line currents. Thus, as long as the system is balanced, we need only analyze one phase. We may do this even if the neutral line is absent, as in the three-wire system.

Example 2: Calculate the line currents in the three-wire Y-Y system of Fig.

Solution:

The three-phase circuit in Fig. shown is balanced; we may replace it with its single-phase equivalent circuit such as in Fig.3.6. We obtain $\mathbf{I_a}$ from the single-phase analysis as



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155\angle 21.8^\circ$. Hence,

$$\mathbf{I}_a = \frac{110\angle 0^\circ}{16.155\angle 21.8^\circ} = 6.81\angle -21.8^\circ \text{ A}$$

Since the source voltages in Fig. 12.13 are in positive sequence, the line currents are also in positive sequence:

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 6.81\angle -141.8^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle -240^\circ = 6.81\angle -261.8^\circ \text{ A} = 6.81\angle 98.2^\circ \text{ A}$$

H.W.2: A Y-connected balanced three-phase generator with an impedance of **0.4+j0.3Ω** per phase is connected to a Y-connected balanced load with an impedance of **24+j19Ω** per phase. The line joining the generator and the load has an impedance of **0.6+j0.7Ω** per phase. Assuming a positive sequence for the source voltages and that $V_{an} = 120\angle 30^0\ V$ find: (a) the line voltages, (b) the line currents

Answer: (a) $207.85\angle 60^\circ$ V, $207.85\angle -60^\circ$ V, $207.85\angle -180^\circ$ V,
(b) $3.75\angle -8.66^\circ$ A, $3.75\angle -128.66^\circ$ A, $3.75\angle -111.34^\circ$ A.

3.2) Balanced Wye-Delta Connection

A balanced Y- Δ system consists of a balanced Y-connected source feeding a balanced Δ-connected load.

The balanced Y-delta system is shown in Fig.3.7, where the source is Y-connected and the load is Δ-connected. There is, of course, no neutral connection from source to load for this case. Assuming the positive sequence, the phase voltages are again

$$\left.\begin{aligned} V_{an} &= V_p\angle 0^0 \\ V_{bn} &= V_p\angle -120^0 \\ V_{cn} &= V_p\angle +120^0 \end{aligned}\right\} \qquad (\mathbf{3.19})$$

While the line voltages (as shown in section 3.1),

$$\left.\begin{aligned} V_{ab} &= \sqrt{3}\, V_p\angle 30^0 = V_{AB} \\ V_{bc} &= \sqrt{3}\, V_p\angle -90^0 = V_{BC} \\ V_{ca} &= \sqrt{3}\, V_p\angle -150^0 = V_{CA} \end{aligned}\right\} \qquad (\mathbf{3.20})$$

showing that the line voltages are equal to the voltages across the load impedances for this system configuration. From these voltages, we can obtain the phase currents as

$$\left.\begin{aligned} I_{AB} &= \frac{V_{AB}}{Z_\Delta} \\ I_{BC} &= \frac{V_{BC}}{Z_\Delta} \\ I_{CA} &= \frac{V_{CA}}{Z_\Delta} \end{aligned}\right\} \qquad (\mathbf{3.21})$$

These currents have the same magnitude but are out of phase with each other by $\mathbf{120^0}$.

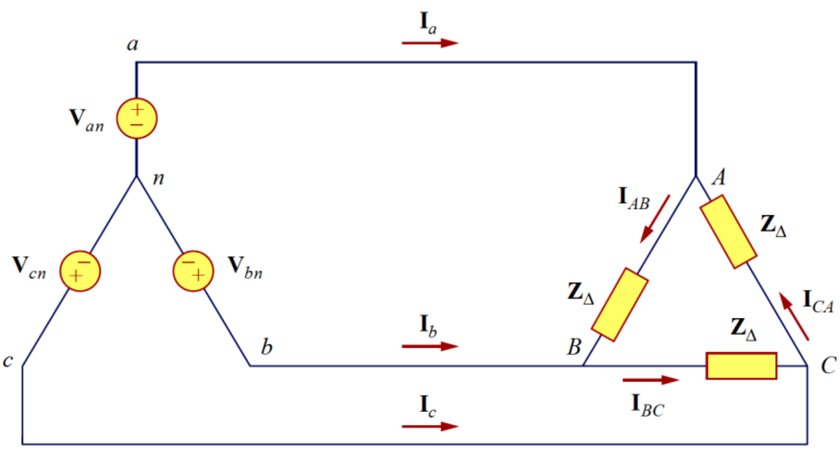


Fig.3.7 Balanced Y-Δ connection

Another way to get these phase currents is to apply KVL. For example, applying KVL around loop ***aABbna*** gives

$$-V_{an} + Z_\Delta I_{AB} + V_{bn} = 0$$

or

$$I_{AB} = \frac{V_{an} - V_{bn}}{Z_\Delta} = \frac{V_{ab}}{Z_\Delta} = \frac{V_{AB}}{Z_\Delta} \qquad (3.22)$$

which is same as Eq. (3.21). This is the more general way of finding the phase currents. The line currents are obtained from the phase currents by applying KCL at nodes *A*, *B*, and *C*. Thus,

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} \qquad \textbf{(3.23)}$$

Since $\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/-240^\circ}$,

$$\begin{aligned}\mathbf{I}_a &= \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\underline{/-240^\circ}) \\ &= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ} \qquad \textbf{(3.24)}\end{aligned}$$

showing that the magnitude I_L of the line current is $\sqrt{3}$ times the magnitude I_p of the phase current, or

$$\boxed{I_L = \sqrt{3}I_p} \qquad \textbf{(3.25)}$$

where

$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c| \qquad \textbf{(3.26)}$$

and

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}| \qquad \textbf{(3.27)}$$

Also, the line currents lag the corresponding phase currents by $\mathbf{30^0}$ assuming the positive sequence. Fig.3.8 is a phasor diagram illustrating the relationship between the phase and line currents.

An alternative way of analyzing the Y-Δ circuit is to transform the Δ-connected load to an equivalent Y-connected load. Using the Y-Δ transformation formula in Eq. (3.8),

$$\boxed{\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}} \qquad \textbf{(3.28)}$$

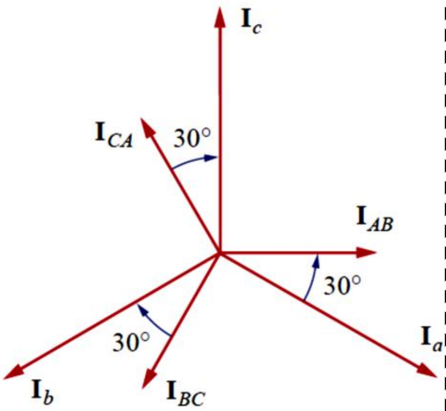


Fig.3.8 Phasor diagram illustrating the relationship between phase and line currents

After this transformation, we now have a Y-Y system as in Fig.3.4. The three-phase Y-Δ system in Fig.3.7 can be replaced by the singlephase equivalent circuit in Fig.3.9. This allows us to calculate only the line currents. The phase currents are obtained using Eq. (3.25) and utilizing the fact that each of the phase currents leads the corresponding line current by $\mathbf{30^0}$.

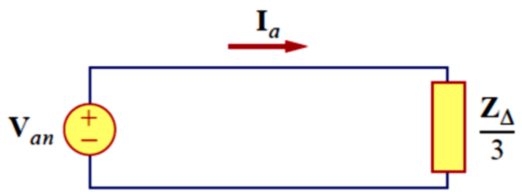


Fig.3.9 A single-phase equivalent circuit of a balanced Y-Δ circuit.

Example 3: A balanced *abc*-sequence Y-connected source with $\mathbf{V_{an}} = \mathbf{100\angle\, 10^0}$ **V** is connected to a Δ-connected balanced load **(8+j4)Ω** per phase. Calculate the phase and line currents.

Solution: This can be resolved in two ways,

METHOD 1 The load impedance is

$$\mathbf{Z}_\Delta = 8 + j4 = 8.944\underline{/26.57^\circ}\ \Omega$$

If the phase voltage $\mathbf{V}_{an} = 100\underline{/10^\circ}$, then the line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\underline{/30^\circ} = 100\sqrt{3}\underline{/10^\circ + 30^\circ} = \mathbf{V}_{AB}$$

or

$$\mathbf{V}_{AB} = 173.2\underline{/40^\circ}\ \text{V}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{173.2\underline{/40^\circ}}{8.944\underline{/26.57^\circ}} = 19.36\underline{/13.43^\circ}\ \text{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\underline{/-120^\circ} = 19.36\underline{/-106.57^\circ}\ \text{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/+120^\circ} = 19.36\underline{/133.43^\circ}\ \text{A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ} = \sqrt{3}(19.36)\underline{/13.43^\circ - 30^\circ}$$

$$= 33.53\underline{/-16.57^\circ}\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 33.53\underline{/-136.57^\circ}\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ} = 33.53\underline{/103.43^\circ}\ \text{A}$$

METHOD 2 Alternatively, using single-phase analysis,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\Delta/3} = \frac{100\underline{/10^\circ}}{2.981\underline{/26.57^\circ}} = 33.54\underline{/-16.57^\circ}\ \text{A}$$

as above. Other line currents are obtained using the *abc* phase sequence.

H.W.3: One line voltage of a balanced Y-connected source is $V_{AB} = \mathbf{240\angle{-20^0}}\ V$. If the source is connected to a Δ-connected load of $\mathbf{20\angle 40^0}\Omega$, find the phase and line currents. Assume the ***abc*** sequence.

Answer:
$12\underline{/-60^\circ}$ A, $12\underline{/-180^\circ}$ A, $12\underline{/60^\circ}$ A, $20.79\underline{/-90^\circ}$ A, $20.79\underline{/-150^\circ}$ A, $20.79\underline{/30^\circ}$ A.

3.3) Balanced Delta-Delta Connection

A balanced Δ-Δ system is one in which both the balanced source and balanced load are Δ-connected.

The source as well as the load may be delta-connected as shown in Fig.3.10.

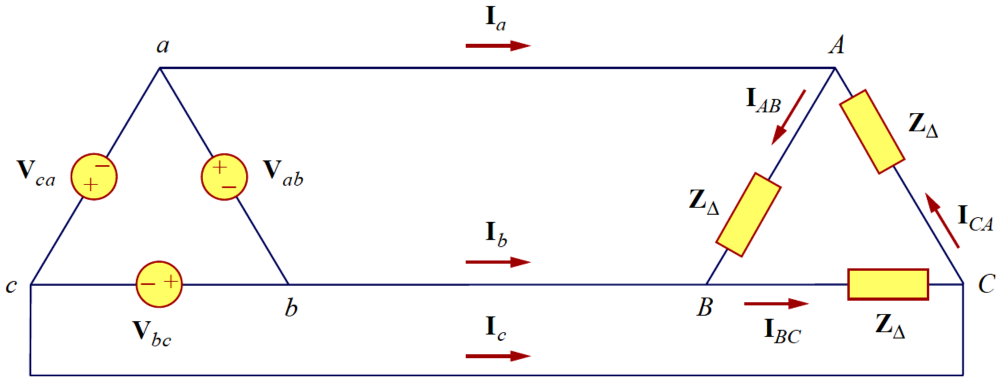


Fig.3.10 A balanced Δ-Δ connection.

Assuming a positive sequence, the phase voltages for a delta-connected source are

$$\left.\begin{aligned} V_{ab} &= V_p\angle 0^0 = V_{AB} \\ V_{bc} &= V_p\angle -120^0 = V_{BC} \\ V_{ca} &= V_p\angle +120^0 = V_{CA} \end{aligned}\right\} \qquad (3.29)$$

The line voltages are the same as the phase voltages. From Fig.3.10, assuming there is no line impedances, the phase voltages of the deltaconnected source are equal to the voltages across the impedances.

The phase currents are,

$$\left.\begin{aligned} I_{AB} &= \frac{V_{AB}}{Z_\Delta} = \frac{V_{ab}}{Z_\Delta} \\ I_{BC} &= \frac{V_{BC}}{Z_\Delta} = \frac{V_{bc}}{Z_\Delta} \\ I_{CA} &= \frac{V_{CA}}{Z_\Delta} = \frac{V_{ca}}{Z_\Delta} \end{aligned}\right\} \qquad (3.30)$$

Since the load is delta-connected just as in the previous section, some of the formulas derived there apply here. The line currents are obtained from the phase currents by applying KCL at nodes *A*, *B*, and *C*, as we did in the previous section:

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} \qquad (3.31)$$

Also, as shown in the last section, each line current lags the corresponding phase current by 30^0; the magnitude I_L of the line current is $\sqrt{3}$ times the magnitude I_n of the phase current,

$$I_L = \sqrt{3} I_p \qquad (3.32)$$

An alternative way of analyzing the Δ-Δ circuit is to convert both the source and the load to their Y equivalents. We already know that $\boldsymbol{Z_Y} = \frac{\boldsymbol{Z_\Delta}}{\boldsymbol{3}}$. To convert a Δ-connected source to a Y-connected source, see the next section.

