University of Diyala

Collage of Engineering

Department of Chemical Engineering

Fundamentals of Electrical Engineering

Second Class

By

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Basic Concepts

1.1 System of Units

The International System of Units is commonly called **SI**. In this system, there are seven principal units from which the units of all other physical quantities can be derived. The seven base units are shown in Table (1.1). The SI units also uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit. Table (1.2) shows the SI prefixes and their symbols.

Table (1.1): The basic SI units

Quantity	Basic Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Light intensity	candela	cd
Charge	coulomb	c

Table (1.2): The SI prefixes

Multiplier	Prefix	Symbol
$\mathbf{10^{18}}$	exa	E
$\mathbf{10^{15}}$	peta	P
$\mathbf{10^{12}}$	tera	T
$\mathbf{10^{9}}$	giga	G
$\mathbf{10^{6}}$	mega	M
$\mathbf{10^{3}}$	kilo	k
$\mathbf{10^{2}}$	hecto	h
10	deka	da
$\mathbf{10^{-1}}$	deci	d
$\mathbf{10^{-2}}$	centi	c
$\mathbf{10^{-3}}$	milli	m
$\mathbf{10^{-6}}$	micro	µ
$\mathbf{10^{-9}}$	nano	n
$\mathbf{10^{-12}}$	pico	p
$\mathbf{10^{-15}}$	femto	f
$\mathbf{10^{-18}}$	atto	a

1.2 Charge and Current

1. **Electric Charge (q)**: The electric charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (**C**). The charge e on an electron is negative and equal in magnitude to ($\mathbf{1.06 \times 10^{-19}}$**C**), while a proton carries a positive charge of the same magnitude as the electron.
2. **Electric Current ($\boldsymbol{i}$)**: The electric current is the time rate of change of charge, measured in amperes (**A**).

$$\boldsymbol{i = \frac{dq}{dt}} \qquad \boldsymbol{q = \int_{t_1}^{t_2} i\,dt}$$

Electric current can be classified into two basic types:

- **Direct Current (dc)**: is a current that remains constant with time. The symbol **(I)** is used to represent such a constant current.

- **Alternating Current (ac)**: is a current that varies sinusoidally with time. The symbol ($\boldsymbol{i}$) is used to represent such a time-varying current.

Example: Determine the total charge entering a terminal between t=1s and t=2s if the current passing the terminal is $i = (3t^2 - t)A$.

Solution:

$$q = \int_{t_1}^{t_2} idt = \int_{1}^{2} (3t^2 - t)dt = (t^3 - \frac{t^2}{2})\Big|_1^2 = (8-2) - \left(1 - \frac{1}{2}\right) = \mathbf{5.5}\boldsymbol{C}$$

Q: If the current flowing through an element is:

$$i = \begin{cases} 4A, & 0 < t < 1 \\ 4t^2 A, & t > 1 \end{cases}$$

Calculate the charge entering the element from t=0s to t=2s. **Answer: 13.333 C.**

1.3 Voltage

The voltage or potential difference ($\boldsymbol{V_{ab}}$) between two points **a** and **b** in an electric circuit is the energy (or work) needed to move a unit charge from a to b. The voltage is measured in volts (**V**). If the potential of point **a** is higher than that of point **b** then:

$$\boxed{\boldsymbol{V_{ab} = V_a - V_b \quad if \;\; V_a > V_b}}$$

1.4 Power and Energy

Power (P): is the time rate of expending or absorbing energy, measured in watts (**W**).

$$\boxed{\boldsymbol{P = \frac{dw}{dt}}}$$

where **P** is power in watts (**W**), **w** is energy in joules (**J**), and **t** is time in seconds (**s**).

$$\boldsymbol{P = \frac{dw}{dt} = \frac{dw}{dq} \times \frac{dq}{dt} = v.i}$$

or

$$\boxed{\boldsymbol{P = vi}}$$

Energy (W): is the capacity to do work, measured in joules (**J**). The energy absorbed or supplied by an element from time $\boldsymbol{t_0}$ to time **t** is:

$$\boxed{\boldsymbol{W = \int_{t_0}^{t} P dt = \int_{t_0}^{t} vi dt}}$$

The electric power utility companies measure energy in watt-hours (**Wh**), where:

$$\boxed{\mathbf{1\ Wh=3600\ J}}$$

Example: How much energy does a 100-W electric bulb consume in two hours?

Solution:

$$W = P.t = 100 \times 2 \times 60 \times 60 = 720000J = 720KJ$$

Or $W = 100 \times 2 = 200Wh$

Q: A stove element draws 15 A when connected to a 240-V line. How long does it take to consume 180 kJ? **Answer: 50s.**

1.5 Circuit Elements

An electric circuit is simply an interconnection of the elements. **Circuit analysis** is the process of determining voltages across (or the currents through) the elements of the circuit. There are two types of elements found in electric circuits which are:

- **Active Elements**: These elements are capable of generating energy such as generators, batteries, and operational amplifiers.
- **Passive Elements**: These elements are incapable of generating energy such as resistors, capacitors, and inductors.

  The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them. There are two kinds of sources:
- **Independent Source**: is an active element that provides a specified voltage or current that is completely independent of other circuit elements.
- **Dependent Source or controlled source**: is an active element in which the source quantity is controlled by another voltage or current.

Basic Laws

2.1Ohm's Law

Ohm's law states that the voltage **v** *across a resistor is directly proportional to the current* ***i*** *flowing through the resistor.*

$$\boldsymbol{V \alpha I}$$

$$\boldsymbol{V = IR}$$

Where **R** is the resistance. The resistance **R** denotes the ability of an element to resist the flow of electric current; it is measured in ohms (Ω).

$$\boldsymbol{R = \frac{V}{I}}$$

$$\boldsymbol{I = \frac{V}{R}}$$

Example: An electric iron draws 2 A at 120 V. Find its resistance.

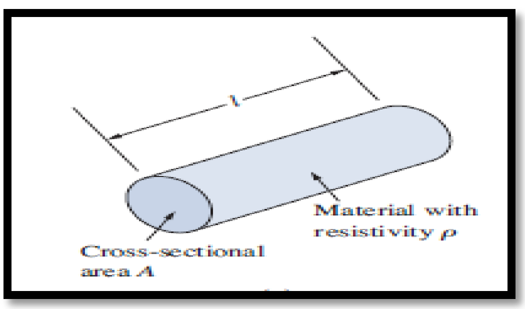
Solution:

$$R = \frac{V}{I} = \frac{120}{2} = \mathbf{60}\Omega$$

Q: The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 15 Ω at 110 V? **Answer: 7.333A.**

For any material, the resistance **R** depends on its physical dimensions as follows:

$$\boldsymbol{R = \rho \frac{l}{A}}$$



Where $\boldsymbol{\rho}$ is known as the resistivity of the material. $\boldsymbol{l}$ and **A** are the length and the cross-sectional area of the material respectively. Good conductors, such as copper and aluminum, have low resistivities, while insulators, such as mica and paper, have high resistivities. The resistivities of common materials are shown in Table (2.1).

Table (2.1): Resistivities of common materials

Material	Resistivity (Ω.m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{-2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

Conductance (G): is the ability of an element to conduct electric current; it is measured in mhos (**℧**) or Siemens (**S**).

$$\boxed{\boldsymbol{G = \frac{1}{R} = \frac{i}{v}}}$$

The conductance is a measure of how well an element will conduct electric current.

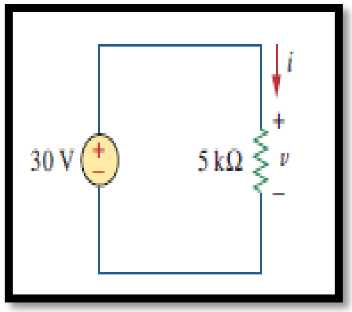
$$\boxed{\boldsymbol{i = Gv}}$$

$$\boxed{\boldsymbol{P = vi = i^2 R = \frac{v^2}{R}}}$$

Or

$$\boxed{\boldsymbol{P = vi = v^2 G = \frac{i^2}{G}}}$$

<u>Example</u>: In the circuit shown, calculate the current **i**, the conductance **G**, and the power **P**.



<u>Solution</u>:

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = \boldsymbol{6mA}$$

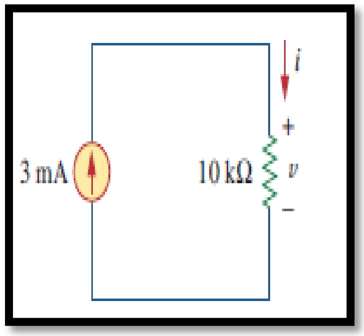
$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = \mathbf{0.2}\boldsymbol{mS}$$

$$P = vi = 30 \times 6 \times 10^{-3} = \mathbf{180}\boldsymbol{mW}$$

Or $P = i^2R = (6 \times 10^{-3})^2 \times 5 \times 10^3$=**180mW**

Or $P = v^2G = (30)^2 \times 0.2 \times 10^{-3} = \mathbf{180}\boldsymbol{mW}$

Q: For the circuit shown, calculate the voltage **v**, the conductance **G**, and the power **P**.



Answer: 30 V, 100 mS, 90 mW.

2.2 Nodes, Branches, and Loops

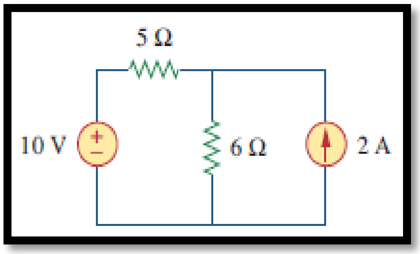
- **<u>A branch</u>** represents a single element such as a voltage source or a resistor.
- **<u>A node</u>** is the point of connection between two or more branches.
- **<u>A loop</u>** is any closed path in a circuit.

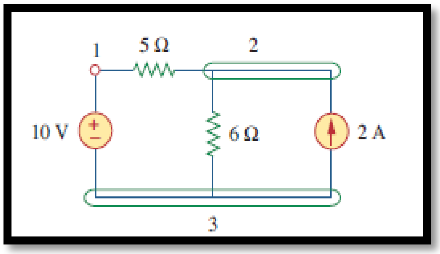
A network with **b** branches, **n** nodes, and $\boldsymbol{l}$ independent loops will satisfy the fundamental theorem of network topology:

$$\boxed{\boldsymbol{b = l + n - 1}}$$

Example: For the circuit shown, determine the number of branches, nodes and independent loops.

Solution:



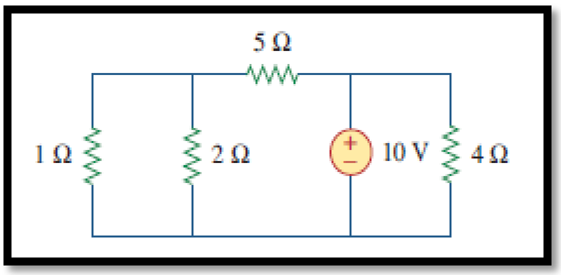


$l = \mathbf{2}, b = \mathbf{4}, n = \mathbf{3}$

$\boldsymbol{b = l + n - 1}$

$4 = 2 + 3 - 1 = 4 \quad \Rightarrow \quad 4 = 4$

Q: For the circuit shown, determine the number of branches, nodes and independent loops.



Answer: $\boldsymbol{l = 3, b = 5, n = 3}$

2.3 Kirchhoff's Laws

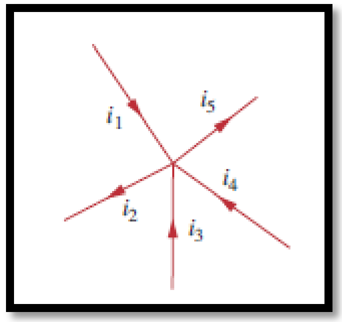
Ohm's law by itself is not sufficient to analyze circuits. However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits. Kirchhoff's laws were first introduced in 1847 by the German physicist **Gustav Robert Kirchhoff** (1824–1887). These laws are formally known as Kirchhoff's current law (**KCL**) and Kirchhoff's voltage law (**KVL**).

❖ **<u>Kirchhoff's current law (KCL)</u>** states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

Mathematically, KCL implies that:

$$\boxed{\sum_{n=1}^{N} \boldsymbol{i_n} = \mathbf{0}}$$

Where **N** is the number of branches connected to the node and $\boldsymbol{i_n}$ is the nth current entering (or leaving) the node. By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa as shown:



$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

By rearranging the terms, we get:

$$i_1 + i_3 + i_4 = i_2 + i_5$$

In other word, Kirchhoff's current law states that:

(**The sum of the currents entering a node is equal to the sum of the currents leaving the node**).

❖ **<u>Kirchhoff's Voltage Law (KVL)</u> states** that the algebraic sum of all voltages around a closed path (or loop) is zero.

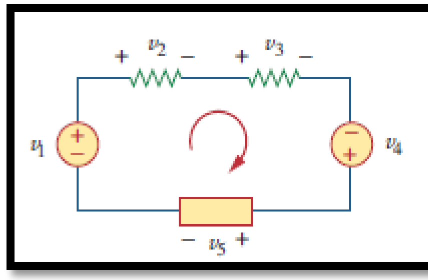
Expressed mathematically, KVL states that:

$$\boxed{\sum_{m=1}^{M} \boldsymbol{V_m} = \mathbf{0}}$$

Where **M** is the number of voltages in the loop (or the number of branches in the loop) and $\boldsymbol{v_m}$ is the mth voltage.

To illustrate KVL, consider the circuit in figure shown below.

$$v_1 - v_2 - v_3 + v_4 - v_5 = 0$$



By rearranging the terms, we get:

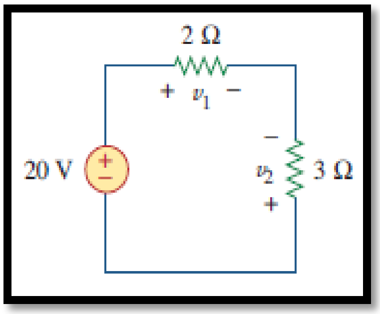
$$v_1 + v_4 = v_2 + v_3 + v_5$$

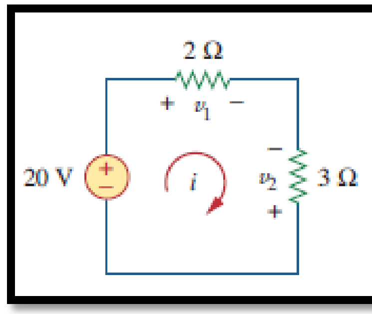
This may be interpreted as:

(Sum of voltage drops = Sum of voltage rises)

<u>Example</u>: For the circuit shown, find voltages $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$.

<u>Solution</u>:





To find $\boldsymbol{v_1}$and $\boldsymbol{v_2}$, we apply Ohm's law and Kirchhoff's voltage law.

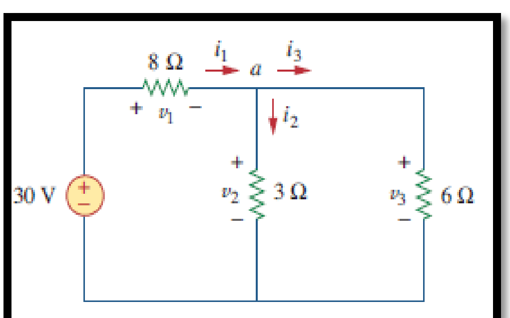
$$20 - v_1 - v_2 = 0 \;\Rightarrow\; 20 - 2i - 3i = 0 \;\Rightarrow\; 20 - 5i = 0$$

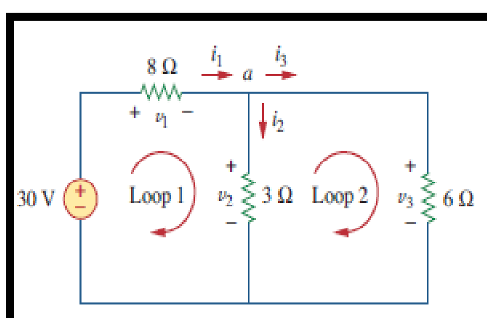
$$20 = 5i \;\Rightarrow\; i = \boldsymbol{4A}$$

$$v_1 = 2i = 2 \times 4 = \boldsymbol{8V} \qquad v_2 = 3i = 3 \times 4 = \boldsymbol{12V}$$

$$E = v_1 + v_2 = 8 + 12 = 20V \;\Rightarrow\; \boldsymbol{20 = 20}$$

<u>Example</u>: Find currents and voltages in the circuit shown by using Kirchhoff's laws.





