Lecture (5)

Capacitors and Inductors

Introduction:

The capacitor and the inductor. Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time. For this reason, capacitors and inductors are called storage elements.

Capacitors

A capacitor is a passive element designed to store energy in its electric field. Capacitors are used extensively in electronics, communications, computers, and power systems. For example, they are used in the tuning circuits of radio receivers and as dynamic memory elements in computer systems.

A capacitor consists of two conducting plates separated by an insulator (or dielectric).

In many practical applications, the plates may be aluminum foil while the dielectric may be air, ceramic, paper, or mica.

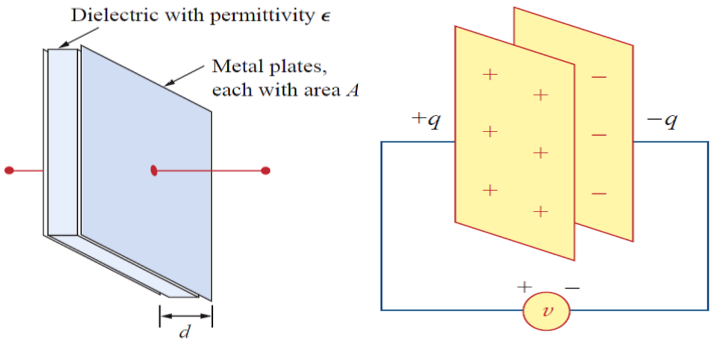
When a voltage source v is connected to the capacitor, as in **Figure (5.1) (b)**, the source deposits a positive charge (q) on one plate and a negative charge (- q) on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q, is directly proportional to the applied voltage v so that,

$$\boxed{q = Cv} \qquad (5.1)$$

Where C, the constant of proportionality, is known as the ***capacitance*** of the capacitor.

Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

The unit of capacitance is the farad (F) **(**1 farad = 1 coulomb/volt).



(a) A typical capacitor. (b) A capacitor with applied voltage v.

Figure (5.1)

Although the capacitance C of a capacitor is the ratio of the charge q per plate to the applied voltage v, it does not depend on q or v. It depends on the physical dimensions of the capacitor, the capacitance is given by

$$\boxed{C = \frac{\epsilon A}{d}} \tag{5.2}$$

where A is the surface area of each plate, d is the distance between the plates, and ϵ is the permittivity of the dielectric material between the plates.

The ratio of the flux density to the electric field intensity in the dielectric is called the ***permittivity*** of the dielectric

For a vacuum, the value of e (denoted by ϵ_o) is 8.85×10^{-12} F/m.
The ratio of the permittivity of any dielectric to that of a vacuum is called the **relative permittivity,** ϵ_r. It simply compares the permittivity of the dielectric to that of air. In equation form,

$$\boxed{\epsilon_r = \frac{\epsilon}{\epsilon_0}} \tag{5.3}$$

Capacitors are commercially available in different values and types. Typically, capacitors have values in the picofarad (pF) to microfarad (μF)range.

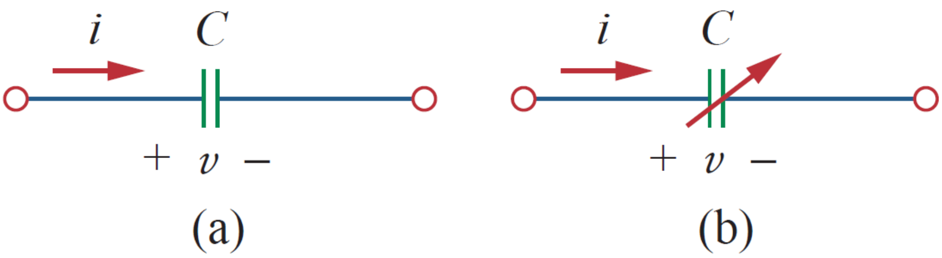


Figure (5.2): Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.

Figure (5.3): Several examples of commercially available capacitors

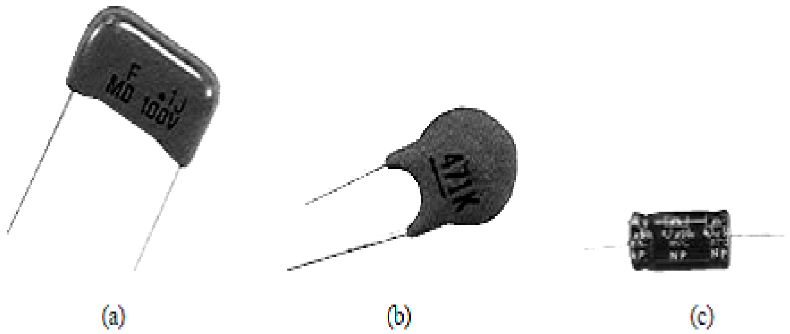


Figure (5.4): Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor.

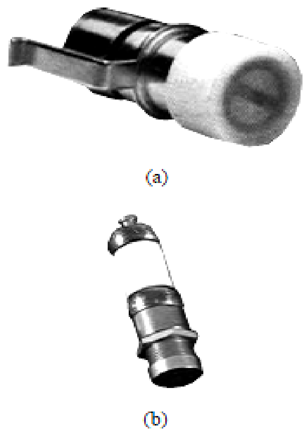


Figure (5.5): Variable capacitors: (a) trimmer capacitor, (b) film trim capacitor.

To obtain the current-voltage relationship of the capacitor, we take the derivative of both sides of Eq. (5.1). Since

$$i = \frac{dq}{dt} \qquad (5.4)$$

differentiating both sides of Eq. (5.1) gives

$$i = C\,\frac{dv}{dt} \qquad (5.5)$$

The voltage-current relation of the capacitor can be obtained by integrating both sides of Eq. (5.5). We get

$$v = \frac{1}{C}\int_{-\infty}^{t} i.\,dt \qquad (5.6)$$

or

$$v = \frac{1}{C}\int_{t_0}^{t} i.\,dt + v(t_0) \qquad (5.7)$$

The instantaneous power delivered to the capacitor is

$$P = vi = Cv\,\frac{dv}{dt} \qquad (5.8)$$

The energy stored in the capacitor is therefore

$$w = \int_{-\infty}^{t} p\,dt = C\int_{-\infty}^{t} v\frac{dv}{dt}dt = C\int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)} \qquad (5.9)$$

We note that $v(-\infty) = 0$, because the capacitor was uncharged at $t = -\infty$. Thus,

$$w = \frac{1}{2} C v^2 \quad \text{or} \quad w = \frac{q^2}{2C} \tag{5.10}$$

1) Note from Eq. (5.5) that when the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. Thus,

> A capacitor is an open circuit to dc.

However, if a battery (dc voltage) is connected across a capacitor, the capacitor charges.

2) The voltage on the capacitor must be continuous.

> The voltage on a capacitor cannot change abruptly.

3) The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.

4) A real, non-ideal capacitor has a parallel-model leakage resistance, as shown in **Figure (5.6)**. The leakage resistance may be as high as 100 M and can be neglected for most practical applications.

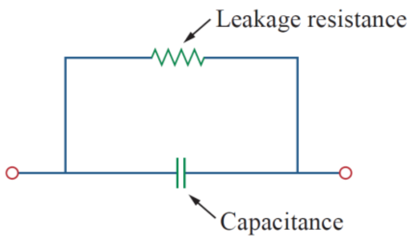


Figure (5.6): Circuit model of a non-ideal capacitor.

Example 1: (a) Calculate the charge stored on a 3-pF capacitor with 20 V across it. (b) Find the energy stored in the capacitor.

Solution: (a) Since $q = Cv$,

$$q = 3 \times 10^{-12} \times 20 = 60 \text{ pC}$$

(b) The energy stored is

$$w = \frac{1}{2} C v^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

H.W.: What is the voltage across a 3-μF capacitor if the charge on one plate is 0.12 mC? How much energy is stored? **Answer: 40 V, 2.4 mJ.**

Example 2: The voltage across a 5-μF capacitor is,

$v(t) = 10 \cos 6000t$ V

Calculate the current through it.

Solution:

By definition, the current is

$$i(t) = C\frac{dv}{dt} = 5 \times 10^{-6}\frac{d}{dt}(10 \cos 6000t)$$
$$= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \text{ A}$$

H.W.: If a 10-μF capacitor is connected to a voltage source with, v(t) = 50 sin 2000t V Determine the current through the capacitor. **Answer: cos 2000t A**.

Example 3:

Determine the voltage across a 2-μF capacitor if the current through it is

$$i(t) = 6e^{-3000t} \text{ mA}$$

Assume that the initial capacitor voltage is zero.

Solution:

Since $v = \frac{1}{C}\int_0^t i\,dt + v(0)$ and $v(0) = 0$,

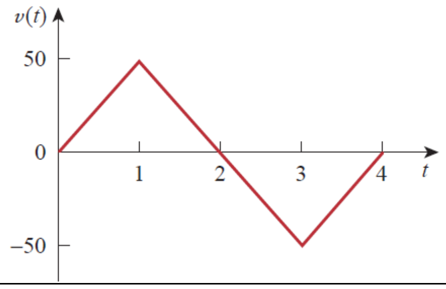
$$v = \frac{1}{2 \times 10^{-6}}\int_0^t 6e^{-3000t}\,dt \cdot 10^{-3}$$
$$= \frac{3 \times 10^3}{-3000}e^{-3000t}\Big|_0^t = (1 - e^{-3000t}) \text{ V}$$

H.W.:

The current through a 100-μF capacitor is $i(t) = 50 \sin 120\pi t$ mA. Calculate the voltage across it at $t = 1$ ms and $t = 5$ ms. Take $v(0) = 0$.

Answer: 93.14 mV, 1.736 V.

Example 4: Determine the current through a 200-μF capacitor whose voltage is shown in Figure.



Solution:

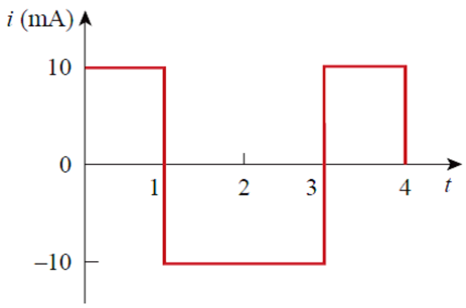
The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Since $i = C\,dv/dt$ and $C = 200\ \mu\text{F}$, we take the derivative of v to obtain

$$i(t) = 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10 \text{ mA} & 0 < t < 1 \\ -10 \text{ mA} & 1 < t < 3 \\ 10 \text{ mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

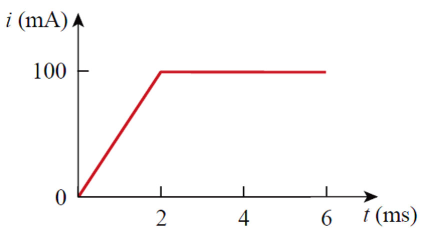


Thus the current waveform is as shown in Fig.

H.W.

An initially uncharged 1-mF capacitor has the current shown in Fig. 6.11 across it. Calculate the voltage across it at $t = 2$ ms and $t = 5$ ms.

Answer: 100 mV, 400 mV.



Example 5: Obtain the energy stored in each capacitor in Figure under dc conditions.

Solution:

Under dc conditions, we replace each capacitor with an open circuit, as shown in Fig. 6.12(b). The current through the series combination of the 2-kΩ and 4-kΩ resistors is obtained by current division as

$$i = \frac{3}{3+2+4}(6 \text{ mA}) = 2 \text{ mA}$$

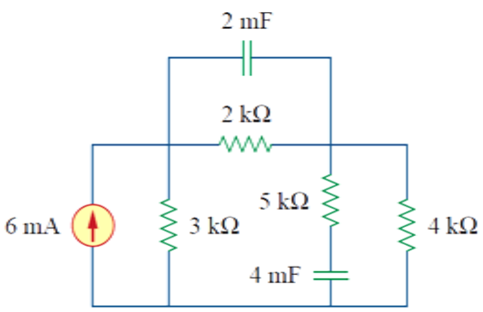
Hence, the voltages v_1 and v_2 across the capacitors are

$$v_1 = 2000i = 4 \text{ V} \qquad v_2 = 4000i = 8 \text{ V}$$

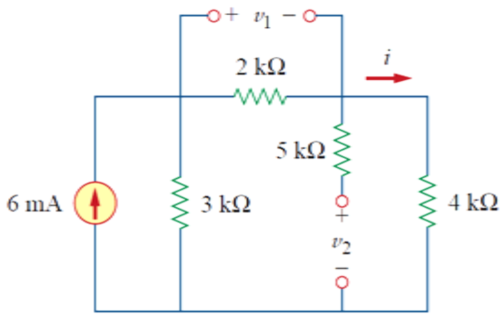
and the energies stored in them are

$$w_1 = \frac{1}{2}C_1 v_1^2 = \frac{1}{2}(2 \times 10^{-3})(4)^2 = 16 \text{ mJ}$$

$$w_2 = \frac{1}{2}C_2 v_2^2 = \frac{1}{2}(4 \times 10^{-3})(8)^2 = 128 \text{ mJ}$$



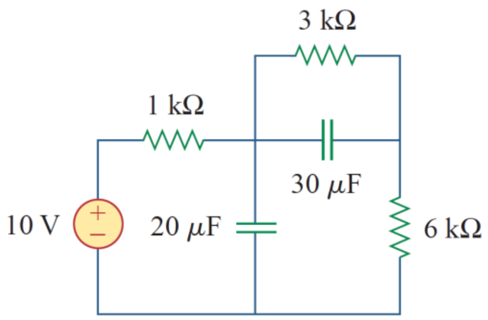
(a)



(b)

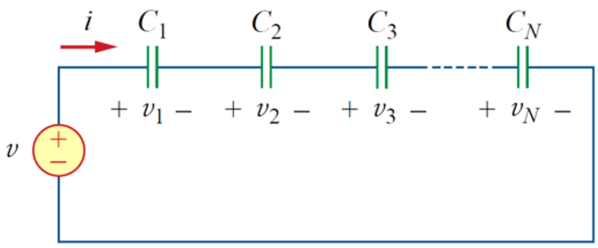
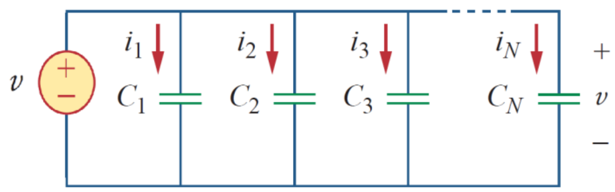
H.W.: Under dc conditions, find the energy stored in the capacitors in Fig.