Example 4: A balanced Δ-connected load having an impedance **(20 – j15) Ω** is connected to a Δ-connected, positive-sequence generator having $\mathbf{V_{ab}} = \mathbf{330\angle 0^0\ V}$ .Calculate the phase currents of the load and the line currents.

Solution:

The load impedance per phase is

$$\mathbf{Z}_\Delta = 20 - j15 = 25\angle{-36.87^\circ}\ \Omega$$

Since $\mathbf{V}_{AB} = \mathbf{V}_{ab}$, the phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{330\angle{0^\circ}}{25\angle{-36.87}} = 13.2\angle{36.87^\circ}\ \text{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle{-120^\circ} = 13.2\angle{-83.13^\circ}\ \text{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle{+120^\circ} = 13.2\angle{156.87^\circ}\ \text{A}$$

For a delta load, the line current always lags the corresponding phase current by 30° and has a magnitude $\sqrt{3}$ times that of the phase current. Hence, the line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle{-30^\circ} = (13.2\angle{36.87^\circ})(\sqrt{3}\angle{-30^\circ})$$

$$= 22.86\angle{6.87^\circ}\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle{-120^\circ} = 22.86\angle{-113.13^\circ}\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle{+120^\circ} = 22.86\angle{126.87^\circ}\ \text{A}$$

H.W.4: A positive-sequence, balanced Δ-connected source supplies a balanced Δ-connected load. If the impedance per phase of the load is **(18 + j12) Ω** and $\boldsymbol{I_a} = \mathbf{19.202\angle 35^0}\ A$. Find $\boldsymbol{I_{AB}}$ and $\boldsymbol{V_{AB}}$

Answer:

$11.094\angle{65^\circ}$ A, $240\angle{98.69^\circ}$ V.

3.4) Balanced Delta-Wye Connection

A balanced Δ-Y system consists of a balanced Δ-connected source feeding a balanced Y-connected load.

Consider the Δ-Y circuit in Fig.3.11. Again, assuming the ***abc* sequence**, the phase voltages of a delta-connected source are

$$\left.\begin{aligned} V_{ab} &= V_p\angle 0^0 \\ V_{bc} &= V_p\angle -120^0 \\ V_{ca} &= V_p\angle +120^0 \end{aligned}\right\} \qquad (3.33)$$

These are also the line voltages as well as the phase voltages.

We can obtain the line currents in many ways. One way is to apply KVL to loop *aANBba* in Fig.3.11, writing

$$-\mathbf{V}_{ab} + \mathbf{Z}_Y\mathbf{I}_a - \mathbf{Z}_Y\mathbf{I}_b = 0$$

or

$$\mathbf{Z}_Y(\mathbf{I}_a - \mathbf{I}_b) = \mathbf{V}_{ab} = V_p\underline{/0^\circ}$$

Thus,

$$\mathbf{I}_a - \mathbf{I}_b = \frac{V_p\underline{/0^\circ}}{\mathbf{Z}_Y} \qquad (3.34)$$

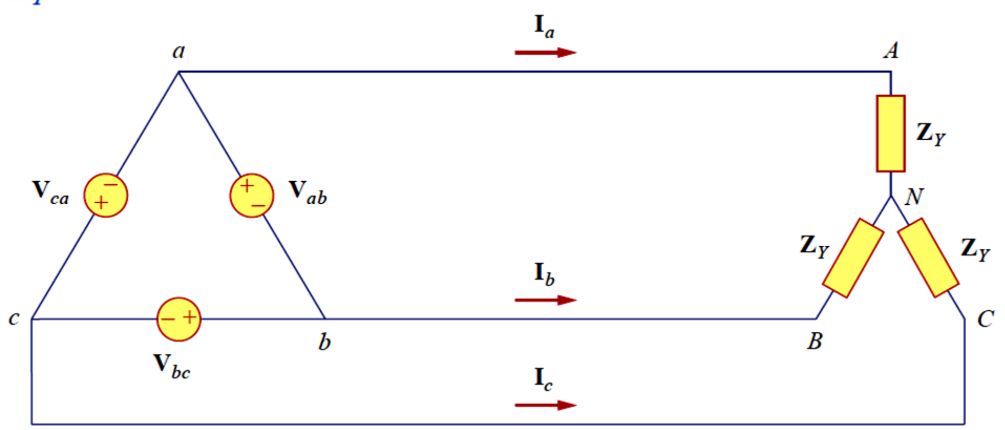


Fig.3.11 A balanced Δ-Y connection

But $\mathbf{I_b}$ lags $\mathbf{I_a}$ by $\mathbf{120^0}$, since we assumed the *abc* sequence; that is $I_b = I_a\angle -120^0$,Hence,

$$\begin{aligned} \mathbf{I}_a - \mathbf{I}_b &= \mathbf{I}_a(1 - 1\underline{/-120^\circ}) \\ &= \mathbf{I}_a\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \mathbf{I}_a\sqrt{3}\underline{/30^\circ} \end{aligned} \qquad (3.35)$$

Substituting eq.(3.35) into Eq. (3.34) gives,

$$\mathbf{I}_a = \frac{V_p/\sqrt{3}\underline{/-30^\circ}}{\mathbf{Z}_Y} \qquad (3.36)$$

From this, we obtain the other line currents $\mathbf{I}_b$ and $\mathbf{I}_c$ using the positive phase sequence, i.e., $\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ}$, $\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ}$. The phase currents are equal to the line currents.

Another way to obtain the line currents is to replace the deltaconnected source with its equivalent wye-connected source, as shown in Fig.3.12. In Section 3.1, we found that the line-to-line voltages of a wye-connected source lead their corresponding phase voltages by $\mathbf{30^0}$. Therefore, we obtain each phase voltage of the equivalent wyeconnected source by dividing the corresponding line voltage of the delta-connected source by $\sqrt{3}$ and shifting its phase by $\mathbf{-30^0}$. Thus, the equivalent wye-connected source has the phase voltages

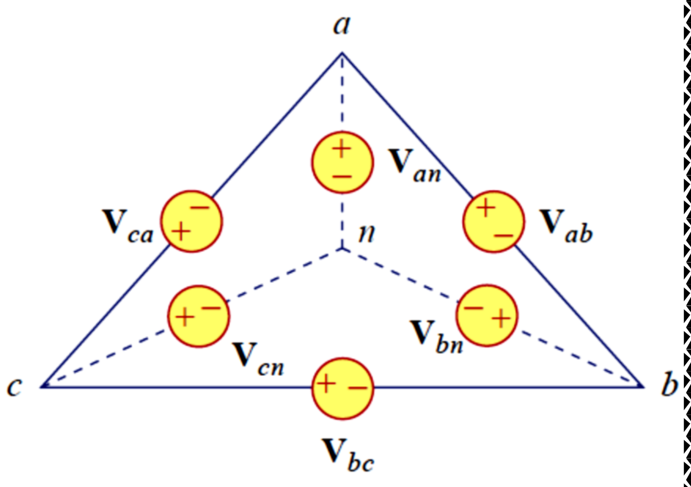


Fig.3.12 Transforming a Δ-connected source to an equivalent Y-connected source.

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\underline{/-30^\circ}$$

$$\mathbf{V}_{bn} = \frac{V_p}{\sqrt{3}}\underline{/-150^\circ}, \qquad \mathbf{V}_{cn} = \frac{V_p}{\sqrt{3}}\underline{/+90^\circ} \qquad \textbf{(3.37)}$$

If the delta-connected source has source impedance $\mathbf{Z}_s$ per phase, the equivalent wye-connected source will have a source impedance of $\mathbf{(Z_s/3)}$ per phase.

Once the source is transformed to wye, the circuit becomes a wyewye system. Therefore, we can use the equivalent single-phase circuit shown in Fig.3.13, from which the line current for phase *a* is

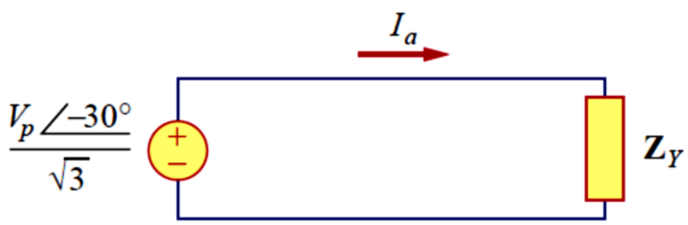


Fig.3.12 The single-phase equivalent circuit

$$\mathbf{I}_a = \frac{V_p/\sqrt{3}\underline{/-30^\circ}}{\mathbf{Z}_Y} \qquad \textbf{(3.38)}$$

which is the same as Eq. (3.37).

Alternatively, we may transform the wye-connected load to an equivalent delta-connected load. This results in a Δ-Δ system, which can be analyzed as in Section 3.3. Note that

$$\mathbf{V}_{AN} = \mathbf{I}_a\mathbf{Z}_Y = \frac{V_p}{\sqrt{3}}\underline{/-30^\circ}$$

$$\mathbf{V}_{BN} = \mathbf{V}_{AN}\underline{/-120^\circ}, \qquad \mathbf{V}_{CN} = \mathbf{V}_{AN}\underline{/+120^\circ} \qquad \textbf{(3.39)}$$

As stated earlier, the delta-connected load is more desirable than the wye-connected load. It is easier to alter the loads in any one phase of the delta-connected loads, as the individual loads are connected directly across the lines. However, the delta-connected

source is hardly used in practice, because any slight imbalance in the phase voltages will result in unwanted circulating currents.
Table 3.1 presents a summary of the formulas for phase currents and voltages and line currents and voltages for the four connections. Students are advised not to memorize the formulas but to understand how they are derived. The formulas can always be

TABLE 3.1

Summary of phase and line voltages/currents for balanced three-phase systems.[1]

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0°$	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30°$
	$\mathbf{V}_{bn} = V_p\angle -120°$	$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120°$
	$\mathbf{V}_{cn} = V_p\angle +120°$	$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120°$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0°$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30°$
	$\mathbf{V}_{bn} = V_p\angle -120°$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120°$
	$\mathbf{V}_{cn} = V_p\angle +120°$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120°$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0°$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120°$	
	$\mathbf{V}_{ca} = V_p\angle +120°$	
	$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$
	$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
	$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0°$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120°$	
	$\mathbf{V}_{ca} = V_p\angle +120°$	
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\angle -30°}{\sqrt{3}\mathbf{Z}_Y}$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120°$

[1] Positive or abc sequence is assumed.

Example 5: A balanced Y-connected load with a phase impedance of **(40 + j25) Ω** is supplied by a balanced, positive sequence Δ-connected source with a line voltage of **210 V**. Calculate the phase currents. Use $\mathbf{V}_{ab}$ as reference.

Solution:

The load impedance is

$$\mathbf{Z}_Y = 40 + j25 = 47.17\underline{/32^\circ}\ \Omega$$

and the source voltage is

$$\mathbf{V}_{ab} = 210\underline{/0^\circ}\ \text{V}$$

When the Δ-connected source is transformed to a Y-connected source,

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\underline{/-30^\circ} = 121.2\underline{/-30^\circ}\ \text{V}$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2\underline{/-30^\circ}}{47.12\underline{/32^\circ}} = 2.57\underline{/-62^\circ}\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 2.57\underline{/-178^\circ}\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/120^\circ} = 2.57\underline{/58^\circ}\ \text{A}$$

which are the same as the phase currents.

H.W.5: In balanced Δ-Y circuit, $\boldsymbol{V_{ab}} = \mathbf{240\angle 15^0}\ V$ and $\mathbf{Z}_Y$ = **(12 + j15) Ω.** Calculate the line currents.

Answer:

$7.21\underline{/-66.34^\circ}$ A, $7.21\underline{/-173.66^\circ}$ A, $7.21\underline{/53.66^\circ}$ A.