<u>Solution</u>:

Using Ohm's law:

$v_1 = i_1 R_1 = 8i_1$, $\quad v_2 = i_2 R_2 = 3i_2$, $\quad v_3 = i_3 R_3 = 6i_3$

❖ Applying **KCL** at node a:

$i_1 = i_2 + i_3 \quad \Longrightarrow \quad \boxed{i_1 - i_2 - i_3 = 0}$ Eq. (1)

❖ Applying **KVL** to loop (**1**):

$E - v_1 - v_2 = 0 \quad \Longrightarrow \quad 30 - 8i_1 - 3i_2 = 0 \quad \Longrightarrow \quad \boxed{i_1 = \dfrac{30 - 3i_2}{8}}$ Eq. (2)

❖ Applying **KVL** to loop (2):

$v_2 - v_3 = 0 \quad \Longrightarrow \quad v_2 = v_3 \quad \Longrightarrow \quad 3i_2 = 6i_3 \quad \Longrightarrow \quad \boxed{i_3 = \dfrac{i_2}{2}}$ Eq. (3)

Substituting Eqs. (2) and (3) into (1) gives:

$\dfrac{30 - 3i_2}{8} - i_2 - \dfrac{i_2}{2} = 0 \qquad i_2 = \mathbf{2A}$

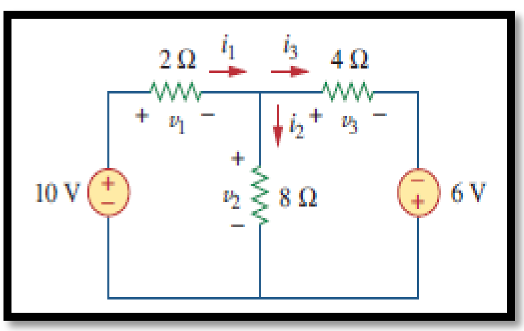
$i_1 = \dfrac{30 - 3(2)}{8} = \mathbf{3A}, \qquad i_3 = \dfrac{i_2}{2} = \dfrac{2}{2} = \mathbf{1A}$

$v_1 = 8i_1 = 8 \times 3 = \mathbf{24V}, \quad v_2 = 3i_2 = 3 \times 2 = \mathbf{6V}, \quad v_3 = 6i_3 = 6 \times 1 = \mathbf{6V}$

$i_1 = i_2 + i_3 = 2 + 1 = \mathbf{3A}$, **3A=3A**

$E = v_1 + v_2 = 24 + 6 = \mathbf{30V}, \quad \mathbf{30V = 30V}$

Q: Find the currents and voltages in the circuit shown by using Kirchhoff's laws.

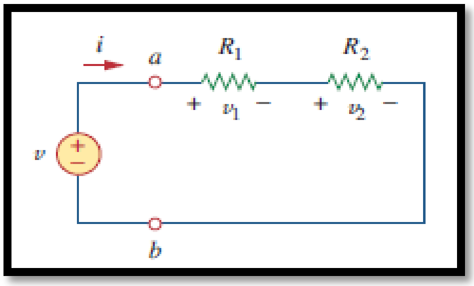


Answer: $v_1 = 6V,\ v_2 = 4V,\ v_3 = 10V,\ i_1 = 3A,\ i_2 = 500mA,\ i_3 = 1.25A$

2.4 Circuit Transformations

- **<u>Series Circuits</u>:** Two or more elements are in **series** if they have only one point in common and thus they carry the same current.

For the circuit shown, if we apply **KVL** to the loop, we have:



$$v - v_1 - v_2 = 0$$

$$v = v_1 + v_2 \Longrightarrow v = i_1R_1 + i_2R_2$$

$$v = i(R_1 + R_2)$$

$$i = \frac{v}{R_1 + R_2}$$

$$\boxed{i = \frac{v}{R_T} = \frac{v}{R_{eq}}}$$

In general, <u>the equivalent resistance of any number of resistors connected in series is the sum of the individual resistances</u>.

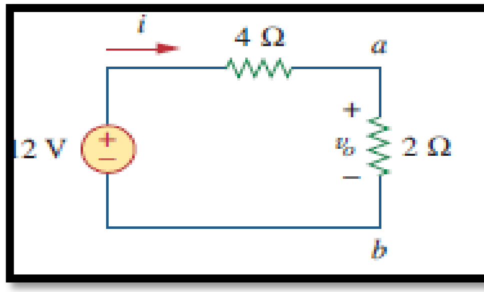
For **N** resistors in **series** then:

1. $$\boxed{R_T = R_1 + R_2 + R_3 + \cdots + R_N}$$

2. $$\boxed{i = i_1 = i_2 = i_3 = \cdots = i_N}$$

3. $$\boxed{V = E = v_1 + v_2 + v_3 + \cdots + v_N}$$

Example: For the circuit shown, find:



a. The total resistance.
b. The total current.
c. The voltage drop across each resistance.

Solution:

a. $R_T = R_1 + R_2 = 2 + 4 = \mathbf{6\Omega}$

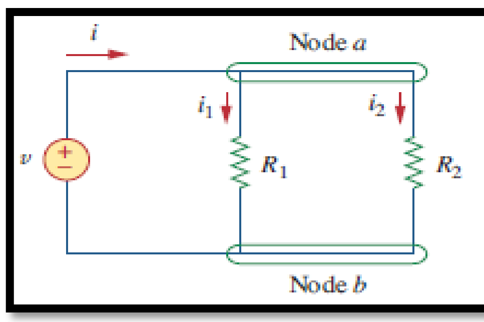
b. $i = \frac{E}{R_T} = \frac{12}{6} = \mathbf{2A}$

c. $v_1 = iR_1 = 2 \times 4 = \mathbf{8V,} \qquad v_2 = iR_2 = 2 \times 2 = \mathbf{4V}$

$$E = v_1 + v_2 = 8 + 4 = \mathbf{12V}$$

- **Parallel Circuits:** Two or more elements are in **parallel** if they have two points in common and thus they have the same voltage across them.

  For the circuit shown, if we apply **KCL** at node (a), we have:



$$i = i_1 + i_2 = \frac{v_1}{R_1} + \frac{v_2}{R_2} = \frac{v}{R_1} + \frac{v}{R_2} = v(\frac{1}{R_1} + \frac{1}{R_2})$$

$$\frac{v}{R_{eq}} = v(\frac{1}{R_1} + \frac{1}{R_2})$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\boxed{\boldsymbol{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}}$$

In general, <u>the equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum</u>.

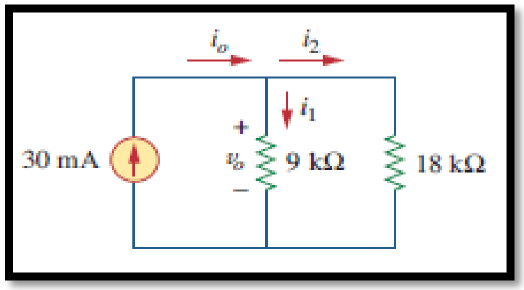
For **N** resistors in **parallel** then:

1. $$\boxed{\frac{\mathbf{1}}{\boldsymbol{R_T}} = \frac{\mathbf{1}}{\boldsymbol{R_1}} + \frac{\mathbf{1}}{\boldsymbol{R_2}} + \frac{\mathbf{1}}{\boldsymbol{R_3}} + \cdots + \frac{\mathbf{1}}{\boldsymbol{R_N}}}$$

2. $$\boxed{\boldsymbol{V = E = v_1 = v_2 = v_3 = \cdots = v_N}}$$

3. $$\boxed{\boldsymbol{i = i_1 + i_2 + i_3 + \cdots + i_N}}$$

<u>**Example**</u>: For the circuit shown, find:



a. The total resistance.
b. The total voltage.
c. The current for each resistance.
d. Determine the power to each resistive load.
e. Determine the power delivered by the source and compare it with the total power dissipated by the resistive elements.

<u>**Solution**</u>:

a. $R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{9 \times 18}{9 + 18} = \boldsymbol{6k}\Omega$

b. $E = V = iR_T = 30 \times 10^{-3} \times 6 \times 10^3 = \mathbf{180}\boldsymbol{V} = v_1 = v_2$

c. $i_1 = \frac{v}{R_1} = \frac{180}{9 \times 10^3} = \mathbf{20}\boldsymbol{mA},\qquad i_2 = \frac{v}{R_2} = \frac{180}{18 \times 10^3} = \mathbf{10}\boldsymbol{mA}$

$$i_0 = i_1 + i_2 = 20 + 10 = \mathbf{30}\boldsymbol{A}$$

d. $P_1 = i_1 v_1 = 20 \times 10^{-3} \times 180 = 3600 \times 10^{-3} = \mathbf{3.6}\boldsymbol{W}$

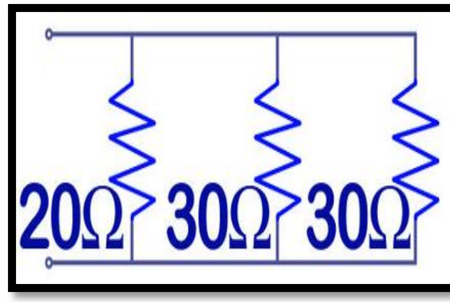
$P_2 = i_2 v_2 = 10 \times 10^{-3} \times 180 = \mathbf{1.8}\boldsymbol{W}$

e. $P_T = i_0 \times V = 30 \times 10^{-3} \times 180 = \mathbf{5.4}\boldsymbol{W}$

$P_T = P_1 + P_2 = 3.6 + 1.8 = \mathbf{5.4}\boldsymbol{W}$

Example: Find the total resistance for the network shown:

Solution:



$$\frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}=\frac{1}{20}+\frac{1}{30}+\frac{1}{30}=\frac{3+2+2}{60}$$

$$\frac{1}{R_T}=\frac{7}{60}$$

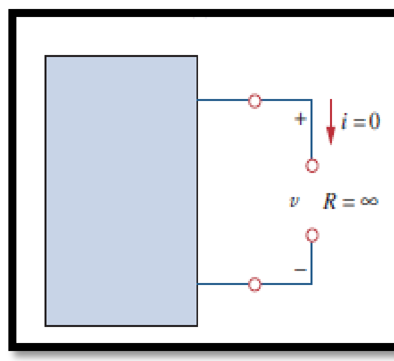
$$R_T=\frac{60}{7}=\mathbf{8.57}\Omega$$

2.5 Open and Short Circuits

We often need to apply the open circuit or short circuit in the analysis of electric network.

❖ **Open Circuit**: An open circuit is simply two isolated terminals not connected by any element of any kind. **An open circuit is a circuit element with resistance approaching infinity**.

For the circuit shown:

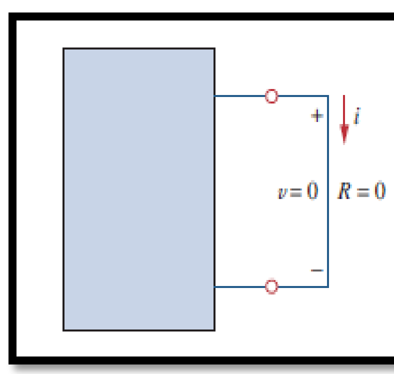


$$i=\frac{v}{R}=\frac{v}{\infty}=\mathbf{0\ A}$$

In general, an open circuit can have a voltage (potential difference) across its terminals but the current is always Zero.

❖ **Short Circuit**: A short circuit is a direct connection between two elements or combination of elements. **A short circuit is a circuit element with resistance approaching zero.**

For the circuit shown:

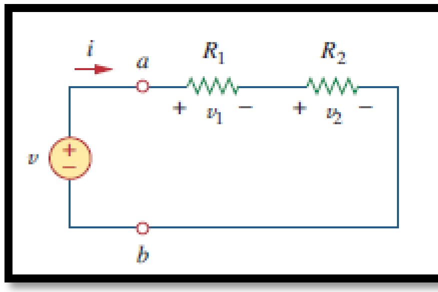


$$v=iR=\mathbf{0\ V}$$

In general, <u>a short circuit can carry a current of any level but the voltage (potential difference) across its terminals is always Zero</u>.

2.6 Voltage Divider Rule in Series Circuits

Consider the series circuit shown, we have:



$$R_T = R_1 + R_2$$

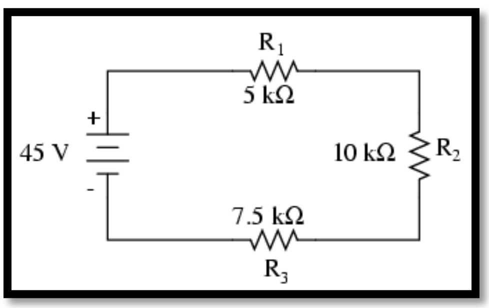
$$i = \frac{v}{R_T}$$

$$v_1 = iR_1 = \left(\frac{v}{R_T}\right)R_1 = v(\frac{R_1}{R_1 + R_2})$$

$$v_2 = iR_2 = \left(\frac{v}{R_T}\right)R_2 = v\left(\frac{R_2}{R_1 + R_2}\right)$$

$$\boxed{\boldsymbol{v_1 = v(\frac{R_1}{R_1 + R_2})}} \qquad \boxed{\boldsymbol{v_2 = v(\frac{R_2}{R_1 + R_2})}}$$

<u>Example</u>: Find the voltage across the three resistances for the circuit shown.



<u>Solution</u>:

$$v_1 = v\left(\frac{R_1}{R_1 + R_2 + R_3}\right) = \frac{45 \times (5 \times 10^3)}{(5 + 10 + 7.5) \times 10^3} = \boldsymbol{10V}$$

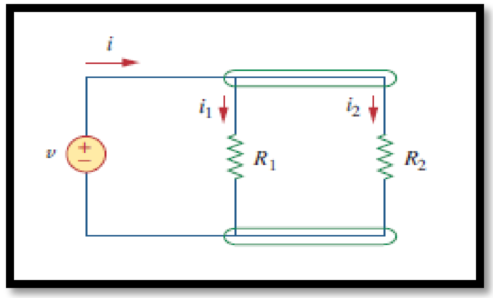
$$v_2 = v\left(\frac{R_2}{R_1 + R_2 + R_3}\right) = \frac{45 \times (10 \times 10^3)}{(5 + 10 + 7.5) \times 10^3} = \boldsymbol{20V}$$

$$v_3 = v\left(\frac{R_3}{R_1 + R_2 + R_3}\right) = \frac{45 \times (7.5 \times 10^3)}{(5 + 10 + 7.5) \times 10^3} = \boldsymbol{15V}$$

$E = v_1 + v_2 + v_3 = 10 + 20 + 15 = \mathbf{45V}$

2.7 Current Divider Rule in Parallel Circuits

Consider the parallel circuit shown, we have:



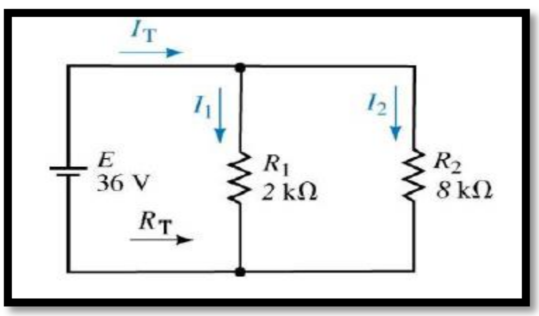
$$i = \frac{v}{R_T}, \quad v = iR_T, \qquad R_T = \frac{R_1R_2}{R_1 + R_2}$$

$$i_1 = \frac{v}{R_1} = \frac{iR_T}{R_1} = \frac{i\left(\frac{R_1R_2}{R_1 + R_2}\right)}{R_1} = i\left(\frac{R_2}{R_1 + R_2}\right)$$

$$i_2 = \frac{v}{R_2} = \frac{iR_T}{R_2} = \frac{i\left(\frac{R_1R_2}{R_1 + R_2}\right)}{R_2} = i\left(\frac{R_1}{R_1 + R_2}\right)$$

$$\boxed{\boldsymbol{i_1 = i(\frac{R_2}{R_1 + R_2})}} \qquad \boxed{\boldsymbol{i_2 = i(\frac{R_1}{R_1 + R_2})}}$$

Example: Determine the currents $\boldsymbol{I_1}$ *and* $\boldsymbol{I_2}$ in the circuit shown.