Answer: 810 μJ, 135 μJ.



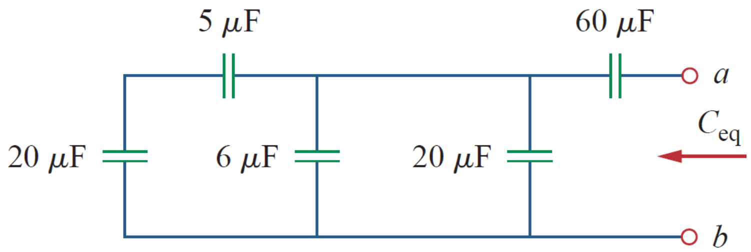
Series and Parallel Capacitors

Table (5.1)

NO.	Series Circuit	Parallel Circuit
1	The connection is as shown 	The connection is as shown 
2	The same current flows through each capacitor. $I = I_1 = I_2 = \dots = I_n$	The same voltage exists across all the capacitor in parallel. $V = V_1 = V_2 = \dots = V_n$
3	The voltage across each capacitor is different.	The current through each capacitor is different.
4	The sum of the voltages across all the capacitor is the supply voltage. $V = V_1 + V_2 + \dots + V_n$ $v = \frac{1}{C_1}\int_{t_0}^{t} i(t)\,dt + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(t)\,dt + v_2(t_0) + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(t)\,dt + v_N(t_0)$ $= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(t)\,dt + v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0)$ $= \frac{1}{C_{eq}}\int_{t_0}^{t} i(t)\,dt + v(t_0)$	The sum of the currents through all the capacitor is the supply current. $I = I_1 + I_2 + \dots + I_n$ $i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$ $= \left(\sum_{k=1}^{N} C_k\right)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$
5	The equivalent capacitance is, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$	The equivalent capacitance is, $C_{eq} = C_1 + C_2 + \dots + C_n$
6	The equivalent capacitance is the smaller than the smallest of all the capacitance in parallel.	The equivalent capacitance is the largest than each of the capacitance in series. $C_{eq} > C_1,\ C_{eq} > C_2 \dots,\ C_{eq} > C_n$

Example 6: Find the equivalent capacitance seen between terminals *a* and *b* of the circuit

Solution:



The 20-μF and 5-μF capacitors are in series; their equivalent capacitance is

$$\frac{20 \times 5}{20 + 5} = 4\ \mu\text{F}$$

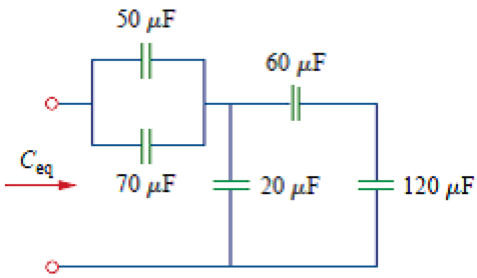
This 4-μF capacitor is in parallel with the 6-μF and 20-μF capacitors; their combined capacitance is

$$4 + 6 + 20 = 30\ \mu\text{F}$$

This 30-μF capacitor is in series with the 60-μF capacitor. Hence, the equivalent capacitance for the entire circuit is

$$C_{eq} = \frac{30 \times 60}{30 + 60} = 20\ \mu\text{F}$$

H.W.: Find the equivalent capacitance seen between terminals *a* and *b* of the circuit.



Answer: 40 μF.

Example 7: For the circuit in Fig., find the voltage across each capacitor

Solution:

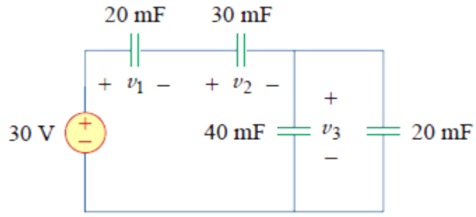


Figure a

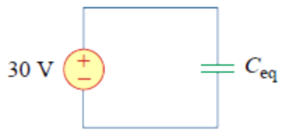


Figure b

Solution:

We first find the equivalent capacitance C_{eq}, shown in Fig. **b** The two parallel capacitors in Fig. **a** can be combined to get 40 + 20 = 60 mF. This 60-mF capacitor is in series with the 20-mF and 30-mF capacitors. Thus,

$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}} \text{ mF} = 10 \text{ mF}$$

The total charge is

$$q = C_{eq}v = 10 \times 10^{-3} \times 30 = 0.3 \text{ C}$$

This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-V source. (A crude way to see this is to imagine that charge acts like current, since $i = dq/dt$.) Therefore,

$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15 \text{ V} \qquad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10 \text{ V}$$

Having determined v_1 and v_2, we now use KVL to determine v_3 by

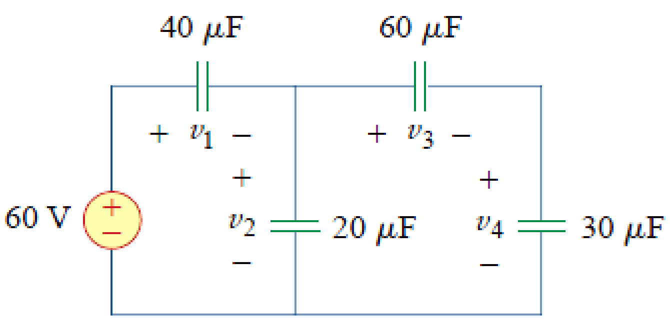
$$v_3 = 30 - v_1 - v_2 = 5 \text{ V}$$

Alternatively, since the 40-mF and 20-mF capacitors are in parallel, they have the same voltage v_3 and their combined capacitance is 40 + 20 = 60 mF. This combined capacitance is in series with the 20-mF and 30-mF capacitors and consequently has the same charge on it. Hence,

$$v_3 = \frac{q}{60 \text{ mF}} = \frac{0.3}{60 \times 10^{-3}} = 5 \text{ V}$$

H.W.: Find the voltage across each of the capacitors in Fig.

Answer: v_1 = 30 V, v_2 = 30 V, v_3 = 10 V, v_4 = 20 V.



Inductors

An inductor is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems. They are used in power supplies, transformers, radios, TVs, radars, and electric motors.

Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire, as shown in **Figure (5.7).**

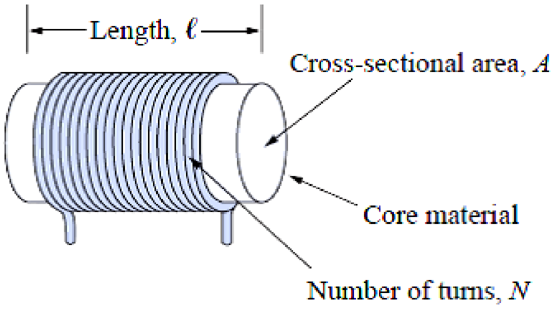


Figure (5.7): Typical form of an inductor

An inductor consists of a coil of conducting wire.

$$\boxed{v = L\frac{di}{dt}} \tag{5.11}$$

where L is the constant of proportionality called the *inductance* of the inductor. The unit of inductance is the henry (H),(1 henry equals 1 volt-second per ampere).

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

Like capacitors, commercially available inductors come in different values and types. Typical practical inductors have inductance values ranging from a few microhenrys (μH), as in communication systems, to tens of henrys (H) as in power systems. Inductors may be fixed or variable. The core may be made of iron, steel, plastic, or air. The terms *coil* and *choke* are also used for inductors. Common inductors are shown in **Figure (5.8)**. The circuit symbols for inductors are shown in **Figure (5.9)**, following the passive sign convention.

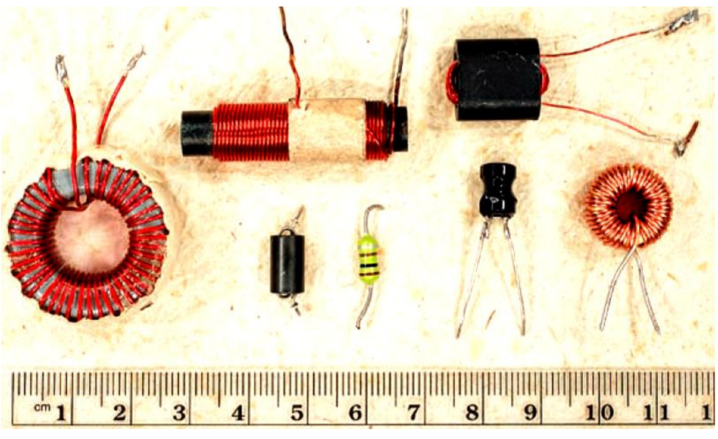


Figure (5.8): Several examples of commercially available inductors

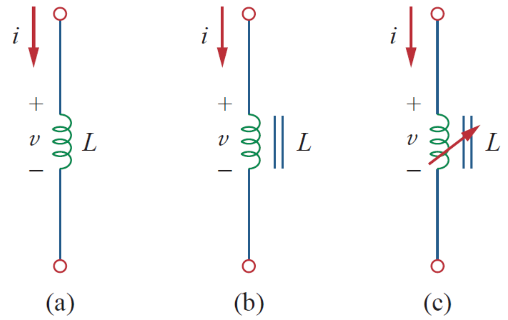


Figure (5.9): Circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core.

The current-voltage relationship is obtained from Eq. (**5.11**) as

$$di = \frac{1}{L} v \, dt$$

Integrating gives

$$i = \frac{1}{L} \int_{-\infty}^{t} v(t) \, dt \qquad \textbf{(5.12)}$$

or

$$\boxed{i = \frac{1}{L} \int_{t_0}^{t} v(t) \, dt + i(t_0)} \qquad \textbf{(5.13)}$$

Power delivered to the inductor is,

$$p = vi = \left(L \frac{di}{dt} \right) i \qquad \textbf{(5.14)}$$

The energy stored is

$$\begin{aligned} w &= \int_{-\infty}^{t} p \, dt = \int_{-\infty}^{t} \left(L \frac{di}{dt} \right) i \, dt \\ &= L \int_{-\infty}^{t} i \, di = \frac{1}{2} L i^2(t) - \frac{1}{2} L i^2(-\infty) \end{aligned} \qquad \textbf{(5.15)}$$

Since $i(-\infty) = 0$,

$$\boxed{w = \frac{1}{2}Li^2} \qquad \textbf{(6.24)}$$

1) Note from Eq. (5.11) that the voltage across an inductor is zero when the current is constant. Thus,

An inductor acts like a short circuit to dc.

2) An important property of the inductor is its opposition to the change in current flowing through it.

The current through an inductor cannot change instantaneously.

Example 8:

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

Solution:

Since $v = L\,di/dt$ and $L = 0.1$ H,

$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t) \text{ V}$$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t} \text{ J}$$

H.W.:

If the current through a 1-mH inductor is $i(t) = 20\cos 100t$ mA, find the terminal voltage and the energy stored.

Answer: $-2\sin 100t$ mV, $0.2\cos^2 100t$ μJ.

Example 9:

Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at $t = 5$ s. Assume $i(v) > 0$.

Solution:

Since $i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$ and $L = 5$ H,

$$i = \frac{1}{5}\int_0^t 30t^2\,dt + 0 = 6 \times \frac{t^3}{3} = 2t^3 \text{ A}$$

The power $p = vi = 60t^5$, and the energy stored is then

$$w = \int p\,dt = \int_0^5 60t^5\,dt = 60\frac{t^6}{6}\Big|_0^5 = 156.25 \text{ kJ}$$

or

$$w\big|_0^5 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li(0) = \frac{1}{2}(5)(2 \times 5^3)^2 - 0 = 156.25 \text{ kJ}$$

as obtained before.

H.W.:

The terminal voltage of a 2-H inductor is $v = 10(1 - t)$ V. Find the current flowing through it at $t = 4$ s and the energy stored in it at $t = 4$ s. Assume $i(0) = 2$ A.

Answer: -18 A, 320 J.

Example 10: Consider the circuit in Fig. (a). Under dc conditions, find: (a) i, v_c and i_L (b) the energy stored in the capacitor and inductor.

Solution:

(a) Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit, as in Fig. (b). It is evident from Fig.(b) that

$$i = i_L = \frac{12}{1+5} = 2 \text{ A}$$

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

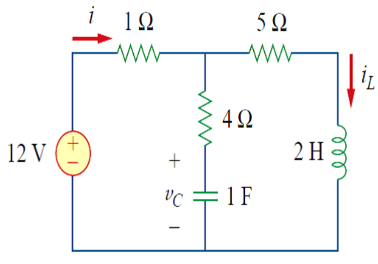
$$v_C = 5i = 10 \text{ V}$$

(b) The energy in the capacitor is

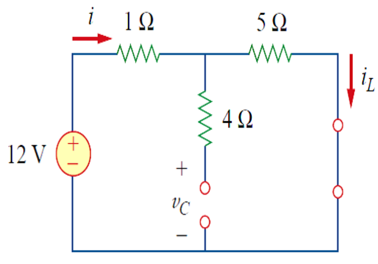
$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J}$$

and that in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$



(a)

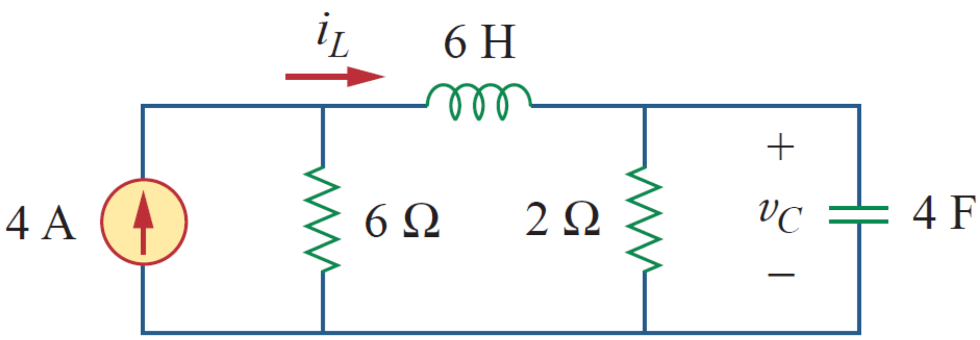


(b)

H.W.:

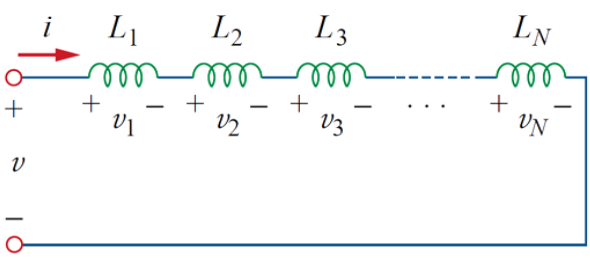
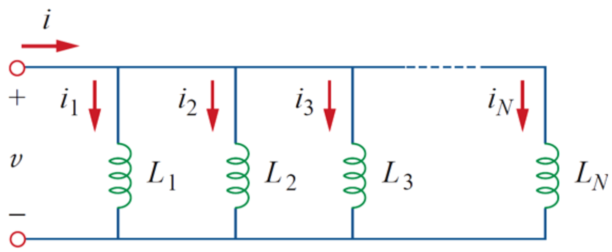
Determine v_C, i_L, and the energy stored in the capacitor and inductor in the circuit of Fig. under dc conditions.