4) Power in a Balanced System

Let us now consider the power in a balanced three-phase system. We begin by examining the instantaneous power absorbed by the load. This requires that the analysis be done in the time domain. For a Y-connected load, the phase voltages are

$$v_{AN} = \sqrt{2}V_p \cos\omega t, \qquad v_{BN} = \sqrt{2}V_p \cos(\omega t - 120^\circ)$$
$$v_{CN} = \sqrt{2}V_p \cos(\omega t + 120^\circ) \qquad \textbf{(4.1)}$$

where the factor $\sqrt{2}$ is necessary because $\boldsymbol{V_p}$ has been defined as the rms value of the phase voltage. If $Z_Y = Z\angle\theta$,the phase currents lag behind their corresponding phase voltages by θ. Thus,

$$i_a = \sqrt{2}I_p \cos(\omega t - \theta), \qquad i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120^\circ) \qquad \textbf{(4.2)}$$
$$i_c = \sqrt{2}I_p \cos(\omega t - \theta + 120^\circ)$$

where $\boldsymbol{I_p}$ is the rms value of the phase current. The total instantaneous power in the load is the sum of the instantaneous powers in the three phases; that is,

$$\begin{aligned} p &= p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c \\ &= 2V_pI_p[\cos\omega t\cos(\omega t - \theta) \\ &\qquad + \cos(\omega t - 120^\circ)\cos(\omega t - \theta - 120^\circ) \\ &\qquad + \cos(\omega t + 120^\circ)\cos(\omega t - \theta + 120^\circ)] \end{aligned} \qquad \textbf{(4.3)}$$

Applying the trigonometric identity

$$\cos A\cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)] \qquad \textbf{(4.4)}$$

gives

$$\begin{aligned} p &= V_pI_p[3\cos\theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240^\circ) \\ &\qquad + \cos(2\omega t - \theta + 240^\circ)] \\ &= V_pI_p[3\cos\theta + \cos\alpha + \cos\alpha\cos 240^\circ + \sin\alpha\sin 240^\circ \\ &\qquad + \cos\alpha\cos 240^\circ - \sin\alpha\sin 240^\circ] \qquad \textbf{(4.5)} \\ &\qquad \text{where } \alpha = 2\omega t - \theta \\ &= V_pI_p\left[3\cos\theta + \cos\alpha + 2\left(-\frac{1}{2}\right)\cos\alpha\right] = 3V_pI_p\cos\theta \end{aligned}$$

Thus the total instantaneous power in a balanced three-phase system is constant—it does not change with time as the instantaneous power of each phase does. This result is true whether the load is Y- or Δ-connected.
This is one important reason for using a three-phase system to generate and distribute power. We will look into another reason a little later. Since the total instantaneous power

is independent of time, the average power per phase P_p for either the Δ-connected load or the Y-connected load is $p/3$ or

$$P_p = V_p I_p \cos\theta \qquad \textbf{(4.6)}$$

and the reactive power per phase is

$$Q_p = V_p I_p \sin\theta \qquad \textbf{(4.7)}$$

The apparent power per phase is

$$S_p = V_p I_p \qquad \textbf{(4.8)}$$

The complex power per phase is

$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^* \qquad \textbf{(4.9)}$$

where $\mathbf{V_p}$ and $\mathbf{I_p}$ are the phase voltage and phase current with magnitudes V_p and I_p respectively. The total average power is the sum of the average powers in the phases:

$$P = P_a + P_b + P_c = 3P_p = 3V_p I_p \cos\theta = \sqrt{3} V_L I_L \cos\theta \qquad \textbf{(4.10)}$$

For a Y-connected load, $I_L = I_p$ but $V_L = \sqrt{3}\, V_p$ whereas for a Δ-connected load, $I_L = \sqrt{3}\, I_p$ but $V_L = V_p$. Thus, Eq. (4.10) applies for both Y-connected and Δ-connected loads. Similarly, the total reactive power is

$$Q = 3V_p I_p \sin\theta = 3Q_p = \sqrt{3} V_L I_L \sin\theta \qquad \textbf{(4.11)}$$

and the total complex power is

$$\boxed{\mathbf{S} = 3\mathbf{S}_p = 3\mathbf{V}_p \mathbf{I}_p^* = 3I_p^2 \mathbf{Z}_p = \frac{3V_p^2}{\mathbf{Z}_p^*}} \qquad \textbf{(4.12)}$$

Where $Z_p = Z_p\angle\theta$ is the load impedance per phase. (Z_p could be Z_Y or Z_Δ). Alternatively, we may write Eq.(4.12) as

$$\boxed{\mathbf{S} = P + jQ = \sqrt{3} V_L I_L \underline{/\theta}} \qquad \textbf{(4.13)}$$

Remember that $\mathbf{V_p}$, $\mathbf{I_p}$, $\mathbf{V_L}$ and $\mathbf{I_L}$ are all rms values and that θ is the angle of the load impedance or the angle between the phase voltage and the phase current.

A second major advantage of three-phase systems for power distribution is that the three-phase system uses a lesser amount of wire than the single-phase system for the same line voltage $\mathbf{V_L}$ and the same absorbed power $\boldsymbol{P_L}$. We will compare these cases and assume in both that the wires are of the same material (e.g., copper with resistivity ρ), of the same length ℓ, and that the loads are resistive (i.e., unity power factor). For the two-wire single-phase system in Fig.4.1(a), $I_L = \frac{P_L}{V_L}$ so the power loss in the two wires is

$$P_{\text{loss}} = 2I_L^2 R = 2R\frac{P_L^2}{V_L^2} \qquad \textbf{(4.14)}$$

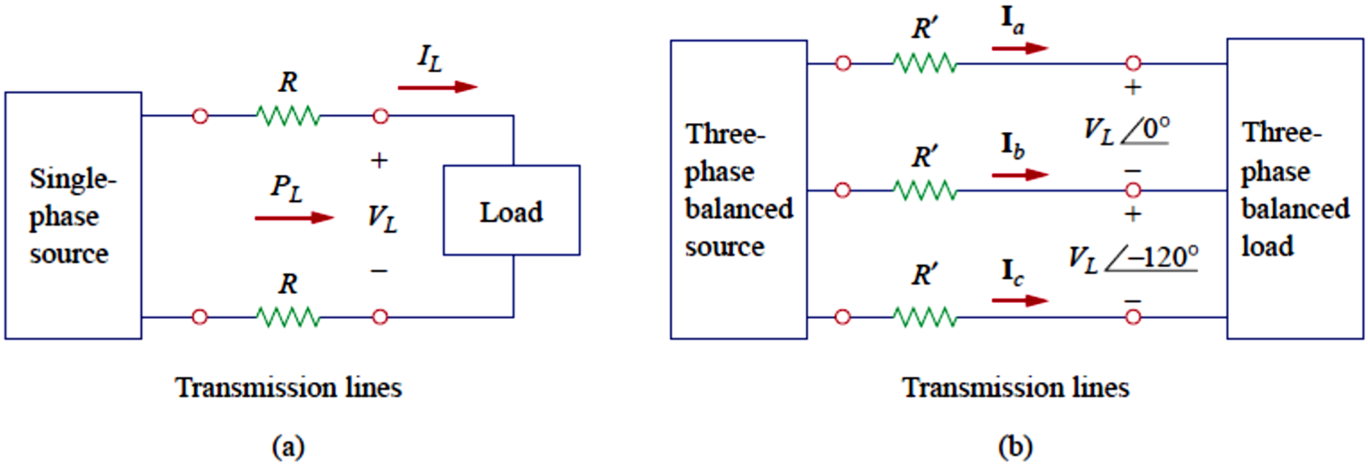


Fig.4.1 Comparing the power loss in (a) a single-phase system, and (b) a three-phase system.

For the three-wire three-phase system in Fig.4.1(b), $I_L' = |I_a| = |I_b| = |I_c| = \frac{P_L}{\sqrt{3}\,V_L}$ from Eq.(4.10). The power loss in the three wires is

$$P'_{\text{loss}} = 3(I_L')^2 R' = 3R'\frac{P_L^2}{3V_L^2} = R'\frac{P_L^2}{V_L^2} \qquad \textbf{(4.15)}$$

Equations (4.14) and (4.15) show that for the same total power delivered P_L and same line voltage V_L,

$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2R}{R'} \qquad \textbf{(4.16)}$$

But $R = \frac{\rho\ell}{\pi r^2}$ and $R' = \frac{\rho\ell}{\pi r'^2}$. Where r and r' are the radii of the wires. Thus,

$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2r'^2}{r^2} \qquad \textbf{(4.17)}$$

If the same power loss is tolerated in both systems, then $r^2 = 2\ r'^2$. The ratio of material required is determined by the number of wires and their volumes, so

$$\frac{\text{Material for single-phase}}{\text{Material for three-phase}} = \frac{2(\pi r^2 \ell)}{3(\pi r'^2 \ell)} = \frac{2r^2}{3r'^2} = \frac{2}{3}(2) = 1.333 \qquad \textbf{(4.18)}$$

since $r^2 = 2\ r'^2$. Equation (4.18) shows that the single-phase system uses 33 percent more material than the three-phase system or that the threephase system uses only 75 percent of the material used in the equivalent single-phase system. In other words, considerably less mate-rial is needed to deliver the same power with a three-phase system than is .required for a single-phase system

Example 6: For **Example2**. Determine the total average power, reactive power, and complex power at the source and at the load.

Solution:

It is sufficient to consider one phase, as the system is balanced. For phase a,

$$\mathbf{V}_p = 110\underline{/0^\circ}\text{ V} \qquad \text{and} \qquad \mathbf{I}_p = 6.81\underline{/-21.8^\circ}\text{ A}$$

Thus, at the source, the complex power absorbed is

$$\begin{aligned}\mathbf{S}_s = -3\mathbf{V}_p\mathbf{I}_p^* &= -3(110\underline{/0^\circ})(6.81\underline{/21.8^\circ}) \\ &= -2247\underline{/21.8^\circ} = -(2087 + j834.6)\text{ VA}\end{aligned}$$

The real or average power absorbed is -2087 W and the reactive power is -834.6 VAR.

At the load, the complex power absorbed is

$$\mathbf{S}_L = 3|\mathbf{I}_p|^2\mathbf{Z}_p$$

where $\mathbf{Z}_p = 10 + j8 = 12.81\underline{/38.66^\circ}$ and $\mathbf{I}_p = \mathbf{I}_a = 6.81\underline{/-21.8^\circ}$. Hence,

$$\begin{aligned}\mathbf{S}_L &= 3(6.81)^2 12.81\underline{/38.66^\circ} = 1782\underline{/38.66} \\ &= (1392 + j1113)\text{ VA}\end{aligned}$$

The real power absorbed is 1391.7 W and the reactive power absorbed is 1113.3 VAR. The difference between the two complex powers is absorbed by the line impedance $(5 - j2)\ \Omega$. To show that this is the case, we find the complex power absorbed by the line as

$$\mathbf{S}_\ell = 3|\mathbf{I}_p|^2\mathbf{Z}_\ell = 3(6.81)^2(5 - j2) = 695.6 - j278.3 \text{ VA}$$

which is the difference between $\mathbf{S}_s$ and $\mathbf{S}_L$; that is, $\mathbf{S}_s + \mathbf{S}_\ell + \mathbf{S}_L = 0$, as expected.

H.W.6: For the Y-Y circuit in **H.W.2**, calculate the complex power at the source and at the load.

Answer:

$-(1054 + j843.3)$ VA, $(1012 + j801.6)$ VA.

Example 7: A three-phase motor can be regarded as a balanced Y-load. A threephase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor.

Solution:

The apparent power is

$$S = \sqrt{3}V_L I_L = \sqrt{3}(220)(18.2) = 6935.13 \text{ VA}$$

Since the real power is

$$P = S\cos\theta = 5600 \text{ W}$$

the power factor is

$$\text{pf} = \cos\theta = \frac{P}{S} = \frac{5600}{6935.13} = 0.8075$$

H.W.7: Calculate the line current required for a 30-kW three-phase motor having a power factor of 0.85 lagging if it is connected to a balanced source with a line voltage of 440 V.

Answer:

46.31 A.

Example 8: Two balanced loads are connected to a 240-kV rms 60-Hz line, as shown in Fig.(a). Load 1 draws 30 kW at a power factor of 0.6 lagging, while load 2 draws 45 kVAR at a power factor of 0.8 lagging. Assuming the *abc* sequence, determine: (a) the complex, real, and reactive powers absorbed by the combined load, (b) the line currents, and (c) the kVAR rating of the three capacitors Δ-connected in parallel with the load that will raise the power factor to 0.9 lagging and the capacitance of each capacitor.