Solution:

$$R_T = \frac{R_1R_2}{R_1 + R_2} = \frac{2 \times 8}{2 + 8} = \frac{16}{10} = \mathbf{1.6k}\Omega$$

$$I_T = \frac{E}{R_T} = \frac{36}{1.6 \times 10^3} = \mathbf{22.5mA}$$

$$I_1 = I\left(\frac{R_2}{R_1 + R_2}\right) = \frac{22.5 \times 10^{-3} \times 8 \times 10^3}{(2 + 8) \times 10^3} = 0.018A = \mathbf{18mA}$$

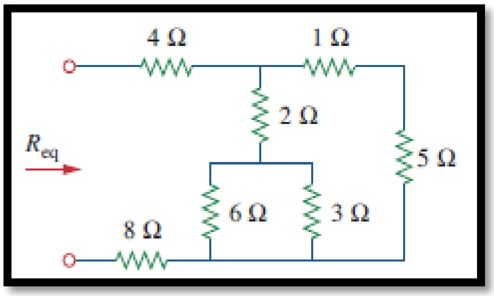
$$I_2 = I\left(\frac{R_1}{R_1 + R_2}\right) = \frac{22.5 \times 10^{-3} \times 2 \times 10^3}{(2+8) \times 10^3} = 0.0045A = \mathbf{4.5mA}$$

$$I = I_1 + I_2 = 18 + 4.5 = \mathbf{22.5mA}$$

2.8 Series-Parallel Circuits

Series-parallel circuit is a combination of series and parallel resistors.

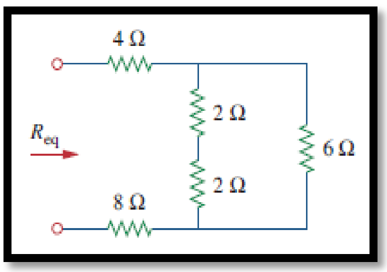
Example: Find $\boldsymbol{R_{eq}}$ for the circuit shown.

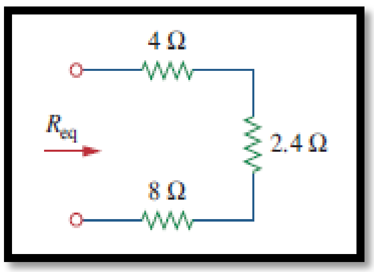


Solution:

$$\frac{6 \times 3}{6+3} = \frac{18}{9} = 2\Omega\,, \qquad 1 + 5 = 6\Omega, \quad 2 + 2 = 4\Omega, \quad \frac{4 \times 6}{4+6} = \frac{24}{10} = 2.4\Omega$$

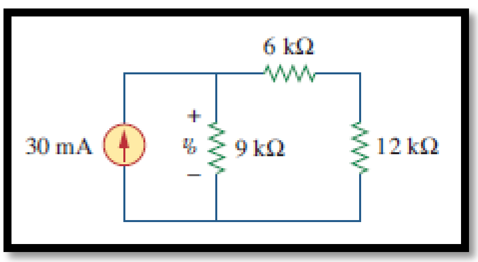
,



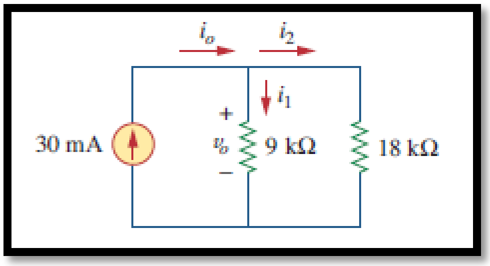


$$R_{eq} = 4 + 2.4 + 8 = \mathbf{14.4}\Omega$$

Example: For the circuit shown, determine: (a) the voltage $\boldsymbol{v_0}$ (b) the power supplied by the current source, (c) the power absorbed by each resistor.



Solution:



a.

$$i_1 = i_0\left(\frac{R_2}{R_1+R_2}\right) = \frac{30\times10^{-3}\times18\times10^{3}}{(9+18)\times10^{3}} = \boldsymbol{20mA}$$

$$i_2 = i_0\left(\frac{R_1}{R_1+R_2}\right) = \frac{30\times10^{-3}\times9\times10^{3}}{(9+18)\times10^{3}} = \boldsymbol{10mA}$$

$v_0 = 9\times10^{3}\times20\times10^{-3} = \boldsymbol{180V},\qquad v_0 = 18\times10^{3}\times10\times10^{-3} = \boldsymbol{180V}$

b. $P_0 = v_0 i_0 = 180\times30\times10^{-3} = \boldsymbol{5.4W}$

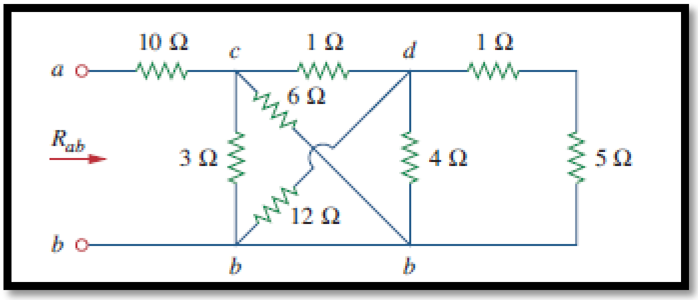
c. $P_{12k} = vi_2 = i_2^2R = (10\times10^{-3})^2\times12\times10^{3} = \boldsymbol{1.2W}$

$P_{6k} = i_2^2R = (10\times10^{-3})^2\times6\times10^{3} = \boldsymbol{0.6W}$

$P_{9k} = i_1^2\times R = (20\times10^{-3})^2\times9\times10^{3} = \boldsymbol{3.6W}$

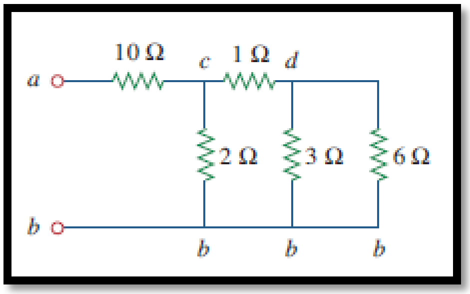
$P_0 = 1.2+0.6+3.6 = \boldsymbol{5.4W}$

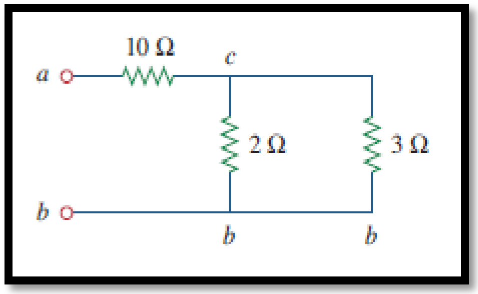
Example: Calculate the equivalent resistance $\boldsymbol{R_{ab}}$ in the circuit shown.



Solution:

$$1+5=6\Omega,\quad \frac{4\times12}{4+12}=3\Omega,\quad \frac{3\times6}{3+6}=2\Omega,\quad 2+1=3\Omega$$

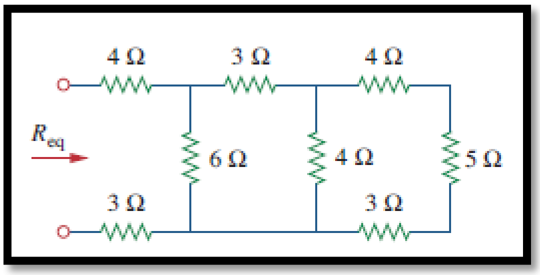




$$\frac{3 \times 2}{3+2} = 1.2\Omega, \qquad R_{ab} = 10 + 1.2 = \mathbf{11.2}\Omega$$

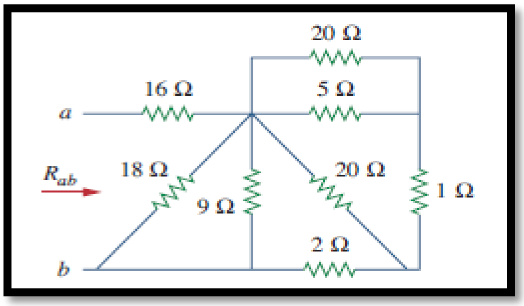
Problems

Q: Find $\boldsymbol{R_{eq}}$ for the circuit shown.



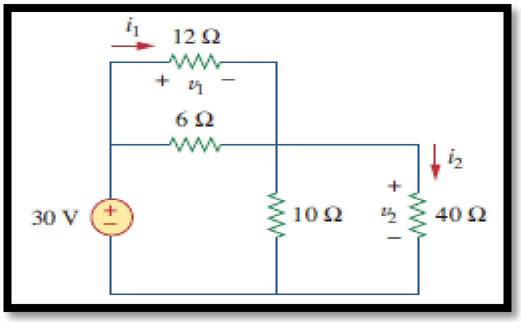
Answer: 10Ω

Q: Find $\boldsymbol{R_{ab}}$ for the circuit shown



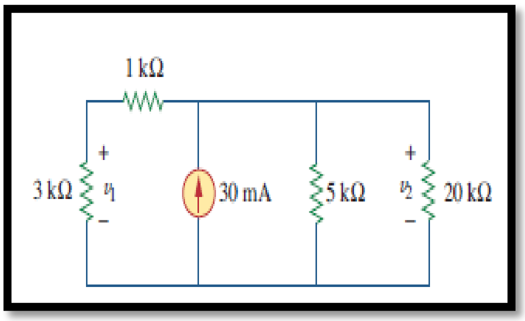
Answer: 19Ω

Q: Find $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ in the circuit shown. Also calculate $\boldsymbol{i_1}$ and $\boldsymbol{i_2}$ and the power dissipated in the **12 Ω** and **40 Ω** resistors.



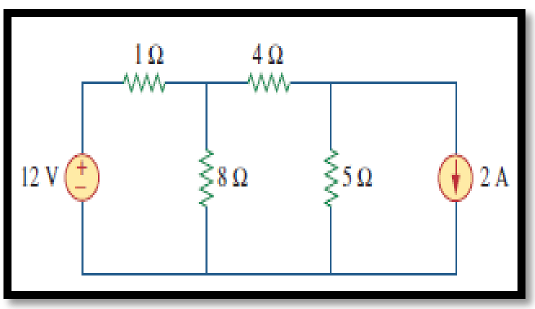
Answer: $v_1 = 10V,\ i_1 = 833.3mA,\ P_1 = 8.333W,\ v_2 = 20V,\ i_2 = 500mA,\ P_2 = 10W.$

Q: For the circuit shown, find: (a) $v_1$and v_2(b) the power dissipated in the **3kΩ** and 20kΩ resistors, and (c) the power supplied by the current source.

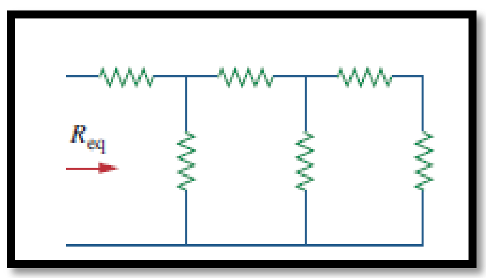


Answer: (a) 45 V, 60 V, (b) 675 mW, 180 mW, (c) 1.8 W.

Q: Determine the number of branches, nodes, and independent loops in the circuit shown.

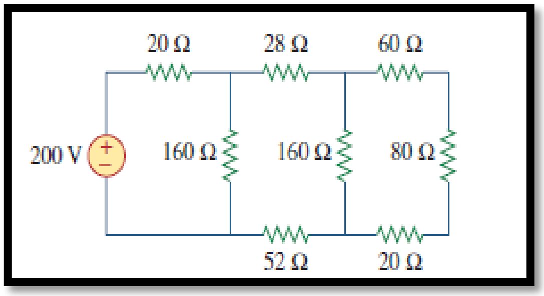


Q: All resistors in the Fig. shown are **5Ω** each. Find R_{eq}.



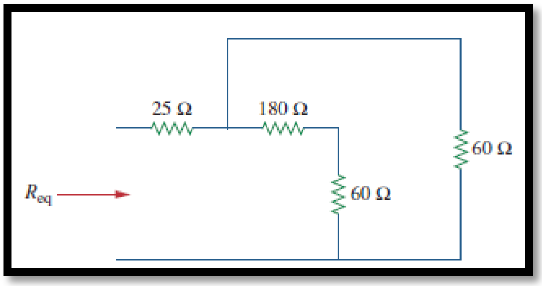
Answer: 8.125Ω.

Q: Using series/parallel resistance combination, find the equivalent resistance seen by the source in the circuit of Fig. shown. Find the overall absorbed power by the resistor network.



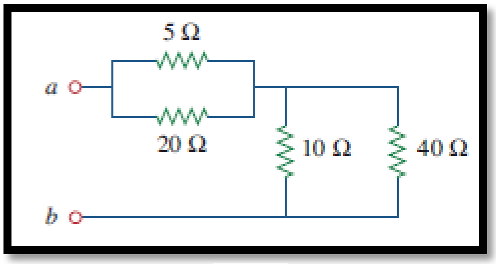
Answer: 100Ω, P=400W.

Q: Find $\boldsymbol{R_{eq}}$ for the circuit shown.

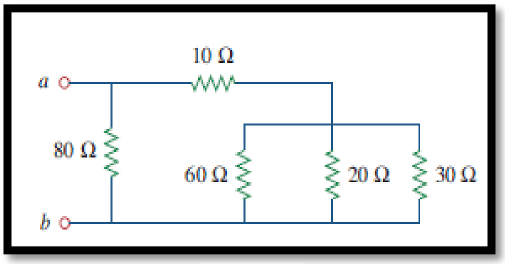


Answer: 73Ω.

Q: Calculate the equivalent resistance $\boldsymbol{R_{ab}}$ at terminals a-b for each of the circuits in Fig shown.



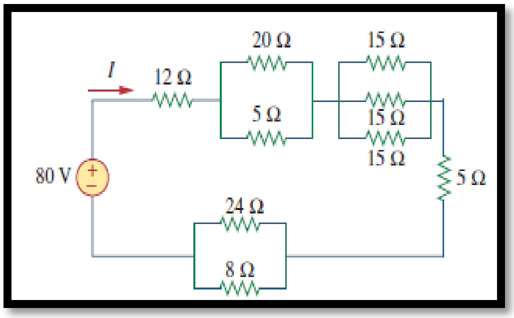
(a)



(b)

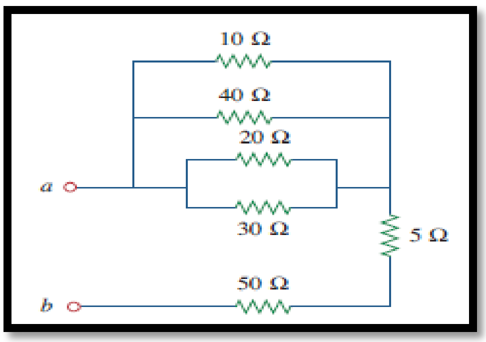
Answer: (a) 12Ω, (b) 16Ω.

Q: Find **I** in the circuit shown.



Answer: 2.5A.

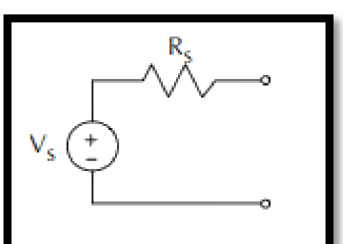
Q: Find the equivalent resistance at terminals a-b of in the circuit shown.



Answer: 59.8Ω.

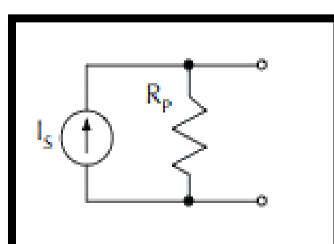
2.9 Source Transformation

It is often necessary to have a voltage source rather than a current source or a current source rather than a voltage source. In the circuit shown in Fig. (a), we have a voltage source connected to a series resistance (R_s). We can replace the voltage source by equivalent current source connected to a parallel resistance (R_p) as shown in Fig. (b) and vice versa.

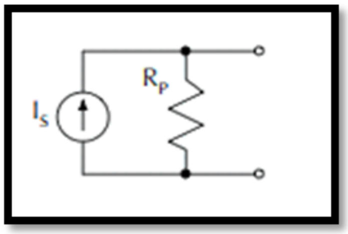


$$\boldsymbol{I_s = \frac{V_s}{R_s}}$$

$$\boldsymbol{R_s = R_p}$$

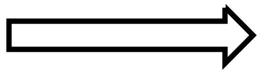


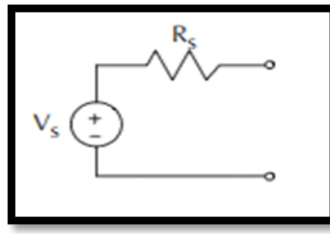
(a) **(b)**



$$V_s = I_s R_p$$

$$R_s = R_p$$

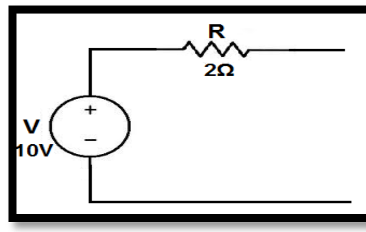




(a) **(b)**

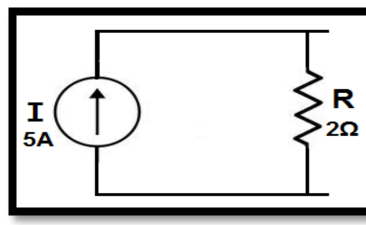
In general, <u>a voltage source and series resistor can be replaced by a current source and parallel resistor</u>.

