Lecture ( )

Representation of Alternating Quantities

1. Introduction

A vector is a physical quantity which has magnitude as well as direction. Vectors graphically represented by straight lines. The length of the line represents the magnitude of the alternating quantity, the inclination of the line with respect to some axis of reference gives the direction of that quantity and an arrow-head placed at one end indicates the direction in which that quantity acts.

In fact, vectors are a shorthand for the representation of alternating voltages and currents and their use greatly simplifies the problems in a.c. work.

The alternating voltages and currents are represented by such vectors rotating counter-clockwise with the same frequency as that of the alternating quantity as shown in **Figure (1.1)**.

Notes:

1) that the length of vector represents the maximum value of sinusoidal quantity.
2) Instead of using maximum values as above, it is very common practice to draw vector diagrams using r.m.s. values of alternating quantities. But it should be understood that in that case, the projection of the rotating vector on the Y-axis does not give the instantaneous value of that alternating quantity.

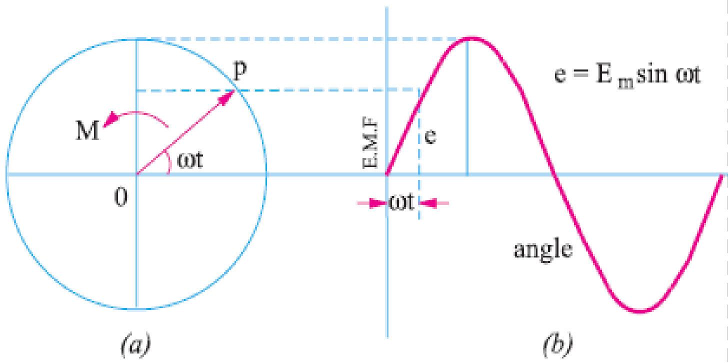


Figure (1.1)

2. Vector Diagrams of Sine Waves of Same Frequency

Two or more sine waves of the same frequency can be shown on the same vector diagram because the various vectors representing different waves all rotate counter-clockwise at the same frequency and maintain a fixed position relative to each other as shown in **Figure (2.1).**

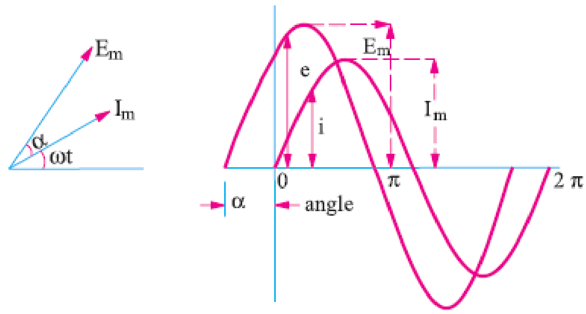


Figure (2.1)

$i = I_m sin\omega t$

$e = E_m \sin(\omega t + \alpha)$

Sine wave of different frequencies cannot be represented on the same vector diagram in a still picture because due to difference in speed of different vectors, the phase angles between them will be continuously changing.

3. Addition of Two Alternating Quantities

The sum of the two sine waves of the same frequency is another sine wave of the same frequency but of a different maximum value and phase. The value of the instantaneous resultant value, at any instant is obtained by algebraically adding the projections of the two vectors on the *Y*-axis

Example 1: *Add the following currents as waves and as vectors.*
$i_1 = 7$ sinωt and $i_2 = 10 \sin(\omega t + \pi/3)$

Solution:

As Waves

$$i_r = i_1 + i_2 = 7 \sin\omega t + 10 \sin(\omega t + 60^o)$$
$$= 7 \sin \omega t + 10 \sin \omega t \cos 60^o + 10 \cos\omega t \sin 60^o$$
$$= 12 \sin \omega t + 8.66 \cos \omega t$$

Dividing both sides by $\sqrt{(12^2 + 866^2)} = 14.8$, we get

$$i_r = 14.8\left((\frac{12}{14.8} \sin \omega t) + (\frac{8.66}{14.8} \cos \omega t)\right)$$

$$= 14.8(\cos \alpha \sin \omega t + \sin \alpha \cos \omega t)$$

where $\cos \alpha = 12/14.8$ and $\alpha = 8.66/14.8$ —as shown in **Figure 1**

$$\therefore i_r = 14.8 \sin(\omega t + \alpha)$$

where $\tan \alpha = 8.66/12$ or $\alpha = \tan^{-1}(8.66/12) = 35.8^o$

$$\therefore i_r = 14.8 \sin(\omega t + 35.8^o)$$

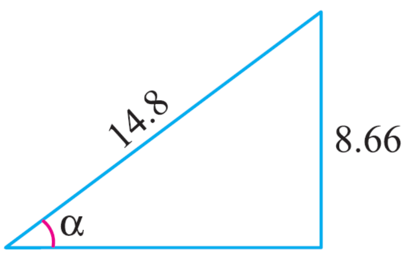


Figure 1

As Vectors

Vector diagram is shown in **Figure 2**. Resolving the vectors into their horizontal and vertical components, we have

X-component $= 7 + 10\cos 60^o = 12$

Y-component $= 0 + 10\sin 60^o = 8.66$

Resultant $= \sqrt{(12^2 + 866^2)} = 14.8A$

and $\alpha = \tan^4(8.66/12) = 35.8^o$

Hence, the resultant equation can be written as

$i_r = 14.8\sin(\omega t + 35.8^o)$

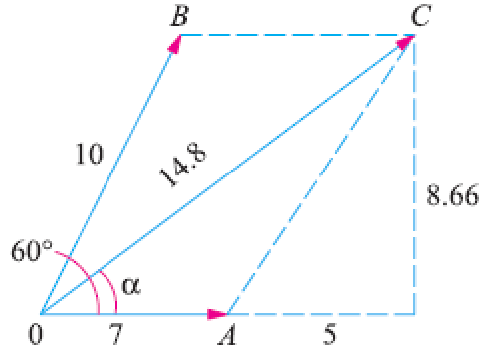


Figure 2

As Complex Numbers

$i_1 = 7\sin\omega t \quad \Rightarrow \quad i_1 = 7\angle 0^o$

$i_2 = 10\sin(\omega t + \pi/3) \quad \Rightarrow \quad i_2 = 10\angle 60^o$

$\therefore i_r = i_1 + i_2 = 7\angle 0^o + 10\angle 60^o = 14.8\angle 35.8^o$

Notes:

1) **If the sinusoidal quantities are given in (sin) and (cos) then convert all quantities to (sin) and add them or convert all quantities to (cos) and then add them.**

   $$\boldsymbol{e_1 = 20\sin\omega t;\ e_2 = 30\sin\left(\omega t - \frac{\pi}{4}\right),\ e_3 = 40\cos\left(\omega t + \frac{\pi}{6}\right)}$$

   $$\boldsymbol{e_3 = 40\cos\left(\omega t + \frac{\pi}{6}\right) = 40\sin\left(\omega t + \frac{\pi}{6} + \frac{\pi}{2}\right) = 40\sin\left(\omega t + \frac{4\pi}{6}\right)}$$

   $$\boldsymbol{\therefore\ e_r = e_1 + e_2 + e_3}$$

2) **If difference of two vectors is required, then one of the vectors is reversed and this reversed vector is then compounded with the other vector as usual.**

Example 2: *Two currents* i_1 *and* i_2 *are given by the expressions*
$i_1 = 10sin(314t + \pi/4)$ *amperes and* $i_2 = 8sin(313t - \pi/3)$ *amperes*
Find (*a*) $i_1 + i_2$ *and* (*b*) $i_1 - i_2$. *Express the answer in the form* $i = I_m sin(314t \pm \varphi)$

Solution:

(*a*) The current vectors representing maximum values of the two currents are shown in **Figure1 (a)**. Resolving the currents into their X-and Y-components, we get

X- component $= 10 \cos 45^o + 8 \cos 60^o = 10/\sqrt{2} + 8/2 = 11.07A$

Y- component $= 10 \sin 45^o - 8 \sin 60^o = 0.14A$

$I_m = \sqrt{1107^2 + 014^2} = 11.08A$

$\tan \varphi = (0.14/11.07) = 0.01265 \therefore \varphi = 44'$

$\therefore i = 11.08 \sin (314t + 44')$ amperes

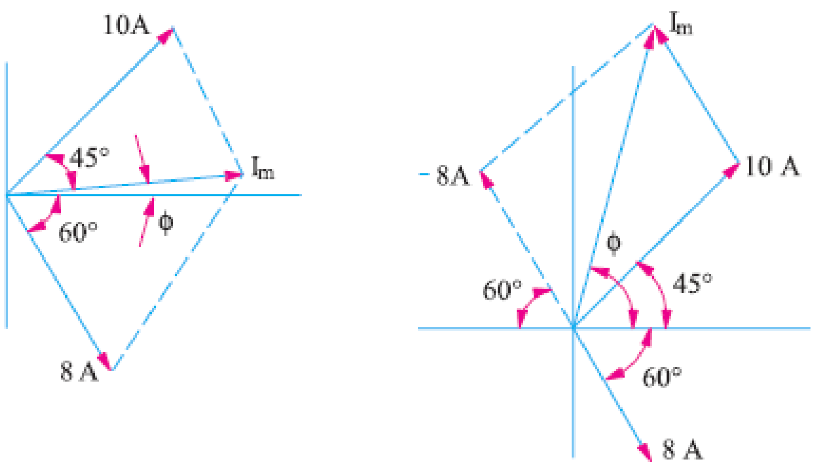
(*b*) X -component $= 10 \cos 45^o - 8 \cos 60^o = 3.07A$

Y -component $= 10 \sin 45^o + 8 \sin 60^o = 14A$

$I_m = \sqrt{307^2 + 14^2} = 14.33A$ $\cdots$ **Figure1 (b)**

$\varphi = \tan^{-1}(14/3.07) = 77^0 38'$

$\therefore i = 14.33 \sin$ (314+77o 38') amperes



(a) *(b)*

Figure1

Example 3: *The maximum values of the alternating voltage and current are 400 Vand20 A respectively in a circuit connected to a 50 Hz supply. The instantaneous values of voltage and current are 283 V and 10 A respectively at time $t = 0$, both increasing positively (i) Write down the expression for voltage and current at time t. (ii) Determine the power consumed in the circuit. Take the voltage and current to be sinusoidal*

Solution:

$V_m = 400$, $I_m = 20$, $v_{inst.} = 283$, $i_{inst.} = 10$, $\omega = 314$rad. / sec

(i) Let the expressions be as follows:

$$v(t) = V_m \sin(\omega t + \Theta_1) = 400 \sin(314t + \Theta_1)$$

$$I(t) = I_m \sin(\omega t + \Theta_2) = 20 \sin(314t + \Theta_2)$$

where Θ_1 and Θ_2 indicate the concerned phase-shifts with respect to some reference. Substituting the given instantaneous values at $t = 0$,

$$v(t) = 400 \sin(314t + \Theta_1) \Rightarrow 283 = 400 \sin(0 + \Theta_1) \Rightarrow \Theta_1 = 45^0$$

$$I(t) = 20 \sin(314t + \Theta_2) \Rightarrow 10 = 20 \sin(0 + \Theta_2) \Rightarrow \Theta_2 = 30^o$$

The required expressions are:

$$V(t) = 400 \sin(314t + 45^o)$$

$$i(t) = 20 \sin(314t + 30^o)$$

(ii)

Thus, the voltage leads the current by 15^o.

V = RMS voltage = $400/\sqrt{2} = 283V$

I = RMS voltage = $20/\sqrt{2} = 14.14A$

Power-factor, $\cos\varphi = \cos 15^o = 0.966$ lagging, since current lags behind the voltage.

Power = V I $\cos\varphi = 283 \times 14.14 \times 0.966 = 3865$ watts

Lecture (2)

Sinusoidal Waves

Sine & Cosine Waves

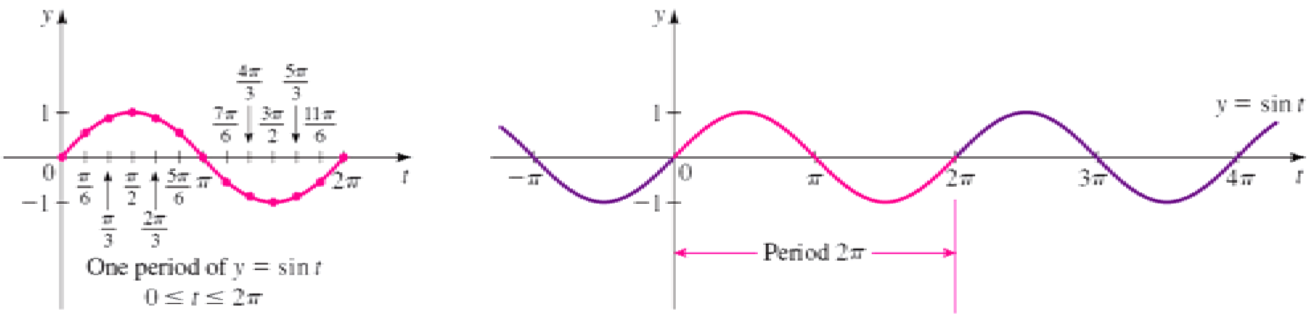


Figure (2.1): Sine wave (sin(t))

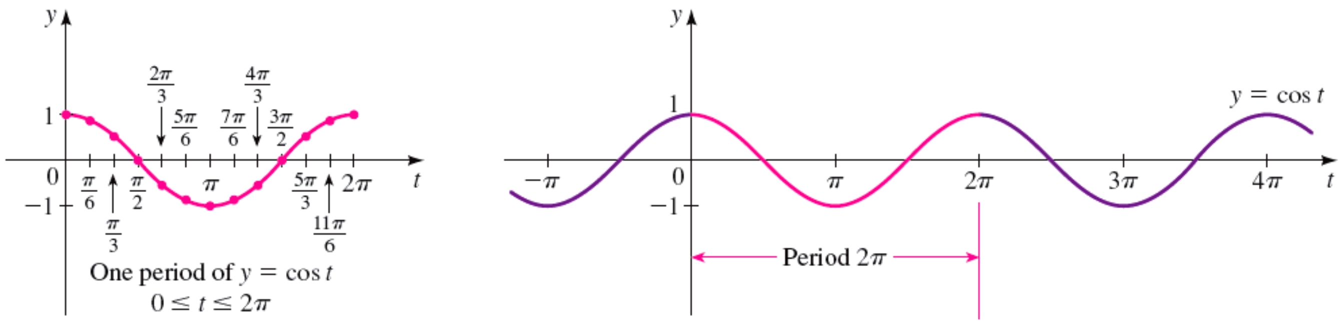


Figure (2.2): Cosine wave (cos (t))

Shifting of functions

Horizontal Shift

(1) Left shift (shifting function **a** unit to the left) $f(x) = f(x+a)$

(2) Right shift (shifting function **a** unit to the right) $f(x) = f(x - a)$

Vertical Shift

(1) Up shift (shifting function **a** units to the up) $f(x) = f(x) + a$

(2) Down shift (shifting function **a** units to the down) $f(x) = f(x) - a$

Note: - a is a positive number.