Answer: 6 V, 3 A, 72 J, 27 J.



Series and Parallel Inductors

Table (5.2)

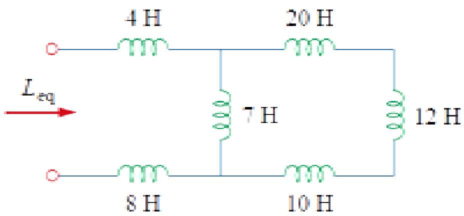
NO.	Series Circuit	Parallel Circuit
1	The connection is as shown  	The connection is as shown 
2	The same current flows through each inductor. $I = I_1 = I_2 = \ldots = I_n$	The same voltage exists across all the inductor in parallel. $V = V_1 = V_2 = \ldots = V_n$
3	The voltage across each inductor is different.	The current through each inductor is different.
4	The sum of the voltages across all the inductor is the supply voltage. $V = V_1 + V_2 + \ldots + V_n$ $v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \cdots + L_N\frac{di}{dt}$ $= (L_1 + L_2 + L_3 + \cdots + L_N)\frac{di}{dt}$ $= \left(\sum_{k=1}^{N} L_k\right)\frac{di}{dt} = L_{eq}\frac{di}{dt}$	The sum of the currents through all the inductor is the supply current. $I = I_1 + I_2 + \ldots + I_n$ $i = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v\,dt + i_2(t_0)$ $+ \cdots + \frac{1}{L_N}\int_{t_0}^{t} v\,dt + i_N(t_0)$ $= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right)\int_{t_0}^{t} v\,dt + i_1(t_0) + i_2(t_0)$ $+ \cdots + i_N(t_0)$ $= \left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt + i(t_0)$
5	The equivalent inductor is, $L_{eq} = L_1 + L_2 + \ldots + L_n$	The equivalent inductor is, $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}$
6	The equivalent inductor is the smaller than the smallest of all the inductor in parallel.	The equivalent inductor is the largest than each of the inductor in series. $L_{eq} > L_1, L_{eq} > L_2 \ldots, L_{eq} > L_n$

Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit

Example 11: Find the equivalent inductance of the circuit shown in Fig.

Solution:



The 10-H, 12-H, and 20-H inductors are in series; thus, combining them gives a 42-H inductance. This 42-H inductor is in parallel with the 7-H inductor so that they are combined, to give

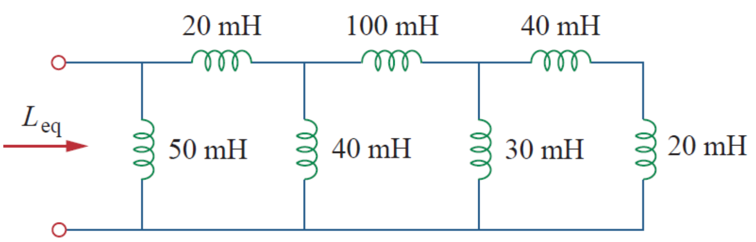
$$\frac{7 \times 42}{7 + 42} = 6\text{ H}$$

This 6-H inductor is in series with the 4-H and 8-H inductors. Hence,

$$L_{eq} = 4 + 6 + 8 = 18\text{ H}$$

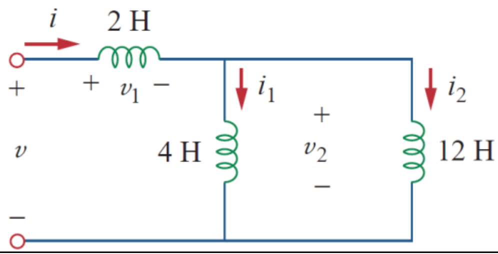
H.W.:

Calculate the equivalent inductance for the inductive ladder network in Fig. 6.32.



Answer: 25 mH.

Example 12: For the circuit in Fig. , $i(t) = 4(2 - e^{-10t})$ mA. If $i_2(0) = -1$ mA, find: (a) $i_1(0)$; (b) $v(t)$, $v_1(t)$, and $v_2(t)$; (c) $i_1(t)$ and $i_2(t)$.



Solution:

(a) From $i(t) = 4(2 - e^{-10t})$ mA, $i(0) = 4(2 - 1) = 4$ mA. Since $i = i_1 + i_2$,

$$i_1(0) = i(0) - i_2(0) = 4 - (-1) = 5 \text{ mA}$$

(b) The equivalent inductance is

$$L_{eq} = 2 + 4 \,\|\, 12 = 2 + 3 = 5 \text{ H}$$

Thus,

$$v(t) = L_{eq}\frac{di}{dt} = 5(4)(-1)(-10)e^{-10t} \text{ mV} = 200e^{-10t} \text{ mV}$$

and

$$v_1(t) = 2\frac{di}{dt} = 2(-4)(-10)e^{-10t} \text{ mV} = 80e^{-10t} \text{ mV}$$

Since $v = v_1 + v_2$,

$$v_2(t) = v(t) - v_1(t) = 120e^{-10t} \text{ mV}$$

(c) The current i_1 is obtained as

$$i_1(t) = \frac{1}{4}\int_0^t v_2\, dt + i_1(0) = \frac{120}{4}\int_0^t e^{-10t}\, dt + 5 \text{ mA}$$
$$= -3e^{-10t}\Big|_0^t + 5 \text{ mA} = -3e^{-10t} + 3 + 5 = 8 - 3e^{-10t} \text{ mA}$$

Similarly,

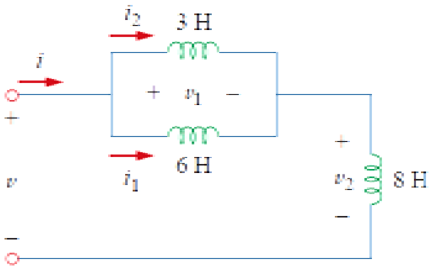
$$i_2(t) = \frac{1}{12}\int_0^t v_2\, dt + i_2(0) = \frac{120}{12}\int_0^t e^{-10t}\, dt - 1 \text{ mA}$$
$$= -e^{-10t}\Big|_0^t - 1 \text{ mA} = -e^{-10t} + 1 - 1 = -e^{-10t} \text{ mA}$$

Note that $i_1(t) + i_2(t) = i(t)$.

H.W.:

In the circuit of Fig., $i_1(t) = 0.6e^{-2t}$ A. If $i(0) = 1.4$ A, find:
(a) $i_2(0)$; (b) $i_2(t)$ and $i(t)$; (c) $v_1(t)$, $v_2(t)$, and $v(t)$.

Answer: (a) 0.8 A, (b) $(-0.4 + 1.2e^{-2t})$ A, $(-0.4 + 1.8e^{-2t})$ A,
(c) $-36e^{-2t}$ V, $-7.2e^{-2t}$ V, $-28.8e^{-2t}$ V.



Lecture (7)

SERIES A.C. CIRCUITS

Let
V = r.m.s. value of the applied voltage,
I = r.m.s. value of the resultant current

1) *A.C. Through Resistance and Inductance (R-L)*

A pure resistance **R** and a pure inductive coil of inductance **L** are shown connected in series in Figure(7.3).

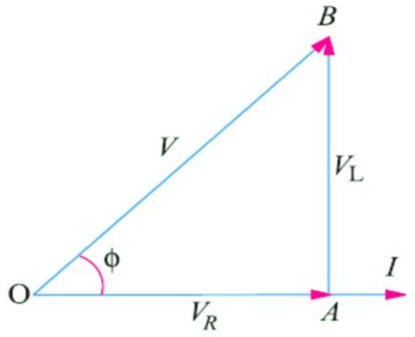
$V = \sqrt{V_R^2 + V_L^2}$

$V_R = V\cos\phi$

$V_L = V\sin\phi$

Phase difference angle $\phi = tan^{-1}\frac{V_L}{V_R}$

Power **F**actor **PF** ($\cos\phi = \frac{V_R}{V}$).



Figure(7.1):Voltage triangle

If we divide the ***voltage triangle*** by current (I), we get ***impedance triangle***.

V/I=Z, where Z is the circuit ***impedance*** in (Ω).

V_R/I=R, where R is the circuit ***resistance*** in (Ω).

V_L/I=X_L, where X_L is the ***inductive reactance*** in (Ω).

($X_L = \omega L = 2\pi fL$)

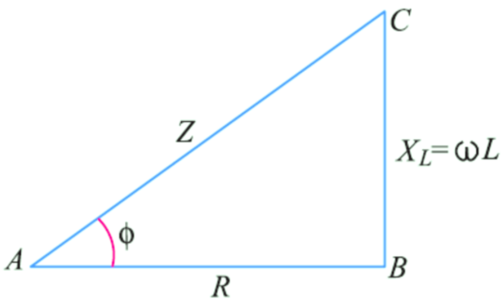
$Z = \sqrt{R^2 + X_L^2}$

$R = Z\cos\phi$

$X_L = Z\sin\phi$

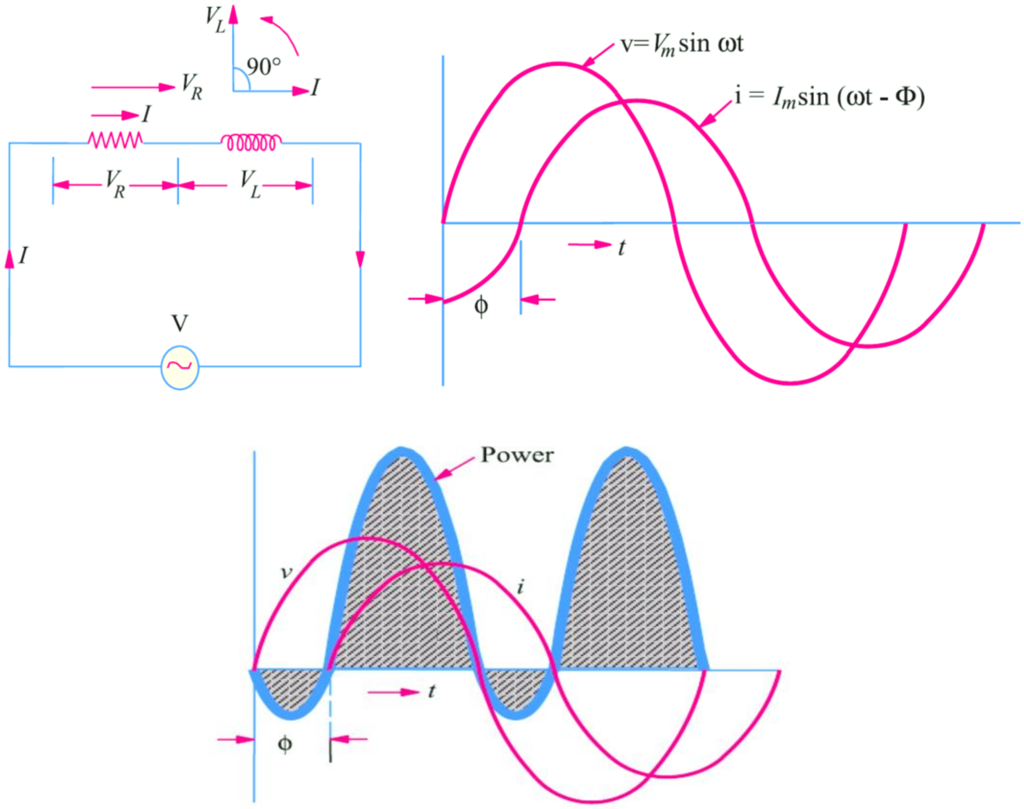
Phase difference angle $\phi = tan^{-1}\frac{X_L}{R}$

Power **F**actor **PF** ($\cos\phi = \frac{R}{Z}$) lag or lagging.



Figure(7.2):Impedance triangle

Notes:
1)V = IZ − voltage drop across *Z* (ahead of *I* by angle less than 90°),
2)V_R = IR − voltage drop across *R* (in phase with *I*),
3)V_L = I . X_L –voltage drop across coil (ahead of *I* by 90°)
4) Power factor is lagging because the circuit current lags the total voltage by 0≤ ϕ ≤ 90).
$v = V_m \sin \omega t$
$i = I_m \sin(\omega t - \phi)$ lag voltage by 0≤ ϕ ≤ 90)



Figure(7.3):Series R-L circuit, phasor and vector diagram

Instantaneous power

$p = vi = V_m I_m \sin(\omega t)\sin(\omega t - \phi) = \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t - \phi)]$

Note:

- $\sin(A)\sin(B) = \frac{1}{2}\cos(A\text{-}B) - \frac{1}{2}\cos(A+B)$
- $\cos(A)\cos(B) = \frac{1}{2}\cos(A\text{-}B) + \frac{1}{2}\cos(A+B)$
- $\sin(A)\cos(B) = \frac{1}{2}\sin(A\text{-}B) + \frac{1}{2}\sin(A+B)$

Power consists of:

1) A constant part $\frac{V_m I_m}{2}[\cos\phi]$ which is the real power.
2) A pulsating part $\frac{V_m I_m}{2}[\cos(2\omega t - \phi)]$of frequency ***double*** that of voltage and current waves. It does not contribute to actual power since its average value over a complete cycle is zero.