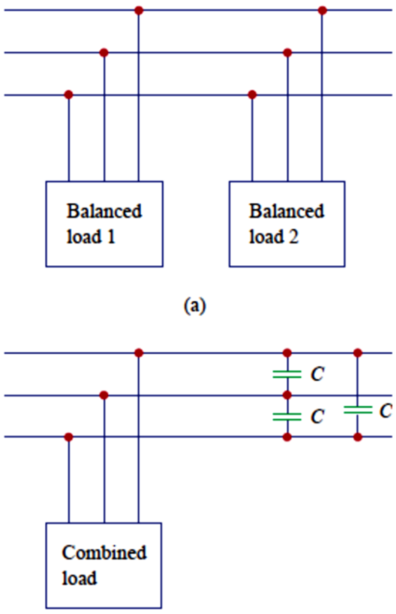
Solution:

(a) For load 1, given that $P_1 = 30$ kW and $\cos\theta_1 = 0.6$, then $\sin\theta_1 = 0.8$. Hence,

$$S_1 = \frac{P_1}{\cos\theta_1} = \frac{30\text{ kW}}{0.6} = 50\text{ kVA}$$

and $Q_1 = S_1 \sin\theta_1 = 50(0.8) = 40$ kVAR. Thus, the complex power due to load 1 is

$$\mathbf{S}_1 = P_1 + jQ_1 = 30 + j40\text{ kVA} \qquad \textbf{(1)}$$



(a)

(b)

For load 2, if $Q_2 = 45$ kVAR and $\cos\theta_2 = 0.8$, then $\sin\theta_2 = 0.6$. We find

$$S_2 = \frac{Q_2}{\sin\theta_2} = \frac{45\text{ kVA}}{0.6} = 75\text{ kVA}$$

and $P_2 = S_2\cos\theta_2 = 75(0.8) = 60$ kW. Therefore the complex power due to load 2 is

$$\mathbf{S}_2 = P_2 + jQ_2 = 60 + j45\text{ kVA} \qquad \textbf{(2)}$$

From Eqs. (1) and (2), the total complex power absorbed by the load is

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = 90 + j85\text{ kVA} = 123.8\underline{/43.36^\circ}\text{ kVA} \qquad \textbf{(3)}$$

which has a power factor of $\cos 43.36^\circ = 0.727$ lagging. The real power is then 90 kW, while the reactive power is 85 kVAR.

(b) Since $S = \sqrt{3}V_L I_L$, the line current is

$$I_L = \frac{S}{\sqrt{3}V_L} \qquad \textbf{(4)}$$

We apply this to each load, keeping in mind that for both loads, $V_L = 240$ kV. For load 1,

$$I_{L1} = \frac{50{,}000}{\sqrt{3}\,240{,}000} = 120.28\text{ mA}$$

Since the power factor is lagging, the line current lags the line voltage by $\theta_1 = \cos^{-1}0.6 = 53.13^\circ$. Thus,

$$\mathbf{I}_{a1} = 120.28\underline{/-53.13^\circ}$$

For load 2,

$$I_{L2} = \frac{75{,}000}{\sqrt{3}\,240{,}000} = 180.42 \text{ mA}$$

and the line current lags the line voltage by $\theta_2 = \cos^{-1} 0.8 = 36.87°$. Hence,

$$\mathbf{I}_{a2} = 180.42\underline{/-36.87°}$$

The total line current is

$$\begin{aligned}\mathbf{I}_a = \mathbf{I}_{a1} + \mathbf{I}_{a2} &= 120.28\underline{/-53.13°} + 180.42\underline{/-36.87°} \\ &= (72.168 - j96.224) + (144.336 - j108.252) \\ &= 216.5 - j204.472 = 297.8\underline{/-43.36°} \text{ mA}\end{aligned}$$

Alternatively, we couldobtain the current from the total complex power using Eq. (4) as

$$I_L = \frac{123{,}800}{\sqrt{3}\,240{,}000} = 297.82 \text{ mA}$$

and

$$\mathbf{I}_a = 297.82\underline{/-43.36°} \text{ mA}$$

which is the same as before. The other line currents, $\mathbf{I}_{b2}$ and $\mathbf{I}_{ca}$, can be obtained according to the *abc* sequence (i.e., $\mathbf{I}_b = 297.82\underline{/-163.36°}$ mA and $\mathbf{I}_c = 297.82\underline{/76.64°}$ mA).

(c) We can find the reactive power needed to bring the power factor to 0.9 lagging using Eq. (11.59),

$$Q_C = P(\tan\theta_{\text{old}} - \tan\theta_{\text{new}})$$

where $P = 90$ kW, $\theta_{\text{old}} = 43.36°$, and $\theta_{\text{new}} = \cos^{-1} 0.9 = 25.84°$. Hence,

$$Q_C = 90{,}000(\tan 43.36° - \tan 25.84°) = 41.4 \text{ kVAR}$$

This reactive power is for the three capacitors. For each capacitor, the rating $Q'_C = 13.8$ kVAR. From Eq. (11.60), the required capacitance is

$$C = \frac{Q'_C}{\omega V_{\text{rms}}^2}$$

Since the capacitors are Δ-connected as shown in Fig.(b), V_{rms} in the ,above formula is the line-to-line or line voltage, which is 240 kV. Thus

$$C = \frac{13{,}800}{(2\pi 60)(240{,}000)^2} = 635.5 \text{ pF}$$

H.W.8: Assume that the two balanced loads in Fig.(a) for **Example 8** are supplied by an 840-V rms 60-Hz line. Load 1 is Y-connected with **(30 + j40) Ω** per phase, while load 2 is a balanced three-phase motor drawing 48 kW at a power factor of 0.8 lagging. Assuming the *abc* sequence, calculate: (a) the complex power absorbed by the combined load, (b) the Kvar rating of each of the three capacitors Δ-connected in parallel with the load to raise the power factor to unity, and (c) the current drawn from the supply at unity power factor condition.

Answer:

(a) $56.47 + j47.29$ kVA, (b) 15.7 kVAR, (c) 38.813 A.

5) Unbalanced Three-Phase Systems

An unbalanced system is caused by two possible situations:
(1) the source voltages are not equal in magnitude and/or differ in phase by angles that are unequal.
(2) load impedances are unequal.

An unbalanced system is due to unbalanced voltage sources or an unbalanced load.

To simplify analysis, we will assume balanced source voltages, but an unbalanced load.
Unbalanced three-phase systems are solved by direct application of mesh and nodal analysis. Fig.5.1 shows an example of an unbalanced three-phase system that consists of balanced source voltages (not shown in the figure) and an unbalanced Y-connected load (shown in the figure). Since the load is unbalanced, $\mathbf{Z_A}$, $\mathbf{Z_B}$ and $\mathbf{Z_C}$ are not equal. The line currents are determined by Ohm's law as

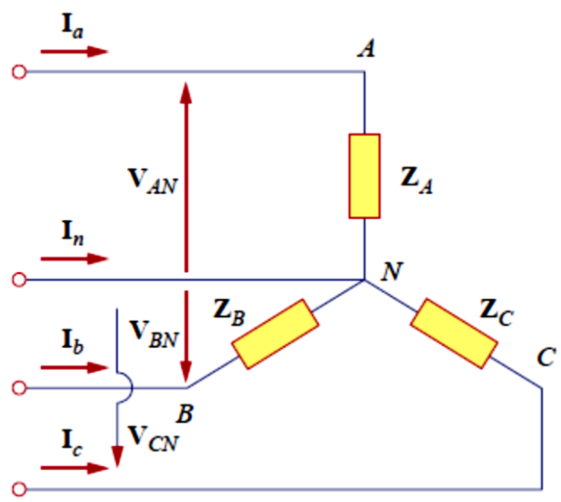


Fig.5.1 Unbalanced three-phase Y -connected load.

$$\boxed{\mathbf{I}_a = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_A}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_B}, \qquad \mathbf{I}_c = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_C}} \qquad (5.1)$$

This set of unbalanced line currents produces current in the neutral line, which is not zero as in a balanced system. Applying KCL at node *N* gives the neutral line current as

$$\boxed{\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c)} \qquad (5.2)$$

In a three-wire system where the neutral line is absent, we can still find the line currents $\mathbf{I_a}$, $\mathbf{I_b}$ and $\mathbf{I_c}$ using mesh analysis. At node *N*, KCL must be satisfied so that $\mathbf{I_a} + \mathbf{I_b} + \mathbf{I_c} = 0$ in this case. The same could be done for an unbalanced Δ -Y, Y-Δ or Δ-Δ three-wire system. As mentioned earlier, in long distance power transmission, conductors in multiples of three (multiple three-wire systems) are used, with the earth itself acting as the neutral conductor.

To calculate power in an unbalanced three-phase system requires that we find the power in each phase using Eqs. (4.6) to (4.9). The total power is not simply three times the power in one phase but the sum of the powers in the three phases.

Example 9: The unbalanced Y-load of Fig.5.1 has balanced voltages of 100 V and the *acb* sequence. Calculate the line currents and the neutral current. Take , $\mathbf{Z_A} = \mathbf{15\ \Omega}$, $\mathbf{Z_B} = \mathbf{10 + j5\ \Omega}$, $\mathbf{Z_C} = \mathbf{6 - j8\ \Omega}$

Solution:

Using Eq.(5.1), the line currents are

$$\mathbf{I}_a = \frac{100\angle 0^\circ}{15} = 6.67\angle 0^\circ \text{ A}$$

$$\mathbf{I}_b = \frac{100\angle 120^\circ}{10 + j5} = \frac{100\angle 120^\circ}{11.18\angle 26.56^\circ} = 8.94\angle 93.44^\circ \text{ A}$$

$$\mathbf{I}_c = \frac{100\angle -120^\circ}{6 - j8} = \frac{100\angle -120^\circ}{10\angle -53.13^\circ} = 10\angle -66.87^\circ \text{ A}$$

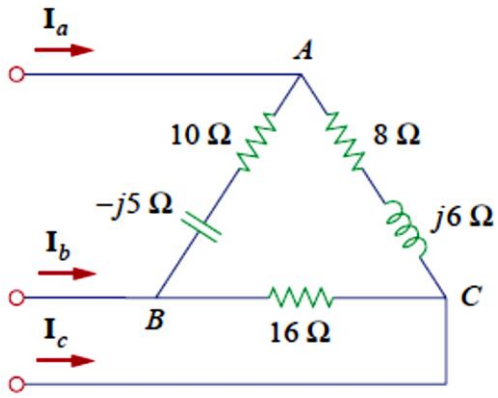
Using Eq.(5.2), the current in the neutral line is

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = -(6.67 - 0.54 + j8.92 + 3.93 - j9.2)$$
$$= -10.06 + j0.28 = 10.06\angle 178.4^\circ \text{ A}$$

H.W.9: The unbalanced -load of Fig. shown is supplied by balanced line-to-line voltages of 240 V in the positive sequence. Find the line currents. Take $\mathbf{V}_{ab}$ as reference.

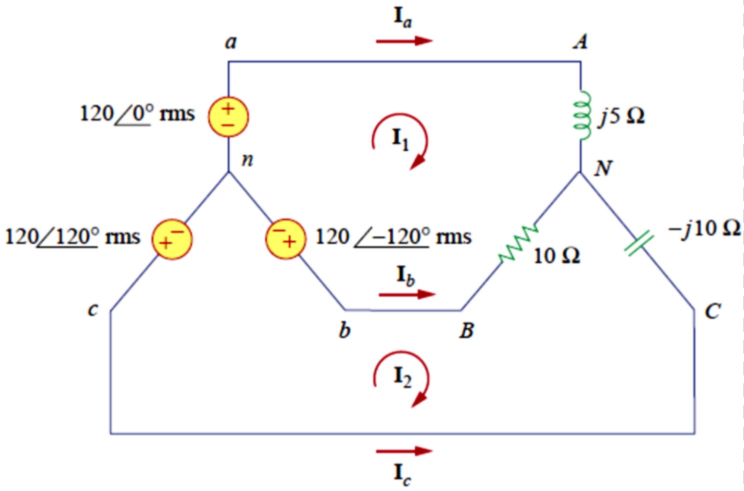
Answer:

$21.66\angle -41.06^\circ$ A, $34.98\angle -139.8^\circ$ A, $38.24\angle 74.27^\circ$ A.



Example 10: For the unbalanced circuit in Fig. shown, find: (a) the line currents, (b) the total complex power absorbed by the load, and (c) the total complex power absorbed by the source.

Solution:



(a) We use mesh analysis to find the required currents.