<u>**Example**</u>: Convert the voltage source in the circuit shown to a current source.

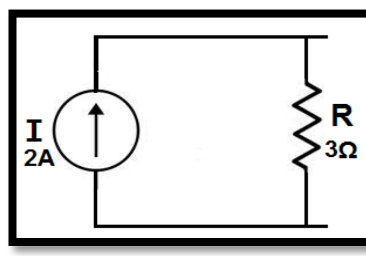


<u>**Solution**</u>:

$$I = \frac{V}{R} = \frac{10}{2} = \mathbf{5A}$$

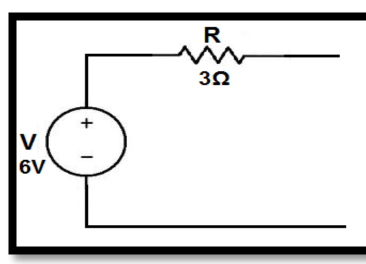


<u>**Example**</u>: Convert the current source in the circuit shown to a voltage source.

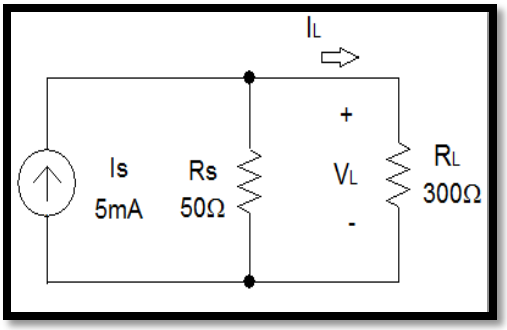


<u>**Solution**</u>:

$$V = IR = 2 \times 3 = \mathbf{6V}$$



<u>**Example**</u>: Convert the current source to a voltage source in the circuit shown, then calculate the current through the load ($\boldsymbol{I_L}$) and ($\boldsymbol{V_L}$) for each source.



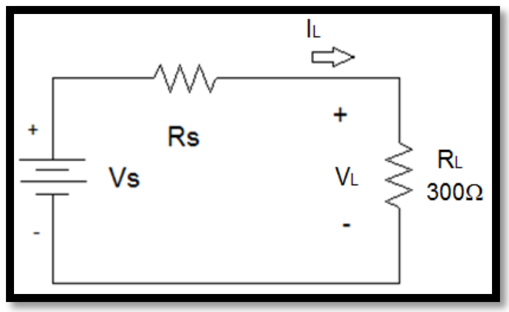
Solution:

For the current source circuit:

$$I_L = I_s\left(\frac{R_s}{R_s + R_L}\right) = \frac{5 \times 10^{-3} \times 50}{50 + 300} = \mathbf{0.714}\boldsymbol{mA}$$

$$V_L = I_L \times R_L = 0.714 \times 10^{-3} \times 300 = \mathbf{0.214}\boldsymbol{V}$$

For the voltage source circuit:

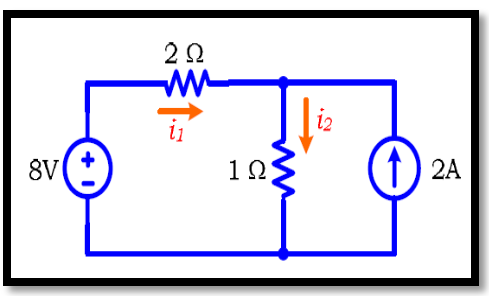


$$V_s = I_s R_s = 5 \times 10^{-3} \times 50 = \mathbf{0.25}\boldsymbol{V}$$

$$I_L = \frac{0.25}{50 + 300} = \mathbf{0.714}\boldsymbol{mA}$$

$$V_L = \frac{0.25 \times 300}{50 + 300} = \mathbf{0.214}\boldsymbol{V}$$

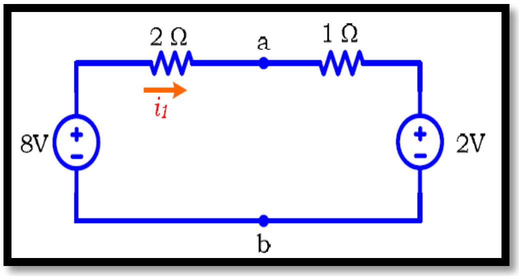
Example: Use source transformation technique to find $\boldsymbol{I_1}$ *and* $\boldsymbol{I_2}$ in the circuit shown.



Solution:

We convert the current source to a voltage source as shown.

$$V = IR = 2 \times 1 = \mathbf{2}\boldsymbol{V}$$



Using **KVL** to the loop, we have:

$8 - 2i_1 - 1i_1 - 2 = 0, \;\; 6 - 3i_1 = 0, \;\; 6 = 3i_1, \quad i_1 = \boldsymbol{2A}$

Applying **KCL** on the original circuit, we have:

$i_1 + 2 = i_2, \qquad 2 + 2 = i_2, \quad i_2 = \boldsymbol{4A}$

Methods of Analysis

Having understood the fundamental laws of circuit theory (Ohm's law and Kirchhoff's laws), we are now prepared to apply these laws to develop two powerful techniques for circuit analysis: **nodal analysis**, which is based on a systematic application of Kirchhoff's current law (**KCL**), and **mesh analysis**, which is based on a systematic application of Kirchhoff's voltage law (**KVL**).

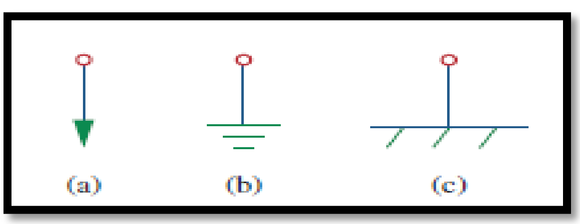
3.1 Nodal Analysis

Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables. Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously.

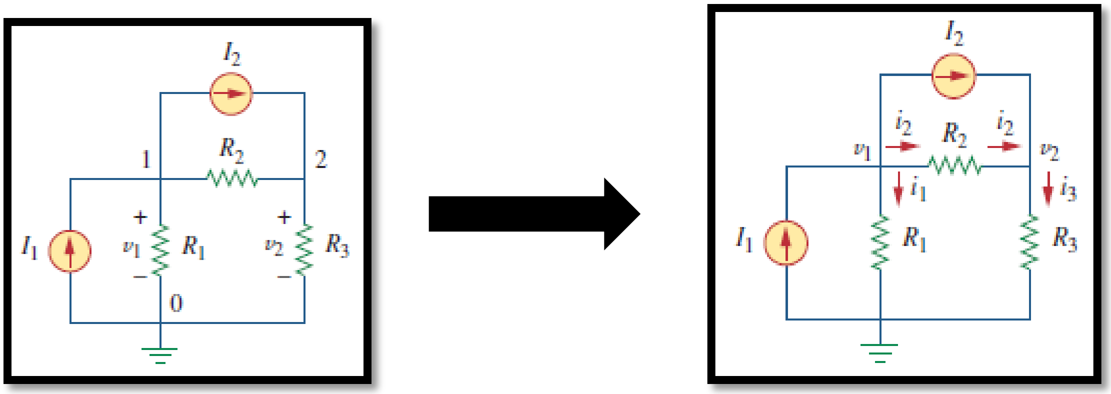
Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages $v_1, v_2, \dots, v_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply **KCL** to each of the $n-1$ nonreference nodes. Use **Ohm's law** to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

- The first step in nodal analysis is selecting a node as the reference or datum node. The reference node is commonly called the ground since it is assumed to have zero potential. A reference node is indicated by any of the three symbols as shown:



Once we have selected a reference node, we assign voltage designations to nonreference nodes. Consider, for example, the circuit shown. Node **0** is the reference node while nodes 1 and 2 are assigned voltages $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ respectively.



(a) **(b)**

Each node voltage is the voltage rise from the reference node to the corresponding nonreference node or simply the voltage of that node with respect to the reference node.

- The second step, we apply **KCL** to each nonreference node in the circuit. We now add $\boldsymbol{i_1}, \boldsymbol{i_2}$ and $\boldsymbol{i_3}$ as the currents through resistors $\boldsymbol{R_1}, \boldsymbol{R_2}$ and $\boldsymbol{R_3}$ respectively. At node **1**, applying **KCL** gives:

$$I_1 = I_2 + i_1 + i_2 \qquad Eq.(1)$$

At node **2:**

$$i_3 = i_2 + I_2 \qquad Eq.(2)$$

We now apply **Ohm's law** to express the unknown currents $\boldsymbol{i_1}, \boldsymbol{i_2}$ and $\boldsymbol{i_3}$ in terms of node voltages.

<u>Note: Current flows from a higher potential to a lower potential in a resistor.</u>

We can express this principle as:

$$\boldsymbol{i = \frac{v_{higher} - v_{lower}}{R}}$$

$$i_1 = \frac{v_1 - 0}{R_1}, \quad i_2 = \frac{v_1 - v_2}{R_2}, \quad i_3 = \frac{v_2 - 0}{R_3} \qquad Eq.(3)$$

Substituting Eq. (3) in Eqs. (1) and (2) results, respectively, gives:

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} \qquad Eq.(4)$$

$$\frac{v_2}{R_3} = \frac{v_1 - v_2}{R_2} + I_2 \qquad Eq. (5)$$

- The third step in nodal analysis is to solve for the node voltages. If we apply **KCL** to $n-1$ nonreference nodes, we obtain $n-1$ simultaneous equations such as Eqs. (4) and (5). We solve Eqs. (4) and (5) to obtain the node voltages $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ using any standard method, such as the **substitution method**, the **elimination method**, **Cramer's rule**, or **matrix inversion**. To use either of the last two methods, one must cast the simultaneous equations in **matrix form**. For example, Eqs. (4) and (5) after we simplify them, can be cast in matrix form as:

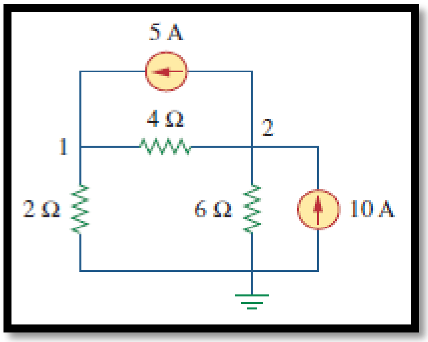
$$v_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - v_2\left(\frac{1}{R_2}\right) = I_1 - I_2 \qquad \boldsymbol{Eq. (6)}$$

$$v_2\left(\frac{1}{R_3} + \frac{1}{R_2}\right) - v_1\left(\frac{1}{R_2}\right) = I_2 \qquad \boldsymbol{Eq. (7)}$$

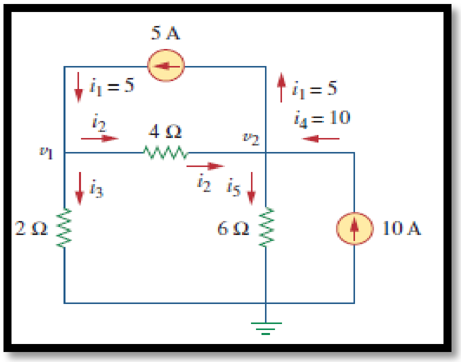
$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2 + R_3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

Which can be solved to get $\boldsymbol{v_1}$and $\boldsymbol{v_2}$.

<u>**Example**</u>: Calculate the node voltages in the circuit shown.



<u>**Solution**</u>: The circuit below has been prepared for nodal analysis.



For node (1):

$$\frac{v_1 - 0}{2} + \frac{v_1 - v_2}{4} = 5, \qquad \frac{v_1}{2} + \frac{v_1}{4} - \frac{v_2}{4} = 5, \qquad v_1\left(\frac{1}{2} + \frac{1}{4}\right) - v_2\left(\frac{1}{4}\right) = 5$$

$$v_1\left(\frac{3}{4}\right) - v_2\left(\frac{1}{4}\right) = 5, \;\; \boldsymbol{3v_1 - v_2 = 20} \qquad \boldsymbol{Eq.(1)}$$

For node (2):

$$\frac{v_2 - v_1}{4} + \frac{v_2 - 0}{6} = 10 - 5, \quad \frac{v_2}{4} - \frac{v_1}{4} + \frac{v_2}{6} = 5, \;\; v_1\left(-\frac{1}{4}\right) + v_2\left(\frac{1}{4} + \frac{1}{6}\right) = 5$$

$$v_1\left(-\frac{1}{4}\right) + v_2\left(\frac{5}{12}\right) = 5, \qquad \boldsymbol{-3v_1 + 5v_2 = 60} \qquad \boldsymbol{Eq.(2)}$$

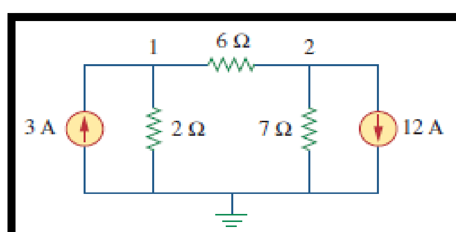
We will use Cramer's rule to solve these two equations:

$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} \boldsymbol{v_1} \\ \boldsymbol{v_2} \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$, We now obtain $\boldsymbol{v_1}$and $\boldsymbol{v_2}$ as:

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix}} = \frac{20 \times 5 - (-1 \times 60)}{3 \times 5 - (-1 \times -3)} = \frac{160}{12} = \boldsymbol{13.333V}$$

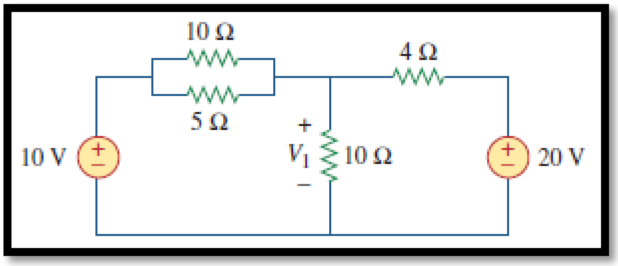
$$v_2 = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{12} = \frac{3 \times 60 - (-20 \times 3)}{12} = \frac{180 + 60}{12} = \boldsymbol{20V}$$

Q: Obtain the node voltages in the circuit shown.



Answer: $\boldsymbol{v_1 = -6V, \;\; v_2 = -42V}$

Example: Find v_1 in the circuit shown using nodal voltage analysis.



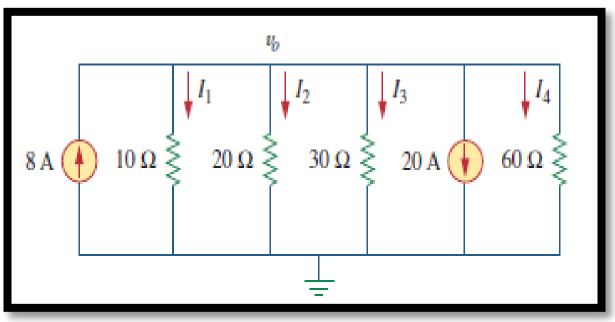
Solution:

$$\frac{v_1-10}{10}+\frac{v_1-10}{5}+\frac{v_1-20}{4}+\frac{v_1-0}{10}=0$$

$$v_1\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{4}+\frac{1}{10}\right)-1-2-5=0$$

$$v_1\left(\frac{2+4+5+2}{20}\right)=8,\quad v_1\left(\frac{13}{20}\right)=8,\quad v_1=\frac{20\times 8}{13}=\mathbf{12.307V}$$

Example: Find the currents I_1 through I_4 and the voltage v_0 in the circuit shown using nodal voltage method.