Hence, the average power consumed is,

$p = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos\phi = V_{RMS} \times I_{RMS} \times \cos\phi = VI\cos\phi$ [watt]

Power Triangle:

1) If we multiply the voltage triangle by current I (or multiply impedance triangle by I^2), we get power triangle,

(*i*) apparent power (*S*)

It is given by the product of r.m.s. values of applied voltage and circuit current.

$\therefore S = VI = I^2Z$ *volt-amperes (VA)*

(*ii*) active power (*P* or *W*)

It is the power which is actually dissipated in the circuit resistance.

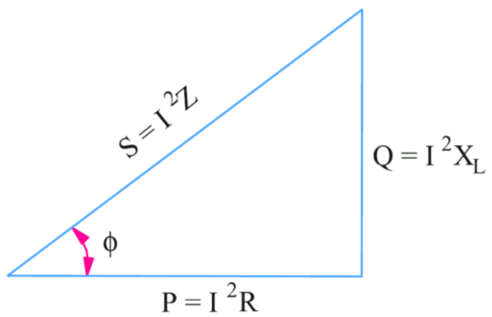
$P = V_RI = VI\cos\varphi = I^2R$ *watts*

(*iii*) reactive power (Q_L)

It is the power developed in the inductive reactance of the circuit.

$Q_L = V_LI = VI\sin\varphi = I^2X_L$ *volt-amperes-reactive (VAR)*

$S = \sqrt{P^2 + Q_L^2}$



Figure(7.4):Power triangle

Representation of AC Quantities in Complex Numbers Forms

$V = V_R + j\,V_L$ or $V = V\angle\phi$

$I = I + j\,0$ or $I = I\angle 0$

$Z = R + j\,X_L$ or $Z = Z\angle\phi$

$S = P + j\,Q_L$ or $S = S\angle\phi$

$Z = \frac{V}{I} = \frac{VR + j\,VL}{I + j\,0} = R + j\,X_L$

$S = VI = (V_R + j\,V_L)(I + j\,0) = P + j\,Q_L$

Power Factor

Power **F**actor **PF** ($\cos\phi = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$) lagging

2) A.C. Through Resistance and Capacitance (R-C)

A pure resistance **R** and a pure capacitance **C** are shown connected in series in Figure(7.7). Let

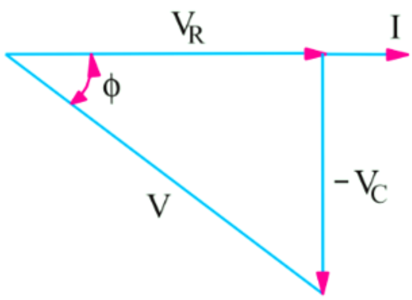
$V = \sqrt{V_R^2 + (-V_C)^2}$

V_R = Vcosϕ

V_C = -Vsinϕ

Phase difference angle $\phi = tan^{-1}\frac{-V_C}{V_R}$

Power **F**actor **PF** (cosϕ = $\frac{V_R}{V}$).



Figure(7.5):Voltage triangle

If we divide the ***voltage triangle*** by current (I), we get ***impedance triangle***.

V/I=Z, where Z is the circuit ***impedance*** in (Ω).

V_R/I=R, where R is the circuit ***resistance*** in (Ω).

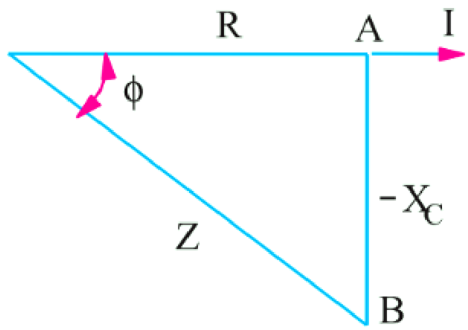
V_c/I=X_c, where X_c is the ***capacitive reactance*** in (Ω).

($X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$)

$Z = \sqrt{R^2 + (-X_C)^2}$

R = Zcosϕ

X_c = -Zsinϕ



Figure(7.6):Impedance triangle

Phase difference angle $\phi = tan^{-1}\frac{-X_C}{R}$

Power **F**actor **PF** (cosϕ = $\frac{R}{Z}$).

Notes:

1)V = IZ = voltage drop across *Z* (ahead of *I* by angle less than 90°),

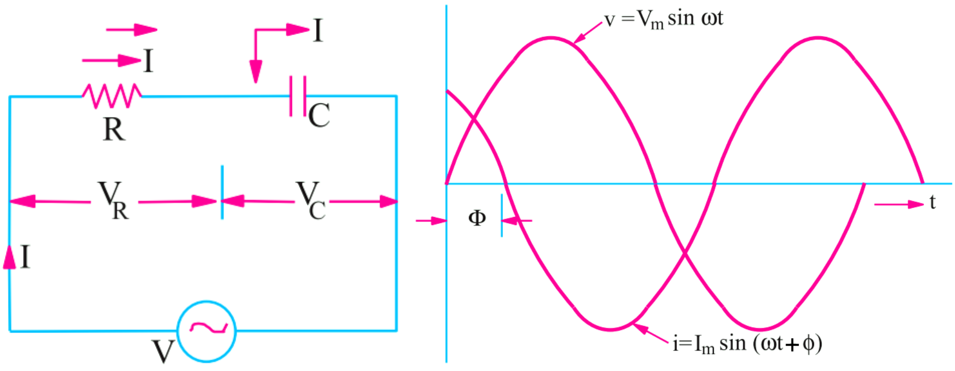
2)V_R = IR= voltage drop across *R* (in phase with *I*),

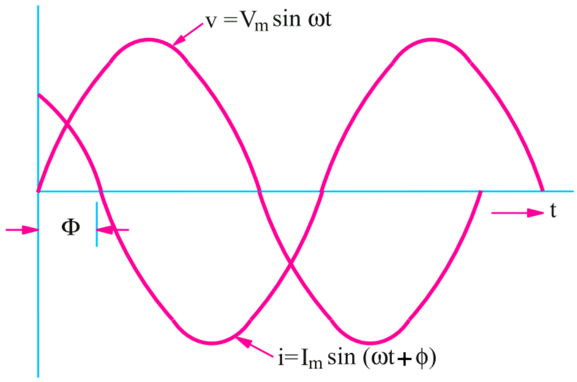
3)V_c = IX_C =voltage drop across capacitance (lag *I* by 90°)

4) Power factor is leading because the circuit current leads the total voltage by 0≤ ϕ ≤ 90).

$v = V_m \sin \omega t$

$i = I_m \sin (\omega t + \phi)$ lead voltage by 0≤ ϕ ≤ 90)





Figure(7.7):Series R-L circuit, phasor and vector diagram

Instantaneous power

$p = vi = V_m I_m \sin(\omega t)\sin(\omega t + \phi) = \frac{V_m I_m}{2}\,[\cos\phi - \cos(2\omega t + \phi)]$

Power consists of:

3) A constant part $\frac{V_m I_m}{2}\,[\cos\phi]$ which is the real power.
4) A pulsating part $\frac{V_m I_m}{2}\,[\cos(2\omega t + \phi)]$of frequency ***double*** that of voltage and current waves. It does not contribute to actual power since its average value over a complete cycle is zero.

Hence, the average power consumed is,

$p = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos\phi = V_{RMS} \times I_{RMS} \times \cos\phi = VI\cos\phi$ [watt]

Power Triangle:

1) If we multiply the voltage triangle by current I (or multiply impedance triangle by I^2), we get power triangle,

(*i*) apparent power (*S*)

It is given by the product of r.m.s. values of applied voltage and circuit current.

$\therefore S = VI = I^2Z$ *volt-amperes (VA)*

(*ii*) active power (*P* or *W*)

It is the power which is actually dissipated in the circuit resistance.

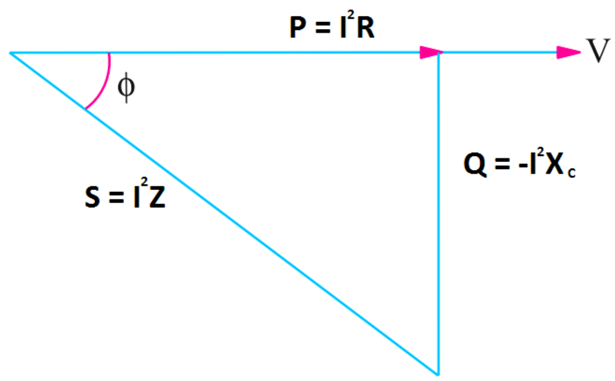
$P = V_R I = VI \cos \varphi = I^2R$ *watts*

(*iii*) reactive power (Q_C)

It is the power developed in the inductive reactance of the circuit.

$Q_C = -V_C I = -VI \sin \varphi = -I^2X_C$ *volt-amperes-reactive (VAR)*

$S = \sqrt{P^2 + (-Q_C)^2}$



Figure(7.8):Power triangle

Representation of AC Quantities in Complex Numbers Forms

$V = V_R - j\,V_C$ or $V = V\angle -\phi$

$I = I + j\,0$ or $I = I\angle 0$

$Z = R - j\,X_C$ or $Z = Z\angle -\phi$

$S = P - j\,Q_C$ or $S = S\angle -\phi$

$Z = \frac{V}{I} = \frac{VR - j\,VC}{I + j\,0} = R - j\,X_C$

$S = VI = (V_R - j\,V_C)(I + j\,0) = P - j\,Q_C$

Power Factor

Power **F**actor **PF** ($\cos\phi = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$) leading

3) A.C. Through Resistance, Inductance and Capacitance (R-L-C)

A pure resistance **R**, pure inductance **L** and a pure capacitance **C** are shown connected in series in Figure(7.11).

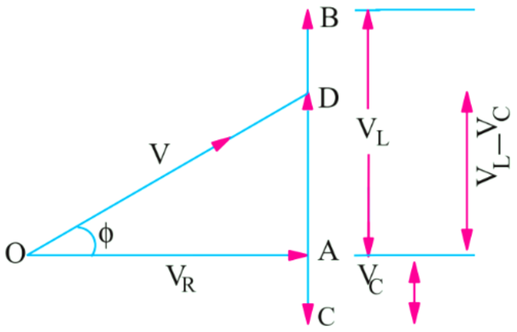
$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$V_R = V\cos\phi$

$(V_L - V_C) = \mp V\sin\phi$

Phase difference angle $\phi = tan^{-1}\frac{(V_L - V_C)}{V_R}$

Power **F**actor **PF** ($\cos\phi = \frac{V_R}{V}$).



Figure(7.9):Voltage triangle

If we divide the ***voltage triangle*** by current (I), we get ***impedance triangle***.

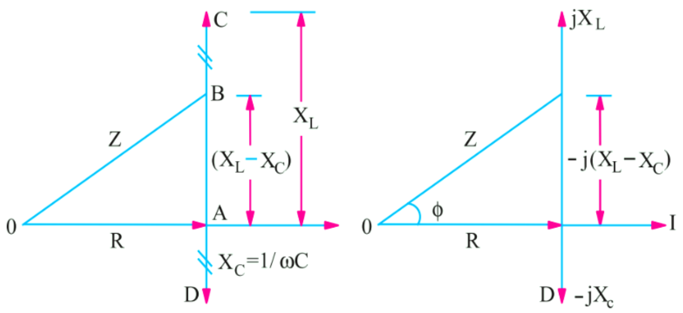
$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$R = Z\cos\phi$

$X_L - X_C = \mp Z\sin\phi$

Phase difference angle $\phi = tan^{-1}\frac{X_L - X_C}{R}$

Power **F**actor **PF** ($\cos\phi = \frac{R}{Z}$).



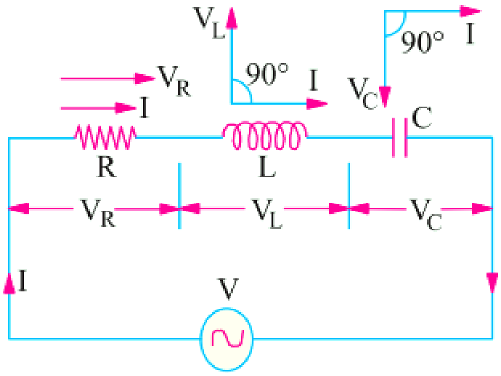
Figure(7.10):Impedance triangles

Notes:

1) If $X_L > X_C$, then the circuit have inductive characteristics, the current lags the voltage by an angle $0 \leq \phi \leq 90$, power factor is lagging and we use the positive sign.
   $v = V_m \sin \omega t$
   $i = I_m \sin(\omega t - \phi)$ lead voltage by $0 \leq \phi \leq 90$)

2) If $X_C > X_L$, then the circuit have capacitive characteristics, the current leads the voltage by an angle $0 \leq \phi \leq 90$, power factor is leading and we use the negative sign.
   $v = V_m \sin \omega t$
   $i = I_m \sin(\omega t + \phi)$ lead voltage by $0 \leq \phi \leq 90$)

3) If $X_C = X_L$, then the circuit have resistive characteristics, the current and voltage are in phase, $\phi = 0$, power factor is unity.
   $v = V_m \sin \omega t$
   $i = I_m \sin \omega t$



Figure(7.11):Series R-L circuit, phasor and vector diagram

Instantaneous power

Depend on wether the circuit characteristics is inductive or capacitive.

Power Triangle:

Depend on wether the circuit characteristics is inductive or capacitive.

Power Factor

Power **F**actor **PF** (cosϕ = $\frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$)

Lecture (7)

SERIES A.C. CIRCUITS

Example 1: In a series circuit containing pure resistance and a pure inductance, the current and the voltage are expressed as:
$i(t) = 5\sin(314t + 2\pi/3)$ and $v(t) = 15\sin(314t + 5\pi/6)$
(a ⟩ What is the impedance of the circuit.(b) What is the value of the resistance? (c) What is the inductance in henrys? (d) What is the average power drawn by the circuit? (e) What is the power factor?