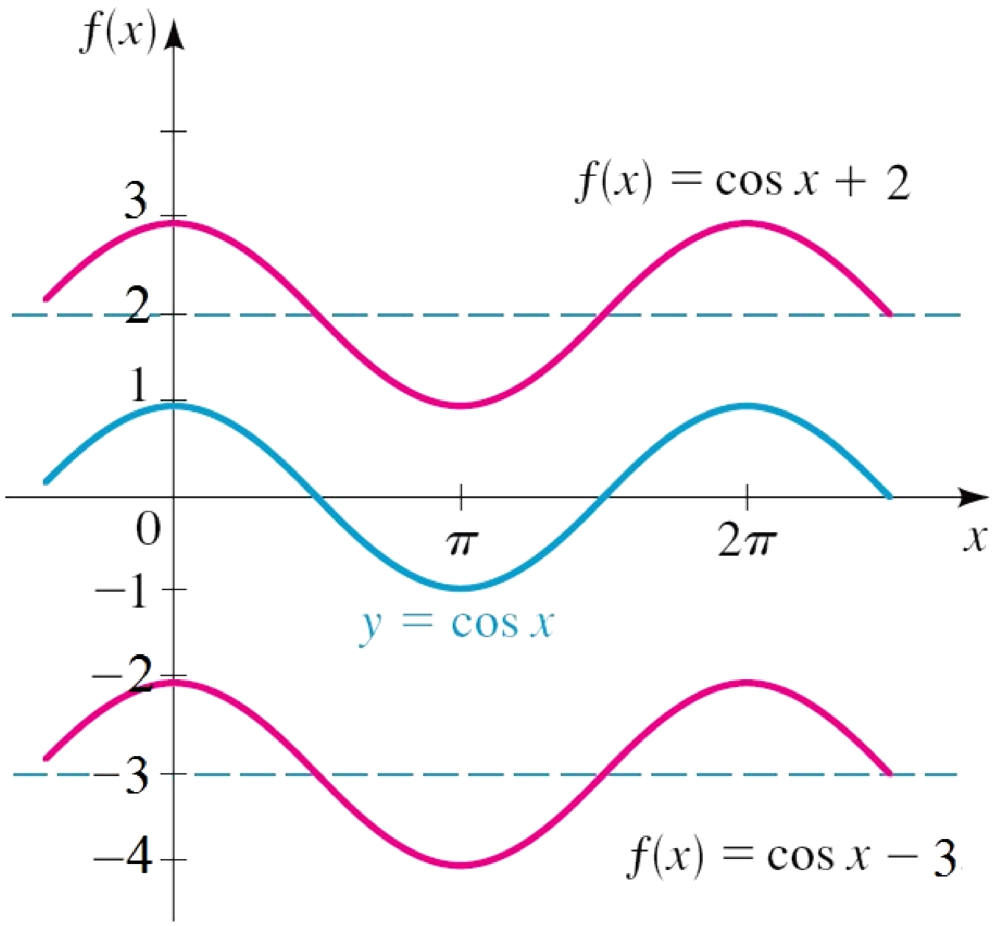


Figure (2.3): Vertical shifting

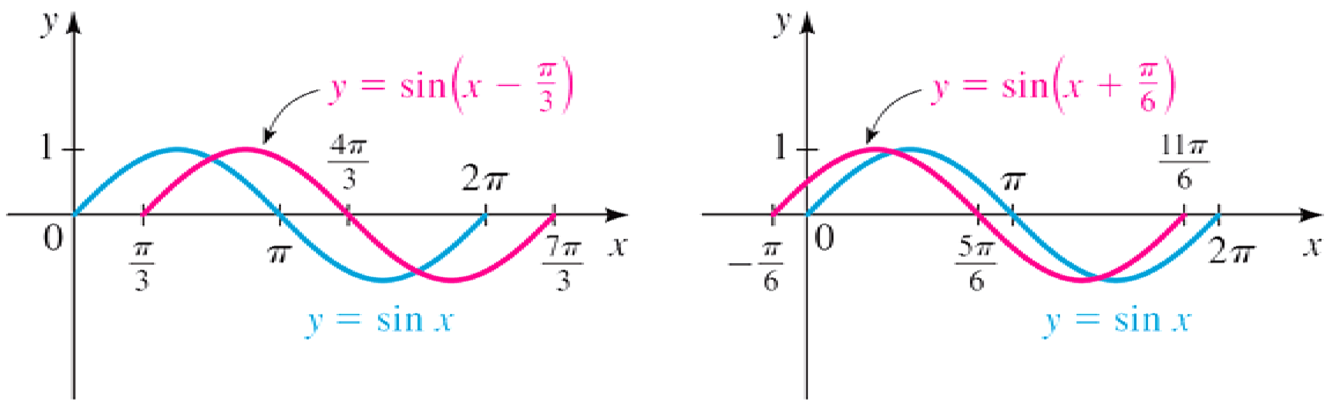
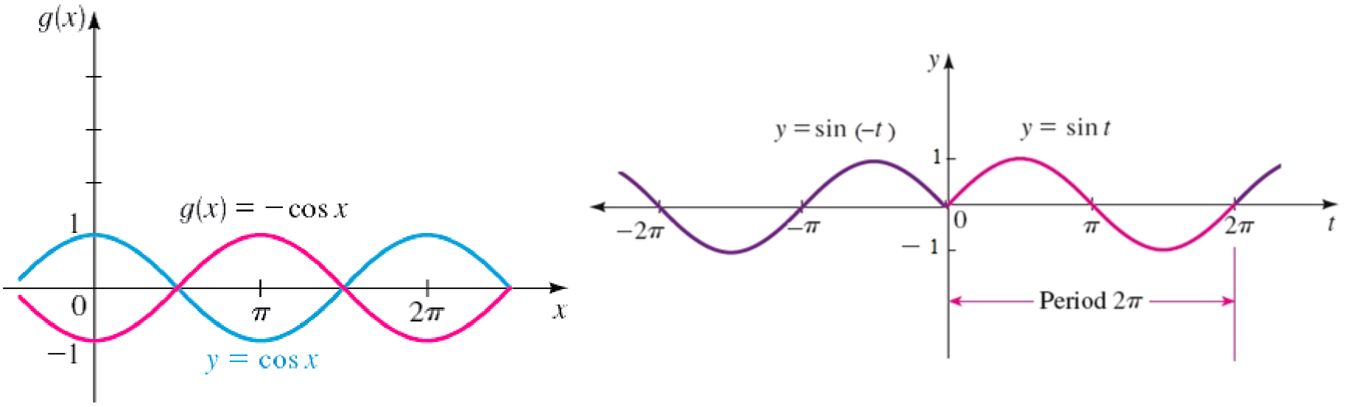


Figure (2.4): Horizontal shifting

Reflection of functions

Reflection about x-axis: $y = -f(x)$

Reflection about y-axis: $y = f(-x)$



(a) x-axis reflection **(b) y-axis reflection**

Figure (2.5): Reflection of functions

Stretching Functions

Vertical stretching (changing amplitude a units): $y = a.f(x)$

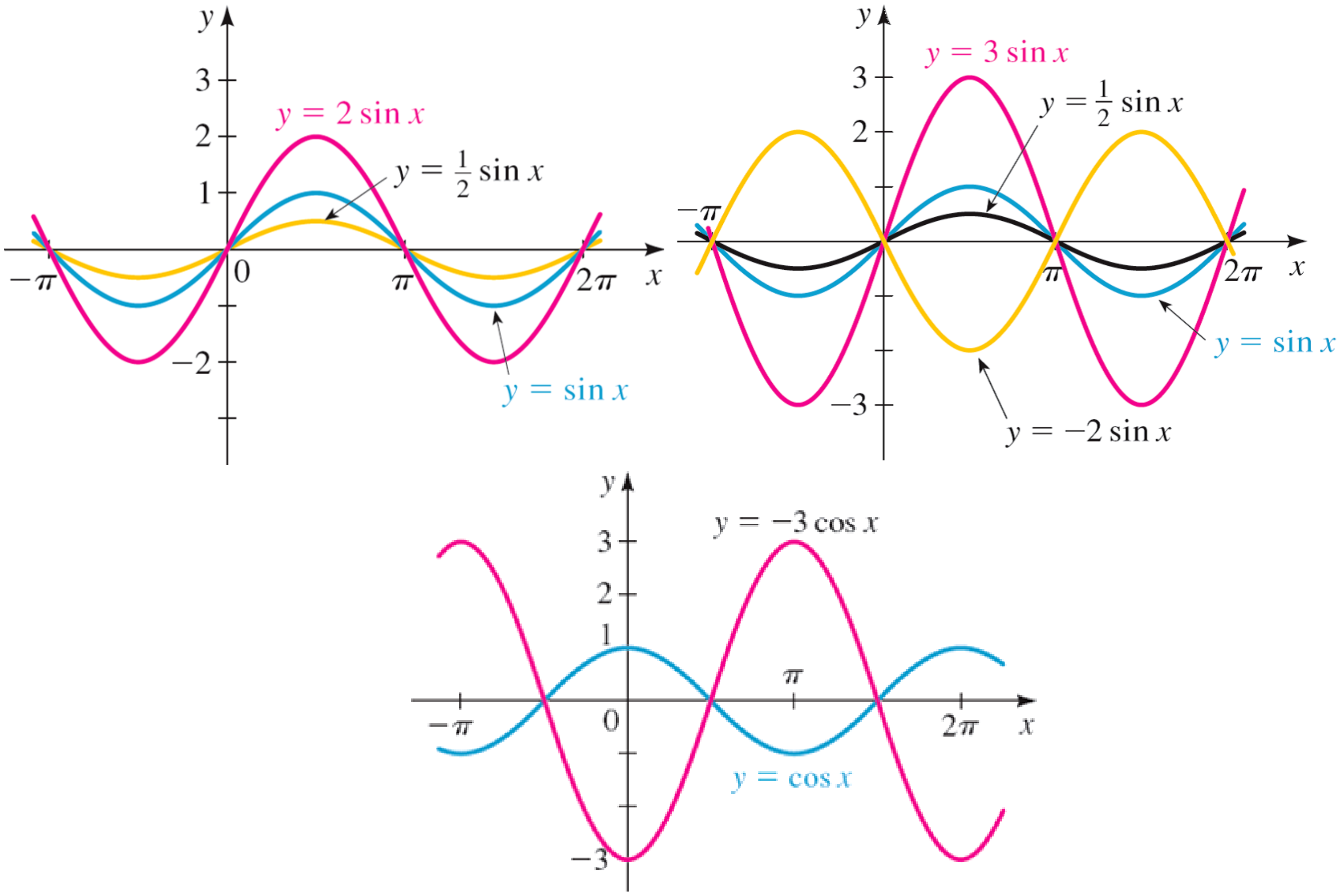


Figure (2.6): Vertical stretching

Horizontal stretching (changing phase α units): $y = f(\alpha.x)$

Note: The period = $\dfrac{x}{\alpha}$

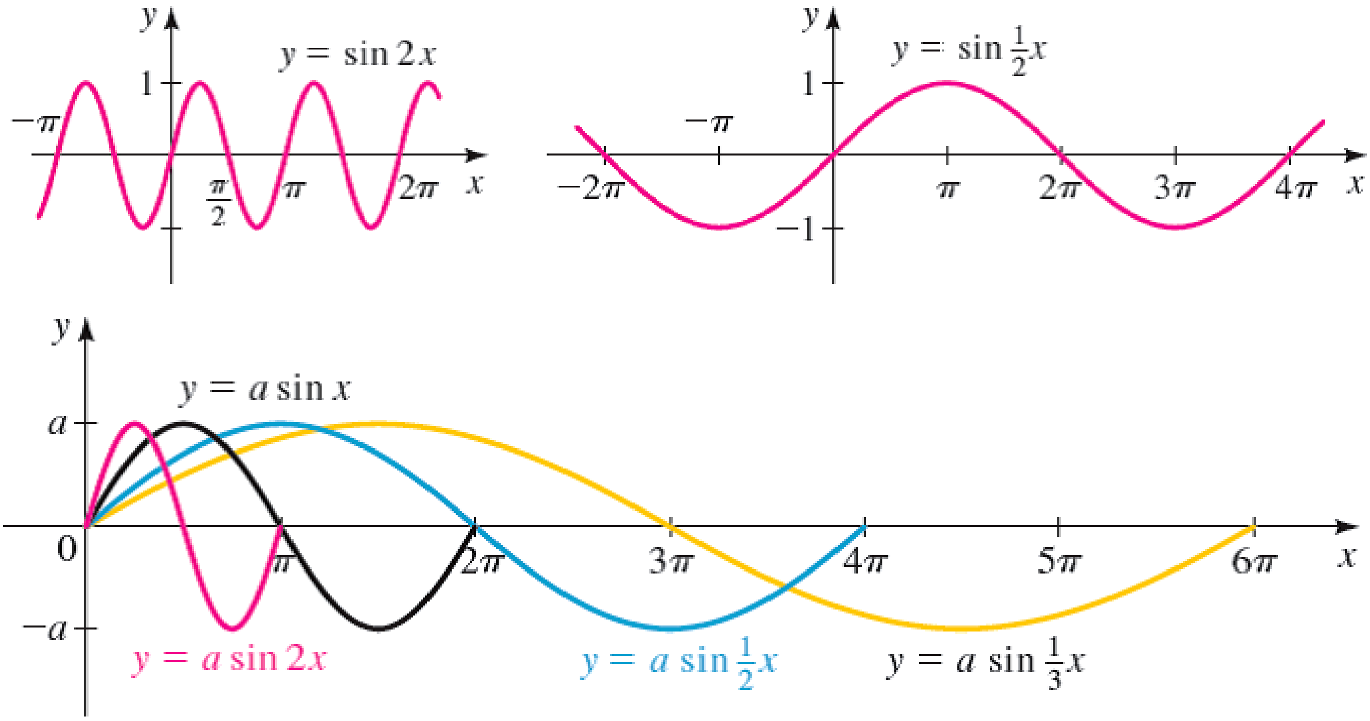


Figure (2.7): Horizontal stretching

Example 1: Find the amplitude and period of each function, and sketch its graph.

(a) y= 4cos3x (b) y= $-2\sin\frac{1}{2}$ x

Solution:

(a) We get the amplitude and period from the form of the function as follows:

amplitude $= |a| = 4$

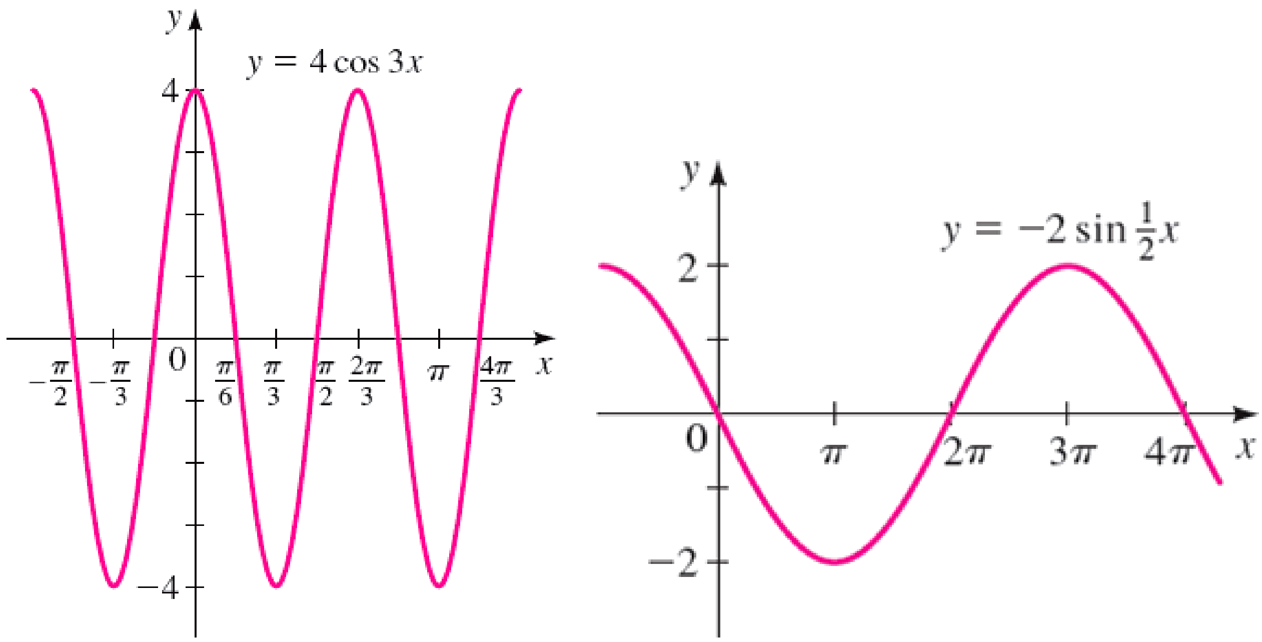
$$y = 4 \cos 3x$$

period $= \dfrac{2\pi}{k} = \dfrac{2\pi}{3}$

(b) For $y = -2 \sin \frac{1}{2}x$,

$$\text{amplitude} = |a| = |-2| = 2$$

$$\text{period} = \frac{2\pi}{\frac{1}{2}} = 4\pi$$



(a) y= 4cos3x (b) y= $-2\sin \frac{1}{2}$ x

Figure (2.8)

Shifted Sine and Cosine Curves

The sine and cosine curves

$$y = a \sin k(x - b) \qquad \text{and} \qquad y = a \cos k(x - b) \qquad (k > 0)$$

have amplitude $|a|$, period $2\pi/k$, and phase shift b.

An appropriate interval on which to graph one complete period is $[b, b + (2\pi/k)]$.

Example 2: Find the amplitude, period and phase shift of each function, and sketch its graph.

(a) y= $3\sin 2\left(x-\frac{\pi}{4}\right)$ (b) y= $\frac{3}{4}\cos\left(2x-\frac{2\pi}{4}\right)$

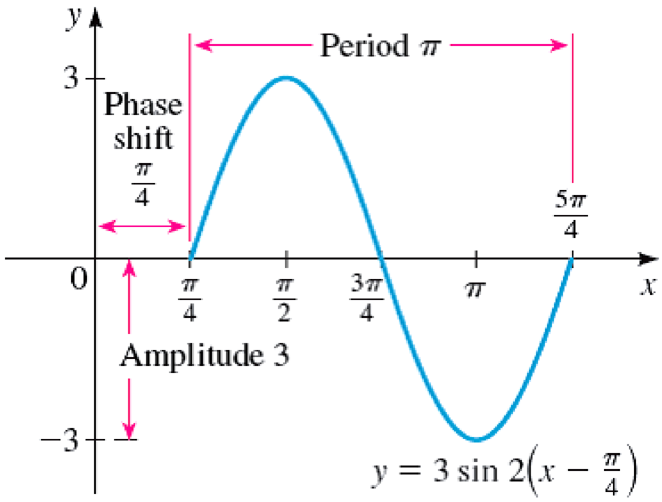
Solution:

(a)

$\text{amplitude} = |a| = 3$ $\text{period} = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$

$$y = 3\sin 2\left(x - \frac{\pi}{4}\right)$$

$\text{phase shift} = \frac{\pi}{4}$ (to the right)



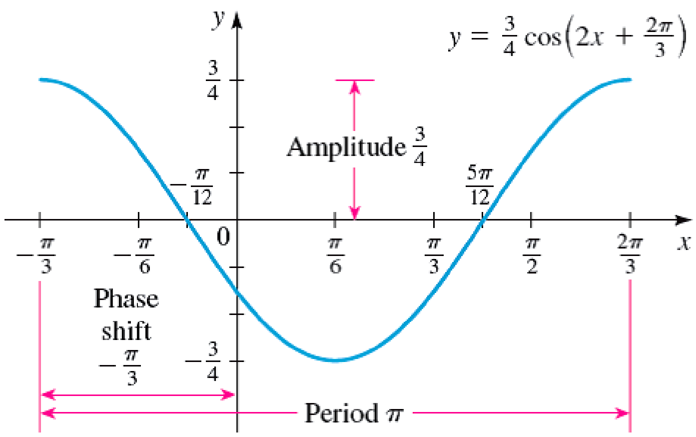
(b)

$$\text{amplitude} = |a| = \frac{3}{4}$$

$$\text{period} = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$$

$$\text{phase shift} = b = -\frac{\pi}{3}$$

Shift $\frac{\pi}{3}$ to the left



Useful Trigonometric identities

TRIGONOMETRY

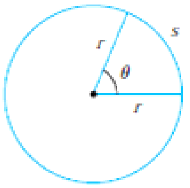
Angle Measurement

π radians $= 180°$

$1° = \dfrac{\pi}{180}$ rad $\qquad 1$ rad $= \dfrac{180°}{\pi}$

$s = r\theta$

(θ in radians)

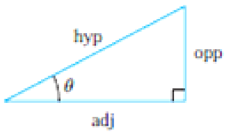


Right Angle Trigonometry

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} \qquad \cot\theta = \dfrac{\text{adj}}{\text{opp}}$

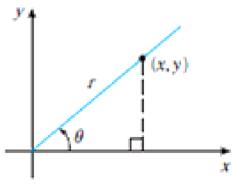


Trigonometric Functions

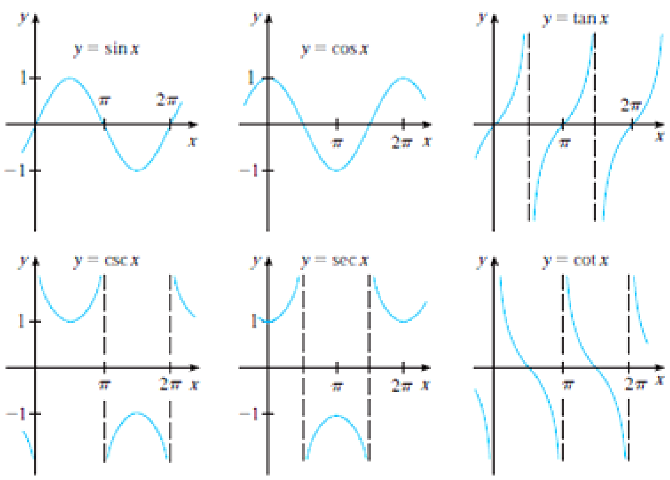
$\sin\theta = \dfrac{y}{r} \qquad \csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r} \qquad \sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x} \qquad \cot\theta = \dfrac{x}{y}$



Graphs of Trigonometric Functions



Trigonometric Functions of Important Angles

θ	radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	—

Fundamental Identities

$\csc\theta = \dfrac{1}{\sin\theta} \qquad \sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} \qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\cot\theta = \dfrac{1}{\tan\theta} \qquad \sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$

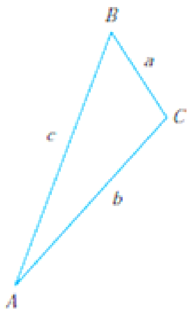
$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta \qquad \sin\left(\dfrac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta \qquad \tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta$

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



The Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Addition and Subtraction Formulas

$\sin(x + y) = \sin x\cos y + \cos x\sin y$

$\sin(x - y) = \sin x\cos y - \cos x\sin y$

$\cos(x + y) = \cos x\cos y - \sin x\sin y$

$\cos(x - y) = \cos x\cos y + \sin x\sin y$

$\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x\tan y}$

$\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x\tan y}$

Double-Angle Formulas

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

Half-Angle Formulas

$\sin^2 x = \dfrac{1 - \cos 2x}{2} \qquad \cos^2 x = \dfrac{1 + \cos 2x}{2}$

Lecture (3)

Triangular Waves

Deriving Triangular and Line Functions

f(t) =m.t + b

Where:

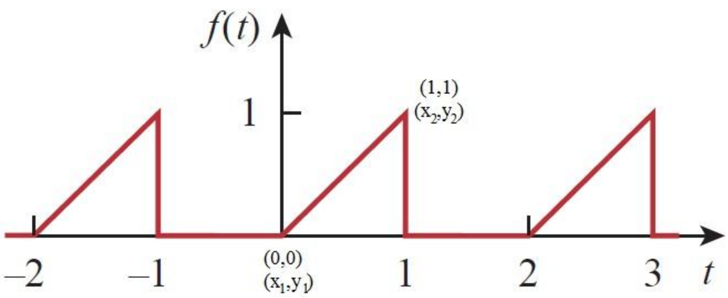
m = slop of lines (m = $\frac{y_2 - y_1}{x_2 - x_1}$)

b = intersection point of line with y-axis

Example 1: Find the function of the following wave form.