Solution:

$$\frac{v_0-0}{10}+\frac{v_0-0}{20}+\frac{v_0-0}{30}+\frac{v_0-0}{60}=8-20$$

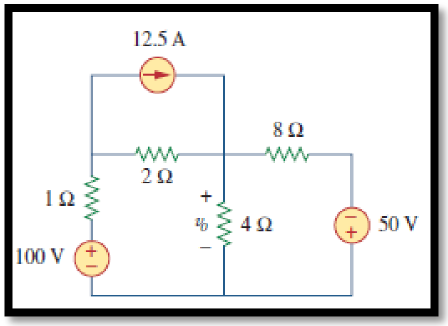
$$v_0\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{30}+\frac{1}{60}\right)=-12,\quad v_0\left(\frac{6+3+2+1}{60}\right)=-12$$

$$v_0\left(\frac{12}{60}\right)=-12,\quad v_0=\frac{-12\times 60}{12}=-\mathbf{60V}$$

$$I_1 = \frac{v_0}{10} = \frac{-60}{10} = -\boldsymbol{6A}, \qquad I_2 = \frac{-60}{20} = -\boldsymbol{3A}, \qquad I_3 = \frac{-60}{30} = -\boldsymbol{2A},$$

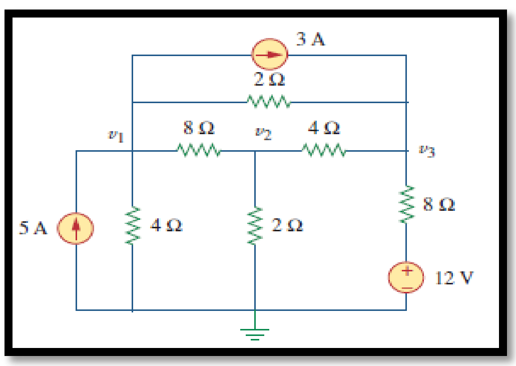
$$I_4 = \frac{-60}{60} = -\boldsymbol{1A}$$

Q: Using nodal analysis, find $\boldsymbol{v_0}$ in the circuit shown.



Answer: $\boldsymbol{v_0 = 67.647V}$

<u>Example</u>: Use nodal analysis to find $\boldsymbol{v_1, v_2}$ $\boldsymbol{and}$ $\boldsymbol{v_3}$ in the circuit shown.



<u>Solution</u>:

For node (1):

$$\frac{v_1 - 0}{4} + \frac{v_1 - v_2}{8} + \frac{v_1 - v_3}{2} = 5 - 3, v_1\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{2}\right) - v_2\left(\frac{1}{8}\right) - v_3\left(\frac{1}{2}\right) = 2$$

$$v_1\left(\frac{7}{8}\right) - v_2\left(\frac{1}{8}\right) - v_3\left(\frac{1}{2}\right) = 2, \qquad \boldsymbol{7v_1 - v_2 - 4v_3 = 16} \qquad \boldsymbol{Eq.(1)}$$

For node (2):

$$\frac{v_2 - v_1}{8} + \frac{v_2 - 0}{2} + \frac{v_2 - v_3}{4} = 0, \quad v_2\left(\frac{1}{8} + \frac{1}{2} + \frac{1}{4}\right) - v_1\left(\frac{1}{8}\right) - v_3\left(\frac{1}{4}\right) = 0$$

$$v_1\left(\frac{-1}{8}\right) + v_2\left(\frac{7}{8}\right) - v_3\left(\frac{1}{4}\right) = 0, \quad -\boldsymbol{v_1} + \mathbf{7}\boldsymbol{v_2} - \mathbf{2}\boldsymbol{v_3} = \mathbf{0} \qquad \boldsymbol{Eq}.(\mathbf{2})$$

For node (3):

$$\frac{v_3 - v_1}{2} + \frac{v_3 - v_2}{4} + \frac{v_3 - 12}{8} = 3$$

$$v_1\left(-\frac{1}{2}\right) + v_2\left(-\frac{1}{4}\right) + v_3\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) = 3 + \frac{12}{8}$$

$$v_1\left(-\frac{1}{2}\right) + v_2\left(\frac{-1}{4}\right) + v_3\left(\frac{7}{8}\right) = 4.5, \qquad -\mathbf{4}\boldsymbol{v_1} - \mathbf{2}\boldsymbol{v_2} + \mathbf{7}\boldsymbol{v_3} = \mathbf{36} \qquad \boldsymbol{Eq}.(\mathbf{3})$$

$$\begin{bmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 0 \\ 36 \end{bmatrix}$$

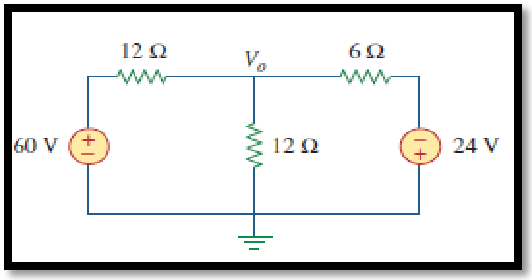
$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 16 & -1 & -4 \\ 0 & 7 & -2 \\ 36 & -2 & 7 \end{vmatrix}\begin{matrix} 16 & -1 \\ 0 & 7 \\ 36 & -2 \end{matrix}}{\begin{vmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{vmatrix}\begin{matrix} 7 & -1 \\ -1 & 7 \\ -4 & -2 \end{matrix}} = \frac{784 + 72 + 0 + 1008 - 64 - 0}{343 - 8 - 8 - 112 - 28 - 7} = \mathbf{10}\boldsymbol{V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 7 & 16 & -4 \\ -1 & 0 & -2 \\ -4 & 36 & 7 \end{vmatrix}\begin{matrix} 7 & 16 \\ -1 & 0 \\ -4 & 36 \end{matrix}}{180} = \frac{0 + 128 + 144 - 0 + 504 + 112}{180} \cong \mathbf{5}\boldsymbol{V}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 7 & -1 & 16 \\ -1 & 7 & 0 \\ -4 & -2 & 36 \end{vmatrix}\begin{matrix} 7 & -1 \\ -1 & 7 \\ -4 & -2 \end{matrix}}{180} = \frac{1764 + 0 + 32 + 448 - 0 - 36}{180}$$

$$v_3 = \mathbf{12.266}\boldsymbol{V}$$

Example: Find the current in the **12Ω** and **6Ω** resistors using nodal voltage analysis for the circuit shown.



Solution:

There is one independent node and a reference node.

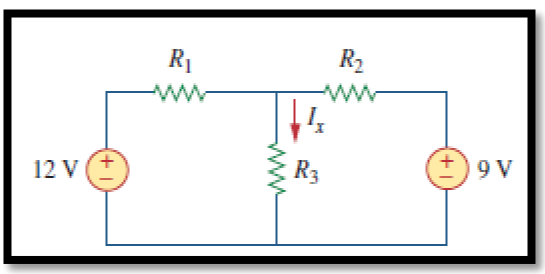
$$\frac{v_0-60}{12}+\frac{v_0-0}{12}+\frac{v_0-24}{6}=0, \quad v_0\left(\frac{1}{12}+\frac{1}{12}+\frac{1}{6}\right)-5-4=0, \quad v_0\left(\frac{1+1+2}{12}\right)=9$$

$$v_0\left(\frac{4}{12}\right)=9, \qquad v_0=\frac{12\times 9}{4}=\boldsymbol{27V}$$

$$I_{12\Omega}=\frac{60-v_0}{12}=\frac{60-27}{12}=\boldsymbol{2.75A}, \quad I_{12\Omega}=\frac{v_0}{12}=\frac{27}{12}=\boldsymbol{2.25A}$$

$$I_{6\Omega}=\frac{24-v_0}{6}=\frac{24-27}{6}=\boldsymbol{-0.5A}$$

Q: For the circuit shown, if $R_1=\mathbf{1}\Omega, R_2=\mathbf{2}\Omega, and\ R_3=\mathbf{3}\Omega$, find $\boldsymbol{I_x}$ using nodal voltage method.



Answer: 3A.

3.2 Mesh Analysis (Maxwell's Loop Current Method)

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. Nodal analysis applies **KCL** to find unknown voltages in a given circuit, while mesh analysis applies **KVL** to find unknown currents.

To understand mesh analysis, we should first explain more about what we mean by a mesh.

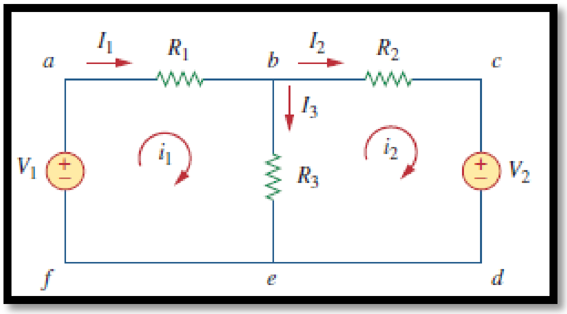
A mesh is a loop which does not contain any other loops within it.

Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, \ldots, i_n$to the n meshes.
2. Apply **KVL** to each of the n meshes. Use **Ohm's law** to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

To illustrate the steps, consider, for example, the circuit shown.

- The first step requires that mesh currents and are assigned to meshes 1 and 2 Although a mesh current may be assigned to each mesh in an arbitrary direction, it is conventional to assume that each mesh current flows clockwise.



- As the second step, we apply **KVL** to each mesh. Applying **KVL** to mesh 1, we obtain:

$$V_1 - i_1R_1 - R_3(i_1 - i_2) = 0, \;\; V_1 - i_1R_1 - i_1R_3 + i_2R_3 = 0$$

$i_1(R_1 + R_3) - i_2R_3 = V_1 \qquad \boldsymbol{Eq.(1)}$

For mesh 2, applying **KVL** gives:

$-i_2R_2 - R_3(i_2 - i_1) - V_2 = 0, \qquad -i_2R_2 - i_2R_3 + i_1R_3 - V_2 = 0$

$i_1R_3 - i_2(R_2 + R_3) = V_2 \qquad \boldsymbol{Eq.(2)}$

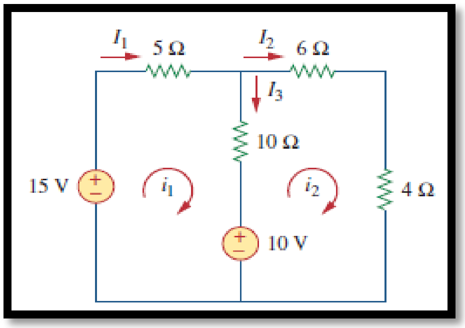
- The third step is to solve for the mesh currents. Putting Eqs. (1) and (2) in matrix form yields:

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ R_3 & -(R_2 + R_3) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Which can be solved to obtain the mesh currents $\boldsymbol{i_1}$and $\boldsymbol{i_2}$. After finding the mesh current:

$$\boldsymbol{I_1 = i_1, I_2 = i_2, \ \ I_3 = i_1 - i_2}$$

Example: For the circuit shown, find the branch currents $\boldsymbol{I_1}, \boldsymbol{I_2}$and $\boldsymbol{I_3}$using mesh analysis.



Solution:

We first obtain the mesh currents using **KVL**:

For loop (1):

$$15 - 5i_1 - 10(i_1 - i_2) - 10 = 0, 15 - 5i_1 - 10i_1 + 10i_2 - 10 = 0$$

$$15i_1 - 10i_2 = 5, \ \ \boldsymbol{3i_1 - 2i_2 = 1 \quad Eq.(1)}$$

For loop (2):

$$-6i_2 - 4i_2 - 10(i_2 - i_1) + 10 = 0, -10i_2 - 10i_2 + 10i_1 + 10 = 0$$

$$-20i_2 + 10i_1 = -10, \ \ 10i_1 - 20i_2 = -10, \ \ \boldsymbol{i_1 - \ 2i_2 = -1 \quad Eq.(2)}$$

We will use Cramer’s rule to solve these two equations:

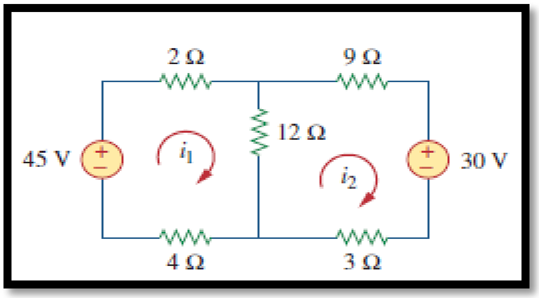
$\begin{bmatrix} 3 & -2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, We now obtain $\boldsymbol{i_1}$and $\boldsymbol{i_2}$as:

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 1 & -2 \\ -1 & -2 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 1 & -2 \end{vmatrix}} = \frac{-2-2}{-6+2} = \frac{-4}{-4} = \mathbf{1A}$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix}}{-4} = \frac{-3-1}{-4} = \frac{-4}{-4} = \mathbf{1A}$$

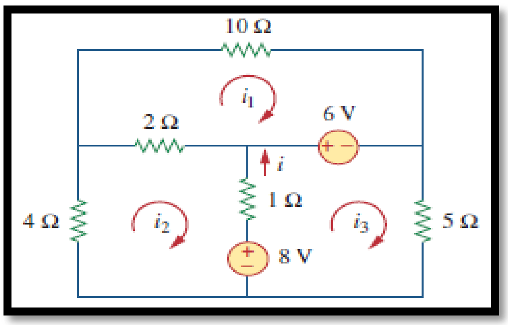
$I_1 = i_1 = \mathbf{1A}, \quad I_2 = i_2 = \mathbf{1A}, \quad I_3 = i_1 - i_2 = 1 - 1 = \mathbf{0A}$

Q: Calculate the mesh currents $\boldsymbol{i_1}$and $\boldsymbol{i_2}$ of the circuit shown.



Answer: $\boldsymbol{i_1 = 2.5A, \; i_2 = 0A.}$

<u>Example</u>: Apply mesh analysis to find $\boldsymbol{i}$ in the circuit shown.



<u>Solution</u>:

For loop (1):

$-10i_1 + 6 - 2(i_1 - i_2) = 0, -10i_1 + 6 - 2i_1 + 2i_2 = 0$

$12i_1 - 2i_2 = 6, \qquad \boldsymbol{6i_1 - i_2 = 3} \quad \boldsymbol{Eq.(1)}$

For loop (2):

$-4i_2 - 2(i_2 - i_1) - 1(i_2 - i_3) - 8 = 0, -4i_2 - 2i_2 + 2i_1 - i_2 + i_3 - 8 = 0$

$-7i_2 + 2i_1 + i_3 = 8,\quad \boldsymbol{2i_1 - 7i_2 + i_3 = 8} \qquad \boldsymbol{Eq.(2)}$

For loop (3):

$-5i_3 + 8 - 1(i_3 - i_2) - 6 = 0, -5i_3 + 8 - i_3 + i_2 - 6 = 0$

$\boldsymbol{i_2 - 6i_3 = -2} \qquad \boldsymbol{Eq.(3)}$

$$\begin{bmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ -2 \end{bmatrix}$$

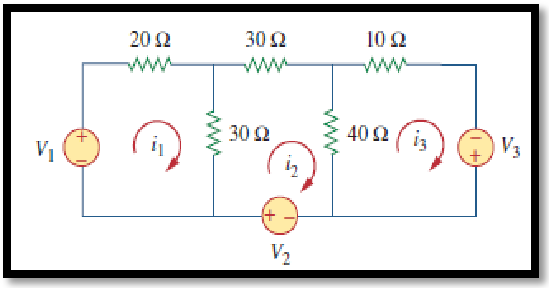
$$i_2 = \frac{\Delta_2}{\Delta} = \frac{\left|\begin{matrix} 6 & 3 & 0 \\ 2 & 8 & 1 \\ 0 & -2 & -6 \end{matrix}\right| \begin{matrix} 6 & 3 \\ 2 & 8 \\ 0 & -2 \end{matrix}}{\left|\begin{matrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & 1 & -6 \end{matrix}\right| \begin{matrix} 6 & -1 \\ 2 & -7 \\ 0 & 1 \end{matrix}} = \frac{-288 + 0 + 0 - 0 + 12 + 36}{252 + 0 + 0 - 0 - 6 - 12} = \boldsymbol{-1.025A}$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{\left|\begin{matrix} 6 & -1 & 3 \\ 2 & -7 & 8 \\ 0 & 1 & -2 \end{matrix}\right| \begin{matrix} 6 & -1 \\ 2 & -7 \\ 0 & 1 \end{matrix}}{234} = \frac{84 + 0 + 6 - 0 - 48 - 4}{234} = \boldsymbol{0.136A}$$

$i = i_2 - i_3 \ or \ i = i_3 - i_2 = -1.025 - 0.136 = \boldsymbol{-1.161A} \ or \ i = \boldsymbol{1.161A}$

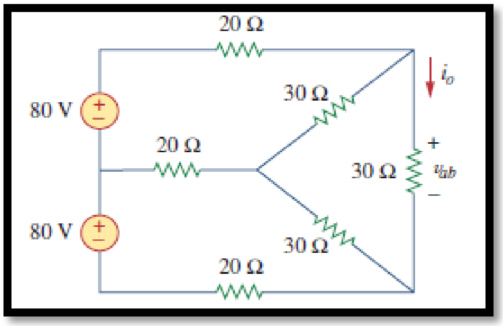
There is no need to find $\boldsymbol{i_1}$.