For mesh 1,

$$120\angle{-120^\circ} - 120\angle{0^\circ} + (10 + j5)\mathbf{I}_1 - 10\mathbf{I}_2 = 0$$

or

$$(10 + j5)\mathbf{I}_1 - 10\mathbf{I}_2 = 120\sqrt{3}\angle{30^\circ} \qquad \textbf{(1)}$$

For mesh 2,

$$120\angle{120^\circ} - 120\angle{-120^\circ} + (10 - j10)\mathbf{I}_2 - 10\mathbf{I}_1 = 0$$

or

$$-10\mathbf{I}_1 + (10 - j10)\mathbf{I}_2 = 120\sqrt{3}\angle{-90^\circ} \qquad \textbf{(2)}$$

Equations (1) and (2) form a matrix equation:

$$\begin{bmatrix} 10 + j5 & -10 \\ -10 & 10 - j10 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 120\sqrt{3}\angle{30^\circ} \\ 120\sqrt{3}\angle{-90^\circ} \end{bmatrix}$$

The determinants are

$$\Delta = \begin{vmatrix} 10 + j5 & -10 \\ -10 & 10 - j10 \end{vmatrix} = 50 - j50 = 70.71\angle{-45^\circ}$$

$$\Delta_1 = \begin{vmatrix} 120\sqrt{3}\angle{30^\circ} & -10 \\ 120\sqrt{3}\angle{-90^\circ} & 10 - j10 \end{vmatrix} = 207.85(13.66 - j13.66)$$

$$= 4015\angle{-45^\circ}$$

$$\Delta_2 = \begin{vmatrix} 10 + j5 & 120\sqrt{3}\angle{30^\circ} \\ -10 & 120\sqrt{3}\angle{-90^\circ} \end{vmatrix} = 207.85(13.66 - j5)$$

$$= 3023.4\angle{-20.1^\circ}$$

The mesh currents are

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{4015.23\angle{-45^\circ}}{70.71\angle{-45^\circ}} = 56.78\ \text{A}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{3023.4\angle{-20.1^\circ}}{70.71\angle{-45^\circ}} = 42.75\angle{24.9^\circ}\ \text{A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_1 = 56.78 \text{ A}, \qquad \mathbf{I}_c = -\mathbf{I}_2 = 42.75\underline{/-155.1^\circ} \text{ A}$$
$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_1 = 38.78 + j18 - 56.78 = 25.46\underline{/135^\circ} \text{ A}$$

(b) We can now calculate the complex power absorbed by the load. For phase A,

$$\mathbf{S}_A = |\mathbf{I}_a|^2\mathbf{Z}_A = (56.78)^2(j5) = j16{,}120 \text{ VA}$$

For phase B,

$$\mathbf{S}_B = |\mathbf{I}_b|^2\mathbf{Z}_B = (25.46)^2(10) = 6480 \text{ VA}$$

For phase C,

$$\mathbf{S}_C = |\mathbf{I}_c|^2\mathbf{Z}_C = (42.75)^2(-j10) = -j18{,}276 \text{ VA}$$

The total complex power absorbed by the load is

$$\mathbf{S}_L = \mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C = 6480 - j2156 \text{ VA}$$

(c) We check the result above by finding the power absorbed by the source. For the voltage source in phase a,

$$\mathbf{S}_a = -\mathbf{V}_{an}\mathbf{I}_a^* = -(120\underline{/0^\circ})(56.78) = -6813.6 \text{ VA}$$

For the source in phase b,

$$\mathbf{S}_b = -\mathbf{V}_{bn}\mathbf{I}_b^* = -(120\underline{/-120^\circ})(25.46\underline{/-135^\circ})$$
$$= -3055.2\underline{/105^\circ} = 790 - j2951.1 \text{ VA}$$

For the source in phase c,

$$\mathbf{S}_c = -\mathbf{V}_{bn}\mathbf{I}_c^* = -(120\underline{/120^\circ})(42.75\underline{/155.1^\circ})$$
$$= -5130\underline{/275.1^\circ} = -456.03 + j5109.7 \text{ VA}$$

The total complex power absorbed by the three-phase source is

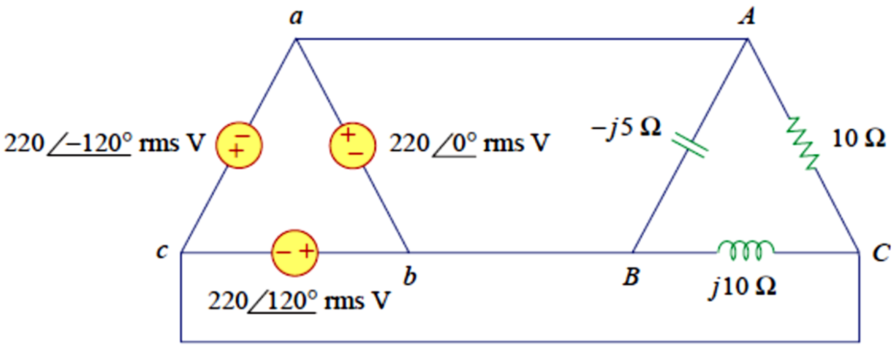
$$\mathbf{S}_s = \mathbf{S}_a + \mathbf{S}_b + \mathbf{S}_c = -6480 + j2156 \text{ VA}$$

showing that $\mathbf{S}_s + \mathbf{S}_L = 0$ and confirming the conservation principle of ac power.

H.W.10: Find the line currents in the unbalanced three-phase circuit of Fig. shown and the real power absorbed by the load.

Answer:

$64\underline{/80.1^\circ}$ A, $38.1\underline{/-60^\circ}$ A, $42.5\underline{/225^\circ}$ A, 4.84 kW.



6) Three-Phase Power Measurement

A single wattmeter can measure the average power in a three-phase system that is balanced, so that $P_1 = P_2 = P_3$; the total power is three times the reading of that one wattmeter. However, two or three single-phase wattmeter are necessary to measure power if the system is unbalanced. However, there are two methods for measuring the power in unbalanced three-phase systems;

1) Three-wattmeter method.
2) Two-wattmeter method.

6.1) Three-wattmeter method

The *three wattmeter method* of power measurement, shown in Fig.6.1, will work regardless of whether the load is balanced or unbalanced, wye or delta-connected. The three-wattmeter method is well suited for power measurement in a three-phase system where the power factor is constantly changing. The total average power is the algebraic sum of the three wattmeter readings,

$$P_T = P_1 + P_2 + P_3 \qquad \ldots(6.1)$$

where P_1 , P_2 , and P_3 correspond to the readings of wattmeters W_1 , W_2 , and W_3 and respectively. Notice that the common or reference point o in Fig.6.1 is selected arbitrarily. If the load is wye-connected, point o can be connected to the neutral point n. For a delta-connected load, point o can be connected to any point. If point o is connected to point b, for example, the voltage coil in wattmeter W_2 reads zero and $P_2 = 0$ indicating that wattmeter W_2 is not necessary. Thus, two wattmeters are sufficient to measure the total power.

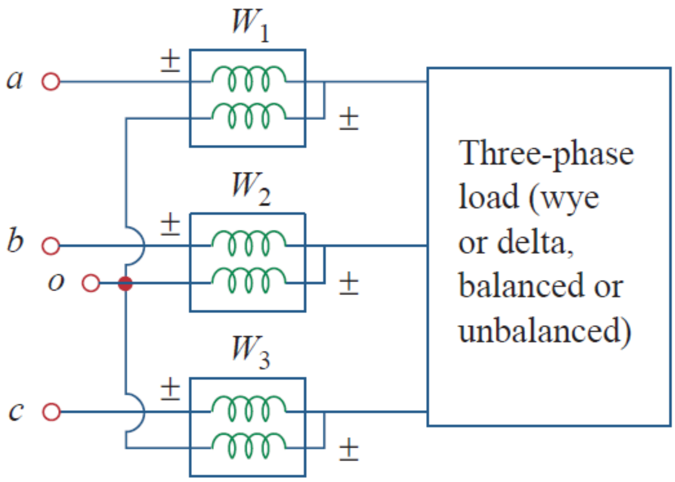


Fig.6.1 Three-wattmeter method for measuring three-phase power.

6.1) Two-wattmeter method

For many load configurations, for example, a three-phase motor, the phase current or voltage is inaccessible. We may wish to measure power with a wattmeter connected to each phase. However, because the phases are not available, we measure the line currents and the line-to-line voltages.

The *two-wattmeter method* is the most commonly used method for three-phase power measurement. The two wattmeters must be properly connected to any two phases, as shown typically in Fig.6.2. Notice that the current coil of each wattmeter measures the line current, while the respective voltage coil is connected between the line and the third line and measures the line voltage. Also notice that the $\pm$ terminal of the voltage coil is connected to the line to which the corresponding current coil is connected. Although the individual wattmeters no longer read the power taken by any particular phase, the algebraic sum of the two wattmeter readings equals the total average power absorbed by the load, regardless of whether it is wye- or delta-connected, balanced or unbalanced. The total real power is equal to the algebraic sum of the two wattmeter readings,

$$P_T = P_1 + P_2 \qquad \ldots(6.2)$$

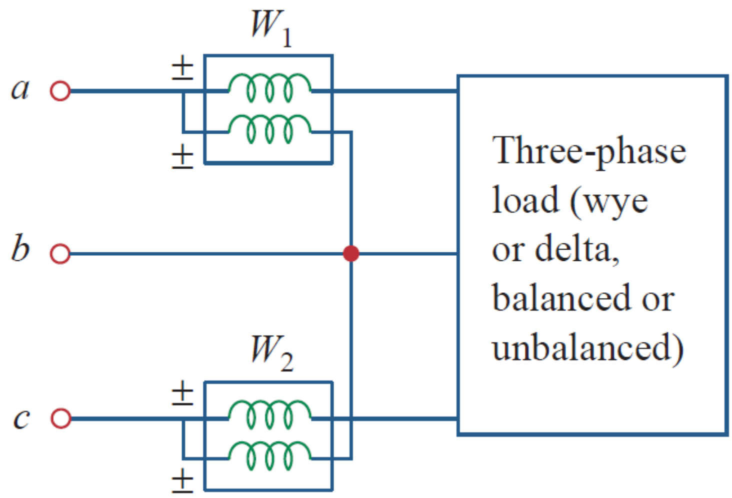


Fig.6.2 Two-wattmeter method for measuring three-phase power.

Now will show that the method works for a balanced three-phase system.
Consider the balanced, wye-connected load in Fig.6.3. Our objective is to apply the two-wattmeter method to find the average power absorbed by the load. Assume the source is in the *abc* sequence and the load impedance $Z_Y = Z_Y \angle\theta$. Due to the load impedance, each voltage coil leads its current coil by θ, so that the power factor is $COS\theta$. We recall that each line voltage leads the corresponding phase voltage by 30^0. Thus, the total phase difference between the phase current and line voltage $\mathbf{V}_{ab}$ is $\theta + 30^0$, and the average power read by wattmeter W_1 and W_2 are,

$$P_1 = V_{AB} I_A \cos\theta_1 \qquad (6\text{-}3)$$

$$P_2 = V_{CB} I_C \cos\theta_2 \qquad (6\text{-}4)$$

For the abc phase sequence for a balanced load,

$\theta_1 = \theta + 30^\circ$

$\theta_2 = \theta - 30^\circ$ (6-5)

$$\begin{aligned} P = P_1 + P_2 &= V_L I_L \cos(\theta + 30^\circ) + V_L I_L \cos(\theta - 30^\circ) \\ &= V_L I_L [\cos\theta \cos 30^\circ - \sin\theta \sin 30^\circ + \cos\theta \cos 30^\circ + \sin\theta \sin 30^\circ \\ &= 2\, V_L I_L \cos\theta \cos 30^\circ = \sqrt{3}\, V_L I_L \cos\theta \end{aligned} \quad (6\text{-}6)$$

$$\begin{aligned} P = P_1 + P_2 &= V_L I_L [\cos(\theta + 30^\circ) + \cos(\theta - 30^\circ)] \\ &= V_L I_L\, 2\cos\theta \cos 30^\circ \end{aligned} \quad (6\text{-}7)$$

Similarly,

$$P_1 - P_2 = V_L I_L(-2\sin\theta \sin 30^\circ) \quad 6\text{-}8)$$

Note that the difference of the wattmeter readings is proportional to the total reactive power, or

$$\boxed{Q_T = \sqrt{3}(P_2 - P_1)} \quad \textbf{(6.9)}$$

$$S_T = \sqrt{P_T^2 + Q_T^2} \quad \textbf{...(6.10)}$$

Divide Equ.(6.6) on (6.8),

$$\frac{P_1 + P_2}{P_1 - P_2} = \frac{2\cos\theta \cos 30^\circ}{-2\sin\theta \sin 30^\circ} = \frac{-\sqrt{3}}{\tan\theta}$$

$$\tan\theta = \frac{Q_T}{P_T} = \sqrt{3}\frac{P_2 - P_1}{P_2 + P_1} \quad \textbf{(6.11)}$$

from which we can obtain the power factor as Thus, the two-wattmeter method not only provides the total real and reactive powers, it can also be used to compute the power factor. From Eqs. (6.7), (6.9), and (6.11), we conclude that:

1. If $P_2 = P_1$, the load is resistive.
2. If $P_2 > P_1$, the load is inductive.
3. If $P_2 < P_1$, the load is capacitive.

Although these results are derived from a balanced wye-connected load, they are equally valid for a balanced delta-connected load. However, the two-wattmeter method cannot be used for power measurement in a three-phase four-wire system unless the current through the neutral line is zero. We use the three-wattmeter method to measure the real power in a three-phase four-wire system.