Solution: $\omega = 314\, rad/s$

$$i(t) = 5\sin\left(314t + \frac{2\pi}{3}\right) = 5\angle\frac{2\pi}{3} = -2.5 + j\,4.33$$

$$v(t) = 15\sin\left(314t + \frac{5\pi}{6}\right) = 15\angle\frac{5\pi}{6} = -13 + j7.5$$

$$Z = \frac{v(t)}{i(t)} = \frac{-13 + j7.5}{-2.5 + j\,4.33} = 2.6 + j1.5 = 3\angle 30^o\ \Omega$$

$$\therefore Z = R + j\,X_L \quad = \quad Z\angle\emptyset$$

$$\therefore Z = 3\,\Omega\,, R = 2.6\,, \qquad X_L = 1.5 \ \Rightarrow L = \frac{X_L}{\omega} = \frac{1.5}{314} = 4.78\,mH$$

(d) $P = I^2R = \left(\frac{5}{\sqrt{2}}\right)^2 2.6 = 32.5\,W$

(e) $PF = cos\emptyset = COS\,30^0 = 0.866\, lagging$ ($PF = cos\emptyset = \frac{R}{Z} = \frac{2.6}{3} 0.866\, lagging$)

Alternative solution

$$Z = \frac{V_m}{I_m} = \frac{15}{5} = 3\,\Omega$$

Hence, current lags behind voltage by 30°. It means that it is an R-L circuit.

$$\emptyset = \frac{5\pi}{6} - \frac{2\pi}{3} = \frac{\pi}{6} = 30°$$

$$R = Zcos\emptyset = 3COS\,30° = 2.6\,\Omega$$

$$X_L = Zsin\emptyset = 3sin\,30° = 1.5\,\Omega \ \Rightarrow L = \frac{X_L}{\omega} = \frac{1.5}{314} = 4.78\,mH$$

$$P = v\,i\,cos\emptyset = \left(\frac{15}{\sqrt{2}}\right)\left(\frac{5}{\sqrt{2}}\right) cos30^0 = 32.5\,W$$

$$PF = cos\emptyset = COS\,30^0 = 0.866\,lagging$$

Example 2: The potential difference measured across a coil is 4.5 V, when it carries a direct current of 9 A. The same coil when carries an alternating current of 9 A at 25 Hz, the potential difference is 24 V. Find the current, the power and the power factor when it is supplied by 50 V, 50 Hz supply.

Solution:

At 9A dc

There is no inductive reactance (because for dc sources f =0) only resistance found,

$$R = \frac{V}{I} = \frac{4.5}{9} = 0.5\Omega$$

At 9A ac

There is resistance (same as previous) as well as inductive reactance,

At 25 Hz

$$Z_{(25Hz)} = \frac{V}{I} = \frac{24}{9} = 2.67\Omega$$

$$X_{L(25Hz)} = \sqrt{{Z_{(25Hz)}}^2 - R^2} = \sqrt{2.67^2 - 0.5^2} = 2.623\Omega \Rightarrow L = \frac{X_L}{\omega} = \frac{2.623}{2\pi \times 25} = 0.0167H$$

At 50 Hz

$$X_{L(50Hz)} = \mathbf{2}X_{L(25Hz)} = 2 \times 2.623 = \mathbf{5.246}\Omega$$

$$Z_{(50Hz)} = \sqrt{R^2 + {X_{L(50Hz)}}^2} = \sqrt{0.5^2 + \mathbf{5.246}^2} = 5.27\Omega$$

$$I_{(50Hz)} = \frac{V_{(50Hz)}}{Z_{(50Hz)}} = \frac{50}{5.27} = 9.49A$$

$$P_{(50Hz)} = \left(I_{(50Hz)}\right)^2 R = 9.49^2 \times 0.5 = 45W$$

H.W.1: In a particular R-L series circuit a voltage of 10 V at 50 Hz produces a current of 700 mA while the same voltage at 75 Hz produces 500 mA. What are the values of R andL in the circuit.

Answer: 40 mH., 6.9 Ω.

H.W.2: A series circuit consists of a resistance of6 Ω and an inductive reactance of 8 Ω A potential difference of141.4 V(rm. s) is applied to it. At a certain instant the applied voltage is+ 100 V and is increasing Calculate at this current, (i) the current (ii) the voltage drop across the resistance and (iii) voltage drop across inductive reactance.

Answer: $= \mathbf{-7.847}A, \mathbf{-47}V, \mathbf{147.2}V.$

,

Example 3: In an alternating circuit, the impressed voltage is given by $V = (100 - j50)$ volts and the current in the circuit is $I = (3 - j4)$ A. Determine the real and reactive power in the circuit

Solution:

Power will be found by the conjugate method. Using current conjugate, we have

$$S_{VA} = (100 - j50)(3 + j4) = 300 + j400 - j150 + 200 = 500 + j250 = 559\angle 26.56^0$$
$$= P + j\,Q_L$$

Alternative solution

$V = (100 - j50) = 111.8\angle - 26.56^0\ V, \&$ $\quad I = (3 - j4) = 5\angle - 53.13^0\ A$

$\emptyset = -26.56^0 + 53.13^0 = 26.57^0$

$$Z = \frac{v}{i} = \frac{100 - j50}{3 - j4} = 20 + j10$$
$$= R + jX_L$$

$P = I^2R = 5^2 \times 20 = 500\ W\ (P = vi\,cos\emptyset = 111.8 \times 5 \times COS26.57^0 = 500\ W)$

$Q_L = I^2X_L = 5^2 \times 10 = 250\ VAR(Q_L = vi\,sin\emptyset = 111.8 \times 5 \times sin26.57^0 = 250\ VAR)$

$S = VI = 111.8 \times 5 = 559\ VA$

H.W.3: A choke coil takes a current of 2 A lagging 60° behind the applied voltage of 200 V at 50 Hz. Calculate the inductance, resistance and impedance of the coil. Also, determine the power consumed when it is connected across 100-V 25-Hz supply

Answer: $\mathbf{L = 0.275H, R = 50\Omega, Z_{coil} = 100\Omega, P = 112.5\ W}$

Example 4: When a voltage of 100 V at 50 Hz is applied to a choking coil **A**, the current taken is 8A and the power is 120 W. When applied to a coil **B**, the current is 10A and the power is 500 W What current and power will be taken when 100 V is applied to the two coils connected in series.

Solution:

For coil **A**

$$Z_A = \frac{v}{i} = \frac{100}{8} = 12.5\Omega; P = I^2.R_A \quad \Rightarrow R_A = \frac{P}{I^2} = \frac{120}{8^2} = \frac{15}{8}\Omega$$

$$X_A = \sqrt{Z_A^2 - R_A^2} = \sqrt{125^2 - (\frac{15}{8})^2} = 12.36\Omega$$

For coil **B**

$$Z_B = \frac{v}{i} = \frac{100}{10} = 10\ \Omega;\ P = I^2.R_B \quad \Rightarrow R_B = \frac{P}{I^2} = \frac{500}{10^2} = 5\ \Omega$$

$$X_B = \sqrt{Z_B^2 - R_B^2} = \sqrt{10^2 - 5^2} = 8.66\Omega$$

<u>When coil **A** and coil **B** are in series,</u>

$$R_T = R_A + R_B = \left(\frac{15}{8}\right) + 5 = \frac{55}{9}\ \Omega$$

$$X_T = X_A + X_B = 12.36 + 8.66 = 21.02\ \Omega$$

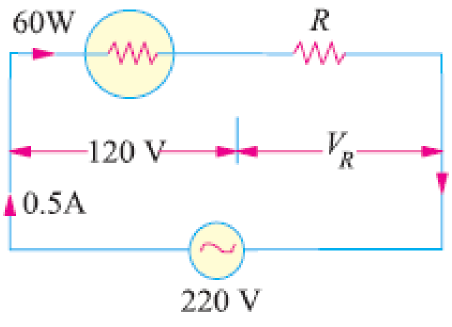
$$Z_T = \sqrt{R_T^2 + X_T^2} = \sqrt{(55/8)^2 + (2102)^2} = 22.1\Omega,$$

$$I = \frac{V}{Z_T} = \frac{100}{22.1} = 4.52A,$$

$$P = I^2 R_T = 4.52^2 \times (55/8) = 140W$$

Example 5: A 120-V 60-W lamp is to be operated on 220-V 50-Hz supply mains. Calculate what value of (a) non-inductive resistance (b) pure inductance would be required in order that lamp is run on correct voltage. Which method is preferable and why.

Solution:



(a)

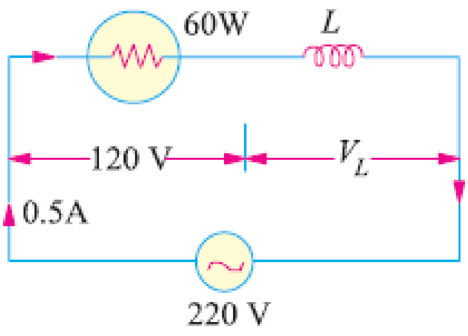
$$I_{bulb} = \frac{P}{V\cos\emptyset} = \frac{60}{120\times\cos 0} = 0.5A$$

(a) $V_R = 220 - 120 = 100\ V,$

$$R = \frac{V_R}{I_{bulb}} = \frac{100}{0.5} = 200\ \Omega$$

$$P_{loss} = {I_{bulb}}^2 \times R = 0.5^2 \times 200 = 50\ W$$

(b) $V_{bulb} = 120V$



(b)

$$V_L = \sqrt{(220^2 - 120^2)} = 184.4V$$

$$V_L = I_{bulb} \times X_L$$

$$184.4 = 0.5 \times (L \times 2\pi \times 50) \Rightarrow L = \frac{184.4}{0.5} \times 3.14 = 1.17H$$

$$P_{loss} = 0\ W$$

Method (b) is preferable to (a) because in method (b) , there is no loss of power.

Example 6: A current of 5 A flows through a non-inductive resistance in series with a choking coil when supplied at 250-V 50-Hz. If the voltage across the resistance is 125 V and across the coil 200 V calculate (a) impedance, reactance and resistance of the coil (b) the power absorbed by the coil and (c) the total power. Draw the vector diagram.

Solution:

From vector diagram,

$$V_R^2 + V_L^2 = V_{coil}^2$$

$$V_R^2 + V_L^2 = 200^2 \qquad (1)$$

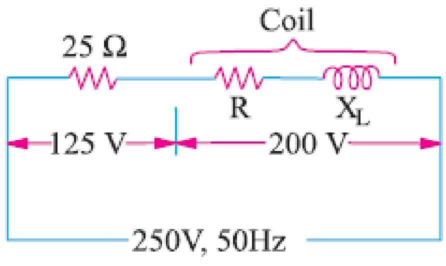
$$(125 + V_R)^2 + V_L^2 = 250^2 \qquad (2)$$

Subtracting Eq. (*1*) from (2) , we get,

$$(125 + V_R)^2 - V_R^2 = 250^2 - 200^2$$

$$V_R = 27.5\ V$$

$$V_L = \sqrt{200^2 - 275^2} = 198.1\ V$$



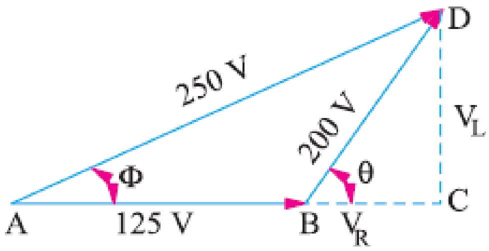
(a)

(*i*) Coil impedance $= \frac{V_{coil}}{I} = \frac{200}{5} = 40\Omega$

$V_R = IR \Rightarrow \therefore R = \frac{27.5}{5} = 5.5\Omega$

$V_L = I.X_L \Rightarrow \therefore X_L = \frac{198.1}{5} = 39.62\Omega$

or $X_L = \sqrt{40^2 - 55^2} = 39.62\Omega$



(b)

(*ii*) $P_{coil} = I^2R = 5^2 \times 5.5 = 137.5W$

Also $P = 200 \times 5 \times 27.5/200 = 137.5W$

(*iii*) $\cos\varphi_{total} = \frac{AC}{AD} = \frac{152.5}{250} = \ \ 0.61\ lagging$

Total power $= VI\cos\varphi = 250 \times 5 \times 0.61 = 762.5W$

The power may also be calculated by using I^2R formula.

Series resistance $= 125/5 = 25\Omega$

Total *circuit* resistance $= 25 + 5.5 = 30.5\Omega$

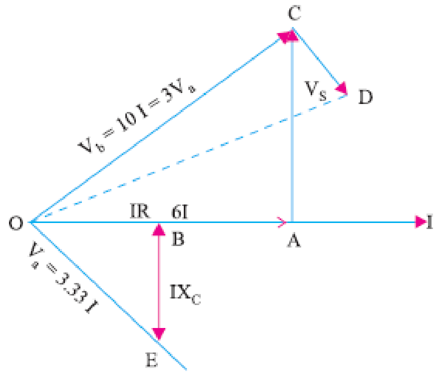
$\therefore$ Total power $= 5^2 \times 30.5 = 762.5W$

H.W.4: Two coils A andB are connected in series across a 240-V 50-Hz supply The resistance of A is 5 Ω and the inductance of B is 0.015 H. If the input from the supply is 3 kW and 2 kVAR,fmd the inductance ofA and the resistance of B. Calculate the voltage across each coil.

Answer: $R_B = 8.3\Omega, L_A = 0.0132H$ **(approx),** $97.5V, 143.5V$

Example 7: For the circuit shown in Fig. fnd the values of R and C so that $V_b = 3V_a$, and V_b and V_a are in phase quadrature. Find also the phase relationships between V_s and V_b, and V_b and I.