Solution: Time period (T = 2)

f(t) =m.t + b



1) For 0 < t < 1

m = $\frac{y_2 - y_1}{x_2 - x_1} = = \frac{1-0}{1-0} = 1,$ b=0

∴ f(t) = t

2) For 1 < t < 2

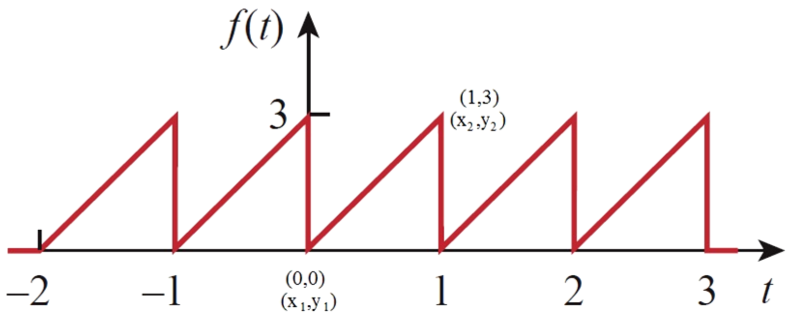
∴ f(t) = 0

$$\therefore f(t) = \begin{cases} t & 0 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$$

Example 2: Find the function of the following wave form.

Solution: Time period (T = 1)

f(t) =m.t + b



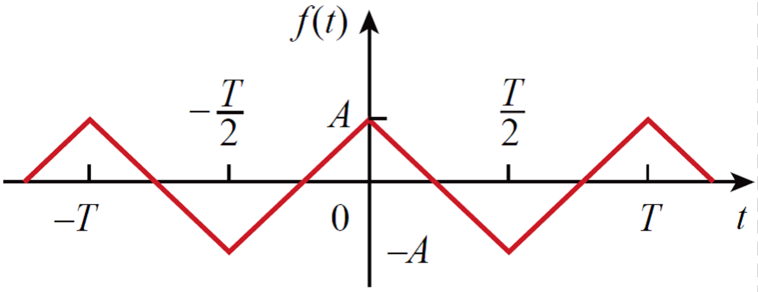
For 0 < t < 1

m = $\frac{y_2 - y_1}{x_2 - x_1} = = \frac{3-0}{1-0} = 3,$ b=0

$\boxed{\therefore f(t) = 3t}$

Example 3: Find the function of the following wave form.

Solution: Time period (T = T)



f(t) =m.t + b

1) For $0 < t < \frac{T}{2}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = = \frac{-A-A}{\frac{T}{2} - 0} = \frac{-4A}{T},$$

b = A

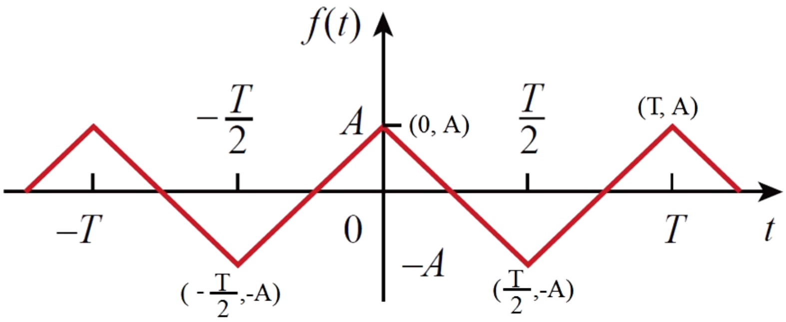
$$\boxed{\therefore\ f_1(t) = \frac{-4A}{T}\ t + A}$$

2) For $-\frac{T}{2} < t < 0$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = = \frac{-A-A}{-\frac{T}{2} - 0} = \frac{4A}{T}$$

b = A

$$\boxed{\therefore\ f_2(t) = \frac{4A}{T}\ t + A}$$



Note:- the function $f_2(t)$ is the reflection of function $f_1(t)$ about y-axis and vice versa, so you can find any reflect function about y-axis directly using this prosperity,

f(t) = f(-t)

putting minus sign(-) in front the x-axis variable (t) in the equation.

So,

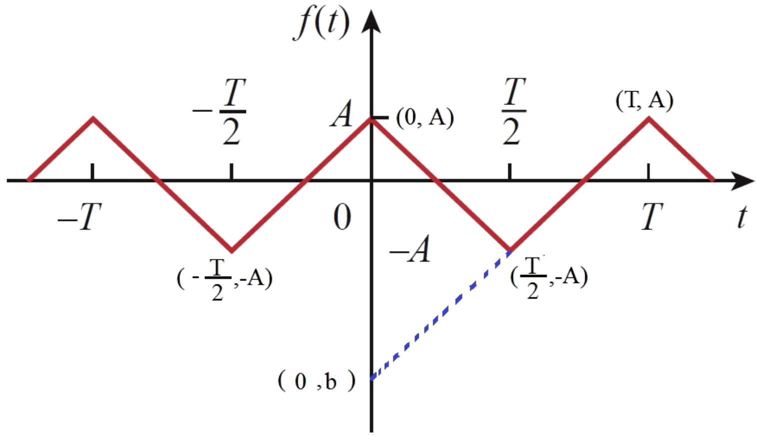
$$\boxed{f_1(t) = \frac{-4A}{T}\ t + A} \quad \& \quad \boxed{f_2(t) = f_1(-t) = \frac{-4A}{T}\ (-t) + A = \frac{4A}{T}\ t + A}$$

$$\therefore\ f(t) = \begin{cases} \frac{-4A}{T}\ t + A & 0 < t < \frac{T}{2} \\ \frac{4A}{T}\ t + A & -\frac{T}{2} < t < 0 \end{cases}$$

Alternative Solution:-

Time period (T = T)

f(t) =m.t + b



1) For $0 < t < \frac{T}{2}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = = \frac{-A-A}{\frac{T}{2} - 0} = \frac{-4A}{T}$$

b = A

$$\boxed{\therefore\ f_3(t) = \frac{-4A}{T}\ t + A}$$

3) For $\frac{T}{2} < t < T$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = = \frac{A-(-A)}{T - \frac{T}{2}} = \frac{4A}{T},$$

To find (b) in this case we use the slop equation a gain,

$$m = \frac{y_2 - y_0}{x_2 - x_0} \quad \text{or} \quad m = \frac{y_1 - y_0}{x_1 - x_0}$$

$$\therefore \frac{4A}{T} = \frac{A-b}{T-0} \Longrightarrow b = -3A$$

$$\boxed{\therefore\ f_4(t) = \frac{4A}{T}\ t - 3A}$$

$$\boxed{\therefore\ f(t) = \begin{cases} \frac{-4A}{T}\ t + A & 0 < t < \frac{T}{2} \\ \frac{4A}{T}\ t - 3A & \frac{T}{2} < t < T \end{cases}}$$

Note:- the function $f_4(t)$ is the right shifting on x-axis of function $f_2(t)$ and vice versa, so you can find any shifted function on x-axis directly using this prosperities,

(1) Left shift (shifting function **a** unit to the left) f(x) = f(x+a)

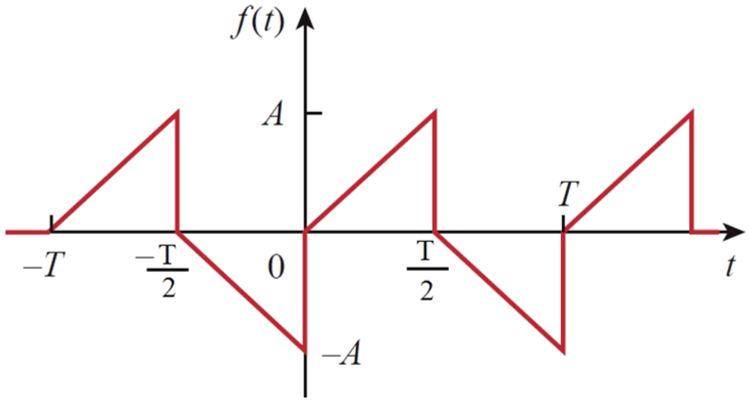
(2) Right shift (shifting function **a** unit to the right) f(x) = f(x −a)

So,

$$\boxed{f_2(t) = \frac{4A}{T}\ t + A \quad \&\ f_4(t) = f_2(t - T) = \frac{4A}{T}\ (t - T) + A = \frac{4A}{T}\ t - 3A}$$

Example 4: Find the function of the following wave form.

Solution: Time period (T = T)



f(t) =m.t + b

1) For $0 < t < \frac{T}{2}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = = \frac{A-0}{\frac{T}{2} - 0} = \frac{2A}{T},$$

b = 0

$$\boxed{\therefore\ f_1(t) = \frac{2A}{T}\ t}$$

2) For $\frac{-T}{2} < t < 0$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = = \frac{0-(-A)}{-\frac{T}{2} - 0} = \frac{-2A}{T}$$

b = $-A$

$$\boxed{\therefore\ f_2(t) = \frac{-2A}{T}\ t - A}$$

$$\boxed{\therefore\ f(t) = \begin{cases} \frac{2A}{T}t & 0 < t < \frac{T}{2} \\ \frac{-2A}{T} - A & \frac{-T}{2} < t < 0 \end{cases}}$$

Note:- the function $f_2(t)$ is the left shifting on x-axis of function $f_1(t)$ and reflected about x-axis, so you can find any shifted function on x-axis directly using this prosperities,

(1) Left shift (shifting function **a** unit to the left) f(x) = f(x+a)

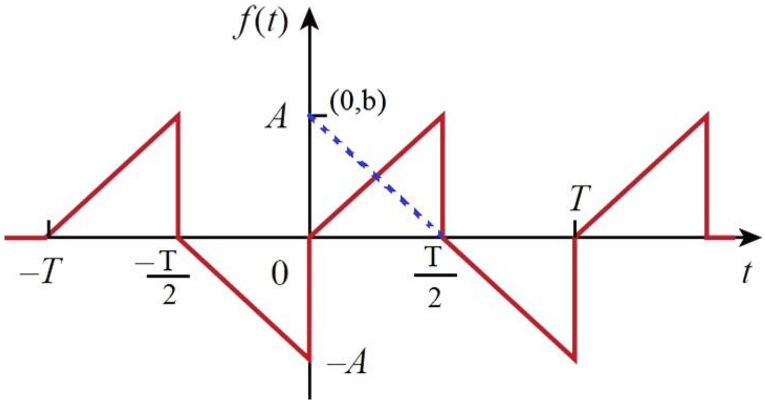
(2) X-axis reflection f(t) = –f(t)

So,

$$\boxed{f_1(t) = \frac{2A}{T}\ t \ \ \&\ \ f_2(t) = -f_1(t + \frac{T}{2}) = -\left(\frac{2A}{T}\ (t + \frac{T}{2})\right) = \frac{-2A}{T}\ t - A}$$

Alternative Solution:-

Time period (T = T)



f(t) =m.t + b

1) For $0 < t < \frac{T}{2}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = = \frac{A - 0}{\frac{T}{2} - 0} = \frac{2A}{T},$$

b = 0

$$\boxed{\therefore\ f_3(t) = \frac{2A}{T}\ t}$$

2) For $\frac{T}{2} < t < T$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = = \frac{-A - 0}{T - \frac{T}{2}} = \frac{-2A}{T}$$

To find (b) in this case we use the slop equation a gain,

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

$$\therefore \frac{-2A}{T} = \frac{-A - b}{T - 0} \Longrightarrow b = A$$

$$\boxed{\therefore\ f_4(t) = \frac{-2A}{T}\ t + A}$$

Note:- the function $f_4(t)$ is the right shifting on x-axis of function $f_3(t)$ and reflected about x-axis, so you can find any shifted function on x-axis directly using this prosperities,

(3) Right shift (shifting function **a** unit to the right) f(x) = f(x − a)

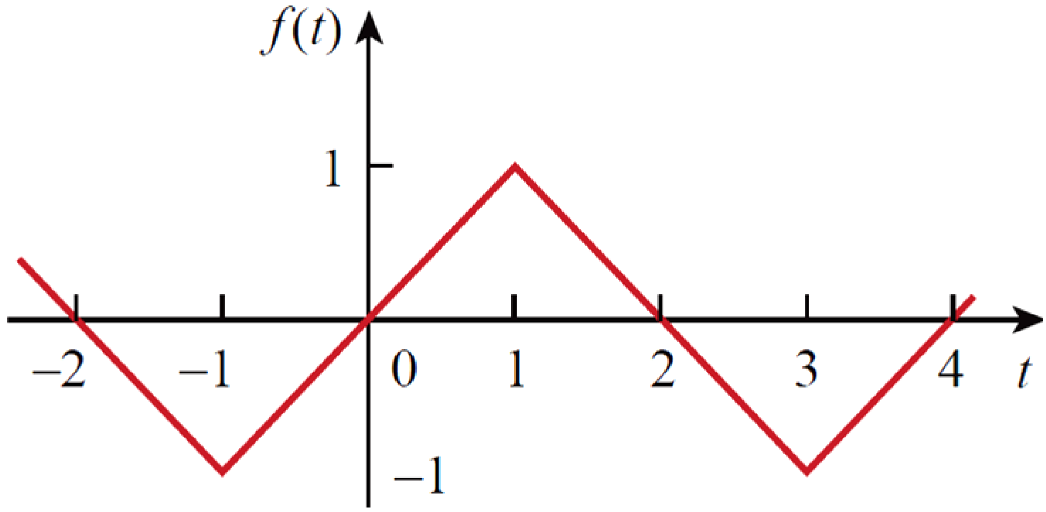
(4) X-axis reflection f(t) = –f(t)

So,

$$\boxed{f_3(t) = \frac{2A}{T}\ t \ \ \&\ \ f_4(t) = -f_3(t - \frac{T}{2}) = -\left(\frac{2A}{T}\ (t - \frac{T}{2})\right) = \frac{-2A}{T}\ t + A}$$

H.W.: Find the function of the following wave form.

Answer [period T = 4, f(t) = t , -1< t < 1 & f(t) = -t+ 2 , 1< t < 3



Lecture (4)

Root-Mean-Square (R.M.S.) & Average Values

Root-Mean-Square (R.M.S.) Value

The r.m.s. value of an alternating current ***is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same* heat *as produced by the alternating current when flowing through the same circuit for the same time.*** It is also known as the ***effective*** or ***virtual*** value of the alternating current, the former term being used more extensively.

For computing the r.m.s. value of symmetrical sinusoidal alternating currents,

1) Graphical method or mid-ordinate method; This can be used for an alternating current having any wave form i.e. sinusoidal, triangular, square, etc.
2) Analytical Method: This is to be used for purely sinusoidally varying alternating current.
3) Area method.

Importance of R.M.S. Value

1) In case of alternating quantities, the r.m.s. values are used for specifying magnitudes of alternating quantities.
2) The ammeters and voltmeters record the r.m.s. values of current and voltage respectively.
3) The heat produced due to a.c. is proportional to square of the r.m.s. value of the current.

Note: In practice r.m.s. values are used to analyze alternating quantities. If the voltage and current doesn't specified, it represent r.m.s. values.

1- Mid-ordinate Method:

In **Figure (4.1)** are shown the positive half cycles for both symmetrical sinusoidal and non-sinusoidal alternating currents.