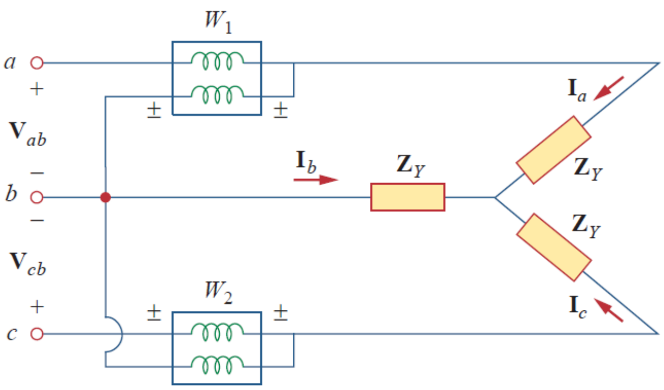
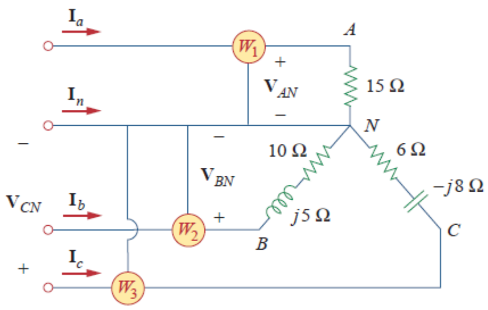


Fig.6.3 Two-wattmeter method applied to a balanced wye load.

Example 11: Three wattmeters W_1, W_2 and W_3 are connected, respectively, to phases *a*, *b*, and *c* to measure the total power absorbed by the unbalanced wye connected load in **Example 9** (a) Predict the wattmeter readings. (b) Find the total power absorbed.



Solution:

,a) From Example 9

$$\mathbf{V}_{AN} = 100\underline{/0^\circ}, \qquad \mathbf{V}_{BN} = 100\underline{/120^\circ}, \qquad \mathbf{V}_{CN} = 100\underline{/-120^\circ}\text{ V}$$

while

$$\mathbf{I}_a = 6.67\underline{/0^\circ}, \qquad \mathbf{I}_b = 8.94\underline{/93.44^\circ}, \qquad \mathbf{I}_c = 10\underline{/-66.87^\circ}\text{ A}$$

We calculate the wattmeter readings as follows:

$$\begin{aligned}
P_1 &= \text{Re}(\mathbf{V}_{AN}\mathbf{I}_a^*) = V_{AN}I_a\cos(\theta_{\mathbf{V}_{AN}} - \theta_{\mathbf{I}_a}) \\
&= 100 \times 6.67 \times \cos(0^\circ - 0^\circ) = 667\text{ W} \\
P_2 &= \text{Re}(\mathbf{V}_{BN}\mathbf{I}_b^*) = V_{BN}I_b\cos(\theta_{\mathbf{V}_{BN}} - \theta_{\mathbf{I}_b}) \\
&= 100 \times 8.94 \times \cos(120^\circ - 93.44^\circ) = 800\text{ W} \\
P_3 &= \text{Re}(\mathbf{V}_{CN}\mathbf{I}_c^*) = V_{CN}I_c\cos(\theta_{\mathbf{V}_{CN}} - \theta_{\mathbf{I}_c}) \\
&= 100 \times 10 \times \cos(-120^\circ + 66.87^\circ) = 600\text{ W}
\end{aligned}$$

(b) The total power absorbed is

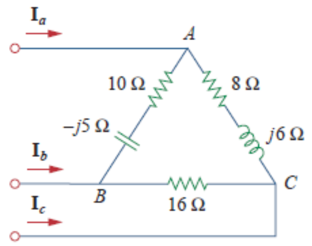
$$P_T = P_1 + P_2 + P_3 = 667 + 800 + 600 = 2067\text{ W}$$

$$\begin{aligned}
P_T &= |I_a|^2(15) + |I_b|^2(10) + |I_c|^2(6) \\
&= 6.67^2(15) + 8.94^2(10) + 10^2(6) \\
&= 667 + 800 + 600 = 2067\text{ W}
\end{aligned}$$

which is exactly the same thing.

H.W.11: Repeat **Example 11** for the network in (**H.W. 9**).
Hint: Connect the reference point *o* in Fig.6.1 to point *B*.

Answer: (a) 3.92 kW, 0 W, 8.895 kW, (b) 12.815 kW.



Example 12: The two-wattmeter method produces wattmeter readings P_1 = 1560 W and P_2 = 2100 W when connected to a delta-connected load. If the line voltage is 220 V, calculate: (a) the per-phase average power, (b) the perphase reactive power, (c) the power factor, and (d) the phase impedance.

Solution:

We can apply the given results to the delta-connected load. (a) The total real or average power is

$$P_T = P_1 + P_2 = 1560 + 2100 = 3660 \text{ W}$$

The per-phase average power is then

$$P_p = \frac{1}{3}P_T = 1220 \text{ W}$$

(b) The total reactive power is

$$Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(2100 - 1560) = 935.3 \text{ VAR}$$

so that the per-phase reactive power is

$$Q_p = \frac{1}{3}Q_T = 311.77 \text{ VAR}$$

(c) The power angle is

$$\theta = \tan^{-1}\frac{Q_T}{P_T} = \tan^{-1}\frac{935.3}{3660} = 14.33^\circ$$

Hence, the power factor is

$$\cos\theta = 0.9689 \text{ (lagging)}$$

It is a lagging pf because Q_T is positive or $P_2 > P_1$.

(c) The phase impedance is $\mathbf{Z}_p = Z_p\underline{/\theta}$. We know that θ is the same as the pf angle; that is, $\theta = 14.33°$.

$$Z_p = \frac{V_p}{I_p}$$

We recall that for a delta-connected load, $V_p = V_L = 220$ V.

$$P_p = V_p I_p \cos\theta \qquad \Rightarrow \qquad I_p = \frac{1220}{220 \times 0.9689} = 5.723 \text{ A}$$

Hence,

$$Z_p = \frac{V_p}{I_p} = \frac{220}{5.723} = 38.44\ \Omega$$

and

$$\mathbf{Z}_p = 38.44\underline{/14.33°}\ \Omega$$

H.W.12: Let the line voltage V_L = 208 V and the wattmeter readings of the balanced system in Fig.6.2 be P_1 = -560 W and P_2 = 800 W. Determine:
(a) the total average power
(b) the total reactive power
(c) the power factor
(d) the phase impedance
Is the impedance inductive or capacitive?

Answer: (a) 240 W, (b) 2355.6 VAR, (c) 0.1014, (d) 18.25∠84.18^0 Ω, inductive.

Example 13: The three-phase balanced load in Fig.6.2 has impedance per phase of Z_Y = 8 + j6 Ω. If the load is connected to 208-V lines, predict the readings of the wattmeters W_1 and W_2. Find P_T and Q_T.

Solution:

The impedance per phase is

$$\mathbf{Z}_Y = 8 + j6 = 10\underline{/36.87°}\ \Omega$$

so that the pf angle is 36.87°. Since the line voltage $V_L = 208$ V, the line current is

$$I_L = \frac{V_p}{|\mathbf{Z}_Y|} = \frac{208/\sqrt{3}}{10} = 12 \text{ A}$$

Then

$$P_1 = V_L I_L \cos(\theta + 30°) = 208 \times 12 \times \cos(36.87° + 30°)$$
$$= 980.48 \text{ W}$$
$$P_2 = V_L I_L \cos(\theta - 30°) = 208 \times 12 \times \cos(36.87° - 30°)$$
$$= 2478.1 \text{ W}$$

Thus, wattmeter 1 reads 980.48 W, while wattmeter 2 reads 2478.1 W. Since $P_2 > P_1$, the load is inductive. This is evident from the load $\mathbf{Z}_Y$ itself. Next,

$$P_T = P_1 + P_2 = 3.459 \text{ kW}$$

and

$$Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(1497.6) \text{ VAR} = 2.594 \text{ kVAR}$$

H.W.13: If the load in Fig.6.2 is delta-connected with impedance per phase of $Z_p = 30 - j40\ \Omega$ and $V_L = 440$ V. predict the readings of the wattmeters W_1 and W_2. Calculate P_T and Q_T.

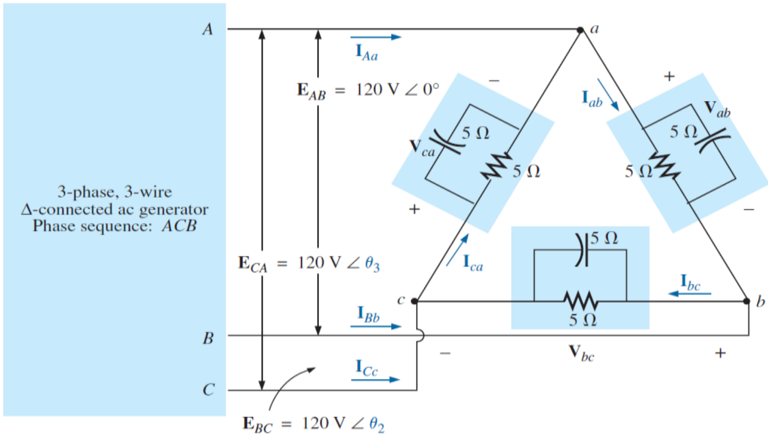
Answer: 6.166 Kw, 0.8021 Kw, 6.968 Kw, -9.291 kVAR.

Lecture ()

Three Phase Circuits

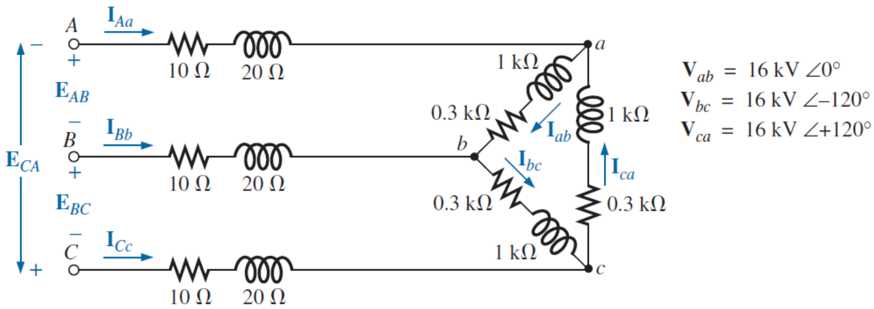
Problems

H.W.(1): For the three-phase $\Delta-\Delta$ system in Figure shown, Find the phase and line currents. Draw phasor diagram.



[Answer: $I_{AB} = 33.9\angle 45^0$ A, $I_{BC} = 33.9\angle 165^0$ A, $I_{CA} = 33.9\angle -75$ A, $I_A = 58.82\angle 75^0$ A, $I_B = 58.82\angle 195^0$ A, $I_C = 58.82\angle -45^0$ A]

H.W.(2): For the Δ-connected load in Figure, find the line voltages, phase currents and line currents. Draw the phasor diagram.



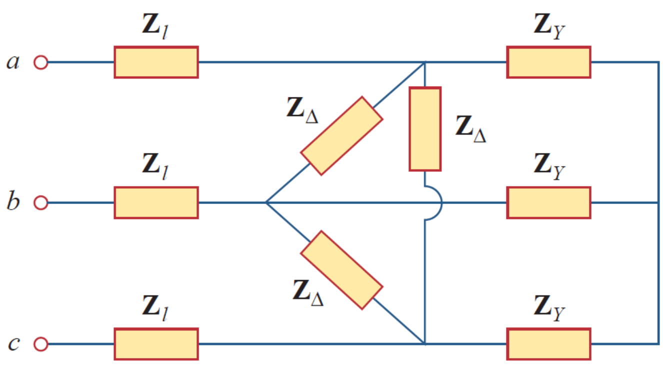
[Answer:]

(a) $I_{ab} = 15.33\ A\ \angle -73.30°$, $I_{bc} = 15.33\ A\ \angle -193.30°$, $I_{ca} = 15.33\ A\ \angle 46.7°$

(b) $I_{Aa} = 26.55\ A\ \angle -103.30°$, $I_{Bb} = 26.55\ A\ \angle 136.70°$, $I_{Cc} = 26.55\ A\ \angle 16.70°$

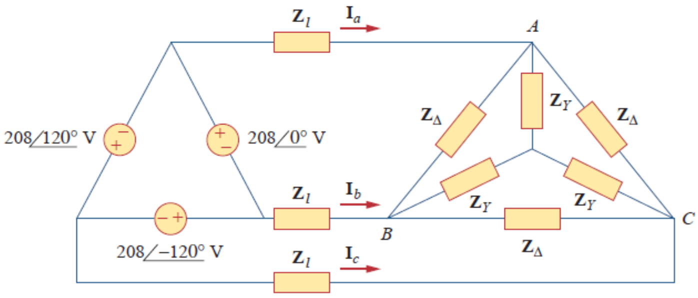
(c) $E_{AB} = 17.01\ kV\ \angle -0.59°$, $E_{BC} = 17.01\ kV\ \angle -120.59°$, $E_{CA} = 17.01\ kV\ \angle 119.41°$

H.W.(3): The circuit in Figure is excited by a balanced three-phase source with a line voltage of 210 V. If $\mathbf{Z}_\ell = 1 + j1\ \Omega$, $\mathbf{Z}_\Delta = 24 - j30\ \Omega$, and $\mathbf{Z}_Y = 12 + j5\ \Omega$, determine the magnitude of the line current of the combined loads.