Solution:



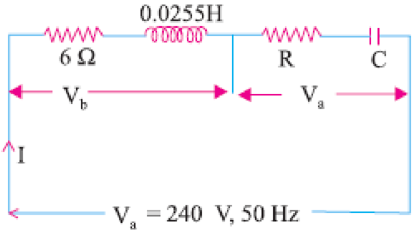
$X_{Lb} = 314 \times 0.0255 = 8$ ohms, $R_b = 6$ ohms

$Z_b = 6 + j8 = 10\angle 53.13^o$ ohms

$$|\emptyset_a| + |\emptyset_b| = 90^0 \Rightarrow |\emptyset_a| = 90^0 - |\emptyset_b| = 90^0 - 53.13^0 = 36.87^o$$

As the circuit in series, hence the same current flow in the circuit.

$$I = \frac{V_b}{Z_b} = \frac{V_a}{Z_a} \;\&\; \because\; V_b = 3V_a$$



$$\therefore \left|\frac{3V_a}{Z_b}\right| = \left|\frac{V_a}{Z_a}\right| \Rightarrow \frac{3}{|Z_b|} = \frac{1}{|Z_a|} \Rightarrow |Z_a| = \frac{|Z_b|}{3}$$

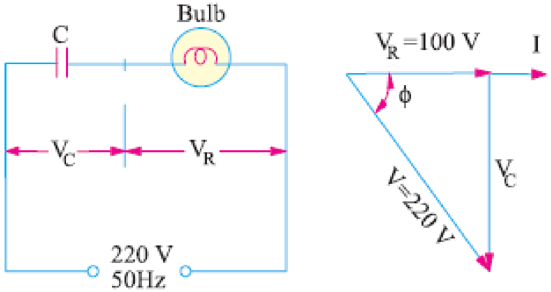
$$|Z_a| = \frac{|Z_b|}{3} = \frac{10}{3} = 3.33\ ohms$$

$$R_a = Z_a \cos\varphi_a = 3.33 \times \cos 36.87^o = 2.66\ ohms$$

$$X_{ca} = Z_a \sin\varphi_a = 3.33 \times \sin 36.87^o = 2\ ohms$$

H.W.5: A tungsten filament bulb rated at 500-W 100-V is to be connected to series with a capacitance across 200-V 50-Hz supply Calculate: (a) the value of capacitor such that the voltage and power consumed by the bulb are according to the rating of the bulb. (b) the power factor of the current drawn from the supply (c) draw the phasor diagram of the circuit.

Answer: 81μF, 0.455 **(lead)**



Example 8: It is desired to operate a 100-W 120-Velectric lamp at its current rating from a 240-V 50-Hz supply Give details of the simplest manner in which this could be done using (a) a resistor (b) a capacitor and (c) an indicator having resistance of 10 Ω. What power factor would be presented to the supply in each case and which method is the most economical of power.

Solution:

$$\mathrm{I}_{bulb} = \frac{P}{V\cos\emptyset} = \frac{100}{120\times\cos 0} = 0.83\text{ A}, R_{bulb} = \frac{p}{\mathrm{I}_{bulb}{}^2} = \frac{100}{0.83^2} = 145\ ohms$$

(a) Using a resistor alone

$$V_R = 240 - 120 = 120\text{V}$$

$$\mathrm{R} = \frac{V_R}{\mathrm{I}_{bulb}} = \frac{120}{0.83} = 145\ \Omega$$

$$\cos\emptyset = unity$$

$$\mathrm{P}_{loss\ total} = I_{bulb}{}^2 \times R_{total} = 0.83^2 \times (145 + 145) = 200\ W$$

(b) Using capacitor alone

$$V_c = \sqrt{V^2 - V_{bulb}^2} = \sqrt{240^2 - 120^2} = 207.9\text{ V}$$

$$\mathrm{X_C} = \frac{V_C}{\mathrm{I}_{bulb}} = \frac{207.9}{0.83} = 250.5\ \Omega \Rightarrow \mathrm{C} = \frac{1}{2\pi f \mathrm{X_C}} = \frac{1}{2\pi\times 50\times 250.5} = 12.7\mu\text{F}$$

$$\cos\emptyset = \frac{V_R}{V} = \frac{120}{240} = 0.5\text{ lead}$$

$$\mathrm{P}_{loss\ total} = I_{bulb}{}^2 \times R_{total} = 0.83^2 \times (145) = 100\ W$$

(c) Using an inductor

$$\mathrm{V_R} = 0.83\times 10 = 8.3\text{ V}$$

$$V_L = \sqrt{V^2 - (V_{bulb} + \mathrm{V_R})^2} = \sqrt{240^2 - (120 + 8.3)^2} = 202.8\text{ V}$$

$$\mathrm{X_L} = \frac{V_L}{\mathrm{I}_{bulb}} = \frac{202.8}{0.83} = 244.3\ \Omega \Rightarrow \mathrm{L} = \frac{\mathrm{X_L}}{2\pi f} = \frac{244.3}{2\pi\times 50} = 0.778\text{H}$$

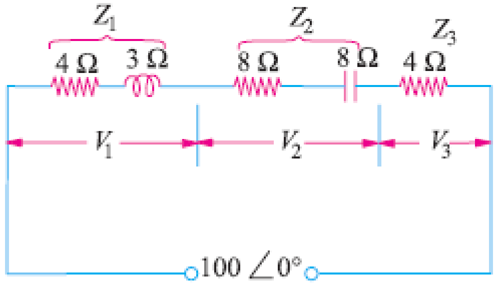
$$\cos\emptyset = \frac{V_{R\ total}}{V} = \frac{120 + 8.3}{240} = 0.535\text{ (lag)}$$

$$\mathrm{P}_{loss\ total} = I_{bulb}{}^2 \times R_{total} = 0.83^2 \times (145 + 10) = 107\ W$$

Method (b) is most economical because it involves least consumption of power.

Example 9: In Fig. shown, calculate (i) current (ii) voltage drops V_1, V_2, and V_3 and (iii) power absorbed by each impedance and total power absorbed by the circuit. Take voltage vector along the reference axis.

Solution:



$Z_1 = (4 + j3)\Omega$, $Z_2 = (6 - j8)\Omega$, $Z_3 = (4 + j0)\Omega$

$$Z = Z_1 + Z_2 + Z_3 = (4 + j3) + (6 - j8) + (4 + j0)$$
$$= (14 - j5)\Omega$$

Taking $V = V\angle 0^o = 100\angle 0^o = (100 + j0)$

$$\therefore I = \frac{V}{Z} = \frac{100}{(14-j5)} = \frac{100(14+j5)}{(14-j5)(14+j5)} = 6.34 + j2.26$$

(i) $I = \sqrt{(6.34^2 + 2.26^2)} = 6.73A$

$(\boldsymbol{ii}) V_1 = IZ_1 = (6.34 + j2.26)(4 + j3) = 18.58 + j\,28.06$

$V_2 = IZ_2 = (6.34 + j2.26)$ (6-78) $= 56.12 - j37.16$

$V_3 = IZ_3 = (6.34 + j2.26)(4 + j0) = 25.36 + j9.04$

$V = 100 + j0$ (check)

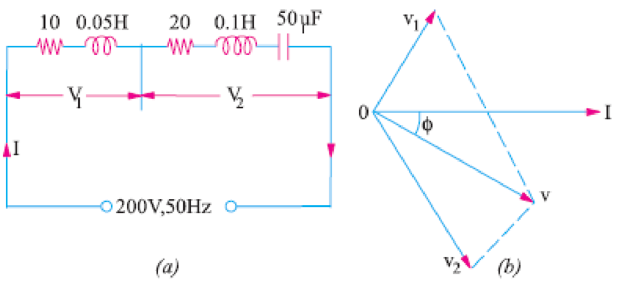
$(\boldsymbol{iii}) P_1 = 6.73^2 \times 4 = 181.13W, P_2 = 6.73^2 \times 6 = 271.74W, P_3 = 6.73^2 \times 4 = 181.13W,$

Total $= 34W$

Otherwise (using current conjugate) $P_{VA} = (100 + j0)$ (6.34-72.26) $= 634 - j226$

H.W.6: for the circuit shown in Fig. indicating the resistance and reactance drops, the terminal voltages V_1 and V_2 and the current. Find the values of (i) the current I(ii)V_1 and V_2 and (iii) pf

Answer: $5.83A, 108.4V, 221\,V, 0.875$ **(lead)**



Lecture (6) A.C. Through Resistance, Inductance and Capacitances

1) A.C. Through Pure Ohmic Resistance Alone

Consider the circuit shown in Figure(6.1).

Let R = ohmic resistance ; i = instantaneous current ; $I_m = \frac{V_m}{R}$

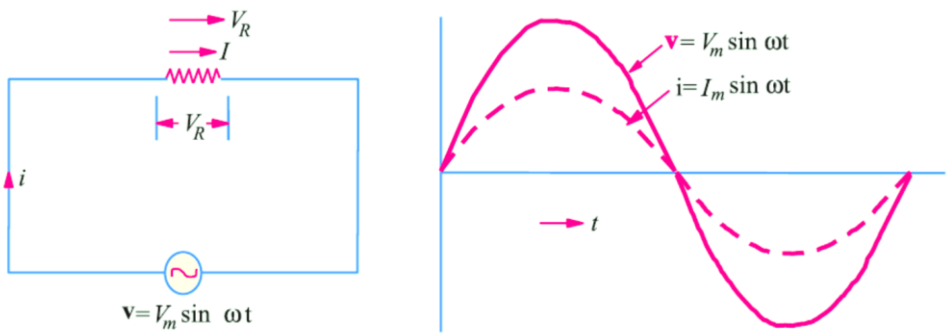
$v = V_m \sin\theta = V_m \sin\omega t$ ...(1)

$$i = \frac{v}{R} = \frac{V_m \sin\omega t}{R} = \frac{V_m}{R}\sin\omega t = I_m \sin\omega t$$

So current and voltage in resistance are in phase (phase difference = 0).

$v = V_m \sin\omega t$

$i = I_m \sin\omega t$



Figure(6.1)

Power. Instantaneous power,

$$p = vi = V_m I_m \sin^2\omega t = \frac{V_m I_m}{2}(1-\cos 2\omega t) = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

Power consists of:

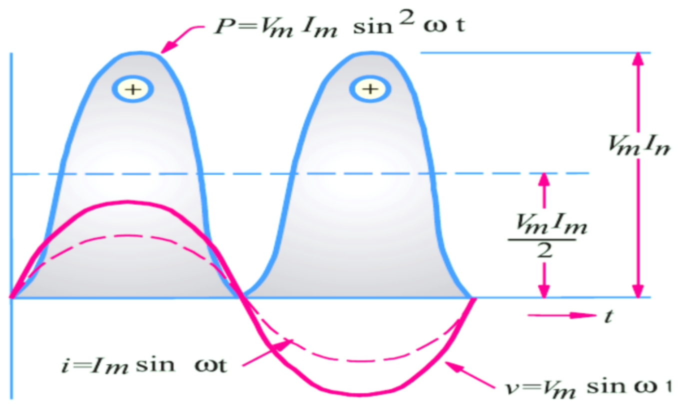
1) A constant part $\frac{V_m I_m}{2}$ and
2) A fluctuating part $\frac{V_m I_m}{2}\cos 2\omega t$ of frequency ***double*** that of voltage and current waves.

For a complete cycle, the average value of $\frac{V_m I_m}{2}\cos 2\omega t = 0$.

Hence, power for the whole cycle is,

$$p = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{RMS} \times I_{RMS} \text{ [watt]}$$

It is seen from Figure(6.2) that no part of the power cycle becomes negative at any time. In other words, in a purely resistive circuit, power is never zero. This is so because the instantaneous values of voltage and current are always either both positive or negative and hence the product is always positive.



Figure(6.2)

2) A.C. Through Pure Inductance Alone

Consider the circuit shown in Figure(6.3).

Let L = self inductance ; i = instantaneous current

$v = V_m \sin \theta = V_m \sin \omega t$

$v_L = L \frac{di}{dt}$

$$i = \frac{1}{L}\int v.dt = \frac{1}{L}\int V_m \sin\omega t.dt = \frac{V_m}{\omega L}(-\cos\omega t) = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= \frac{V_m}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Where: X_L is the inductive rectance (Ω) and equal ($X_L = \omega L$), and $I_m = \frac{V_m}{X_L}$

Note: $-\cos\omega t = \cos(\omega t - \pi), \cos\omega t = \sin\left(\omega t + \frac{\pi}{2}\right),$

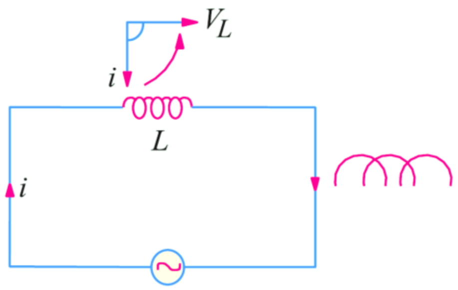
$\therefore -\cos\omega t = \sin\left(\omega t - \pi + \frac{\pi}{2}\right) = \sin\left(\omega t - \frac{\pi}{2}\right)$

So in inductance, the current lags the voltage by 90^0 (phase difference = 90^0).

$v = V_m \sin \omega t$

$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$

Now, ($X_L = \omega L = 2\pi f L$ ohm). It is seen that X_L depends directly on frequency of the voltage. Higher the value of f, greater the reactance offered and ***vice-versa***.