The r.m.s. value of the squares of the instantaneous currents is given by the expression,

$$\therefore \quad I = \sqrt{\left(\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n}\right)}$$

Similarly, the r.m.s. value of alternating voltage is given by the expression,

$$V = \sqrt{\left(\frac{v_1^2 + v_2^2 + \ldots + v_n^2}{n}\right)}$$

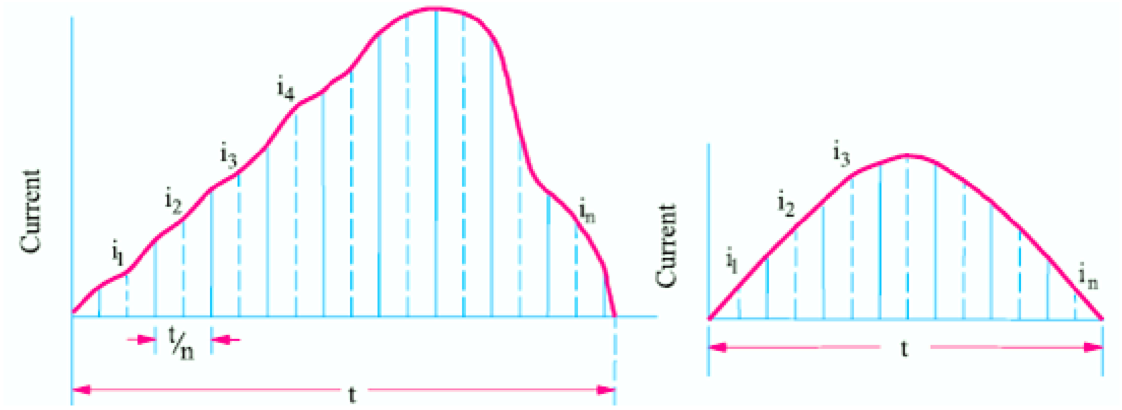


Figure (4.1)

2- Analytical Method

The standard form of a sinusoidal alternating current is,

$$i = I_m \sin \omega t = I_m \sin \theta.$$

The mean of the squares of the instantaneous values of current over one complete cycle is (even the value over half a cycle will do).

$$= \int_0^{2\pi} \frac{i^2 \, d\theta}{(2\pi - 0)}$$

The square root of this value is $= \sqrt{\left(\int_0^{2\pi} \frac{i^2 \, d\theta}{2\pi}\right)}$

Hence, the r.m.s. value of the alternating current is

$$I = \sqrt{\left(\int_0^{2\pi} \frac{i^2 d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m^2}{2\pi}\int_0^{2\pi} \sin^2\theta \, d\theta\right)} \qquad (\text{put } i = I_m \sin\theta)$$

Now, $\cos 2\theta = 1 - 2\sin^2\theta \quad \therefore \ \sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$\therefore \qquad I = \sqrt{\left(\frac{I_m^2}{4\pi}\int_0^{2\pi}(1-\cos 2\theta)\, d\theta\right)} = \sqrt{\left(\frac{I_m^2}{4\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{2\pi}\right)}$$

$$= \sqrt{\frac{I_m^2}{4} \quad 2} \qquad \sqrt{\frac{I_m^2}{2}} \qquad \therefore \ \mathrm{I} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

Hence, we find that for a symmetrical sinusoidal current

r.m.s. value of current = 0.707 × max. value of current

3- Area Method: In analytical method, the integral of i^2 represents the area under the i^2 waveform. Thus,

$$I_{eff} = \sqrt{\frac{\text{area under the } i^2 \text{ curve}}{\text{base}}}$$

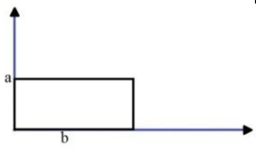
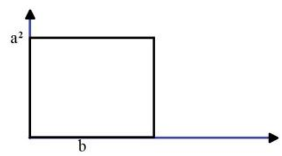
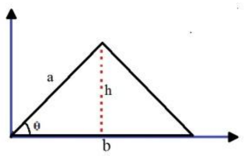
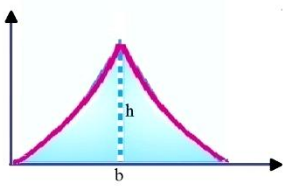
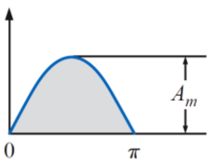
So to compute effective values using this equation, do the following:

Step 1: Square the current (or voltage) curve.
Step 2: Find the area under the squared curve.
Step 3: Divide the area by the length of the curve.
Step 4: Find the square root of the value from Step 3.

Generally:

$$\text{r.m.s. value} = \sqrt{\frac{\text{algebric sum of individual squared shapes area under curve}}{\text{lenght of curve}}}$$

Table (4.1)

	Shapes	Shapes Area	Squared Shapes	Squared Shapes Area
1	Rectangle 	= Length × width = a×b	Rectangle 	= Length × width = (a^2)×b
2	Triangle 	$= \frac{1}{2} \times$ Base × Altitude $= \frac{1}{2} \times b \times h$ $= \frac{1}{2} \times a \times b \times \sin\theta$	Parabolic curve 	$= \frac{1}{3} \times$ Base × Altitude $= \frac{1}{3} \times b \times h$
6	 Sine wave	≅ 2× amplitude ≅ 2× A_m		

R.M.S. Value of a Complex Wave

In their case also, either the mid-ordinate method (when equation of the wave is not known) or analytical method (when equation of the wave is known) may be used.
Suppose a current having the equation,

$$i = 12 \sin \omega t + 6 \sin (3\omega t - \pi/6) + 4 \sin (5\omega t + \pi/3)$$

Then, the r.m.s. value is:

Fundamental .... $(12/\sqrt{2})^2$

3rd harmonic .... $(6/\sqrt{2})^2$

5th harmonic .... $(4/\sqrt{2})^2$

$$I = \sqrt{[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]} = 9.74 \text{ A}$$

If there is a direct current of (say) 5 amperes flowing in the circuit also, the equation of the complex wave, in that case, would be,

$$i = 5 + 12 \sin \omega t + 6 \sin (3\omega t - \pi/6) + 4 \sin (5\omega t + \pi/3)$$

then the r.m.s. value would have been

$$= \sqrt{(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2 + 5^2} = 10.93 \text{ A}$$

Hence, for complex waves the r.m.s. value of a complex current wave is equal to the square root of the sum of the squares of the r.m.s. values of its individual components.

Average Value

The average value I_a of an alternating current is expressed by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.

In the case of a symmetrical alternating current (i.e. one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only. But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.

Importance of Average Value

1) The average value is used for applications like battery charging.
2) The charge transferred in capacitor circuits is measured using average values.
3) The average values of voltages and currents play an important role in analysis the rectifier circuits.
4) The average value is indicated by d.c. ammeters and voltmeters.
5) The average value of purely sinusoidal waveform is always zero.

1- Mid-ordinate Method:

$$I_{av} = \frac{i_1 + i_2 + \dots\dots + i_n}{n}$$

$$V_{av} = \frac{V_1 + V_2 + \dots\dots + V_n}{n}$$

2- Analytical Method

The standard equation of an alternating current is, $i = I_m \sin\theta$

$$I_{av} = \int_0^{\pi} \frac{i\,d\theta}{(\pi - 0)} = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta\, d\theta \qquad \text{(putting value of } i)$$

$$= \frac{I_m}{\pi}\left| -\cos\theta \right|_0^{\pi} = \frac{I_m}{\pi}\left| +1 - (-1) \right| = \frac{2I_m}{\pi} = \frac{I_m}{\pi/2} = \frac{\text{twice the maximum current}}{\pi}$$

$$I_{av} = I_m / \tfrac{1}{2}\pi = 0.637\, I_m$$

∴ **Average value of current = 0.637 × maximum value**

Note. R.M.S. value is always greater than average value except in the case of a rectangular wave when both are equal.

3- Area Method

$$\text{Average value} = \frac{\text{algebric sum of individual area under curve}}{\text{lenght of curve}}$$

Form Factor

It is defined as the ratio, $K_f = \dfrac{\text{r.m.s. value}}{\text{average value}} = \dfrac{0.707\, I_m}{0.637\, I_m} = 1.1.$ (for sinusoidal alternating currents only)

In the case of sinusoidal alternating voltage also, $K_f = \dfrac{0.707\, E_m}{0.637\, E_m} = 1.11$

As is clear, the knowledge of form factor will enable the r.m.s. value to be found from the arithmetic mean value and *vice-versa.*

Crest or Peak or Amplitude Factor

It is defined as the ratio $K_a = \dfrac{\text{maximum value}}{\text{r.m.s. value}} = \dfrac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$ (for sinusoidal a.c. only)

For sinusoidal alternating voltage also, $K_a = \dfrac{E_m}{E_m/\sqrt{2}} = 1.414$

Knowledge of this factor is of importance in dielectric insulation testing, because the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage. The knowledge is also necessary when measuring iron losses, because the iron loss depends on the value of maximum flux.

R.M.S. Value of H.W. Rectified Alternating Current

Half-wave (H.W.) rectified alternating current is one whose one half-cycle has been suppressed as shown in **Figure (4.2)** where suppressed half-cycle is shown dotted.

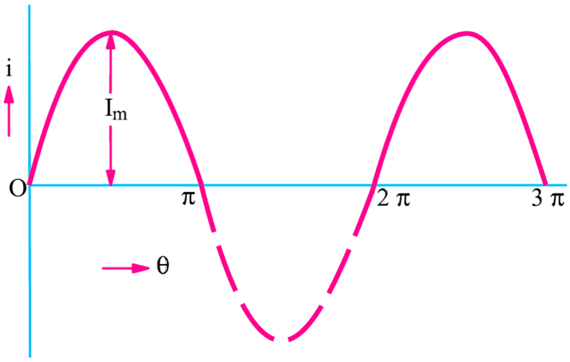


Figure (4.2)

∴ R.M.S. current

$$I = \sqrt{\left(\int_0^{\pi} \frac{i^2 d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2\theta\, d\theta\right)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{\pi}(1-\cos 2\theta)d\theta}$$

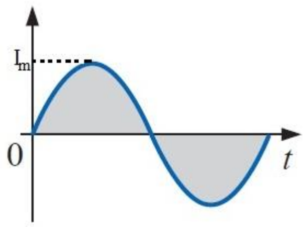
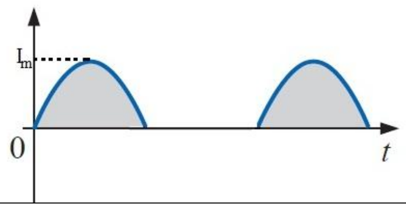
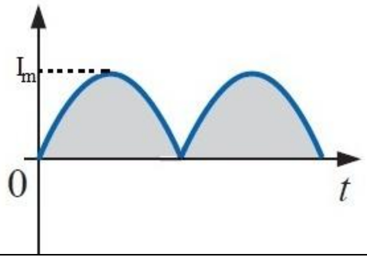
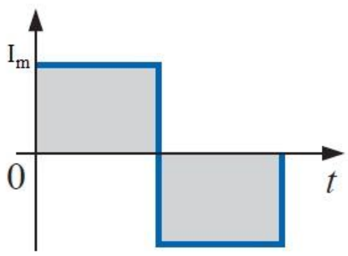
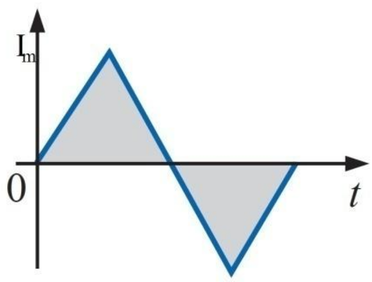
$$= \sqrt{\left(\frac{I_m^2}{4\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{\pi}\right)} = \sqrt{\left(\frac{I_m^2}{4\pi}\times\pi\right)} = \sqrt{\left(\frac{I_m^2}{4}\right)} \quad \therefore\ I = \frac{I_m}{2} = 0.5 I_m$$

Average Value of H.W. Rectified Alternating Current

$$I_{av} = \int_0^{\pi}\frac{i d\theta}{2\pi} = \frac{I_m}{2\pi}\int_0^{\pi}\sin\theta\, d\theta \qquad (\because \quad i = I_m \sin\theta)$$

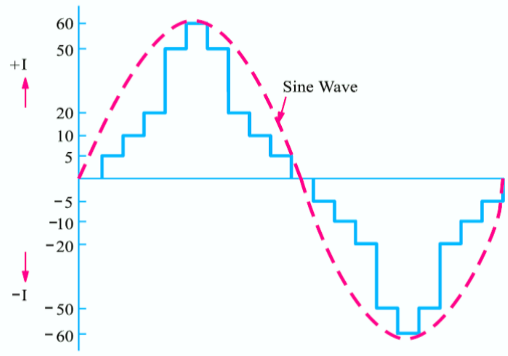
$$= \frac{I_m}{2\pi}\left|-\cos\theta\right|_0^{\pi} = \frac{I_m}{2\pi}\times 2 = \frac{I_m}{\pi}$$

Table (4.2): R.M.S & average values of some wave forms

	Wave form	R.M.S value	Average value	Form factor (K_f) $= \frac{\text{R.M.S. value}}{\text{average value}}$	Peak factor (K_a) $= \frac{\text{Max. value}}{\text{R.M.S. value}}$
1	Sine wave 	$\frac{I_m}{\sqrt{2}}$	$\frac{2I_m}{\pi}$	$K_f = \frac{(I_m/\sqrt{2})}{(2I_m/\pi)}$ $= 1.11$	$K_a = \frac{I_m}{(I_m/\sqrt{2})}$ $= 1.41$
2	Half wave rectified sine wave 	$\frac{I_m}{2}$	$\frac{I_m}{\pi}$	$K_f = \frac{(I_m/2)}{(I_m/\pi)}$ $= 1.57$	$K_a = \frac{I_m}{(I_m/2)}$ $= 2$
3	Full wave rectified sine wave 	$\frac{I_m}{\sqrt{2}}$	$\frac{2I_m}{\pi}$	$K_f = \frac{(I_m/\sqrt{2})}{(2I_m/\pi)}$ $= 1.11$	$K_a = \frac{I_m}{(I_m/\sqrt{2})}$ $= 1.41$
4	Rectangular wave 	I_m	I_m	$K_f = \frac{I_m}{I_m} = 1$	$K_a = \frac{I_m}{I_m} = 1$
5	Triangular wave 	$\frac{I_m}{\sqrt{3}}$	$\frac{I_m}{2}$	$K_f = \frac{(I_m/\sqrt{3})}{(I_m/2)}$ $= 1.16$	$K_a = \frac{I_m}{(I_m/\sqrt{3})}$ $= 1.73$

Example 1: Calculate the r.m.s. value, the form factor and peak factor of a periodic voltage having the following values for equal time intervals changing suddenly from one value to the next : 0, 5, 10, 20, 50, 60, 50, 20, 10, 5, 0,−5, −10 V etc.
What would be the r.m.s value of sine wave having the same peak value ?

Solution:



$$V_{r.m.s.} = \sqrt{\frac{0^2+5^2+10^2+20^2+50^2+60^2+50^2+20^2+10^2+5^2}{10}}$$

$= \mathbf{31\ V\ (approx.)}$

$$V_{average}\ (\text{half-cycle}) = \frac{0+5+10+20+50+60+50+20+10+5}{10}$$

$= \mathbf{23\ V}$

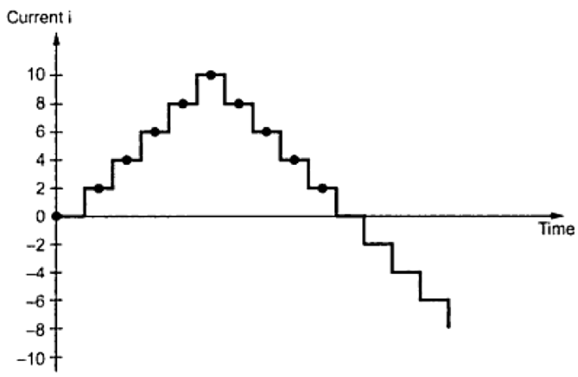
$$\text{Form factor} = \frac{\text{r.m.s.value}}{\text{average value}} = \frac{31}{23} = \mathbf{1.35}$$

$$\text{Peak factor} = \frac{\text{maximum value}}{\text{r.m.s.value}} = \frac{60}{31} = \mathbf{2}$$

R.M.S. value of a sine wave of the same peak value = 0.707 × 60 = **42.2 V.**

Example 2: Calculate the r.m.s. value, average value, form factor and peak factor of a periodic current having the following values for equal time intervals changing suddenly from one value to the next : 0, 2, 4, 6, 8, 10, 8, 6, 4, 2, 0,−2, −4,−6, −8,−10, −8,…

Solution:



$$V_{r.m.s} = \sqrt{\frac{0^2+2^2+4^2+6^2+8^2+10^2+8^2+6^2+4^2+2^2}{10}}$$

$= \mathbf{5.8309\ A\ (approx.)}$

$$V_{average} = \frac{0+2+4+6+8+10+8+6+4+2}{10}$$

$= \mathbf{5\ V}$

$$\text{Form factor} = \frac{\text{r.m.s.value}}{\text{average value}} = \frac{5.8309}{5} = \mathbf{1.1661}$$

$$\text{Peak factor} = \frac{\text{maximum value}}{\text{r.m.s.value}} = \frac{10}{5.8309} = \mathbf{1.715}$$

Example 3: Calculate the reading which will be given by a hot-wire voltmeter if it is connected across the terminals of a generator whose voltage waveform is represented by $v = 200 \sin \omega t + 100 \sin 3\omega t + 50 \sin 5\omega t$

Solution:

Since hot-wire voltmeter reads only r.m.s value, we will have to find the r.m.s. value of the given voltage. Considering one complete cycle,

R.M.S. value $$V = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2 d\theta} \quad \text{where } \theta = \omega t$$

or $$V^2 = \frac{1}{2\pi}\int_0^{2\pi} (200 \sin\theta + 100 \sin 3\theta + 50 \sin 5\theta)^2 \, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (200^2 \sin^2\theta + 100^2 \sin^2 3\theta + 50^2 \sin^2 5\theta$$
$$+ 2\times 200.100 \sin\theta . \sin 3\theta + 2\times 100.50. \sin 3\theta . \sin 5\theta$$
$$+ 2\times 50.200.\sin 5\theta . \sin\theta)\, d\theta$$

$$= \frac{1}{2\pi}\left(\frac{200^2}{2} + \frac{100^2}{2} + \frac{50^2}{2}\right) 2\pi = 26{,}250$$

$$\therefore \quad V = \sqrt{26{,}250} = 162 \text{ V}$$

Alternative Solution

The r.m.s. value of individual components are $(200/\sqrt{2})$, $(100/\sqrt{2})$ and $(50/\sqrt{2})$. Hence, as stated in Art. 11.16,

$$V = \sqrt{V_1^2 + V_2^2 + V_3^2} = \sqrt{(200/\sqrt{2})^2 + (100/\sqrt{2})^2 + (50/\sqrt{2})^2} = 162 \text{ V}$$

Example 4: Find the effective value of a resultant current in a wire which carries simultaneously a direct current of 10 A and alternating current given by,
$i = 12 \sin \omega t + 6 \sin (3\omega t - \frac{\pi}{6}) + 4 \sin (5\omega t + \frac{\pi}{3})$

Solution:

$I_{dc} = 10A$, $I_{m1} = 12A$, $I_{m2} = 6A$, $I_{m3} = 4A$

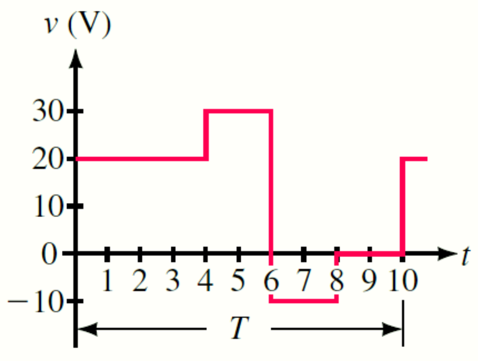
$$I_{r.m.s.} = \sqrt{10^2 + \left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = \mathbf{14.0712 \ A}$$

Example 5: One cycle of a voltage waveform is shown in Figure (a). Determine its effective (rms) value.