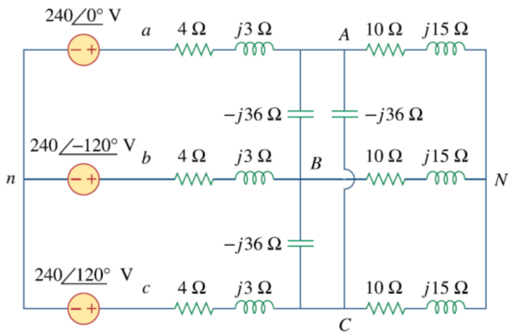
[Answer: $I_a = 13.66\angle 6\cdot 78^0$ A]

H.W.(4): Find the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ in the three-phase network of Figure below. Take $\mathbf{Z}_{\Delta} = 12 - j15\ \Omega$, $\mathbf{Z}_Y = 4 + j6\ \Omega$, and $\mathbf{Z}_\ell = 2\ \Omega$.



[Answer: $I_a = 15.53\angle{-28.4^0}$ A, $I_b = 15.53\angle{-148.4^0}$ A, $I_c = 15.53\angle{91.6^0}$ A]

H.W.(5): Given the circuit in Figure shown, determine currents $\mathbf{I}_{aA}$ and voltage $\mathbf{V}_{BN}$.



[Answer: $I_a = 11.15\angle 37^0$ A, $V_{BN} = 230.8\angle{-133.4^0}$ V,]

H.W.(6): Find the line currents and the power absorbed by the delta-connected load.

[Answer:]

$\mathbf{I}_{aA} = 35.76\angle{-34.74^\circ}$ A rms;

$\mathbf{I}_{bB} = 35.76\angle{-154.74^\circ}$ A rms;

$\mathbf{I}_{cC} = 35.76\angle{85.26^\circ}$ A rms;

$17.29 - j6.92$ kVA.

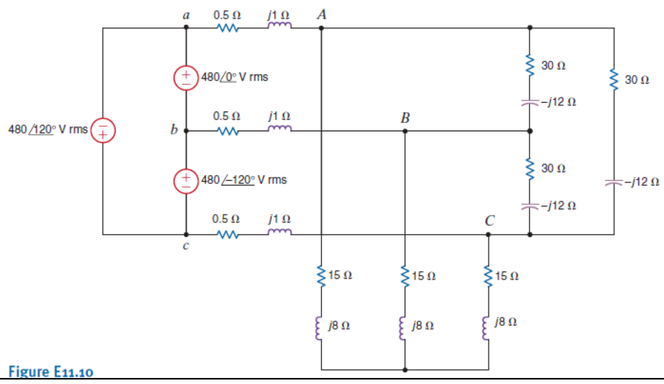


Figure E11.10

H.W.(7): In a balanced three-phase system, the abc phase sequence source is delta connected and $V_{ab} = 120\angle 30^0$ rms. The load consists of two balanced wyes with phase impedances of $(10 + j1)\Omega$ and $(20 + j5)\Omega$. If the line impedance is zero, find the line currents and the load phase voltages using one line equivalent circuit.

[Answer: $V_{an} = 69.28\angle 0^0 V = V_{AN}$, $I_{AN1} = 6.89\angle{-5.81^0}$ A, $I_{AN2} = 3.36\angle{-14.04^0}$ A, $I_a = I_{AN1} + I_{AN2} = 10.23\angle{-8.44^0}$ A, $I_b = 10.23\angle{-128.44^0}$ A, $I_c = 10.23\angle{111.56^0}$ A]

H.W.(8): In a balanced three-phase system, the source has an abc phase sequence and is connected in delta. There are two loads connected in parallel. Load1 is connected in wye and has phase impedance of $(6 + j2)\Omega$. Load2 is connected in delta and has phase impedance of $(9 + j3)\Omega$. The line impedance is $(0.6 + j0.2)\Omega$. Determine the phase voltages of the source if the current in the **a** phase of load1 is $I_{AN1} = 10\angle 30^0$ A rms.

[Answer: $V_{ab} = 142.41\angle 78.43^0 V$. $V_{an} = 82.22\angle 48.43^0 V$. $V_{bn} = 82.22\angle{-71.57^0} V$. $V_{cn} = 82.22\angle 168.43^0 V$]

H.W.(9): In a balanced three-phase wye–wye system, the source is an *abc*-sequence set of voltages. The load voltage on the *a* phase is $V_{AN} = 120\angle 60^0$ V rms, $Z_{line} = (2 + j1.4)\Omega$, and $Z_{load} = (10 + j10)\Omega$. Determine the input voltages.
[Answer: $V_{an} = 140.4\angle 58\cdot 5^0$ V, $V_{bn} = 140.4\angle -61\cdot 5^0$ V, $V_{cn} = 140.4\angle 178\cdot 5^0$ V]

H.W.(10): In a balanced three-phase wye–wye system, the load impedance is $(8 + j4)\Omega$. The source has phase sequence *abc* and $V_{an} = 120\angle 0^0$ V rms .If the load voltage is $V_{AN} = 111.62\angle -1\cdot 33^0$ V rms determine the line impedance.
[Answer: $Z_{line} = (0.5 + j0.5)\Omega$]

H.W.(11): In a balanced three-phase wye–wye system, the total power loss in the lines is 400 W. $V_{AN} = 105.28\angle 31\cdot 56^0$ V rms and the power factor of the load is 0.77 lagging. If the line impedance is $2 + j1\ \Omega$, determine the load impedance.
[Answer: $Z_{load} = (5.74 + j4.75)\Omega$]

H.W.(12): In a balanced 3-phase Y–Y system, $Z_{load} = 20 + j12\ \Omega$. The source has an *abc*-phase sequence and $\mathbf{V}_{an} = 120\angle 0^0$ V rms. If the load voltage is $\mathbf{V}_{AN} = 111.49\angle -0\cdot 2^0$ V rms, determine the magnitude of the line current if the load is suddenly short-circuited.
[Answer: $I_a = 67.42\angle -33\cdot 82^0$ A rms]

H.W.(13): In a balanced three-phase delta–delta system, the source has an *abc*-phase sequence. The phase angle for the source voltage is $\angle \mathbf{V}_{ab} = 40^0$ and $I_{ab} = 4\angle 15^0$ A rms. If the total power absorbed by the load is 1400 W, find the load impedance.
[Answer: $Z_{load\Delta} = 32\cdot 16\angle 25^0\Omega$]

H.W.(14): In a balanced three-phase system, the source has an *abc*-phase sequence and is connected in delta. There are two parallel wye-connected loads. The phase impedance of load 1 and load 2 is $4 + j4\Omega$ and $10 + j4\ \Omega$ respectively. The line impedance connecting the source to the loads is $0.3 + j0.2\ \Omega$. If the current in the *a* phase of load 1 is $I_{AN1} = 10\angle 20^0$ A find the delta currents in the source.
[Answer: $I_{ab} = 8.64\angle 57\cdot 94^0$ A, $I_{bc} = 8.64\angle -62\cdot 1^0$ A, $I_{ca} = 8.64\angle 177\cdot 94^0$ A]

H.W.(15): The magnitude of the complex power (apparent power) supplied by a three-phase balanced Y–Y system is 3600 VA. The line voltage is 208 V rms. If the line impedance is negligible and the power factor angle of the load is 25^0 determine Z_{load}.
[Answer: $Z_{load} = 12\angle 25^0\ \Omega$]

H.W.(16): A three-phase *abc*-sequence wye-connected source supplies 14 kVA with a power factor of 0.75 lagging to a delta load. If the delta load consumes 12 kVA at a power factor of 0.7 lagging and has a phase current of $10\angle 30^0$, determine the per-phase impedance of the load and the line.
[Answer: $Z_{load\Delta} = 40\angle 45\cdot 57^0\ \Omega$, $Z_{line} = 2.43 + j0.776\ \Omega$]

H.W.(17): A balanced three-phase source serves the following loads:
Load 1: 20 kVA at 0.8 pf lagging
Load 2: 10 kVA at 0.7 pf leading
Load 3: 10 kW at unity pf
Load 4: 16 kVA at 0.6 pf lagging
The line voltage at the load is 208 V rms at 60 Hz, and the line impedance is 0.02 + j0.04

Ω. Find the line current, voltage and power factor at the source.

[Answer: : $I_a = 128.1\angle -22\cdot 53^0$ A ,$V_{an} = 249.83\angle -38\cdot 38^0$ A, pf = 0.912 lagging]

H.W.(18): A balanced three-phase source supplies power to three loads. The loads are
Load 1: 24 kVA at 0.6 pf lagging
Load 2: 10 kW at 0.75 pf lagging
Load 3: unknown
If the line voltage at the load is 208 V rms, the magnitude of the total complex power is 35.52 kVA, and the combined power factor at the load is 0.88 lagging, find the unknown load.

[Answer: $S_3 = 13.09\angle -58\cdot 39^0$ kVA]

H.W.(19): Determine the complex power delivered to the three-phase load of a four-wire Y- Y circuit. The phase voltages of the Y-connected source are $V_{an} = 110\angle 0^0$ V rms, $V_{bn} = 110\angle -120^0$ V rms, and $V_{cn} = 110\angle 120^0$ V rms. The load impedances are $Z_A = 50 + j80$ Ω; $Z_B = j50$ Ω, and $Z_C = 100 + j25$ Ω.

[Answer: $S_A = 68 + j109$ VA, $S_B = j242$ VA, $S_C = 114 + j128$ VA, $S_T = 182 + j379$ VA]

H.W.(20): Determine the complex power delivered to the 3-phase load of a four-wire Y-Y circuit. The phase voltages of the Y-connected source are $V_{an} = 110\angle 0^0$ V, $V_{bn} = 110\angle -120^0$ V, and $V_{cn} = 110\angle 120^0$ V. The load impedances are $Z_A = Z_B = Z_C = 50 + j80$ Ω.

[Answer: $S_A = S_B = S_C = 68 + j109$ VA, $S_T = 204 + j327$ VA]

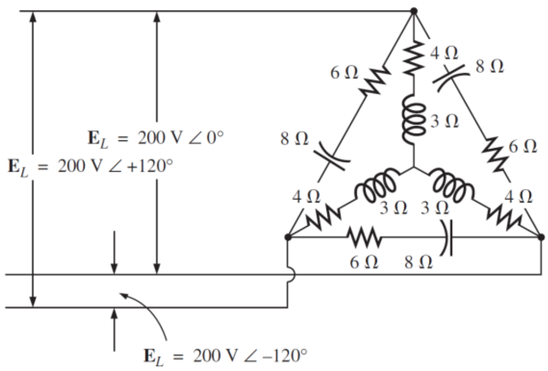
H.W.(21): Determine the complex power delivered to the three-phase load of a three-wire Y- Y circuit. The phase voltages of the Y-connected source are $V_{an} = 110\angle 0^0$ V rms, $V_{bn} = 110\angle -120^0$ V rms, and $V_{cn} = 110\angle 120^0$ V rms. The load impedances are $Z_A = 50 + j80$ Ω; $Z_B = j50$ Ω, and $Z_C = 100 + j25$ Ω.

[Answer: $S_A = 146 + j234$ VA, $S_B = j94$ VA, $S_C = 141 + j35$ VA, $S_T = 287 + j364$ VA]

H.W.(22): Determine the complex power delivered to the three-phase load of a three-wire Y- Y circuit. The phase voltages of the Y-connected source are $V_{an} = 110\angle 0^0$ V rms, $V_{bn} = 110\angle -120^0$ V rms, and $V_{cn} = 110\angle 120^0$ V rms. The load impedances are $Z_A = Z_B = Z_C = 50 + j80$ Ω.

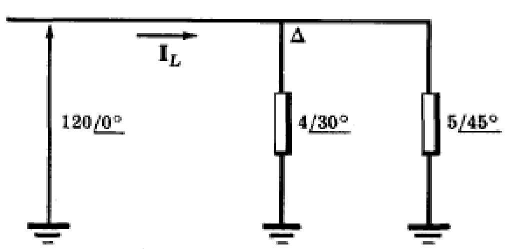
[Answer: $S_A = S_B = S_C = 68 + j109$ VA, $S_T = 204 + j327$ VA]

H.W.(23): For the $\Delta - Y$ connected load in Figure shown. Find the total active, reactive and apparent power, also find the load power factor.