Q: Apply mesh analysis to find $\boldsymbol{i_1}, \boldsymbol{i_2}$ and $\boldsymbol{i_3}$ in the circuit shown if $V_1 = \boldsymbol{10V}, V_2 = \boldsymbol{20V}, and\ V_3 = \boldsymbol{30V}$.



Answer: $\boldsymbol{i_1 = 6.56A, i_2 = 1A, i_3 = 1.4A}$

Example: Use mesh analysis to find $\boldsymbol{v_{ab}}$ and $\boldsymbol{i_0}$ in the circuit shown.



Solution:

For loop (1):

$$80 - 20i_1 - 30(i_1 - i_3) - 20(i_1 - i_2) = 0$$

$$80 - 20i_1 - 30i_1 + 30i_3 - 20i_1 + 20i_2 = 0, 80 - 70i_1 + 20i_2 + 30i_3 = 0$$

$$\boldsymbol{70i_1 - 20i_2 - 30i_3 = 80} \qquad \boldsymbol{Eq.(1)}$$

For loop (2):

$$80 - 20(i_2 - i_1) - 30(i_2 - i_3) - 20i_2 = 0$$

$$80 - 20i_2 + 20i_1 - 30i_2 + 30i_3 - 20i_2 = 0, 80 + 20i_1 - 70i_2 + 30i_3 = 0$$

$$\boldsymbol{20i_1 - 70i_2 + 30i_3 = -80} \qquad \boldsymbol{Eq.(2)}$$

$$-30(i_3 - i_1) - 30i_3 - 30(i_3 - i_2) = 0$$

$$-30i_3 + 30i_1 - 30i_3 - 30i_3 + 30i_2 = 0, \boldsymbol{30i_1 + 30i_2 - 90i_3 = 0} \qquad \boldsymbol{Eq.(3)}$$

$$\begin{bmatrix} 70 & -20 & -30 \\ 20 & -70 & 30 \\ 30 & 30 & -90 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 80 \\ -80 \\ 0 \end{bmatrix}$$

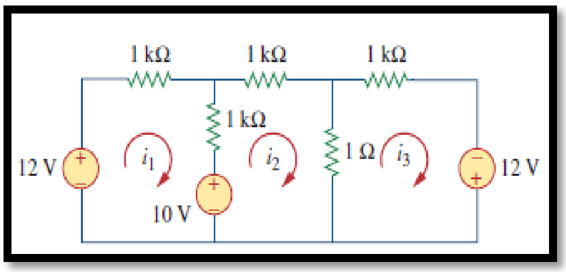
No need to find $\boldsymbol{i_1}$ *and* $\boldsymbol{i_2}$.

$$i_3 = \frac{\left|\begin{array}{ccc|cc} 70 & -20 & 80 & 70 & -20 \\ 20 & -70 & -80 & 20 & -70 \\ 30 & 30 & 0 & 30 & 30 \end{array}\right.}{\left|\begin{array}{ccc|cc} 70 & -20 & -30 & 70 & -20 \\ 20 & -70 & 30 & 20 & -70 \\ 30 & 30 & -90 & 30 & 30 \end{array}\right.}$$

$$i_3 = \frac{0 + 48000 + 48000 + 168000 + 168000 - 0}{441000 - 18000 - 18000 - 63000 - 63000 - 36000} = \mathbf{1.778A}$$

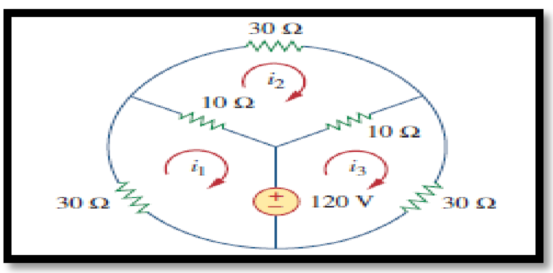
$i_0 = i_3 = \mathbf{1.778A}, \quad v_{ab} = i_0 \times 30 = 1.778 \times 30 = \mathbf{53.34V}$

Q: Find the mesh currents $\boldsymbol{i_1}, \boldsymbol{i_2}$ and $\boldsymbol{i_3}$ in the circuit shown.



Answer: $\boldsymbol{i_1 = 5.25mA, i_2 = 8.5mA, i_3 = 10.25mA}$

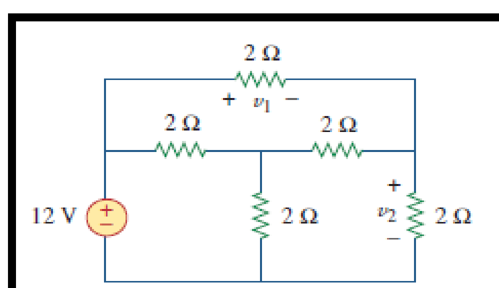
Q: Find the mesh currents $\boldsymbol{i_1}, \boldsymbol{i_2}$ *and* $\boldsymbol{i_3}$ in the circuit shown.



Answer: $\boldsymbol{i_1 = -3A, i_2 = 0A, i_3 = 3A}$

Q: Determine $\boldsymbol{v_1}$ *and* $\boldsymbol{v_2}$ in the circuit shown using mesh analysis.

Answer: $\boldsymbol{v_1 = 6V, v_2 = 6V}$



Circuit Theorems

4.1 Superposition Theorem

If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis. Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the superposition.

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

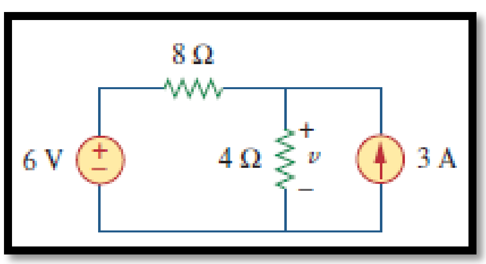
The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.

Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques discussed previously in the last chapters such as Ohm's law, Kirchhoff's laws, current and voltage divider rules, source transformations, mesh and nodal analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

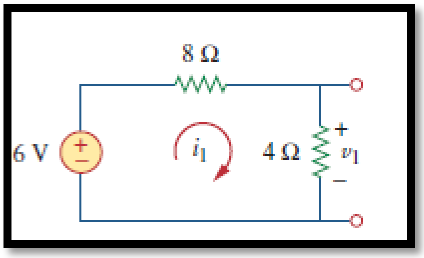
Superposition does help reduce a complex circuit to simpler circuits through replacement of **voltage sources by short circuits** and of **current sources by open circuits.**

Example: Use the superposition theorem to find v in the circuit shown.



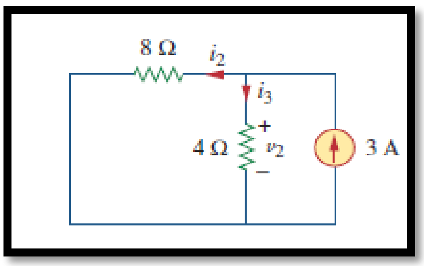
Solution:

The first step is to replace the current source by an open circuit as shown:



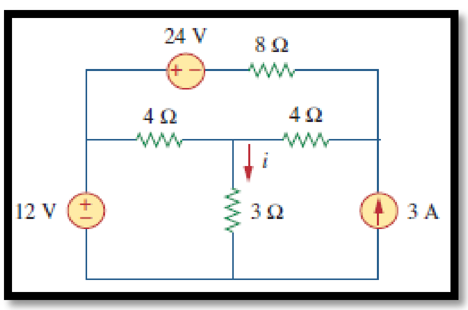
$$v_1 = \frac{6 \times 4}{8 + 4} = \boldsymbol{2V}$$

The second step is to replace the voltage source by a short circuit as shown:



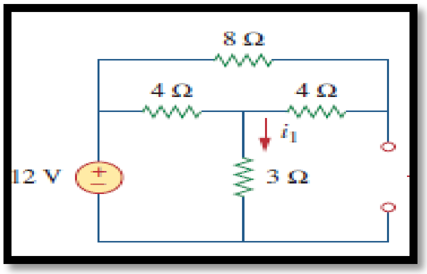
$$i_3 = \frac{3 \times 8}{4 + 8} = 2A, \quad v_2 = 2 \times 4 = \boldsymbol{8V}, \qquad \boldsymbol{v = v_1 + v_2 = 2 + 8 = 10V}$$

Example: For the circuit shown, use the superposition theorem to find $\boldsymbol{i}$.



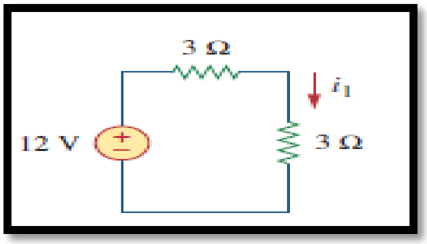
Solution:

The current due to the voltage source **12V**:



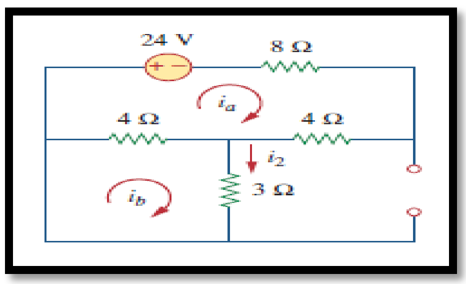
The resistors 4Ω on the right and 8Ω are in series:

$4 + 8 = 12\Omega$, The 12Ω in parallel with 4Ω gives: $\frac{4\times12}{4+12} = 3\Omega$



$$i_1 = \frac{12}{6} = \boldsymbol{2A}$$

The current due to the voltage source **24V**:



Applying mesh analysis gives:

For loop (1):

$$-24 - 8i_a - 4i_a - 4(i_a - i_b) = 0$$

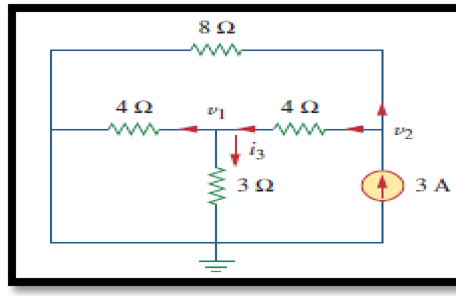
$$\boldsymbol{4i_a - i_b = -6} \quad \boldsymbol{Eq.(1)}$$

For loop (2):

$$-4(i_b - i_a) - 3i_b = 0, \quad \boldsymbol{4i_a - 7i_b = 0} \quad \boldsymbol{Eq.(2)}$$

$$\begin{bmatrix} 4 & -1 \\ 4 & -7 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \end{bmatrix}, \quad i_b = \frac{\begin{vmatrix} 4 & -6 \\ 4 & 0 \end{vmatrix}}{\begin{vmatrix} 4 & -1 \\ 4 & -7 \end{vmatrix}} = \frac{0+24}{-28+4} = \boldsymbol{-1A = i_2}$$

The current due to the current source **3A**:



Using nodal analysis gives:

For node (1):

$$\frac{v_1 - v_2}{4} + \frac{v_1 - 0}{3} + \frac{v_1 - 0}{4} = 0$$

$$v_1\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{4}\right) - \frac{v_2}{4} = 0, \qquad v_1\left(\frac{10}{12}\right) - v_2\left(\frac{1}{4}\right) = 0, \mathbf{10}\boldsymbol{v_1} - \mathbf{3}\boldsymbol{v_2} = \mathbf{0} \quad \boldsymbol{Eq.}(\mathbf{1})$$

For node (2):

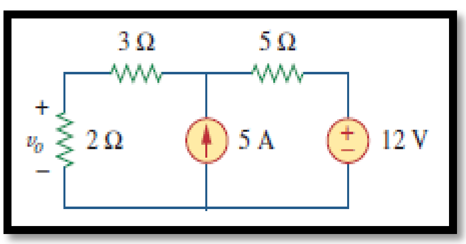
$$\frac{v_2 - v_1}{4} + \frac{v_2 - 0}{8} = 3, \ \ v_2\left(\frac{1}{4} + \frac{1}{8}\right) - v_1\left(\frac{1}{4}\right) = 3, \mathbf{2}\boldsymbol{v_1} - \mathbf{3}\boldsymbol{v_2} = -\mathbf{24} \quad \boldsymbol{Eq.}(\mathbf{2})$$

$$\begin{bmatrix} 10 & -3 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -24 \end{bmatrix}, v_1 = \frac{\begin{vmatrix} 0 & -3 \\ -24 & -3 \end{vmatrix}}{\begin{vmatrix} 10 & -3 \\ 2 & -3 \end{vmatrix}} = \frac{0 - 72}{-30 + 6} = \boldsymbol{3V}$$

There is no need to find v_2.

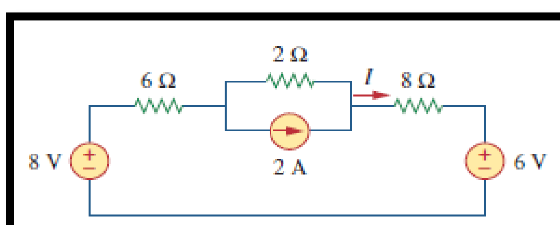
$$i_3 = \frac{v_1}{3} = \frac{3}{3} = \mathbf{1}\boldsymbol{A}, \quad thus\ i = i_1 + i_2 + i_3 = 2 - 1 + 1 = \mathbf{2}\boldsymbol{A}$$

Q: Using the superposition theorem, find $\boldsymbol{v_0}$ in the circuit shown.



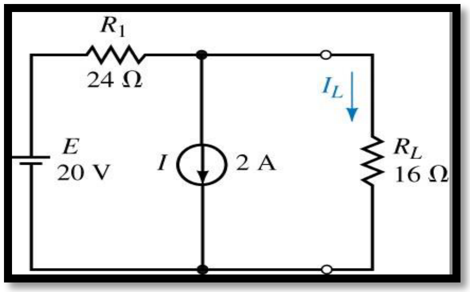
Answer: 7.4V.

Q: Find **I** in the circuit shown using the superposition principle.

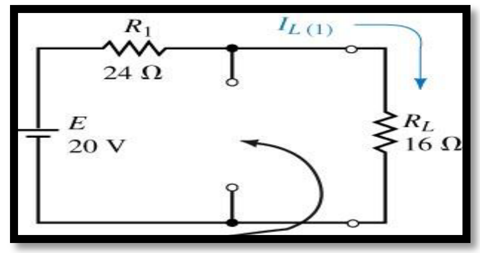


Answer: 375mA.

Example: Use superposition theorem to find $\boldsymbol{I_L}$.

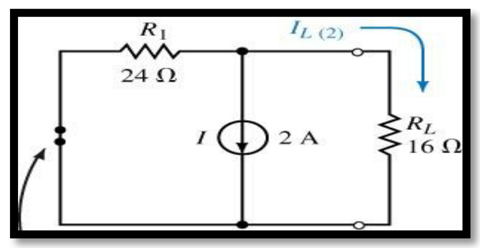


Solution:



The current due to the voltage source **20V**:

$$I_L(1) = I_T = \frac{20}{24 + 16} = \mathbf{0.5}\boldsymbol{A}$$

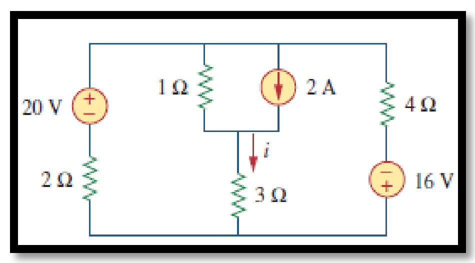


The current due to the current source **2A**:

$$I_L(2) = \frac{2 \times 24}{24 + 16} = \mathbf{1.2}\boldsymbol{A}$$

$$I_L = 1.2 - 0.5 = \mathbf{0.7}\boldsymbol{A}$$

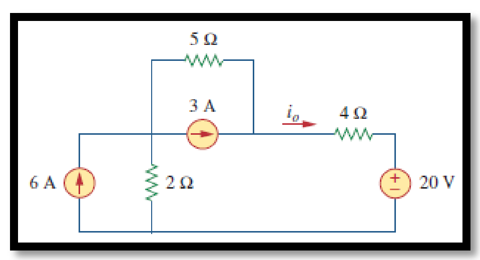
Q: For the circuit shown, use superposition theorem to find $\boldsymbol{i}$. Calculate the power delivered to the **3Ω** resistor.



Answer: $\boldsymbol{i = 1.875A, P = 10.546W}$.

Q: For the circuit shown, use superposition theorem to find $\boldsymbol{i_0}$.