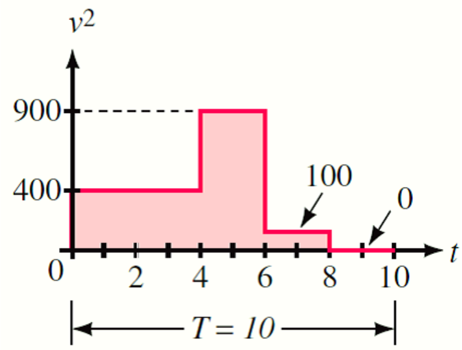
Solution:

Square the voltage waveform point by point is shown in figure (b),

$$V_{\text{eff}} = \sqrt{\frac{(400 \times 4) + (900 \times 2) + (100 \times 2) + (0 \times 2)}{10}} = \sqrt{\frac{3600}{10}} = 19.0 \text{ V}$$



(a) Voltage waveform



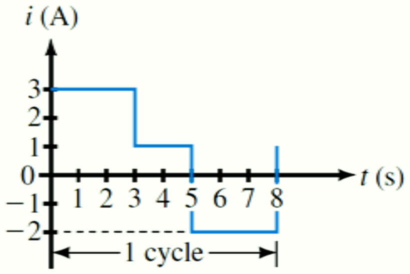
(b) Squared waveform

Example 6: Determine the effective (rms) value of the waveform shown in Figure (a).

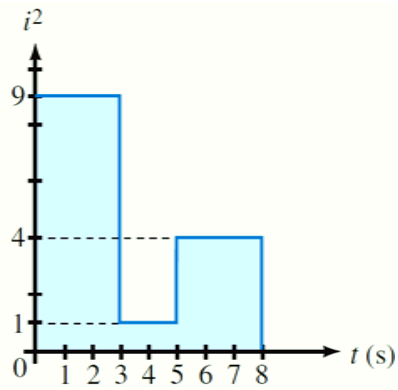
Solution:

Square the voltage waveform point by point is shown in figure (b),

$$I_{\text{eff}} = \sqrt{\frac{(9 \times 3) + (1 \times 2) + (4 \times 3)}{8}} = \sqrt{\frac{41}{8}} = 2.26 \text{ A}$$



(a)



(b)

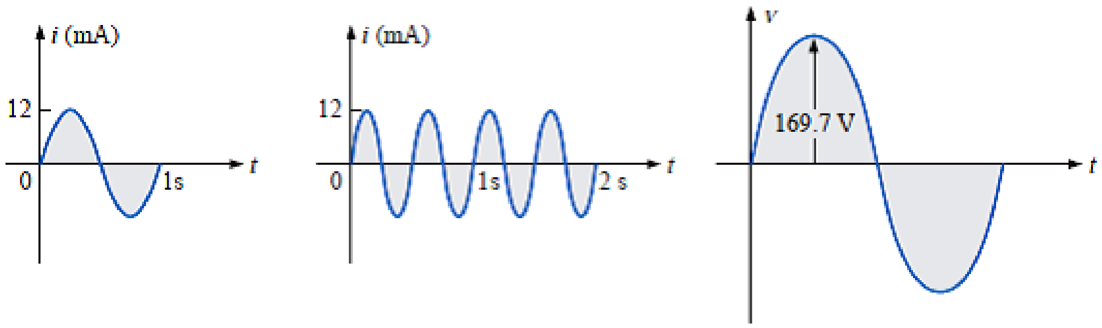
Example 7: Determine the effective (rms) value of the waveform shown in Figure.

Solution:

(a), $I_{\text{rms}} = \dfrac{I_m}{\sqrt{2}} = \dfrac{12}{\sqrt{2}} = \mathbf{8.484\ mA.}$

(b), $I_{\text{rms}} = \dfrac{I_m}{\sqrt{2}} = \dfrac{12}{\sqrt{2}} = \mathbf{8.484\ mA.}$, Note that frequency did not change the effective value

(c), $I_{\text{rms}} = \dfrac{I_m}{\sqrt{2}} = \dfrac{169.7}{\sqrt{2}} \cong \mathbf{120\ V.}$

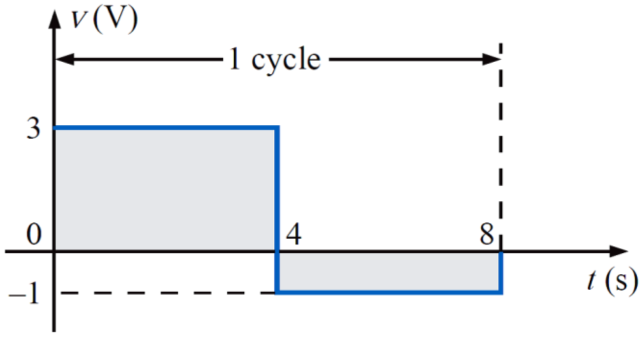


(a) (b) (c)

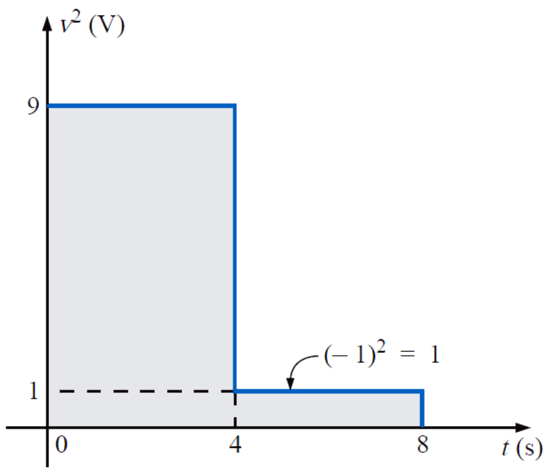
Example 8: Determine the effective (rms) value of the waveform shown in Figure (a).

Solution:

$$V_{\text{rms}} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = \mathbf{2.236\ V}$$



(a)

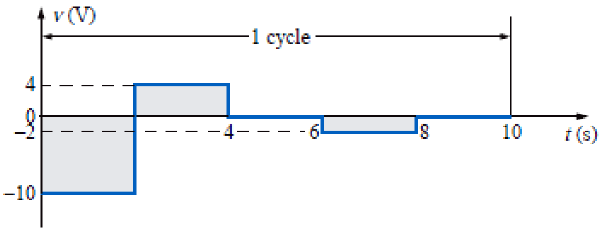


(b) squared shape

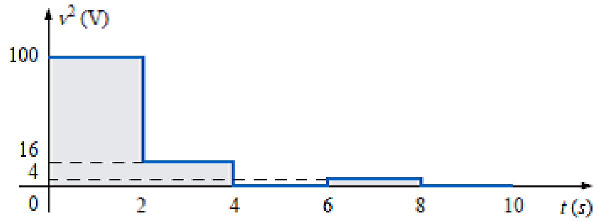
Example 9: Determine the effective (rms) value of the waveform shown in Figure (a).

Solution:

$$V_{rms} = \sqrt{\frac{(100)(2) + (16)(2) + (4)(2)}{10}} = \sqrt{\frac{240}{10}} = \mathbf{4.899\ V}$$



(a)



(b) squared shape

Example 10: Determine the average and effective (rms) value of the waveform shown in Figure (a).

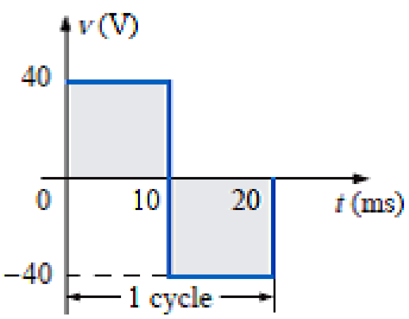
Solution:

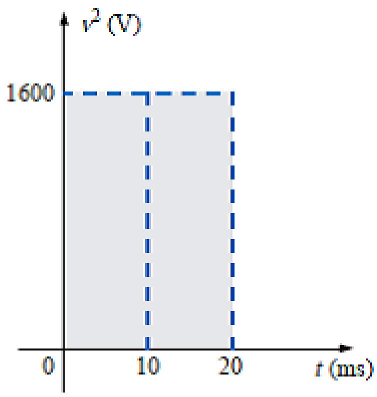
15.00).

$$V_{rms} = \sqrt{\frac{(1600)(10 \times 10^{-3}) + (1600)(10 \times 10^{-3})}{20 \times 10^{-3}}}$$

$$= \sqrt{\frac{32{,}000 \times 10^{-3}}{20 \times 10^{-3}}} = \sqrt{1600}$$

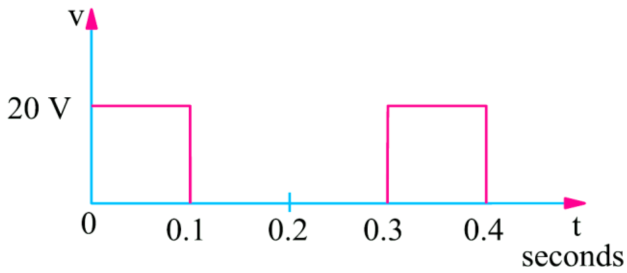
$$V_{rms} = \mathbf{40\ V}$$





Example 11: Compute the average and effective values of the square voltage wave shown.

Solution:



Time period T= 0.3 second

$$V(t) = \begin{cases} 20V & 0 < t < 0.1 \\ 0V & 0.1 < t < 0.3 \end{cases}$$

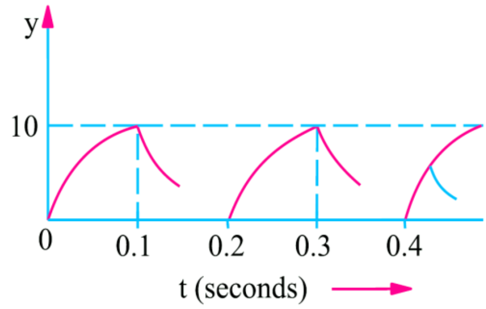
$$V_{av} = \frac{1}{T}\int_0^T v\ dt = \frac{1}{0.3}\int_0^{0.1} 20\ dt$$

$$= \frac{1}{0.3}(20 \times 0.1) = \mathbf{6.67\ V}$$

$$V = \sqrt{\frac{1}{T}\int_0^T v^2 dt} = \sqrt{\frac{1}{0.3}\int_0^{0.1} 20^2\ dt} = \sqrt{\frac{1}{0.3}(400 \times 0.1)} = \sqrt{133.3} = \mathbf{11.5\ V}$$

Example 12: Calculate the RMS value of the function shown in Figure if it is given that for $0 < t < 0.1$, $y = 10(1 - e^{-100t})$ and $0.1 < t < 0.2$, $y = 10\ e^{-50(t-0.1)}$

Solution:



Time period T= 0.2 second

$$Y(t) = \begin{cases} 10(1 - e^{-100t}) & 0 < t < 0.1 \\ 10\ e^{-50(t-0.1)} & 0.1 < t < 0.2 \end{cases}$$

$$Y^2 = \frac{1}{0.2}\left\{\int_0^{0.1} y^2 dt + \int_{0.1}^{0.2} y^2 dt\right\}$$

$$= \frac{1}{0.2}\left\{\int_0^{0.1} 10^2\ (1 - e^{-100t})^2 dt + \int_{0.1}^{0.2} (10e^{-50(t-0.1)})^2 dt\right\}$$

$$= \frac{1}{0.2}\left\{\int_0^{0.1} 100\ (1 + e^{-200t} - 2e^{-100t})dt + \int_{0.1}^{0.2} 100\ e^{-100(t-0.1)} dt\right\}$$

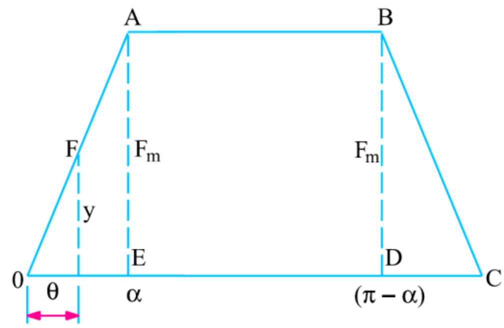
$$= 500\left\{\left[t - 0.005e^{-200t} + 0.02\ e^{-100t}\right]_0^{0.1} + \left[-0.0\ e^{-100(t-0.1)}\right]_{0.1}^{0.2}\right\}$$

$$= 500\left\{\left[\left(0.1 - 0.005e^{-20} + 0.02e^{-10}\right) - (0 - 0.005 + 0.02)\right] + \left[\left(-0.01e^{-10}\right) - (-0.01)\right]\right\}$$

$$= 500 \times 0.095 = 47.5 \quad \therefore \quad Y = \sqrt{47.5} = \mathbf{6.9}$$

Example 13: The half cycle of an alternating signal is as follows: It increases uniformly from zero at 0º to F_m at αº,remains constant from αº (180 − α)º, decreases uniformaly from F_m at (180 − α)º to zero at 180º. Calculate the average and effective values of the signal.

Solution:



average value = (π − α) F_m/π & r.m.s. value = $\boldsymbol{F_m}\sqrt{1-\frac{4\alpha}{3\pi}}$

Example 14: Find the average and r.m.s values of the a.c. voltage whose waveform is given in Figure (a).

Solution:

For finding the average value, $A_1 = 20 \times 1 = 20$ V.s, $A_2 = -5 \times 2 = -10$ V.s

Net area over the full cycle = $A_1 + A_2 = 20 - 10 = 10$ V.s.

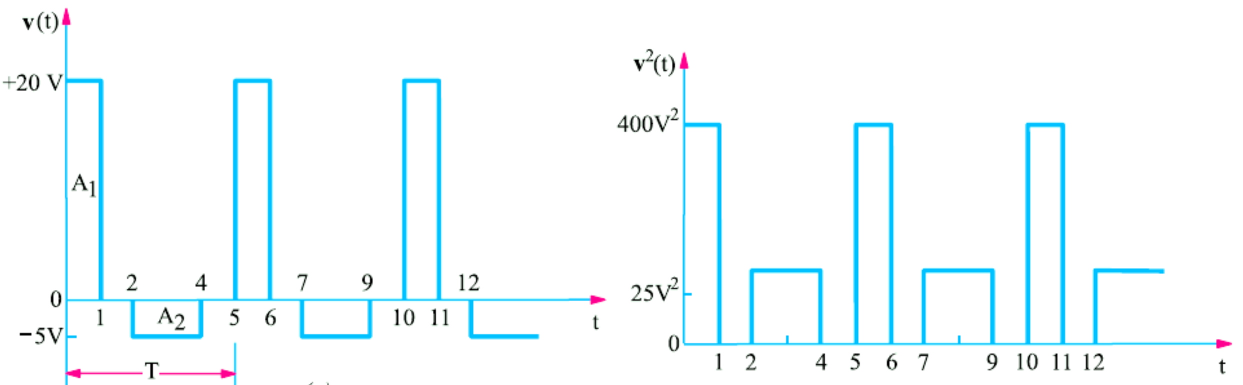
Average value = 10 V.s/5s = 2 V.

Figure (b) shows a graph of v^2 (t).

Average value of the squared area = $400\ V^2 \times 1$ s + $25\ V^2 \times 2$ s = $450\ V^2$.s.

The average value of the sum of the square = $450\ V^2$.s /5s = $90V^2$

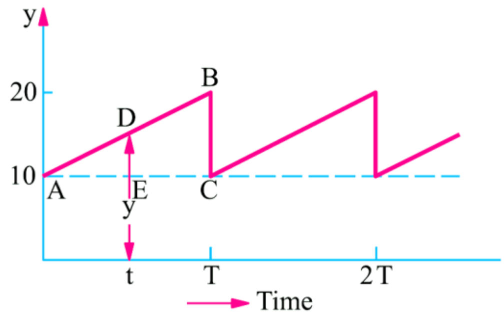
rms value = $\sqrt{90V^2}$ = 9.49 V.



(a) (b)

Example 15: What is the significance of the r.m.s and average values of a wave ? Determine the r.m.s. and average value of the waveform shown

Solution:



Time period T= T second

$$Y(t) = \frac{10}{T}t + 10$$

$$Y_{av} = \frac{1}{T}\int_0^T y\,dt = \frac{1}{T}\int_0^T \left(10 + \frac{10}{T}t\right)dt$$

$$= \frac{1}{T}\int_0^T \left[10.dt + \frac{10}{T}.t.\,dt\right] = \frac{1}{T}\left|10t + \frac{5t^2}{T}\right|_0^T = \mathbf{15}$$

$$\text{Mean square value} = \frac{1}{T}\int_0^T y^2 dt = \int_0^T \left(10 + \frac{10}{T}t\right)^2 dt$$

$$= \frac{1}{T}\int_0^T \left(100 + \frac{100}{T^2}t^2 + \frac{200}{T}t\right)dt = \frac{1}{T}$$

or RMS value = $10\sqrt{7/3}$ = **15.2**

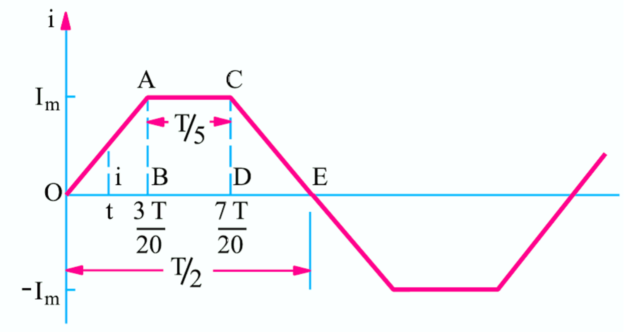
Example 16: For the trapezoidal current wave-form of Figure shown, determine the effective value.