[Answer: $P_{T\Delta}$ = 7200 W, $Q_{T\Delta}$ = 9600 VAR (capacitve), $S_{T\Delta}$ = 12000 VA, P_{TY} = 6414.41 W, Q_{TY} = 4810.81 VAR (inductive), S_{TY} = 8045.76 VA, P_T = 13614.41 W, Q_T = 4789.19 VAR, S_T = 14432.2 VA, $\cos\varphi$ = 0.943 leading]

H.W.(24): Three identical impedances of $\mathbf{12\angle 30^0}$ ohms in a Δ-connection and three identical impedances of $\mathbf{15\angle 45^0}$ ohms in Y-connection are both on the same three-phase, three-wire 208 V system ABC system. Find the line currents and total active power.



[Answer: $I_L = 53.6\angle -36.6^0$ A, P = 15500 W]

H.W.(25): A three-phase source delivers 4800 VA to a wye-connected load with a phase voltage of 208 V and a power factor of 0.9 lagging. Calculate the source line current and the source line voltage.

[Answer: I_L = 7.69 A, V_L = 360.3 V]

H.W.(26): A balanced wye-connected load with a phase impedance of 10 – j16 Ω is connected to a balanced three-phase generator with a line voltage of 220 V. Determine the line current and the complex power absorbed by the load.

[Answer: I_L = 6.732 A, $S = 2565\angle -58^0$ VA]

H.W.(27): A 4200-V, three-phase transmission line has an impedance of 4 + j10 Ω per phase. If it supplies a load of 1 MVA at 0.75 power factor (lagging), find:

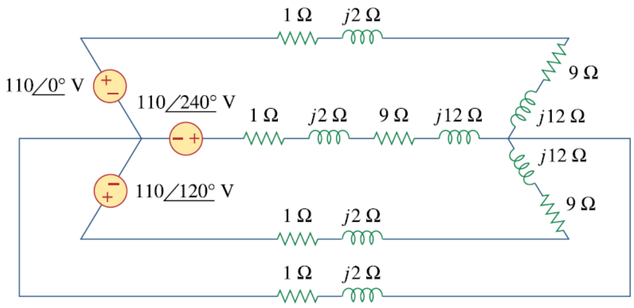
(a) the complex power

(b) the power loss in the line

(c) the voltage at the sending end

[Answer: S = 0.75 + j0.6614 MVA, P_L = 25.19 kW, $V = 4.443\angle -2\cdot 709^0$ V]

H.W.(28): Given the circuit in Figure below, find the total complex power absorbed by the load.



[Answer: S = 551.86 + j735.81 VA]

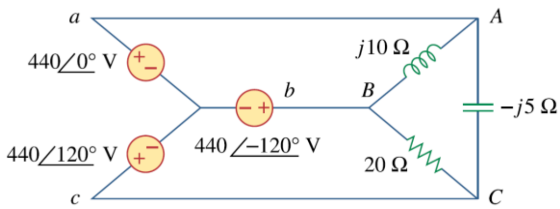
H.W.(29): A three-phase line has an impedance of 1 + j3 Ω per phase. The line feeds a balanced delta-connected load, which absorbs a total complex power of 12 + j5 k VA. If the line voltage at the load end has a magnitude of 240 V, calculate the magnitude of the line voltage at the source end and the source power factor.

[Answer: V = 287.04 V, pf = 0.9451]

H.W.(30): A three-phase supply, with the line voltage 240 V rms positively phased, has an unbalanced delta-connected load as shown in Figure. Find the phase currents and the total complex power.

[Answer: $I_{AB} = 9.6\angle -90^0$ A, $I_{BC} = 6\angle 120^0$ A, $I_{CA} = 8\angle -150^0$ A, S_{AB} = j2304 VA, S_{CA} = 1440 VA, S_{BC} = 1662.77 + j960 VA, S_T = 3102.77 + j3264 VA]

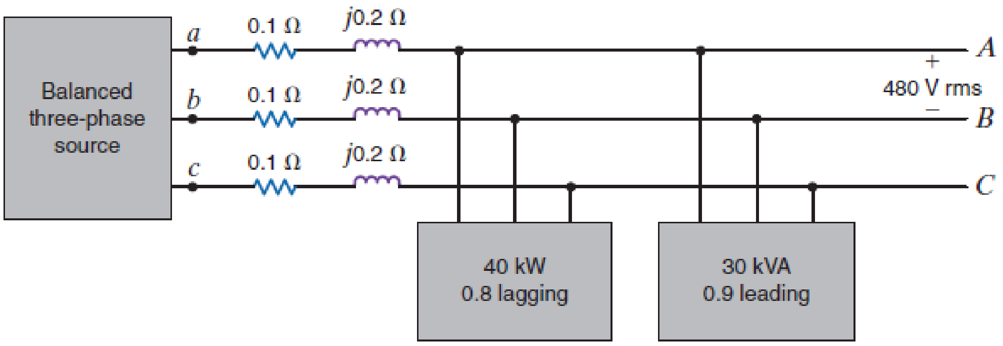
H.W.(31): Refer to Figure. Calculate:
(a) the line currents
(b) the real power absorbed by the load
(c) the total complex power supplied by the source



[Answer: (a) $I_a = 132\angle 30^0$ A, $I_b = 47.23\angle 143\cdot 8^0$ A, $I_c = 120.9\angle 230\cdot 9^0$ A (b) S_{AB} = j58.08 kVA, S_{BC} = 29.04 kVA, S_{CA} = - j116.16 kVA (c) S_T = 29.04 – j58.08 kVA]

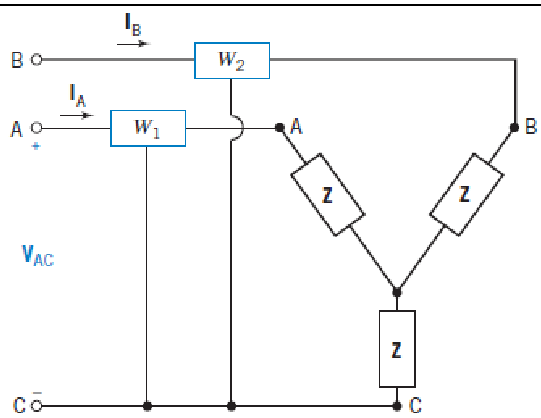
H.W.(32): A balanced three-phase source serves three loads, as follows:
Load 1: 24 kW at 0.6 lagging power factor
Load 2: 10 kW at unity power factor
Load 3: 12 kVA at 0.8 leading power factor
If the line voltage at the loads is 208 V rms at 60 Hz, determine the line current and the combined power factor of the loads.
[Answer: I_L = 139.23 A rms, pf_{load} =0.869 lagging]

H.W.(33): If the line voltage at the load is 480 V rms in Figure, find the line voltage and power factor at the source.



[Answer: V_L = 501.7 V rms, pf_{load} =0.9568 lagging]

H.W.(34): The two-wattmeter method is used, as shown in Figure, to measure the total power delivered to the Y-connected load when Z = $10\angle 45^0$ V and the supply line-to-line voltage is 220 V rms. Determine the reading of each wattmeter and the total power.



[Answer: p_1 = 2698W, p_2 = 723 W, P = P_1 + P_2 = 3421W]

H.W.(35): The two wattmeters in **previous H.W.** read P_1 = 60 kW and P_2 = 180 W, respectively. Find the power factor of the circuit.
[Answer: pf = 0.756 lagging]

H.W.(36): The two-wattmeter method is used to determine the power drawn by a three-phase 440-V rms motor that is a Y-connected balanced load. The motor operates at 20 hp

at 74.6
percent efficiency. The magnitude of the line current is 52.5 A rms. The wattmeters are connected in the A and C lines. Find the reading of each wattmeter. The motor has a lagging power factor.

[Answer:]

H.W.(37): A three-phase system has a line-to-line voltage of 4000 V rms and a balanced Δ -connected load with Z = 40 + j30 V. The phase sequence is abc. Use the two wattmeters connected to lines A and C, with line B as the common line for the voltage measurement. Determine the total power measurement recorded by the wattmeters.

[Answer: P = 768 kW]

H.W.(38): A three-phase system with a sequence abc and a line-to-line voltage of 200 V rms feeds a Y-connected load with $Z = 70.7\angle 45^0$ V. Find the line currents. Find the total power by using two wattmeters connected to lines B and C.

[Answer: P = 400W]

H.W.(39): A three-phase system with a line-to-line voltage of 208 V rms and phase sequence abc is connected to a Y-balanced load with impedance $10\angle -30^0\ \Omega$ and a balanced Δ load with impedance $15\angle 30^0\ \Omega$. Find the line currents and the total power using two wattmeters.

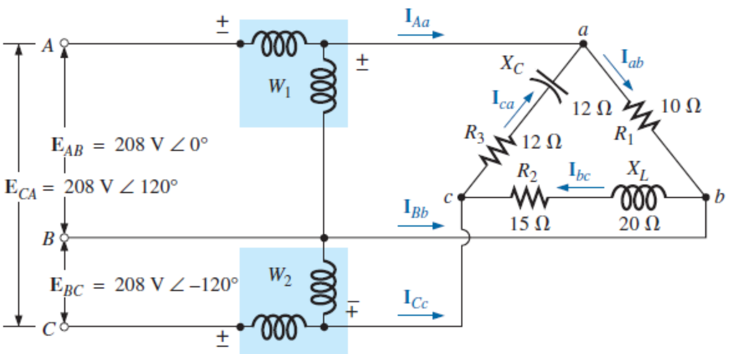
[Answer:]

H.W.(40): The two-wattmeter method is used. The wattmeter in line A reads 920 W, and the wattmeter in line C reads 460 W. Find the impedance of the balanced Δ-connected load. The circuit is a three-phase 120-V rms system with an abc sequence.

[Answer: $Z_\Delta = 27.1\angle 30^0$_ Ω]

H.W.(41): For the unbalanced Δ-connected load in Fig. with two properly connected wattmeters.

a. Determine the magnitude and angle of the phase currents.
b. Calculate the magnitude and angle of the line currents.
c. Determine the power reading of each wattmeter.
d. Calculate the total power absorbed by the load.
e. Compare the result of part (d) with the total power calculated using the phase currents and the resistive elements.

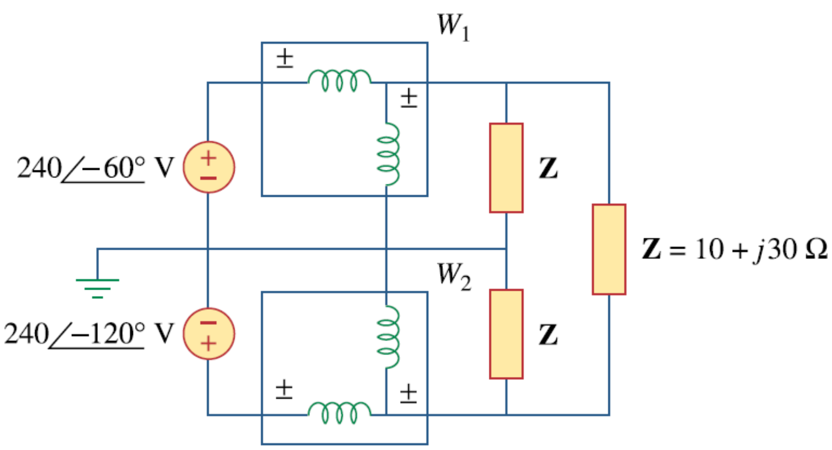


[Answer: (a) $I_{ab} = 20.8\ \angle 0°$ A, $I_{bc} = 8.32\angle -173\cdot 13^0$ A, $I_{ca} = 12.26\ \angle 165°$ A

(b) $I_a = 32.79 \angle -5.55°$ A, $I_b = 29.08 \angle -178.03^0$ A, $I_c = 5.5 \angle 130.65°$ A
(c)$P_1 = 6788.35$ W, $P_2 = 379.1$ W (d) $P_T = 7167.45$ W (e) $P_T = 7168.43$ W (The slight difference is due to the level of accuracy carried through the calculations.)]

H.W.(42): The two-wattmeter method gives $P_1 = 1200$ W and $P_2 = -400$ W for a three-phase motor running on a 240-V line. Assume that the motor load is wye-connected and that it draws a line current of 6 A. Calculate the pf of the motor and its phase impedance.
[Answer: pf = 0.4472 (leading), $Z_p = 40\angle -63.43^0 \Omega$]

H.W.(43): For the circuit displayed in Fig., find the wattmeter readings.



[Answer: $p_1 = 2360$ W, $p_2 = -632.8$ W]