Figure(6.3)

Power. Instantaneous power,

$$p = vi = V_m I_m \sin(\omega t)\sin\left(\omega t - \frac{\pi}{2}\right) = -V_m I_m \sin(\omega t)\cos(\omega t) = -\frac{V_m I_m}{2}\sin 2\omega t$$

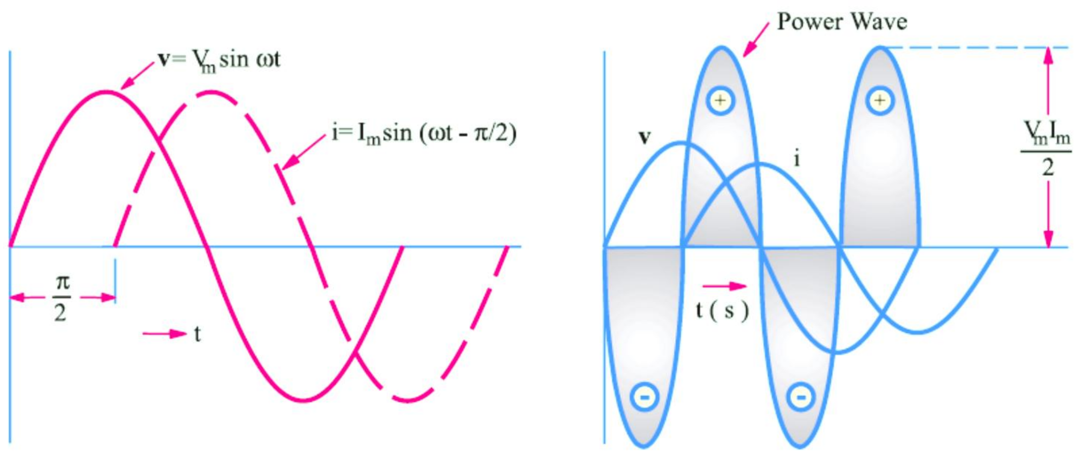
Note: $\sin\left(\omega t - \frac{\pi}{2}\right) = \sin\omega t\cos\frac{\pi}{2} - \cos\omega t\sin\frac{\pi}{2} = -\cos\omega t$,

$\sin 2\omega t = 2\sin(\omega t)\cos(\omega t)$

Power for whole cycle is :

$$p = -\frac{V_m I_m}{2}\int_0^{2\pi}\sin 2\omega t\, dt = 0$$

It is also clear from Figure(6.4) that the average demand of power from the supply for a complete cycle is zero. Here again it is seen that power wave is a sine wave of frequency *double* that of the voltage and current waves. The maximum value of the instantaneous power is $\frac{V_m I_m}{2}$.



Figure(6.4)

Complex Voltage Applied to Pure Inductance

If it is applied voltage has a complex form and is given by :

$V = V_{1m}\sin\omega t + V_{3m}\sin 3\omega t + V_{5m}\sin 5\omega t$

then the reactances offered to the fundamental voltage wave and the harmonics would be different.

For the fundamental wave, $X_1 = \omega L$. For 3rd harmonic ; $X_3 = 3\omega L$. For 5th harmonic ; $X_5 = 5\omega L$.

Hence, the current would be given by the equation.

$$i = \frac{V_{1m}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) + \frac{V_{3m}}{3\omega L}\sin\left(3\omega t - \frac{\pi}{2}\right) + \frac{V_{5m}}{5\omega L}\sin\left(5\omega t - \frac{\pi}{2}\right)$$

3) A.C. Through Pure Capacitance Alone

Let

$v = V_m \sin\theta = V_m \sin\omega t$

q = Charge on plates at that instant.

Then $q = Cv = C\,V_m \sin \omega t$ ...where C is the capacitance, q is Charge on plates

$$i = \frac{dq}{dt} = \frac{d}{dt}(C\,V_m \sin \omega t) = \omega C V_m \cos \omega t = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \cos \omega t = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= \frac{V_m}{X_c} \sin\left(\omega t + \frac{\pi}{2}\right) = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

Where

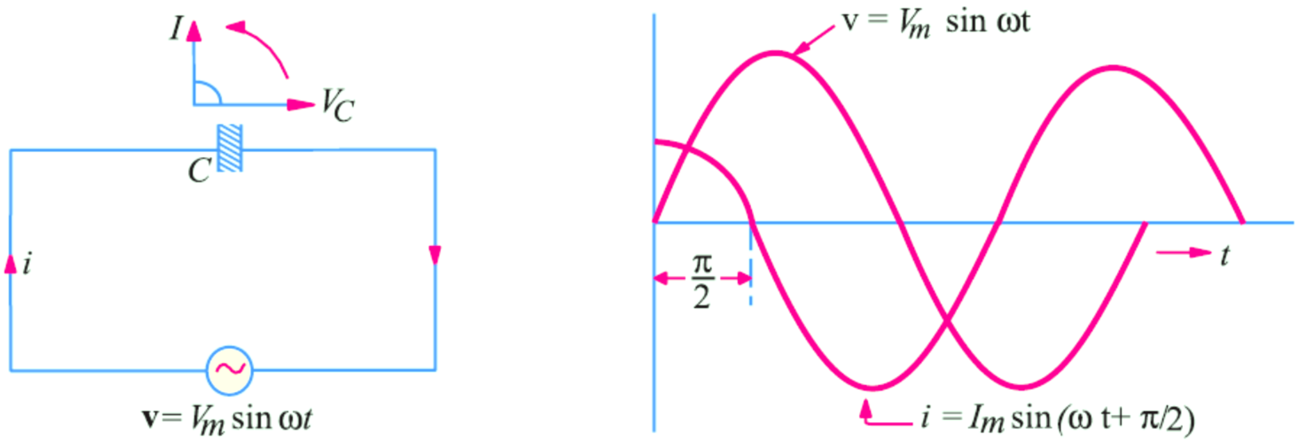
Where: X_c is the capacitive rectance (Ω) and equal ($X_c = \frac{1}{\omega C}$), and $I_m = \frac{V_m}{X_c}$

So in capacitance, the current leads the voltage by 90^0 (phase difference = 90^0).

$v = V_m \sin \omega t$

$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$

Now, ($X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ ohm). It is seen that X_c depends inversly on frequency of the voltage. Higher the value of f, lower the reactance offered and ***vice-versa***.



Figure(6.5)

Power. Instantaneous power,

$p = vi = V_m I_m \sin(\omega t) \sin\left(\omega t + \frac{\pi}{2}\right) = V_m I_m \sin(\omega t)\cos(\omega t) = \frac{V_m I_m}{2} \sin 2\omega t$

Power for whole cycle is :

$p = \frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, dt = 0$

In a purely capacitive circuit, the average demand of power from supply is zero (as in a purely inductive circuit). The power wave is a sine wave of frequency *double* that of the voltage and current waves. The maximum value of the instantaneous power is $\frac{V_m I_m}{2}$.

Example 1: *A 60-Hz voltage of 115 $V(r.m.s.)$ is impressed on a 100 ohm resistance: (i) Write the time equations for the voltage and the resulting current. Let the zero point of the voltage wave be at $t = 0$ (ii) Show the voltage and current on a time diagram. (iii) Show the voltage and current on a phasor diagram.*

Solution:

$(i) V_{\max} = \sqrt{2}v = \sqrt{2} \times 115 = 163V$

$I_{\max} = \frac{V_{\max}}{R} = \frac{163}{100} = 1.63A; \varphi = 0; \omega = 2\pi f = 2\pi \times 60 = 377 rad/s$

The required equations are: $v(t) = 1.63 \sin 377t$ and $i(t) = 1.63 \sin 377t$

(ii) and (iii) These are similar to those shown in Figure(6.1) (a) and (b)

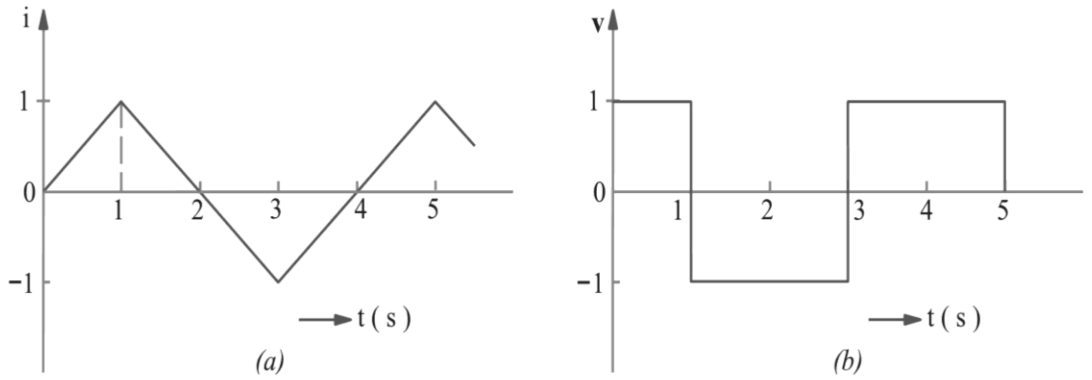
Example 2: *Through a coil of inductance 1 henry, a current of the wave-form shown in Figure(1)(a) is flowing. Sketch the waveform of the voltage across the inductance and calculate the rm.s. value of the voltage.*

Solution:

The instantaneous current $i(t)$ is given by

$$i = \begin{cases} t & 0 < t < 1 \\ 2 - t & 1 < t < 3 \\ t - 4 & 3 < t < 4 \end{cases}$$

$$v = L\frac{di}{dt} = \begin{cases} 1 \times \frac{dt}{dt} = 1 & 0 < t < 1 \\ 1 \times \frac{d(2-t)}{dt} = -1 & 1 < t < 3 \\ 1 \times \frac{d(t-4)}{dt} = 1 & 3 < t < 4 \end{cases}$$



Figure(1)

Example 2: *The voltage applied to a purely inductive coil of self-inductance 15.9 mH is given by the equation, $v = 100sin314t + 75sin942t + 50sin1570t$. Find the equation of the resulting current wave.*

Solution:

Here $\omega = 314rad/s,$

$X_1 = \omega L = (15.9 \times 10^{-3}) \times 314 = 5\Omega$

$X_3 = 3\omega L = 3 \times 5 = 15\Omega.$

$X_5 = 5\omega L = 5 \times 5 = 25\Omega$

Hence, the current equation is

$i = (100/5)\sin(314t - \pi/2) + (75/15)\sin(942t - \pi/2) + (50/25)\sin(1570t - \pi/2)$

or

$i = 20\sin(314t - \pi/2) + 5\sin(942t - \pi/2) + 2\sin(1570\,t - \pi/2)$

Example 3: *The voltage applied across 3-branched circuit of Fig. 11.65 is given by v = 100 sin(5000t + π/4). Calculate the branch currents and total current.*

Solution:

The total instantaneous current is the vector sum o fthe three branch currents.

$i_r = i_R + i_L + i_C.$

Now $i_R = v/R = 100\sin(5000\,t + \pi/4)/25 = 4\sin(5000\,t + \pi/4)$

$$i_L = \frac{1}{L}\int \mathrm{v}\,dt$$

$$= \frac{10^3}{2}\int 1\,00\sin(5000t + \frac{\pi}{4})dt = \frac{10^3 \times 100}{2}\left[\frac{-\cos(5000t + \pi/4)}{5000}\right] = -10\cos(5000\,t + \pi/4)$$

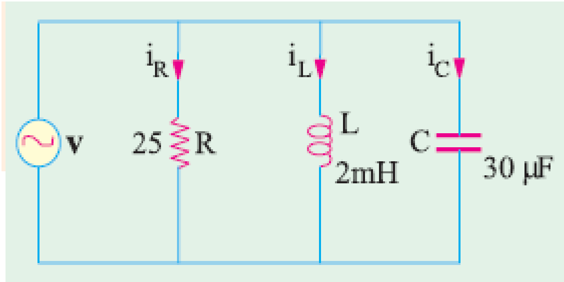
$$i_C = C\frac{dv}{dt}$$

$$= C\frac{d}{dt}\left[100\sin\left(5000\,t + \frac{\pi}{4}\right)\right] = 30 \times 10^{-6} \times 100 \times 5000 \times cos\left(5000\,t + \frac{\pi}{4}\right)$$

$= 15\cos(5000\,t + \pi/4)$

$i_r = 4\sin(5000\,t + \pi/4) - 10\cos(5000\,t + \pi/4) + 15\cos(5000t + \pi/4)$

$$= 4 \sin\ (5000\,t + \pi/4) + 5 \cos\ (5000\,t + \pi/4)$$



Lecture (8)

PARALLEL A.C. CIRCUITS

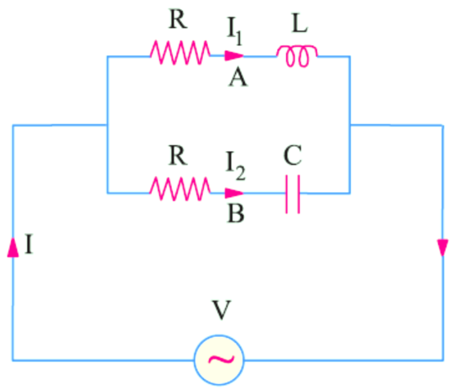
Solving Parallel Circuits

When impedances are joined in parallel, there are three methods available to solve such circuits:

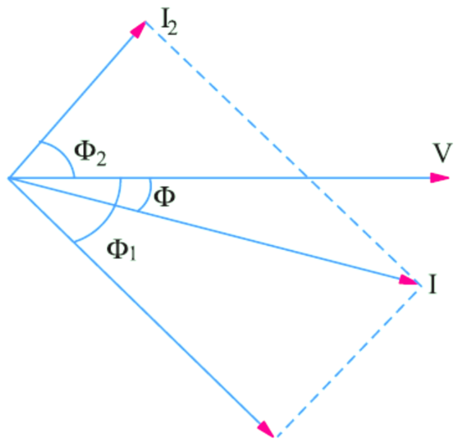
i. ***Vector or phasor Method***
ii. ***Admittance Method and***
iii. ***Vector Algebra***

i. Vector or Phasor Method

Consider the circuits shown in Figure(8.1). Here, two impedances *A* and *B* have been joined in parallel across an r.m.s. supply of *V* volts. The voltage across two parallel branches *A* and *B* is the same, but currents through them are different.



Figure(8.1)



Figure(8.2)

For Branch *A*,

$Z_1 = \sqrt{R_1^2 + X_L^2}\ ,\ I_1 = \frac{V}{Z_1}, cos\emptyset_1 = \frac{R_1}{Z_1}$

Current I_1 lags behind the applied voltage by $\emptyset_1$ (Figure(8.2)).

For Branch B,

$Z_2 = \sqrt{R_2^2 + X_C^2}\ ,\ I_2 = \frac{V}{Z_2}, cos\emptyset_2 = \frac{R_2}{Z_2}$

Current I_2 leads the applied voltage by $\emptyset_2$ (Figure(8.2)).