Solution:

$i(t) = \frac{20I_m}{3T}\, t \qquad \text{for} \quad 0 < t < \frac{3T}{20}$

$i(t) = I_m \qquad \text{for} \quad \frac{3T}{20} < t < \frac{7T}{20}$

$i(t) = -\frac{20I_m}{3T}\, t + \frac{10I_m}{3} \qquad \text{for} \quad \frac{7T}{20} < t < \frac{10T}{20}$



Note: $\int_0^{\frac{3T}{20}} \left(\frac{20I_m}{3T}\, t\right) = \int_{\frac{7T}{20}}^{\frac{10T}{20}} \left(-\frac{20I_m}{3T} t + \frac{10I_m}{3}\right)$, because the integration represent the area under curve and the two areas are equals, so

$$\int_0^{\frac{3T}{20}} \left(\frac{20I_m}{3T}\, t\right) + \int_{\frac{7T}{20}}^{\frac{10T}{20}} \left(-\frac{20I_m}{3T} t + \frac{10I_m}{3}\right) = 2\int_0^{\frac{3T}{20}} \left(\frac{20I_m}{3T}\, t\right)$$

$$= 2\int_{\frac{7T}{20}}^{\frac{10T}{20}} \left(-\frac{20I_m}{3T} t + \frac{10I_m}{3}\right)$$

$$\text{RMS value of current} = \sqrt{\frac{1}{T/2}\left[2\int_0^{3T/20} i^2 dt + \int_{3T/20}^{7T/20} I_m^2 dt\right]}$$

$$= \sqrt{\frac{2}{T}\left[2\left(\frac{20I_m}{3T}\right)^2 \int_0^{3T/20} t^2 dt + I_m^2 \int_{3T/20}^{7T/20} dt\right]} = \frac{3}{5} I_m^2$$

$$\therefore \qquad I = \sqrt{(3/5)}.I_m = \mathbf{0.775\, I_m}$$

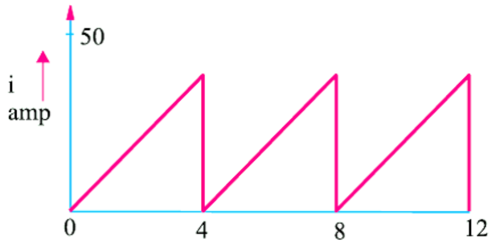
Incidentally, the average value is given by

$$I_{ac} = \frac{2}{T}\left\{2\int_0^{3T/20} i dt + \int_{3T/20}^{7T/20} I_m dt\right\} = \frac{2}{T}\left\{2\int_0^{3T/20}\left(\frac{20I_m}{3T}\right) t dt + I_m \int_{3T/20}^{7T/20} dt\right\}$$

$$= \frac{2}{T}\left\{2\left(\frac{20I_m}{3T}\right) \left| \frac{t^2}{2} \right|_0^{3T/20} + I_m \left| t \right|_{3T/20}^{7T/20}\right\} = \frac{7}{10}.\mathbf{I}_m$$

Example 17: Find the form-factor of the wave form given in fig.

Solution:



Time period T= 4 second

$$i(\mathrm{t}) = \frac{50}{4}\,\mathrm{t} \qquad \text{for } 0 < \mathrm{t} < 4$$

$$\mathrm{I_{av}} = \frac{1}{4}\int_0^4 \left(\frac{50}{4}\mathrm{t}\right).\,\mathrm{dt} = \mathbf{25\ A}$$

$$\mathrm{I_{rms}} = \sqrt{\frac{1}{4}\int_0^4 \left(\frac{50}{4}\mathrm{t}\right)^2.\,\mathrm{dt}} = \sqrt{\frac{1}{4}\times\left(\frac{50}{4}\right)^2\times\left|\frac{t^3}{3}\right|_0^4} = \mathbf{28.87\ A}$$

$$\text{For factor } \mathrm{K_f} = \frac{\text{R.M.S. value}}{\text{average value}} = \frac{28.87\ \mathrm{A}}{25\ \mathrm{A}} = \mathbf{1.1548}$$

Example 18: A half-wave rectifier which prevents current flowing in one direction is connected in series with an a.c. ammeter and a permanent-magnet moving-coil ammeter. The supply is sinusoidal. The reading on the a.c. ammeter is 10 A. Find the reading given by the other ammeter. What should be the readings on the ammeters, if the other half-wave were rectified instead of being cut off ?

Solution:

It should be noted that:-

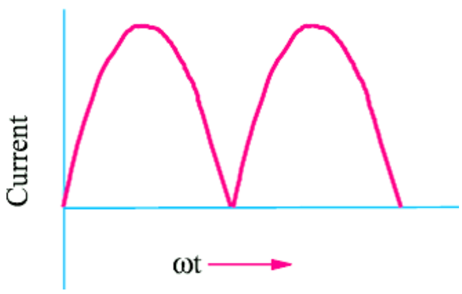
- an a.c. ammeter reads r.m.s. value .
- d.c. ammeter reads the average value of the rectified current.

H.W. rectified alternating current, $I = I_m/2$ and $I_{av} = I_m/\pi$

As a.c. ammeter reads 10 A, hence r.m.s. value of the current is 10 A.

$\therefore\ 10 = I_m/2$ or $I_m = 20$ A

$\therefore\ I_{av} = 20/\pi =$ **6.365 A** − reading of d.c. ammeter.



The full-wave rectified current wave is shown in Figure. In this case mean value of i^2 over a complete cycle is given as

$$= 2\int_0^{\pi}\frac{i^2\,d\theta}{2\pi - 0} = \frac{1}{\pi}\int_0^{\pi} I_m^2\ 2\sin^2\theta\ d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi}(1-\cos 2\theta)\,d\theta = \frac{I_m^2}{2\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{\pi} = \frac{I_m^2}{2}$$

$\therefore\quad I = I_m/\sqrt{2} = 20/\sqrt{2} = 14.14$ A $\therefore$a.c. ammeter w

$$\text{Now, } \mathrm{I_{av}} = \frac{1}{\pi}\int_0^{\pi}(\mathrm{I_m}\sin\theta).\,\mathrm{d}\theta = \frac{\mathrm{I_m}}{\pi}\left|-\cos\theta\right|_0^{\pi} = \mathbf{12.73A}$$

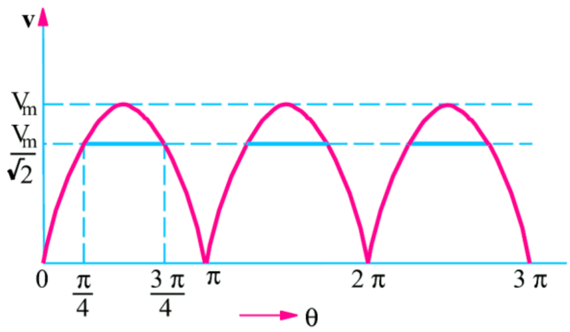
Example 19: A full-wave rectified sinusoidal voltage is clipped at $1/\sqrt{2}$ of its maximum value. Calculate the average and RMS values of such a voltage.

Solution:

$$V = v_m \sin\theta \qquad \text{for} \qquad 0 < \theta < \frac{\pi}{4}$$

$$V = \frac{v_m}{\sqrt{2}} \qquad \text{for} \qquad \frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

$$V = v_m \sin\theta \qquad \text{for} \qquad \frac{3\pi}{4} < \theta < \pi$$



$$V_{av} = \frac{1}{\pi}\left\{\int_0^{\pi/4} v\,d\theta + \int_{\pi/4}^{3\pi/4} v\,d\theta + \int_{3\pi/4}^{\pi} v\,d\theta\right\}$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi/4} V_m \sin\theta\,d\theta + \int_{\pi/4}^{3\pi/4} 0.707\,V_m d\theta + \int_{3\pi/4}^{\pi} V_m \sin\theta\,d\theta\right\}$$

$$= \frac{V_m}{\pi}\left\{|-\cos\theta|_0^{\pi/4} + 0.707\,|\theta|_{\pi/4}^{3\pi/4} + |-\cos\theta|_{3\pi/4}^{\pi}\right\} = \frac{V_m}{\pi}(0.293 + 1.111 + 0.293) = \mathbf{0.54\ V_m}$$

$$V^2 = \frac{1}{\pi}\left\{\int_0^{\pi/4} V_m^2 \sin^2\theta\,d\theta + \int_{\pi/4}^{3\pi/4} (0.707\,V_m)^2 d\theta + \int_{3\pi/4}^{\pi} V_m^2 \sin^2\theta\,d\theta\right\} = 0.341\,V_m^2$$

$$V = \mathbf{0.584\ V_m}$$

Example 20: A delayed full-wave rectified sinusoidal current has an average value equal to half its maximum value. Find the delay angle θ.

Solution:

$$I_{av} = \frac{1}{2} I_m$$

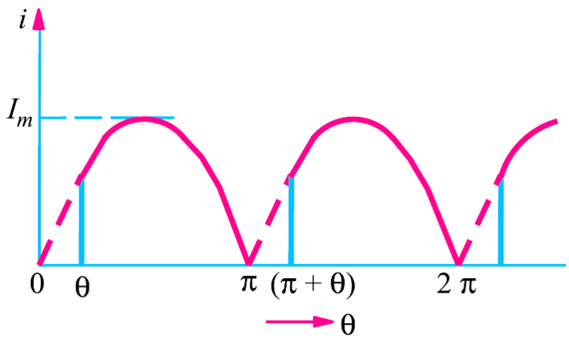
$$I_{av} = \frac{1}{\pi}\int_{\theta}^{\pi} (I_m \sin\theta)\,.\,d\theta$$

$$= \frac{I_m}{\pi}|-\cos\theta|_{\theta}^{\pi}$$

$$= \frac{I_m}{\pi}(1 + \cos\theta)$$

$$\frac{1}{2} I_m = \frac{I_m}{\pi}(1 + \cos\theta)$$

$$\therefore\ \boldsymbol{\theta = 55.25^o}$$



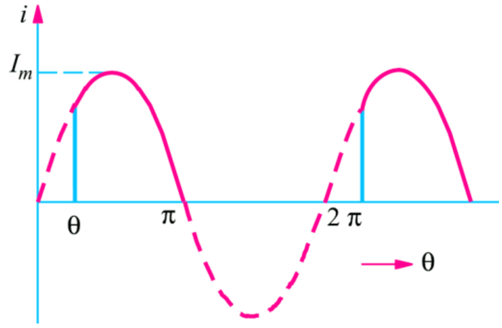
Example 21: The waveform of an output current is as shown in Figure. It consists of a portion of the positive half cycle of a sine wave between the angle θ and 180º. Determine the effective value for θ = 30º.

Solution:

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{\theta}^{\pi}(I_m \sin\theta)^2 . d\theta} = \sqrt{\frac{1}{2\pi}\int_{\frac{\pi}{6}}^{\pi}(I_m \sin\theta)^2 . d\theta}$$

$$= \sqrt{\frac{{I_m}^2}{2\pi}\int_{\frac{\pi}{6}}^{\pi}\left(\frac{1-\cos 2\theta}{2}\right). d\theta} = \sqrt{\frac{{I_m}^2}{4\pi}\left|\left(\theta - \frac{\sin 2\theta}{2}\right)\right|_{\frac{\pi}{6}}^{\pi}}$$

$$= \mathbf{0.492 I_m}$$



Example 22: Calculate the "form factor" and "peak factor" of the sine wave shown.

Solution:

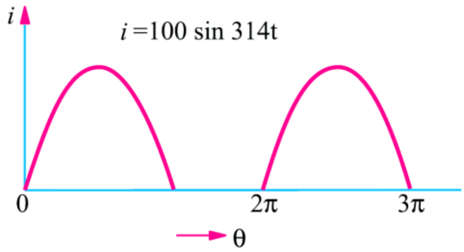
For 0 < θ < π, *i* = 100 sin θ and for π < θ < 2π, *i* = 0. The period is 2

$$\therefore 0 \qquad I_{av} = \frac{1}{2\pi}\left\{\int_0^{\pi} i\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta\right\}$$

$$= \frac{1}{2\pi}\left\{100\int_0^{\pi}\sin\theta\,d\theta\right\} = 31.8 \text{ A}$$

$$I^2 = \frac{1}{2\pi}\int_0^{\pi} i^2 d\theta = \frac{100^2}{2\pi}\int_0^{\pi}\sin^2\theta\,d\theta = \frac{100^2}{4} = 2500\,;\; I = 50\text{A}$$

∴ form factor = 50/31.8 = **1.57 ;** peak factor = 100/50 = **2**

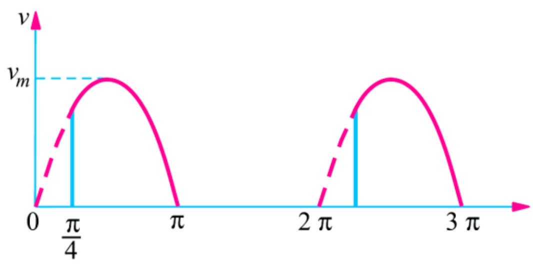


Example 23: Find the average and effective values of voltage of sinusoidal waveform shown in Figure.

Solution:

$$\therefore \qquad V_{av} = \frac{1}{2\pi}\int_{\pi/4}^{\pi} 100\sin\theta\,d\theta = \frac{100}{2\pi}\left|-\cos\theta\right|_{\pi/4}^{\pi} = 27.2 \text{ V}$$

$$V = \sqrt{\frac{1}{2\pi}\int_{\pi/4}^{\pi} 100^2\sin^2\theta\,d\theta} = \sqrt{\frac{100^2}{4\pi}\int_{\pi/4}^{\pi}(1-\cos 2\theta)\,d\theta} = \mathbf{47.7 \text{ V}}$$



Example 24: Find the r.m.s. and average values of the saw tooth waveform shown

Solution:

Graphical Method

The average value can be found by averaging the function from $t = 0$ to $t = 1$ in parts as given below :

$$\text{Average value of } (f) = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{T} \times \text{(net area over one cycle)}$$

Now, area of a right-angled triangle = (1/2) × (base) × (altitude).

Hence, area of the triangle during $t = 0$ to $t = 0.5$ second is

$$A_1 = \frac{1}{2}\times(\Delta t)\times(-2) = \frac{1}{2}\times\frac{1}{2}\times -2 = -\frac{1}{2}$$

Similarly, area of the triangle from $t = 0.5$ to $t = 1$ second is

$$A_2 = \frac{1}{2}\times(\Delta t)\times(+2) = \frac{1}{2}\times\frac{1}{2}\times 2 = \frac{1}{2}$$

Net area from $t = 0$ to $t = 1.0$ second is $A_1 + A_2 = -\frac{1}{2}+\frac{1}{2} = 0$

Hence, average value of $f(t)$ over one cycle is zero.

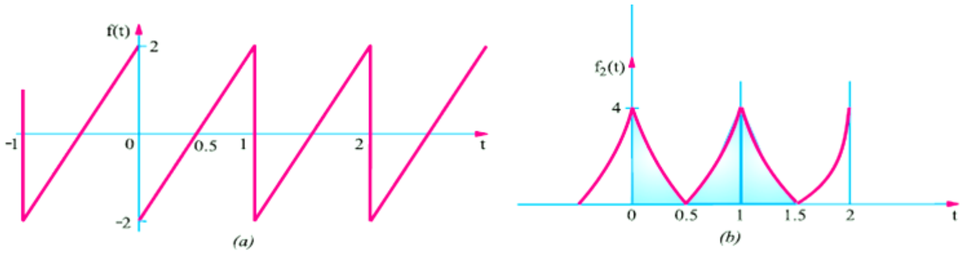
For finding the r.m.s. value, we will first square the ordinates of the given function and draw a new plot for $f^2(t)$ as shown in Fig. 11.34 (*b*). It would be seen that the squared ordinates from a parabola.

Area under parabolic curve = $\frac{1}{3}\times$ base × altitude. The area under the curve from $t = 0$ to $t = 0.5$ second is ; $A_1 = \frac{1}{3}(\Delta t)\times 2^2 = \frac{1}{3}\times\frac{1}{2}\times 4 = \frac{2}{3}$

Similarly, for $t = 0.5$ to $t = 1.0$ second $A_2 = \frac{1}{3}(\Delta t)\times 4 = \frac{1}{3}\times\frac{1}{2}\times 4 = \frac{2}{3}$

Total area $= A_1 + A_2 = \frac{2}{3}+\frac{2}{3} = \frac{4}{3}$. r.m.s. value $= \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} = \sqrt{\text{average of } f^2(t)}$

$\therefore$ r.m.s. value $= \sqrt{4/3}$ = **1.15**



(a) (b)

Analytical Method

The equation of the straight line from $t = 0$ to $t = 1$ in Fig. 11.34 (*a*) is

$$f(t) = 4t - 2;\ f^2(t) = 16t^2 - 16t + 4$$

$$\text{Average value} = \frac{1}{T}\int_0^T (4t-2)\,dt = \frac{1}{T}\left|\frac{4t^2}{2} - 2t\right|_0^T = 0$$

$$\text{r.m.s value} = \sqrt{\frac{1}{T}\int_0^T (16t^2 - 16t + 4)dt} = \sqrt{\frac{1}{T}\left|\frac{16t^3}{3} - \frac{16t^2}{2} + 4t\right|_0^T} = \mathbf{1.15}$$

Example 25: A resultant current wave is made up of two components : a 4A d.c. component and a 50-Hz a.c. component, which is of sinusoidal waveform and which has a maximum value of 4A.
(i) Draw a sketch of the resultant wave.
(ii) Write an analytical expression for the current wave, reckoning t = 0 at a point where the a.c. component is at zero value and when di/dt is positive.
(iii) What is the average value of the resultant current over a cycle ?
(iv) What is the effective or r.m.s. value of the resultant current ?

Solution:

(*i*) Sketch of the resultant wave :

The two current components and the resultant current wave are shown in Fig. **(Ans.)**

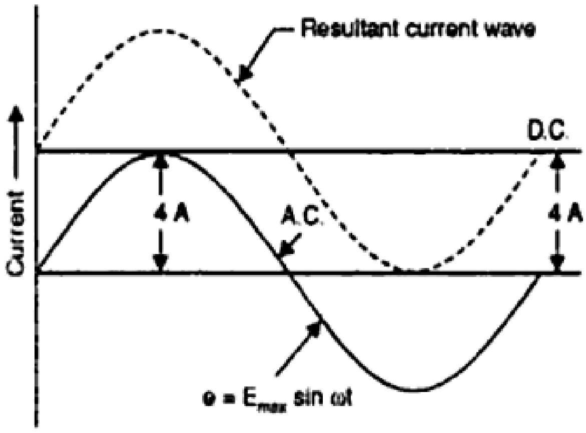


Fig.

(*ii*) **Analytical expression.** The instantaneous value of the resultant current is given by

$$i = (4 + 4 \sin \omega t) = (4 + 4 \sin \theta). \quad \textbf{(Ans.)}$$

(*iii*) **Average value.** Since the average value of the alternating current over one complete cycle is zero, hence the *average value of the resultant current is equal to the value of D.C. component i.e.,* **4A (Ans.)**

(*iv*) **Effective or r.m.s. value :**

Mean value of i^2 over complete cycle is

$$= \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta = \frac{1}{2\pi}\int_0^{2\pi} (4 + 4\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (16 + 32\sin\theta + 16\sin^2\theta)\, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \left[16 + 32\sin\theta + 16\left(\frac{1-\cos 2\theta}{2}\right)\right] d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (24 + 32\sin\theta - 8\cos 2\theta)\, d\theta$$

$$= \frac{1}{2\pi}\left(24\,\theta - 32\cos\theta - 8\times\frac{\sin 2\theta}{2}\right)_0^{2\pi}$$

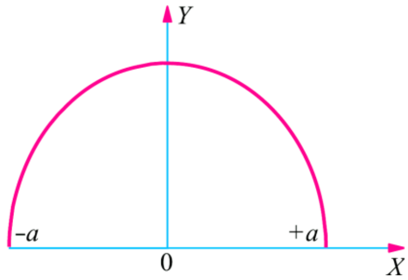
$$= \frac{1}{2\pi}[(48\pi - 32\cos 2\pi - 4\sin 4\pi) - (-32)] = \frac{48\pi}{2\pi} = 24 \text{ A}$$

$\therefore$ R.M.S. value, $\quad I = \sqrt{24} = 4.9$ A. **(Ans.)**

Example 26: Determine the r.m.s. value of a semicircular current wave which has a maximum value of a.