Answer: $\boldsymbol{i_0 = 0.636A}$.

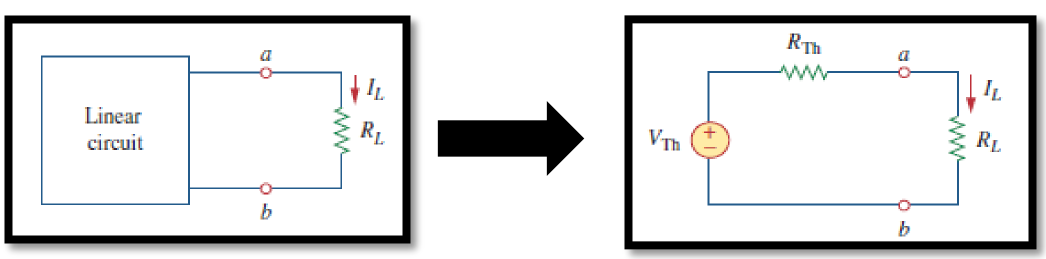


4.2 Thevenin's Theorem

<u>Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.</u>

<u>Steps to Find Thevenin's Equivalent Circuit:</u>

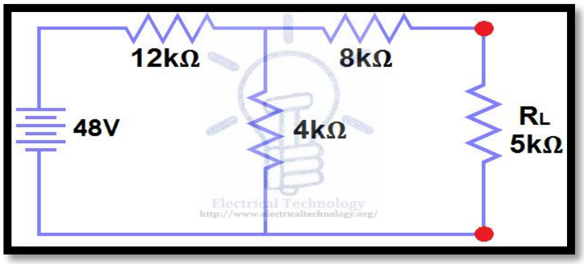
1. Remove the load R_L from the circuit terminals a and b (The load may be a single resistor or another circuit) and redraw the circuit. The two terminals (a and b) have becomes open-circuited.
2. Calculate R_{Th} by first setting all independent sources to zero (voltage sources are replaced by short circuits and current sources are replaced by open circuits), and finding the resultant resistance between the network terminals.
3. Calculate $E_{Th}$by first returning all sources to their original positions and finding the open circuit voltage between the network terminals.
4. Draw the **Thevenin's equivalent circuit** with R_L from where it was previously removed.
5. Finally, calculate the current flowing through the R_L by the following equation:



$$I_L = \frac{V_{Th}}{R_L + R_{Th}}$$

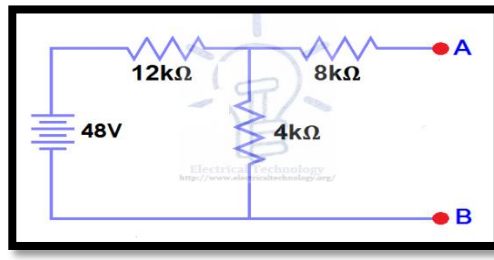
$$V_L = I_L R_L = (\frac{R_L}{R_L + R_{Th}})V_{Th}$$

Example: Using Thevenin's theorem, find the current in the $\boldsymbol{R_L}$ of the network shown.

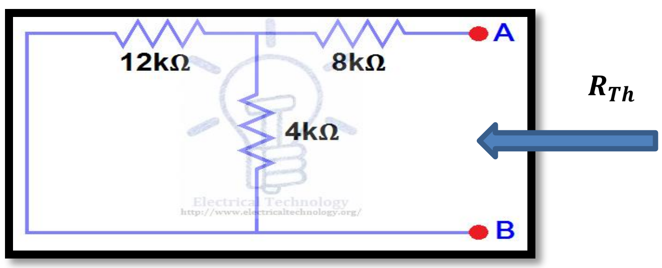


Solution:

Step (1): Remove $\boldsymbol{R_L}$ as shown.

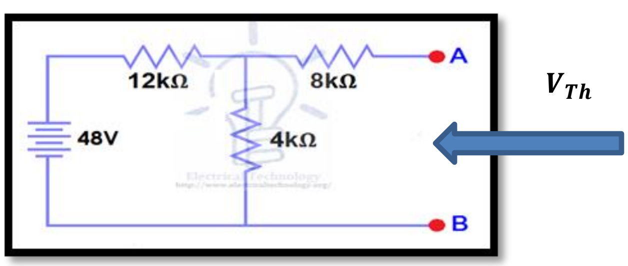


Step (2): Find $\boldsymbol{R_{Th}}$, remove the independent sources (replace the voltage source by short circuit (S. C.) as shown.



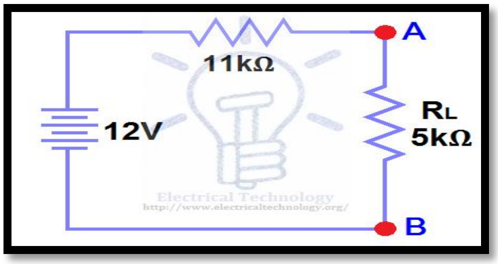
$$R_{Th} = (12 \| 4) + 8 = \frac{12 \times 4}{12 + 4} + 8 = 3 + 8 = \mathbf{11k}\Omega$$

Step (3): Find $\boldsymbol{V_{Th}}$, return all sources to their original positions then determine V_{Th} using any method discussed previously across the open circuit terminals a and b as shown.



$$V_{Th} = \frac{48 \times 4 \times 1000}{(12+4) \times 1000} = \mathbf{12V}$$

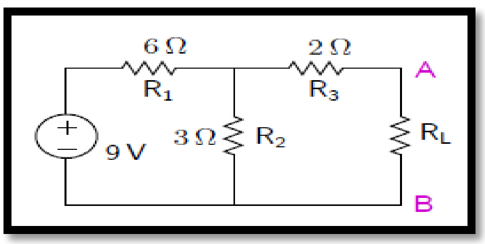
Step (4): Draw the **Thevenin's equivalent circuit** representing the network between points a and b with $\boldsymbol{R_L}$ added.



Step (5): Calculate the current flowing through the $\boldsymbol{R_L}$ as shown.

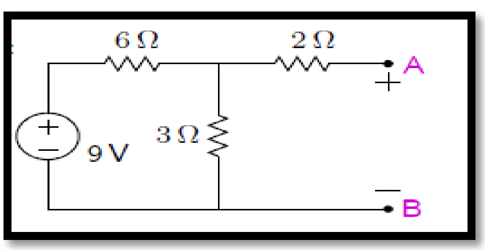
$$I_L = \frac{V_{Th}}{R_L + R_{Th}} = \frac{12}{11+5} = \mathbf{0.75mA}$$

Example: Using Thevenin's theorem, find the current in the $\boldsymbol{R_L} = \mathbf{10}\Omega$ of the network shown.

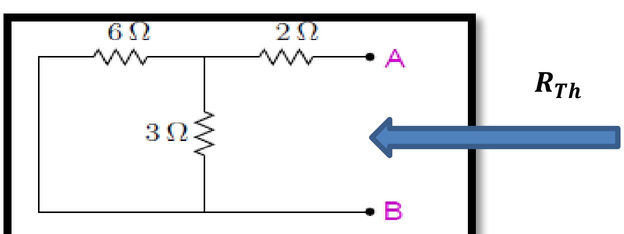


Solution:

Step (1):

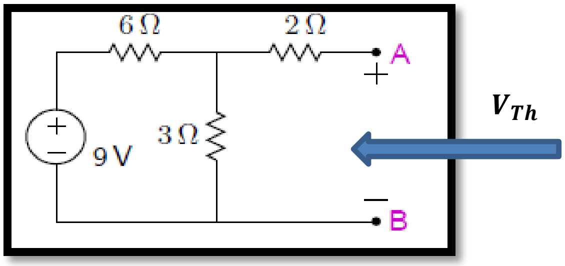


Step (2):



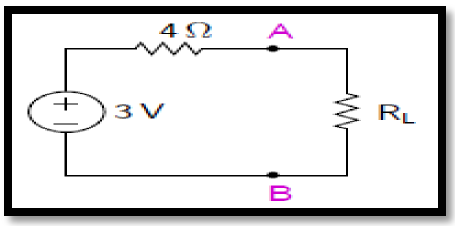
$$R_{Th} = (6\|3) + 2 = \frac{6 \times 3}{6+3} + 2 = \mathbf{4}\Omega$$

Step (3):



$$V_{Th} = \frac{9 \times 3}{3+6} = \boldsymbol{3V}$$

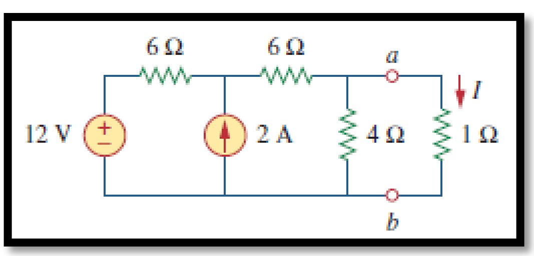
Step (4):



Step (5):

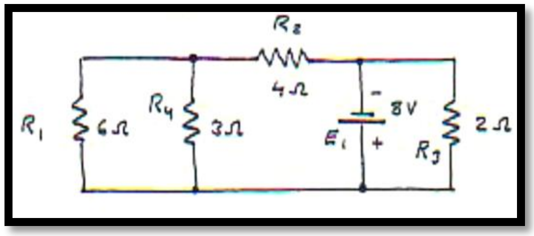
$$I_L = \frac{V_{Th}}{R_L + R_{Th}} = \frac{3}{4+10} = \boldsymbol{0.214A}$$

Q: Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown. Then find **I**.



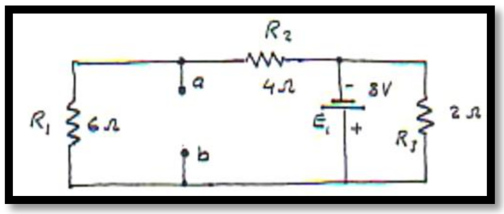
Answer: $\boldsymbol{V_{Th} = 6V, R_{Th}} = 3\Omega, \boldsymbol{I = 1.5A}$.

Example: Find the current in the **3Ω** resistor using Thevenin's theorem in the circuit shown.

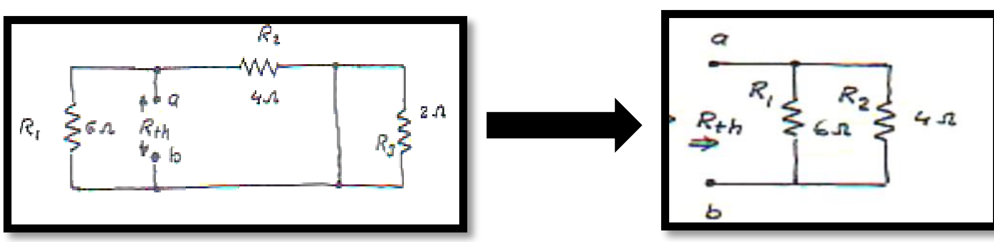


Solution:

Step (1):

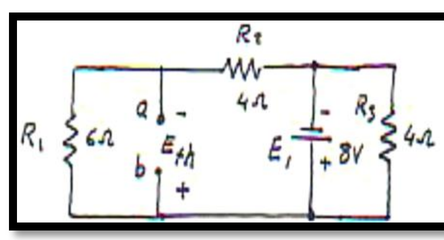


Step (2):



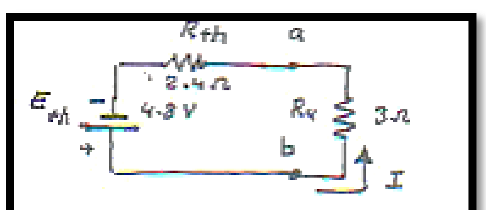
$$R_{Th} = 6 \| 4 = \frac{6 \times 4}{6+4} = \mathbf{2.4}\Omega, \qquad R_3 \; short \; circuited$$

Step (3):



$$E_{Th} = \frac{8 \times 6}{6+4} = \mathbf{4.8V}$$

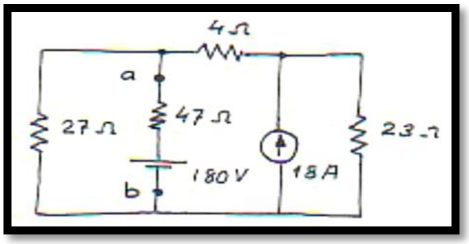
Step (4):



Step (5):

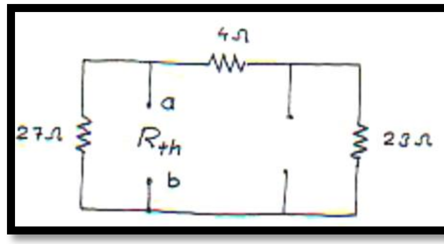
$$I = \frac{E_{Th}}{R_{Th} + R_4} = \frac{4.8}{2.4 + 3} = \mathbf{0.889A}$$

<u>Example</u>: In the circuit shown, find the current through the branch **a-b** using Thevenin's theorem.



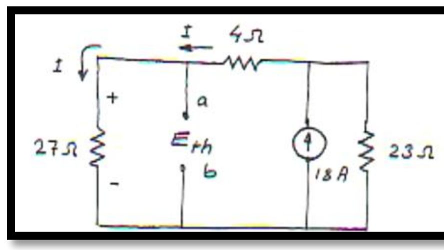
<u>Solution</u>:

$$R_{Th} = (4 + 23)\|27 = 27\|27 = \frac{27 \times 27}{27 + 27} = \mathbf{13.5}\Omega$$



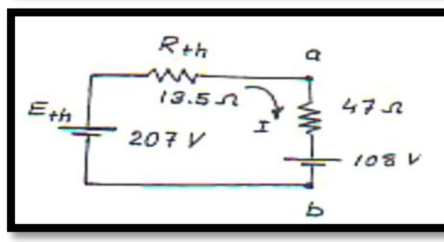
$$I = \frac{18 \times 23}{23 + 4 + 27} = 7.67A,$$

$$E_{Th} = I(27) = 7.67 \times 27 = \mathbf{207.09V}$$



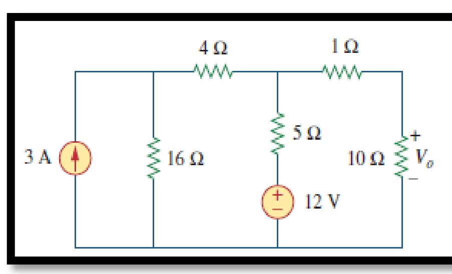
$$I = \frac{E_{Th} - 108}{13.5 + 47}$$

$$I = \frac{207.09 - 108}{13.5 + 47} = \mathbf{1.64A}$$

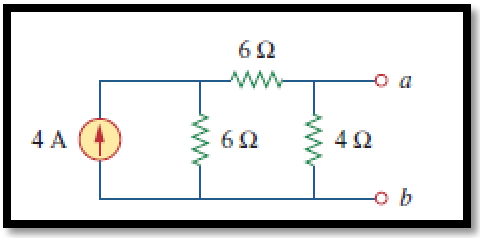


Q: Apply Thevenin's theorem to find $\boldsymbol{V_0}$ in the circuit shown.

Answer: $\boldsymbol{V_0 = 12.8V}$.



Q: Find the Thevenin equivalent circuit to the left of the terminals a and b.



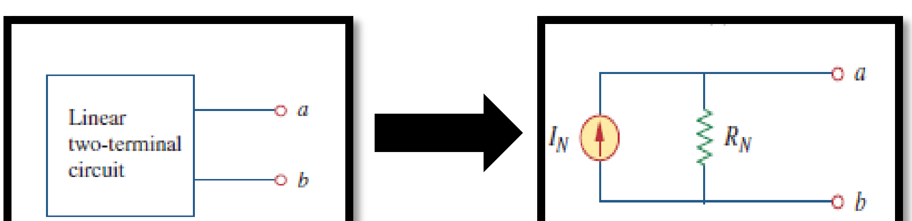
Answer: $\boldsymbol{R_{Th} = 3\Omega, V_{Th} = 6V}$.

4.3 Norton's Theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and RN is the input or equivalent resistance at the terminals when the independent sources are turned off.