Resultant Current I

The resultant circuit current I is the vector sum of the branch currents I_1 and I_2 and can be found by

(*i*) using parallelogram law of vectors, as shown in Figure(8.2)

(*ii*) resolving I_2 into their X- and Y-components (or active and reactive components respectively) and then by combining these components, as shown in Figure(8.3).

Method **(*ii*)** is preferable, as it is quick and convenient.

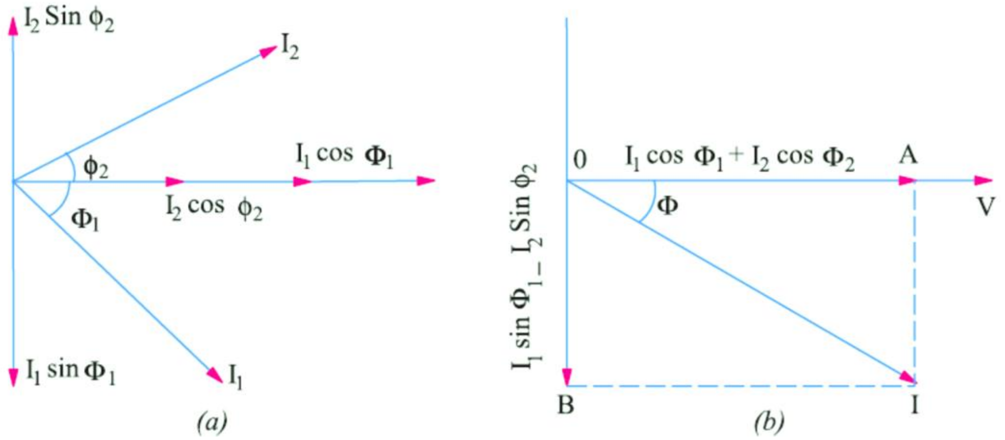
X-components = $I_1 \cos \varphi_1 + I_2 \cos \varphi_2$

Y-components = $I_2 \sin \varphi_2 - I_1 \sin \varphi_1$

$$I_{resultant} = \sqrt{(X - comonent)^2 + (Y - component)^2}$$
$$= \sqrt{(I_1 cos\emptyset_1 + I_2 cos\emptyset_2)^2 + (I_2 sin\emptyset_2 - I_1 sin\emptyset_1)^2}$$

$$tan\emptyset = \frac{Y - component}{X - comonent} = \frac{I_2 sin\emptyset_2 - I_1 sin\emptyset_1}{I_1 cos\emptyset_1 + I_2 cos\emptyset_2}$$
$$cos\emptyset = \frac{X - component}{I_{resultant}} = \frac{I_1 cos\emptyset_1 + I_2 cos\emptyset_2}{I_{resultant}}$$

If tan φ is positive, then current leads and if tan φ is negative, then current lags behind the applied voltage V.



Figure(8.3)

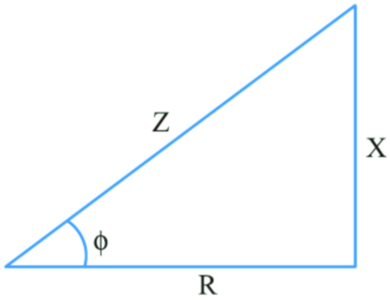
ii. Admittance Method (Y)

Admittance of a circuit is defined as the reciprocal of its impedance . Its symbol is Y.

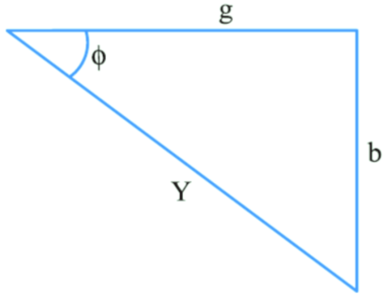
$$\therefore Y = \frac{1}{Z} = \frac{I}{V}$$

Its unit is Siemens (S). A circuit having an impedance of one ohm has an admittance of one Siemens. The old unit was mho (ohm spelled backwards).

As the impedance **Z** of a circuit has two components **(X)** and (**R**) (Figure(8.4)), similarly, admittance **Y** also has two components as shown in Figure(8.5) . The Xcomponent is known as conductance **(g)** and Y-component as susceptance **(b)**.



Figure(8.4)



Figure(8.5)

$$g = Y\cos\varphi = \frac{1}{Z} \times \frac{R}{Z} = \frac{R}{Z^2} = \frac{R}{R^2 + X^2}$$
$$b = Y\sin\varphi = \frac{1}{Z} \times \frac{X}{Z} = \frac{X}{Z^2} = \frac{X}{R^2 + X^2}$$
$$Y = \sqrt{g^2 + b^2}$$

The unit of g, b and Y is Siemens. We will regard the ***capacitive susceptance as positive and inductive susceptance as negative.***

Application of Admittance Method

Consider the 3-branched circuit of Figure(8.5). Total conductance is the sum of the three branches conductances. Similarly, total susceptance is the sum of the three branches susceptances of different branches.

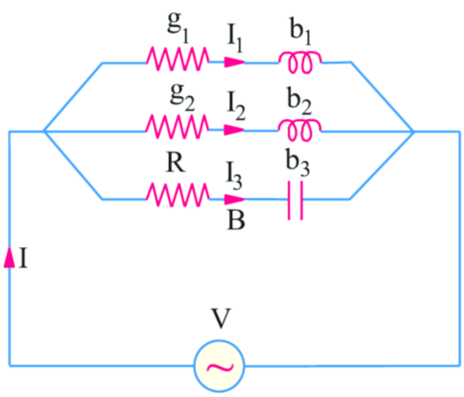
Total conductance $G = g_1 + g_2 + g_3$ ....

Total susceptance $B = (-b_1) + (-b_2) + b_3$ ......

Total admitance $Y = \sqrt{G^2 + B^2}$

Total current I = VY

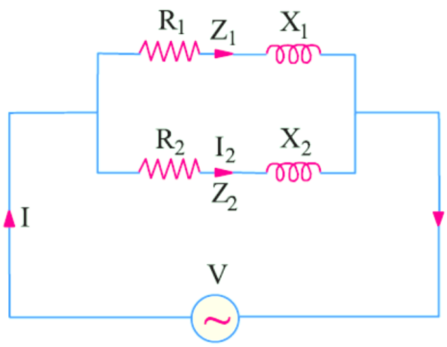
Power factor $\cos\varphi = \frac{G}{Y}$



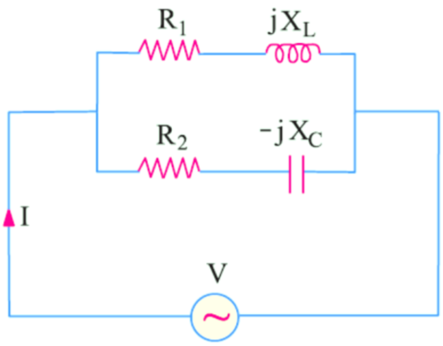
Figure(8.5)

iii. Complex or Phasor Algebra

Consider the parallel circuit shown in Figure(8.6). The two impedances, Z_1 and Z_2, being in parallel, have the same p.d. across them.



Figure(8.6)



Figure(8.7)

$$I_1 = \frac{V}{Z_1} \text{ , } I_2 = \frac{V}{Z_2}$$

$$I = I_1 + \; I_2 = \frac{V}{Z_1} + \frac{V}{Z_2} = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) = V(Y_1 + Y_2) = VY$$

where **Y** = total admittance = **Y1 + Y2**

It should be noted that ***admittances* are *added*** for parallel branches, whereas for branches in series, it is the *impedances* which are *added*. ***However, it is important to remember that since both admittances and impedances are complex quantities, all additions must be in complex form.*** Simple arithmetic additions must not be attempted.

Considering the two parallel branches of Figure(8.7), we have

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L} = \frac{(R_1 - jX_L)}{(R_1 + jX_L) \times (R_1 - jX_L)} = \frac{R_1}{{R_1}^2 + {X_L}^2} - j\frac{X_L}{{R_1}^2 + {X_L}^2}$$

$$= g_1 - jb_1$$

Where

$g_1 = \dfrac{R_1}{{R_1}^2 + {X_L}^2}$, conductance of upper branch

$b_1 = -\dfrac{X_L}{{R_1}^2 + {X_L}^2}$, suseptance of upper branch

Similarly

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_c} = \frac{(R_2 + jX_c)}{(R_2 - jX_c) \times (R_2 + jX_c)} = \frac{R_2}{{R_2}^2 + {X_c}^2} + j\frac{X_c}{{R_2}^2 + {X_c}^2} = g_2 + jb_2$$

Where

$g_2 = \dfrac{R_2}{{R_2}^2 + {X_c}^2}$, conductance of lower branch

$b_2 = \dfrac{X_c}{{R_2}^2 + {X_c}^2}$, suseptance of lower branch

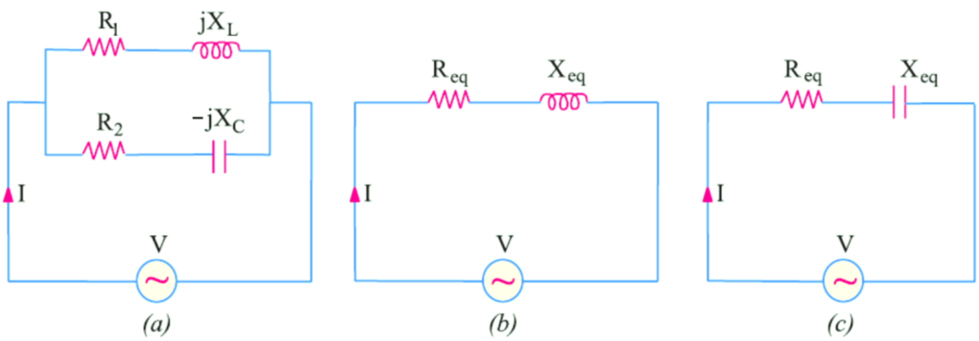
$Y = Y_1 + Y_2 = (g_1 - jb_1) + (g_2 + jb_2) = (g_1 + g_2) - j(b_1 - b_2) = G - jB$

$Y = \sqrt{(g_1 + g_2)^2 + (b_1 - b_2)^2}$

$$\emptyset = tan^{-1}\left(\frac{b_1 - b_2}{g_1 + g_2}\right)$$

Series Equivalent of a Parallel Circuit

Consider the parallel circuit of Figure(8.8)(a).



Figure(8.8)

$$Y_1 = \frac{R_1}{{R_1}^2 + {X_L}^2} - j\frac{X_L}{{R_1}^2 + {X_L}^2} = g_1 - jb_1$$

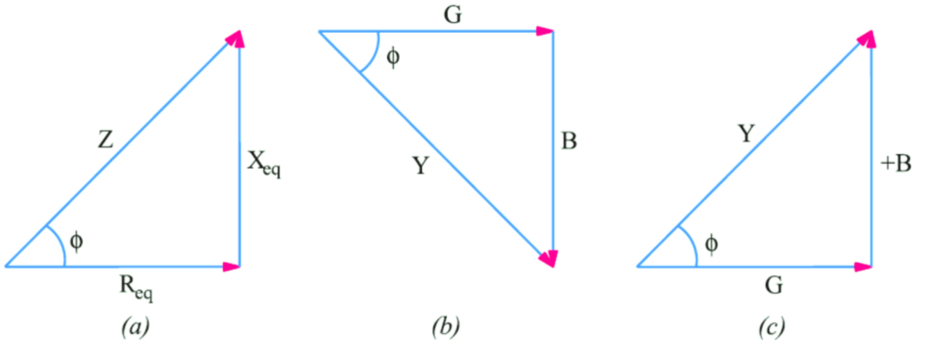
$$Y_2 = \frac{R_2}{{R_2}^2 + {X_c}^2} + j\frac{X_c}{{R_2}^2 + {X_c}^2} = g_2 + jb_2$$

$Y = Y_1 + Y_2 = (g_1 - jb_1) + (g_2 + jb_2) = (g_1 + g_2) + j(b_2 - b_1) = G + jB$

As seen from Figure(8.9)

$R_{eq} = Z\cos\emptyset = \dfrac{1}{Y} \times \dfrac{G}{Y} = \dfrac{G}{Y^2}$, $X_{eq} = Z\sin\emptyset = \dfrac{1}{Y} \times \dfrac{B}{Y} = \dfrac{B}{Y^2}$

Hence, equivalent series circuit is as shown in Fig. Figure(8.8)(b) or (c) depending on whether net susceptance B is negative (inductive) or positive (capacitive). If B is negative, then it is an R-L circuit of Figure(8.8) (b) and if B is positive, then it is an R-C circuit of Figure(8.8)(c).



Figure(8.9)

Parallel Equivalent of a Series Circuit

The two circuits will be equivalent if Y of Figure(8.9)(a) is equal to the Y of the circuit of Figure(8.9)(b).

Series Circuit

$$Y_s = \frac{1}{R_s + jX_s} = \frac{R_s - jX_s}{(R_s + jX_s) \times (R_s - jX_s)} = \frac{R_s - jX_s}{({R_s}^2 + {X_s}^2)}$$

$$= \frac{R_s}{({R_s}^2 + {X_s}^2)} - \boldsymbol{j}\frac{X_s}{({R_s}^2 + {X_s}^2)}$$

Parallel Circuit

$$Y_p = \frac{1}{R_p + j0} + \frac{1}{0 + jX_p} = \frac{1}{R_p} + \frac{1}{jX_p} = \frac{1}{R_p} - j\frac{1}{X_p}$$

$$\therefore \frac{R_s}{({R_s}^2 + {X_s}^2)} - \boldsymbol{j}\frac{X_s}{({R_s}^2 + {X_s}^2)} = \frac{1}{R_p} - j\frac{1}{X_p}$$

$$\therefore \frac{1}{R_p} = \frac{R_s}{({R_s}^2 + {X_s}^2)} \quad or\ R_p = \frac{({R_s}^2 + {X_s}^2)}{R_s}$$

$$\therefore \frac{1}{X_p} = \frac{X_s}{({R_s}^2 + {X_s}^2)} \quad or\ X_p = \frac{({R_s}^2 + {X_s}^2)}{X_s}$$