Solution:

The equation of a semi-circular wave is,



$$x^2 + y^2 = a^2 \quad \text{or} \quad y^2 = a^2 - x^2$$

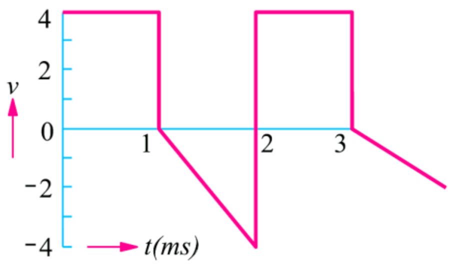
$$\therefore \quad I_{rms} = \sqrt{\frac{1}{2a}\int_{-a}^{+a} y^2 dx} \quad \text{or} \quad I_{rms}^2 = \frac{1}{2a}\int_{-a}^{+a}(a^2 - x^2)\, dx$$

$$= \frac{1}{2a}\int_{-a}^{+a}(a^2\, dx - x^2\, dx) = \frac{1}{2a}\left| a^2 x - \frac{x^3}{3}\right|_{-a}^{+a} = \frac{1}{2a}\left(a\right.$$

$$\therefore \quad I_{rms} = \sqrt{2a^2/3} = \mathbf{0.816\ a}$$

Example 27: Calculate the r.m.s. and average value of the voltage wave shown in Fig.

Solution:



Time periode T = 2

V = 4 for 0 < t < 1

V = -4t +4 for 1 < t < 2

$$\therefore v_{rms} = \sqrt{\frac{1}{2}\left(\int_0^1 v_1^2\, dt + \int_1^2 v_2^2\, dt\right)}$$

$$V_{rms}^2 = \frac{1}{2}\left[\int_0^1 4^2\, dt + \int_1^2 (-4t+4)^2\, dt\right]$$

$$= \frac{1}{2}\left[\left| 16\,t\right|_0^1 + \left|\frac{16\,t^3}{3}\right|_1^2 + \left|16\,t\right|_1^2 - \left|\frac{32\,t^2}{2}\right|_1^2\right]$$

$$= \frac{1}{2}\left[16 + \frac{16\times 8}{3} - \frac{16}{3} + 16\times 2 - 16\times 1 - \frac{32\times 4}{2} + \frac{32\times 1}{2}\right] = \frac{32}{3} \quad \therefore V_{rms} = \sqrt{32/3} = \mathbf{3.265\ volt}$$

$$V_{av} = \frac{1}{2}\left[\int_0^1 v_1 dt + \int_1^2 v_2\, dt\right] = \frac{1}{2}\left[\int_0^1 4dt + \int_1^2 (-4t+4)\, dt\right] = \frac{1}{2}\left[\left|4t\right|_0^1 + \left|-\frac{4t^2}{2} + 4t\right|_1^2\right] = \mathbf{1\ volt}$$

Example 28: Prove that if a dc current of I is superposed in a conductor by an ac current of max. value I, the rms value of the resultant is $\sqrt{\frac{3}{2}}$ I.

Solution:

Let the a current be i=I sinθ where I is the instantaneous value of the ac current and I the dc current.

The rms value of (I + i) over one complete cycle is,

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(I + I\sin\theta)^2\, d\theta} = I\sqrt{\left\{\frac{1}{2\pi}\int_0^{2\pi}\left(1 + 2\sin\theta + \left(\frac{1-\cos 2\theta}{2}\right)\right) d\theta\right\}}$$

$$= I\sqrt{\frac{1}{2\pi}\left|\theta - 2\cos\theta + \frac{\theta}{2} - \left(\frac{\sin 2\theta}{4}\right)\right|_0^{2\pi}} = I\sqrt{\frac{1}{2\pi}(2\pi - 2 + \pi + 2)} = \mathbf{I}\sqrt{\frac{3}{2}}$$

Example 29: compute the rms value of the voltage waveform shown.

Solution:

The waveform is periodic with period $T = 3$ s. The equation for the voltage in the time frame $0 \le t \le 3$ s is

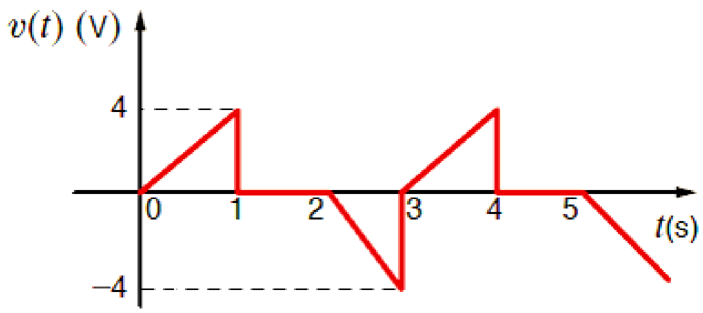
$$v(t) = \begin{cases} 4t \text{ V} & 0 < t \le 1 \text{ s} \\ 0 \text{ V} & 1 < t \le 2 \text{ s} \\ -4t + 8 \text{ V} & 2 < t \le 3 \text{ s} \end{cases}$$

The rms value is

$$V_{\text{rms}} = \left\{\frac{1}{3}\left[\int_0^1 (4t)^2\, dt + \int_1^2 (0)^2\, dt + \int_2^3 (8 - 4t)^2\, dt\right]\right\}^{1/2}$$

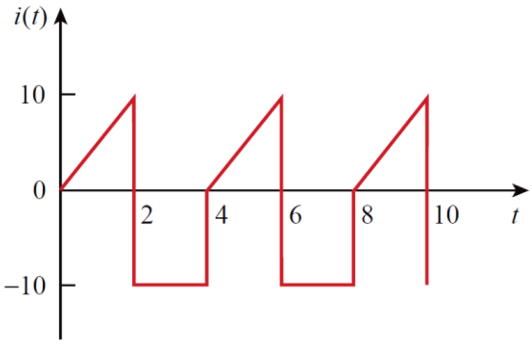
$$= \left[\frac{1}{3}\left(\frac{16t^3}{3}\bigg|_0^1 + \left(64t - \frac{64t^2}{2} + \frac{16t^3}{3}\right)\bigg|_2^3\right)\right]^{1/2}$$

$$= 1.89 \text{ V}$$



Example 30: Determine the rms value of the current waveform in Figure. If the current is passed through a resistor, find the average power absorbed by the resistor.

Solution:



The period of the waveform is $T = 4$. Over a period, we can write the current waveform as

$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

The rms value is

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2\,dt + \int_2^4 (-10)^2\,d\right.}$$

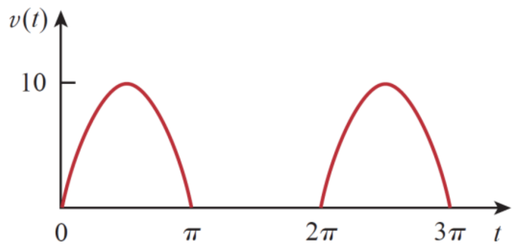
$$= \sqrt{\frac{1}{4}\left[25\frac{t^3}{3}\bigg|_0^2 + 100t\bigg|_2^4\right]} = \sqrt{\frac{1}{4}\left(\frac{200}{3} + 200\right)} =$$

The power absorbed by a 2-Ω resistor is

$$P = I_{\text{rms}}^2 R = (8.165)^2(2) = 133.3 \text{ W}$$

Example 31: The waveform shown in Figure is a half-wave rectified sine wave. Find the rms value and the amount of average power dissipated in a 10Ω resistor.

Solution:



The period of the voltage waveform is $T = 2\pi$, and

$$v(t) = \begin{cases} 10 \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

The rms value is obtained as

$$V_{\text{rms}}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt = \frac{1}{2\pi}\left[\int_0^{\pi} (10 \sin t)^2\,dt + \int_{\pi}^{2\pi} 0^2\right.$$

But $\sin^2 t = \frac{1}{2}(1 - \cos 2t)$. Hence,

$$V_{\text{rms}}^2 = \frac{1}{2\pi}\int_0^{\pi} \frac{100}{2}(1 - \cos 2t)\,dt = \frac{50}{2\pi}\left(t - \frac{\sin 2t}{2}\right)\bigg|$$

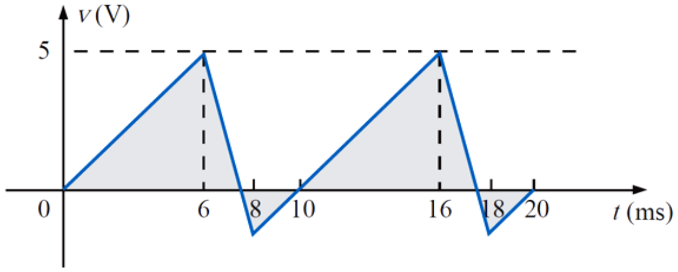
$$= \frac{50}{2\pi}\left(\pi - \frac{1}{2}\sin 2\pi - 0\right) = 25, \qquad V_{\text{rms}} = 5$$

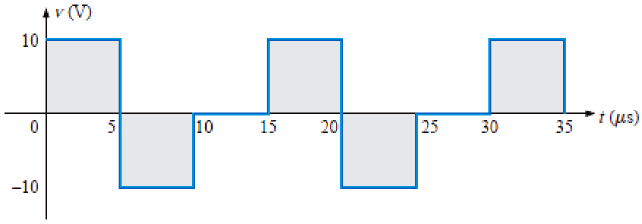
The average power absorbed is

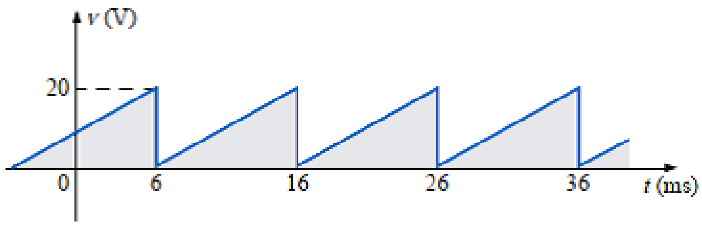
$$P = \frac{V_{\text{rms}}^2}{R} = \frac{5^2}{10} = 2.5 \text{ W}$$

Home Works

[1] For the periodic waveforms,
a. Find the period T.
b. How many cycles are shown?
c. What is the frequency?
d. Determine the positive amplitude and peak-to-peak value.







[2] Find the period of a periodic waveform whose frequency is
a. 25 Hz. **b.** 35 MHz.
c. 55 kHz. **d.** 1 Hz.

[3] Find the frequency of a repeating waveform whose period is
a. 1/60 s. **b.** 0.01 s.
c. 34 ms. **d.** 25 ms.

[4] Find the period of a sinusoidal waveform that completes 80 cycles in 24 ms.

[5] If a periodic waveform has a frequency of 20 Hz, how long (in seconds) will it take to complete five cycles?

[6] What is the frequency of a periodic waveform that completes 42 cycles in 6 s?

[7] Find the amplitude and frequency of the following waves:
a. $20 \sin 377t$ **b.** $5 \sin 754t$
c. $10^6 \sin 10{,}000t$ **d.** $0.001 \sin 942t$
e. $-7.6 \sin 43.6t$ **f.** $(1/42)\sin 6.283t$

[8] If e =300 sin 157t, how long (in seconds) does it take this waveform to complete 1/2 cycle?

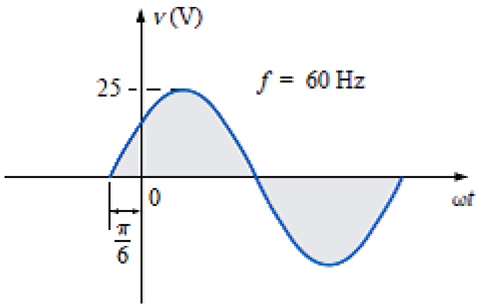
[9] Sketch the following waveforms:

a. $50 \sin(\omega t + 0°)$ **b.** $-20 \sin(\omega t + 2°)$

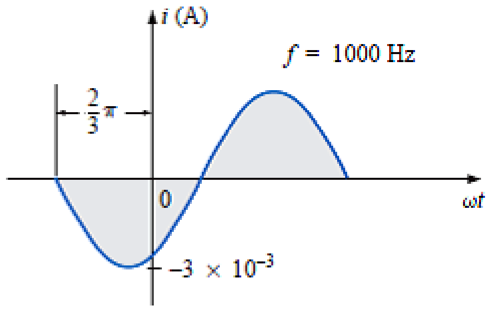
c. $5 \sin(\omega t + 60°)$ **d.** $4 \cos \omega t$

e. $2 \cos(\omega t +10°)$ **f.** $-5 \cos(\omega t + 20°)$

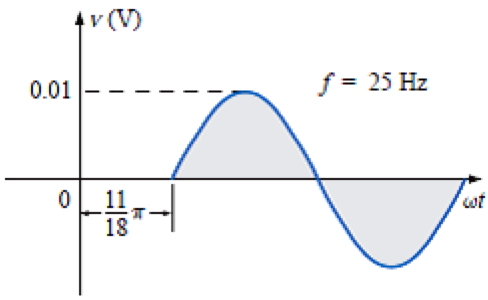
[10] Write the analytical expression for the waveforms with the phase angle in degrees.



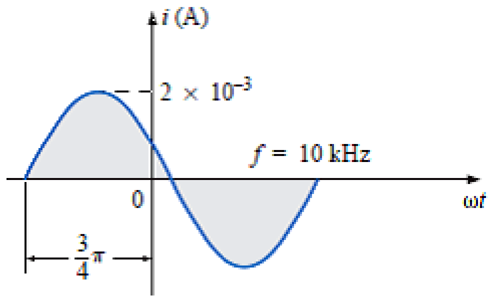
(a)



(b)

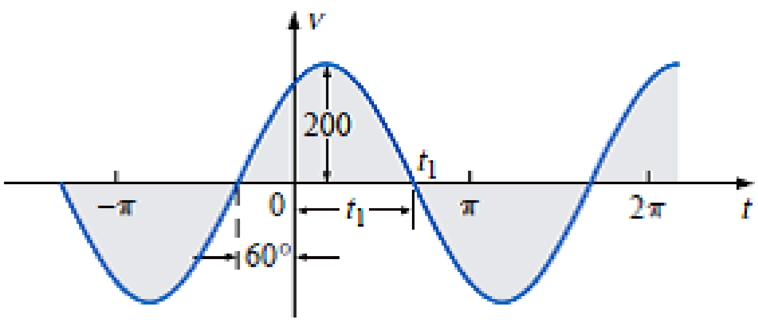


(b)

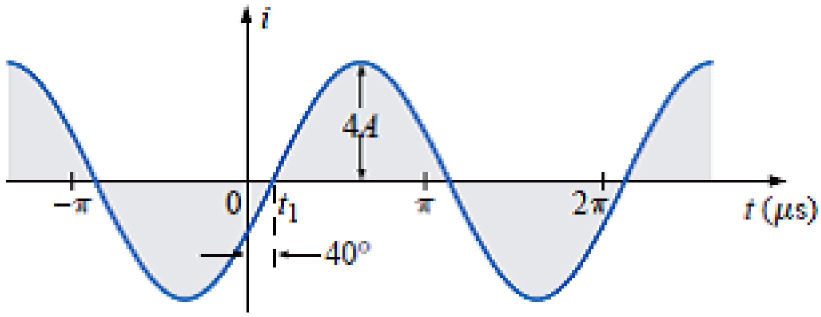


(d)

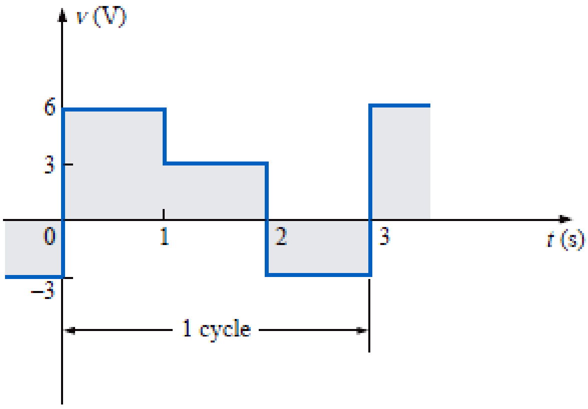
[11] The sinusoidal voltage $v = 200 \sin(2\pi 1000t + 60°)$ is plotted in Figure. Determine the time t_1.

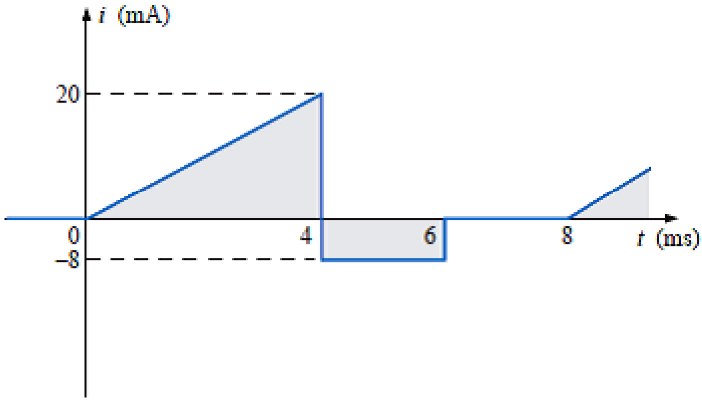


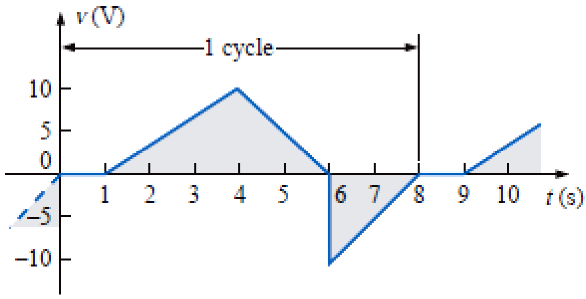
[12] The sinusoidal current i = 4 sin(50,000t − 40°) is plotted in Figure. Determine the time t_1.

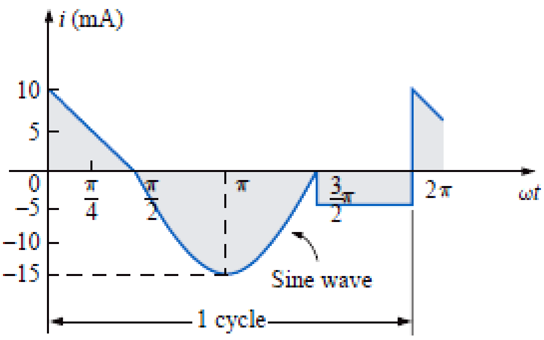


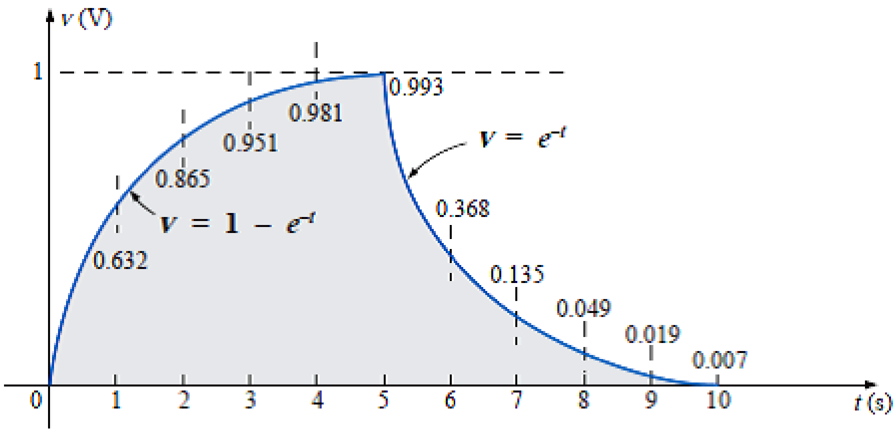
[13] Find the average and rms values for the following wave forms.

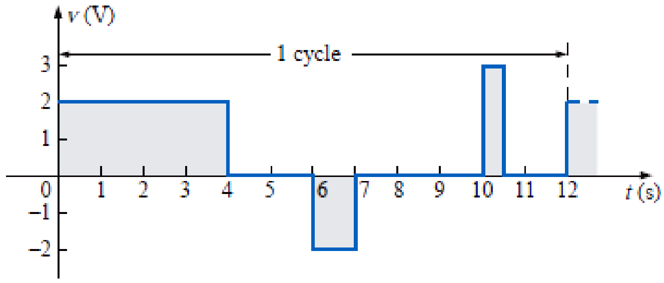


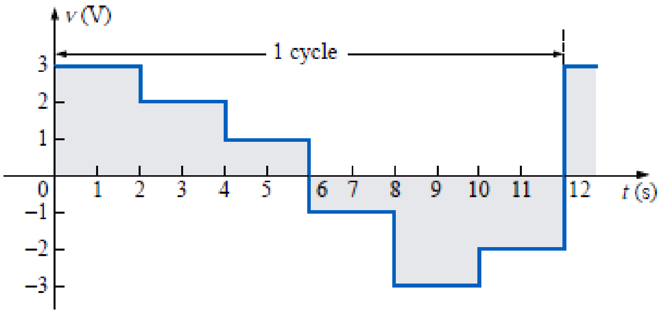


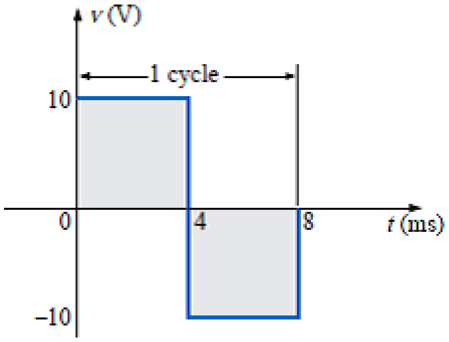




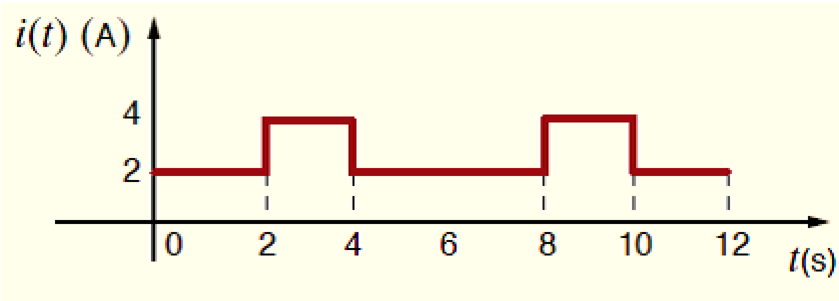




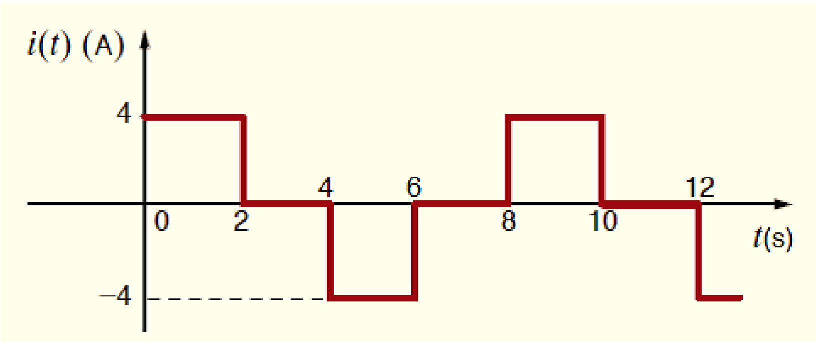




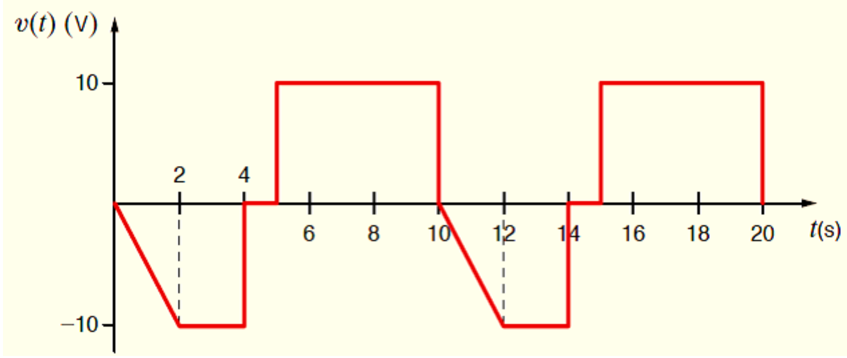
[14] The current waveform in Figure shown is flowing through a 4Ω- resistor. Compute the average power delivered to the resistor. **[Answer: P = 32 W]**



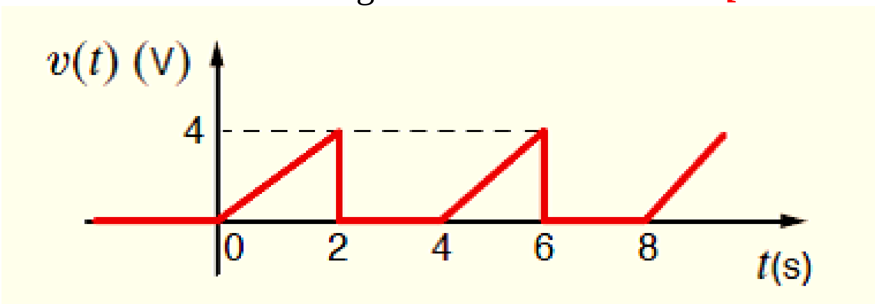
[15] The current waveform in Figure shown is flowing through a 10Ω- resistor. Compute the average power delivered to the resistor. **[Answer: P = 80 W]**



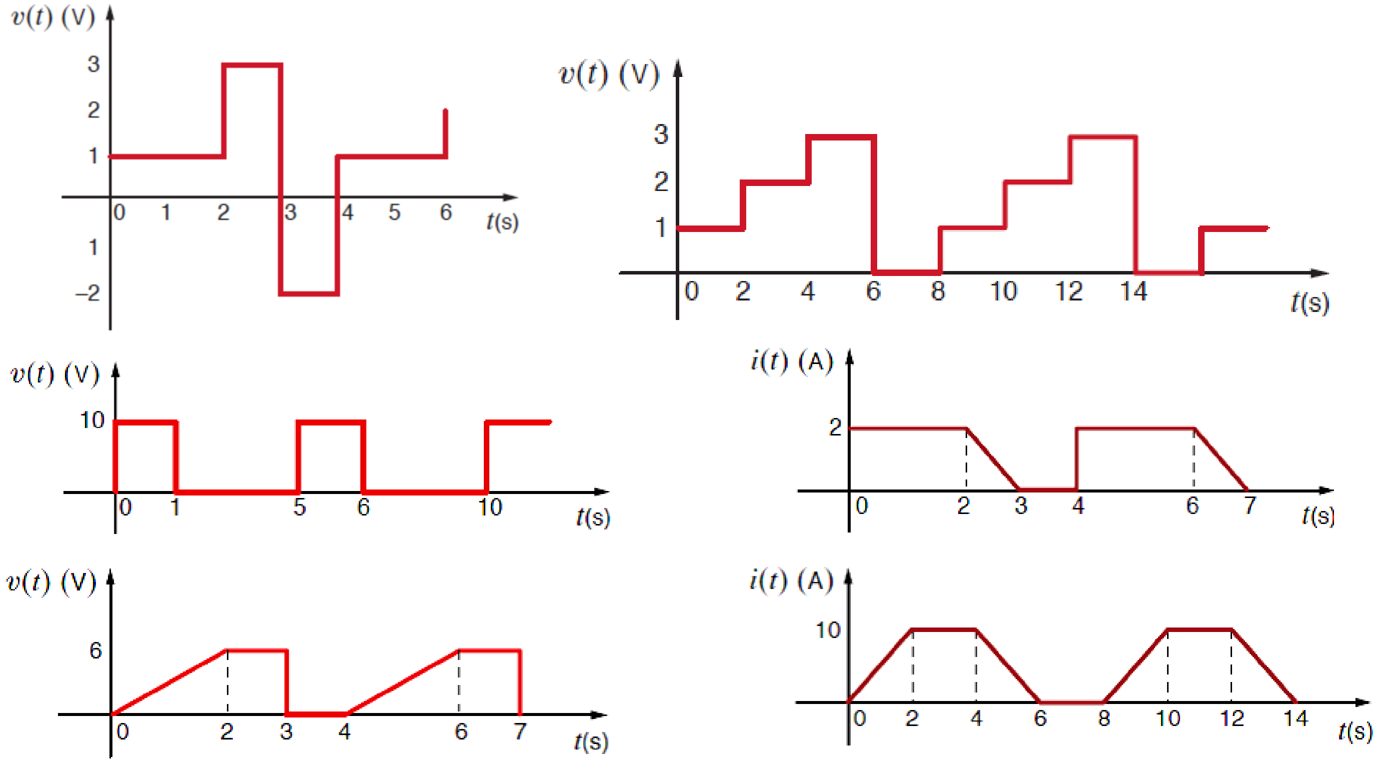
[16] The voltage across a 2-W resistor is given by the waveform in Figure. Find the average power absorbed by the resistor. **[Answer: P = 38.22 W]**

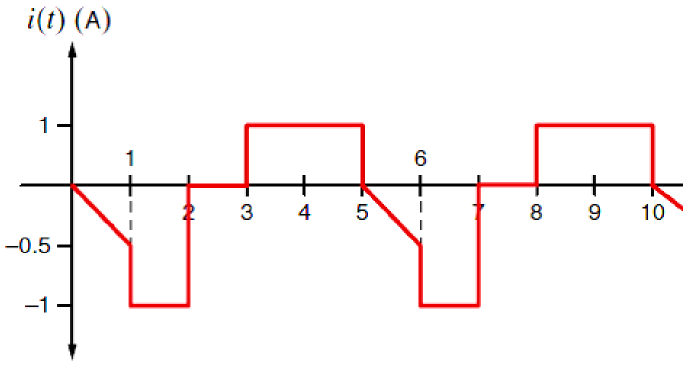


[17] Compute the rms value of the voltage waveform shown. **[Answer: V_{rms} = 1.633 V]**



[18] Find the rms value of the waveforms shown.




i(t) (A)
1
−0.5
−1
1 2 3 4 5 6 7 8 9 10

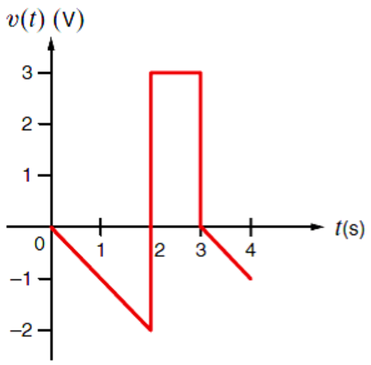
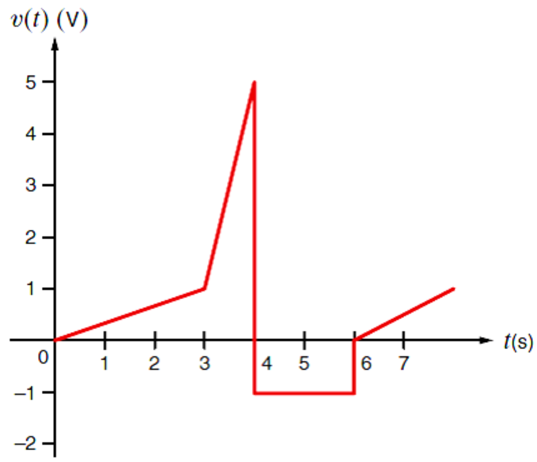
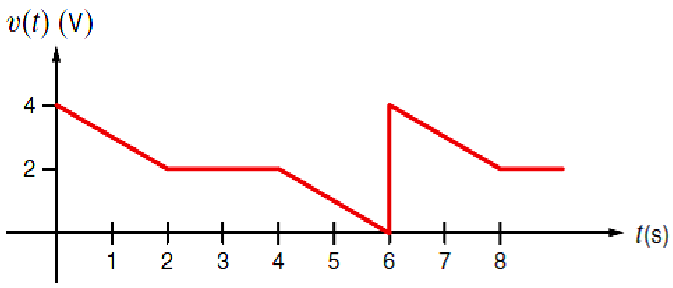
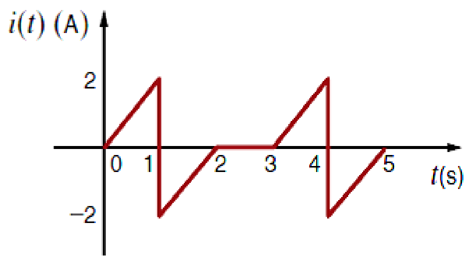
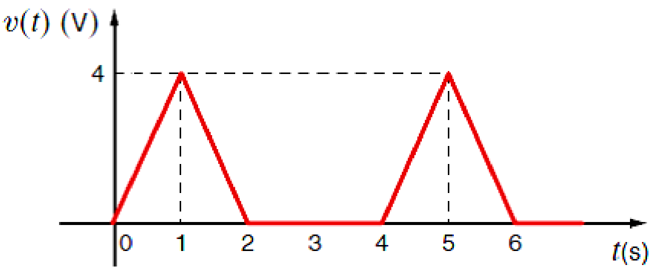

v(t) (V)
3
2
1
0
−1
−2
1 2 3 4
t(s)

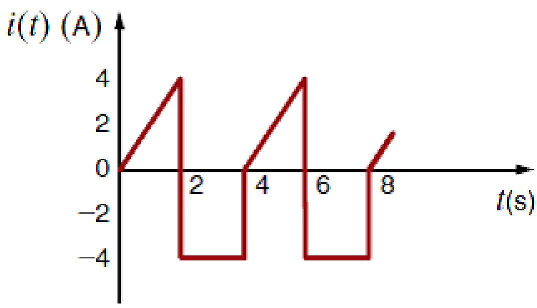
Figure P9.52

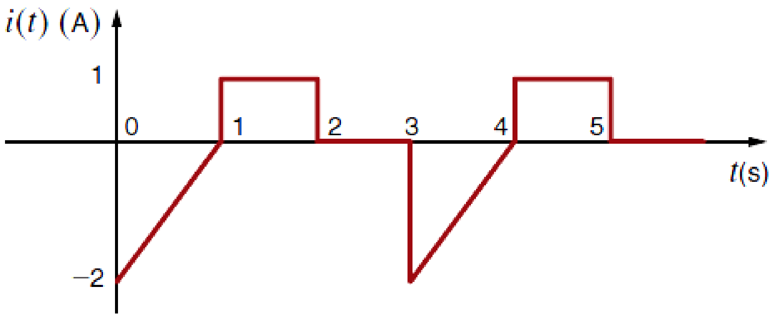

v(t) (V)
5
4
3
2
1
0
−1
−2
1 2 3 4 5 6 7
t(s)


v(t) (V)
4
2
1 2 3 4 5 6 7 8
t(s)


i(t) (A)
2
−2
0 1 2 3 4 5
t(s)

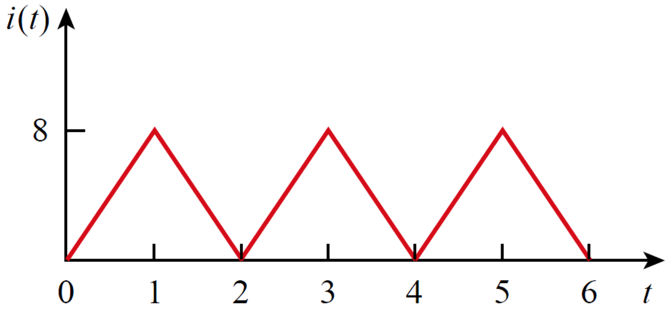

v(t) (V)
4
0 1 2 3 4 5 6
t(s)


i(t) (A)
4
2
0
−2
−4
2 4 6 8
t(s)

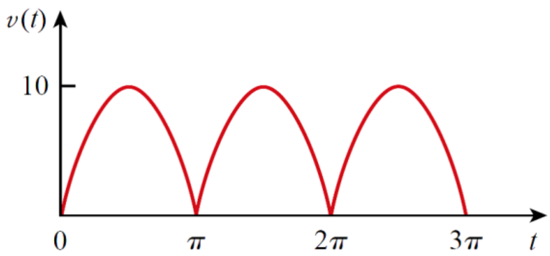

i(t) (A)
1
−2
0 1 2 3 4 5
t(s)

[19] Find the rms value of the current waveform of Figure. If the current flows through a 9Ω resistor, calculate the average power absorbed by the resistor.

[Answer: I=4.318A, P=192W]



[20] Find the rms value of the full-wave rectified sine wave in Figure. Calculate the average power dissipated in a 6Ω resistor. **[Answer: I=7.071A, P=8.333W]**



[21] Find the rms value of the waveforms shown.