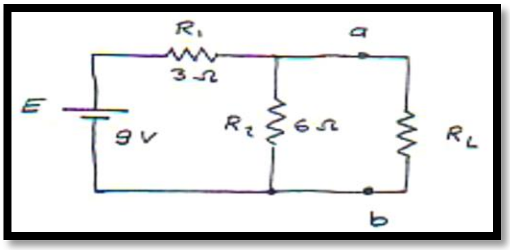
Steps to Find Norton's Equivalent Circuit:

1. Remove the load $\boldsymbol{R_L}$ or any portion of the network across which the Norton's equivalent circuit is found from the circuit terminals a and b.
2. Make the two terminals a and b short circuit.
3. Calculate $\boldsymbol{R_N}$ by first setting all independent sources to zero (voltage sources are replaced by short circuits and current sources are replaced by open circuits), and finding the resultant resistance between the network terminals. In other word, We find $\boldsymbol{R_N}$ in the same way we find $\boldsymbol{R_{Th}}$.
4. Calculate $\boldsymbol{I_N}$ by first returning all sources to their original positions and finding the short circuit current between the network terminals.
5. Draw the **Norton's equivalent circuit** with $\boldsymbol{R_L}$from where it was previously removed.
6. Finally, calculate the current flowing through the $\boldsymbol{R_L}$ by the following equation:



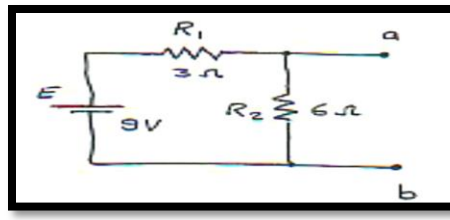
$$\boldsymbol{I_L = \frac{I_N R_N}{R_N + R_L}}$$

<u>Example</u>: For the circuit shown, find the Norton's equivalent circuit for the network to the left of (a-b).

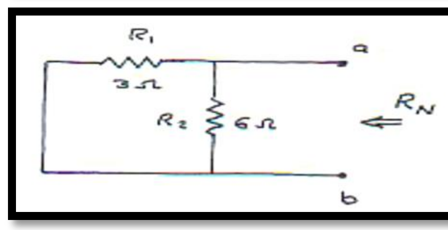


<u>Solution</u>:

Step (1):

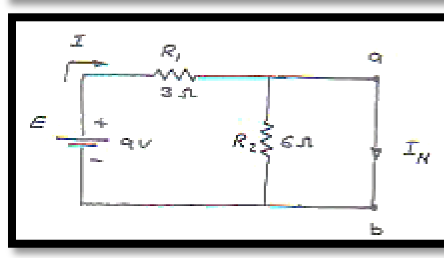


Step (2):



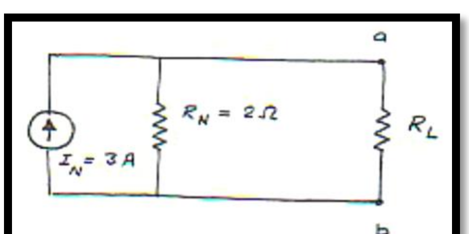
$$R_N = 3\|6 = \frac{3\times 6}{3+6} = \mathbf{2}\Omega$$

Step (3):

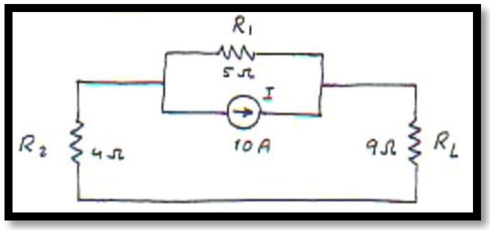


$$I_N = I = \frac{E}{R_1} = \frac{9}{3} = \boldsymbol{3A}, R_2 is\ short\ circuited$$

Step (4): Draw the Norton's equivalent circuit as shown:

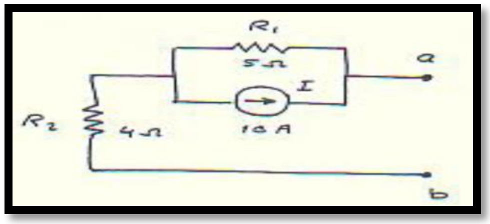


Example: Using Norton's theorem to find the current through the load resistor $\boldsymbol{R_L}$ in the network shown.

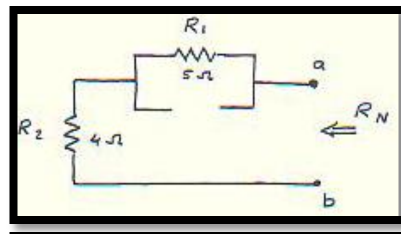


Solution:

Step (1):

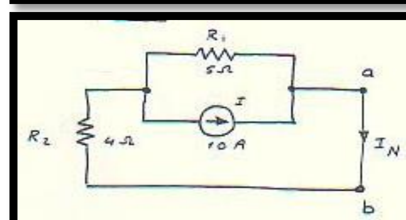


Step (2):



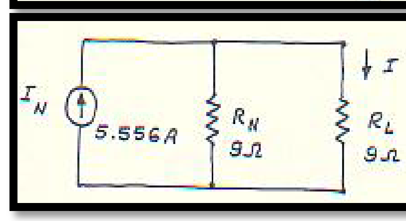
$R_N = 4 + 5 = \mathbf{9}\Omega$

Step (3):



$$I_N = \frac{10 \times 5}{4 + 5} = \frac{50}{9} = \mathbf{5.556}\boldsymbol{A}$$

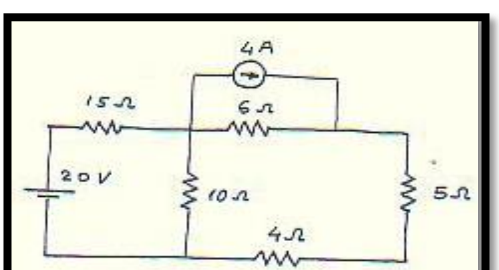
Step (4):



Step (5):

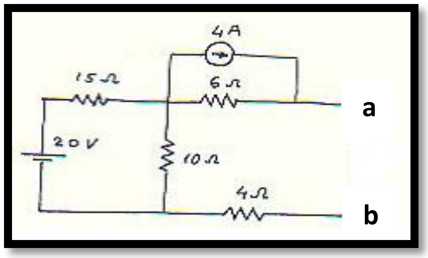
$$I = \frac{I_N}{2} = \frac{5.556}{2} = \mathbf{2.778}\boldsymbol{A}$$

Example: For the circuit shown, find the value of the current passing through the **5Ω** resistor using Norton's theorem. Calculate the power absorbed by this resistor.



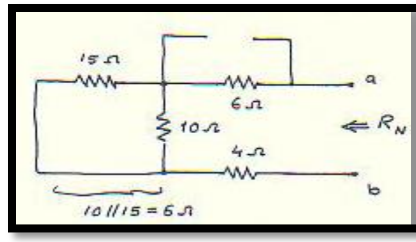
Solution:

Step (1):



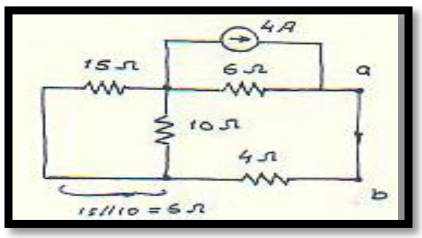
Step (2):

$R_N = (15 \| 10) + 6 + 4 = \mathbf{16}\Omega$

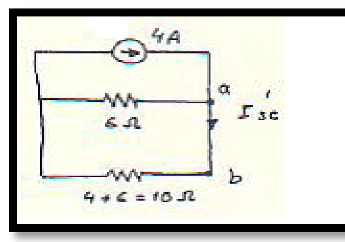


Step (3):

We have two sources, we can use superposition theorem to find the resulting $\boldsymbol{I_{SC}}$.

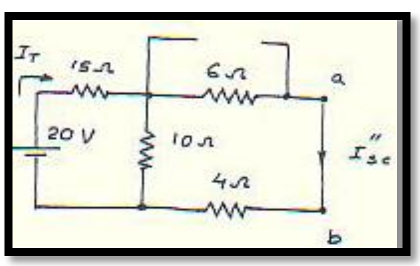


The effect of the source **4A:**



$$15 \| 10 = \frac{15 \times 10}{15 + 10} = \mathbf{6}\Omega, \quad I_N = I_{SC} = \frac{4 \times 6}{6 + 10} = \frac{24}{16} = \mathbf{1.5}\boldsymbol{A}$$

The effect of the source **20V**:

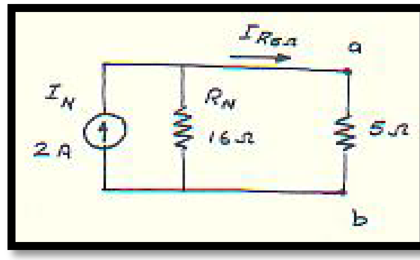


$$R_T = \big((6 + 4) \| 10\big) + 15 = \frac{10 \times 10}{10 + 10} + 15 = \frac{100}{20} + 15 = 5 + 15 = \mathbf{20}\Omega,$$

$$I_T = \frac{20}{20} = \mathbf{1A}, \qquad I_{SC} = \frac{I_T}{2} = \frac{1}{2} = \mathbf{0.5A},$$

$$I_N = I_{SC}(1) + I_{SC}(2) = 1.5 + 0.5 = \mathbf{2A}$$

Step (4):

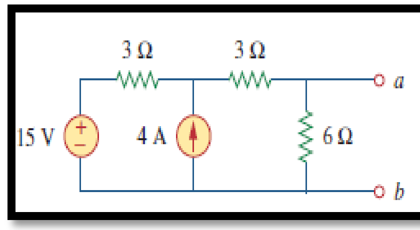


Step (5):

$$I_{5\Omega} = \frac{I_N \times 16}{16+5} = \frac{2 \times 16}{21} = \mathbf{1.52A},$$

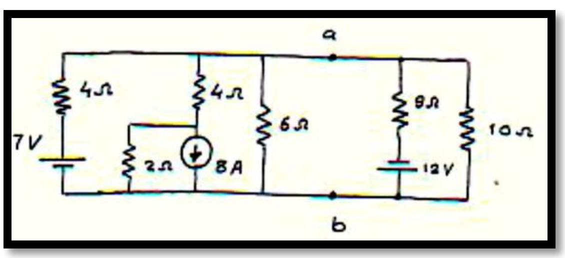
$$P = I_{5\Omega}^2 \times R_{5\Omega} = (1.52)^2 \times 5 = \mathbf{11.6W}$$

Q: Find the Norton equivalent circuit for the circuit shown, at terminals a-b.



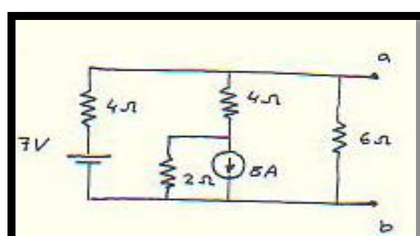
Answer: $\boldsymbol{R_N = 3\Omega, I_N = 4.5A}$.

__**Example**__: Find the Norton equivalent circuit for the portion of the network to the left of (a-b) in the circuit shown.



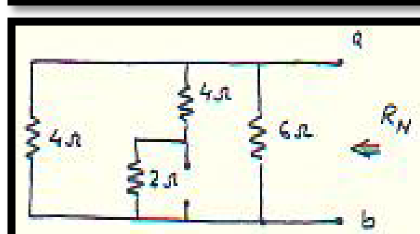
__**Solution**__:

Step (1):



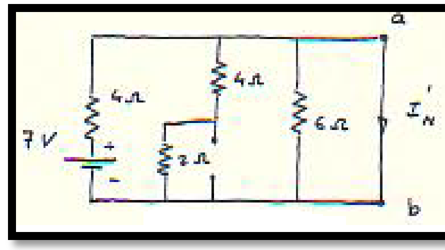
Step (2):

$$R_N = 4 \| 6 \| 6 = \mathbf{1.714\Omega}$$



Step (3):

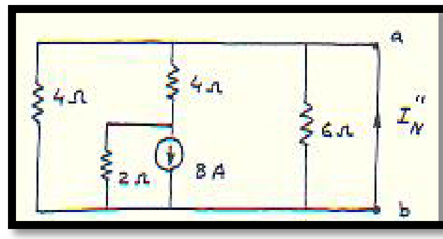
We have two sources, so we use the superposition theorem to find $\boldsymbol{I_N}$.



The effect of the source **7V**:

$$I_N = \frac{7}{4} = \mathbf{1.75}\boldsymbol{A},$$

because the resistors 6Ω *and* 6Ω *are short circuited*.



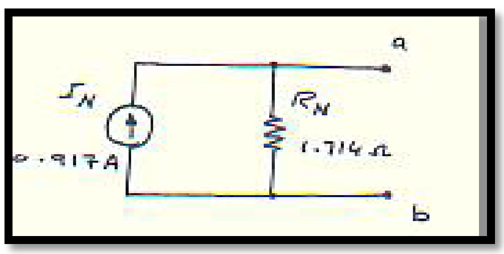
The effect of the source **8A:**

The resistors 4Ω and 6Ω are short circuited.

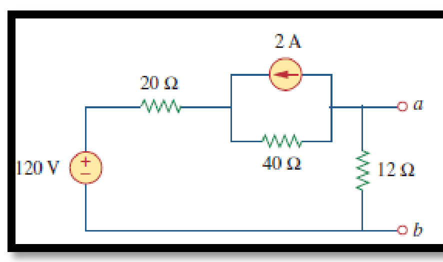
$$I_N = \frac{8 \times 2}{2+4} = \mathbf{2.667}\boldsymbol{A}$$

$$I_N = I_N(1) - I_N(2) = 2.667 - 1.75 = \mathbf{0.917}\boldsymbol{A}$$

Step (4): The Norton equivalent circuit for the portion of the network to the left of (a-b) is:

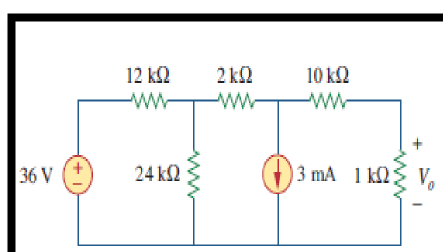


Q: Find the Norton equivalent with respect to terminals a-b in the circuit shown.



Answer: $\boldsymbol{R_N = 10\Omega, I_N = 0.667A}$.

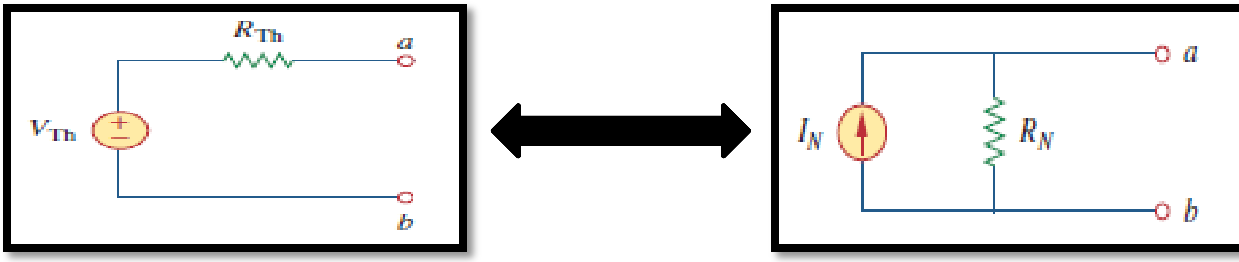
Q: Use Norton's theorem to find $\boldsymbol{V_0}$ in the circuit shown.



Answer: $\boldsymbol{R_N = 20k\Omega, I_N = -0.3mA, V_0 = 0.2857mA}$

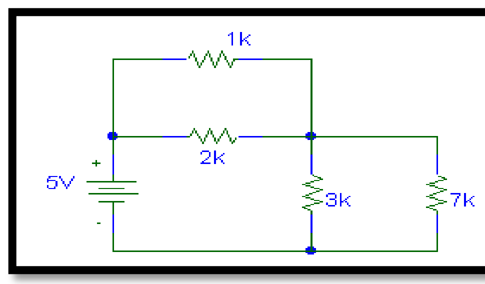
❖ Relation between Thevenin's Equivalent Circuit and Norton Equivalent Circuit

The Thevenin's and Norton equivalent circuits can also be found from each other by using the source transformation technique, as shown:



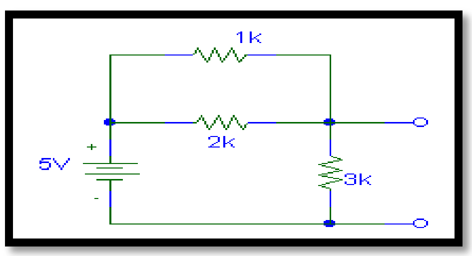
$$\boldsymbol{E_{Th} = I_N R_N} \qquad \boldsymbol{R_{Th} = R_N} \qquad \boldsymbol{I_N = \frac{E_{Th}}{R_{Th}}}$$

Example: Find the Thévenin and Norton equivalent circuits with respect to the **7kΩ** resistor for the circuit shown.

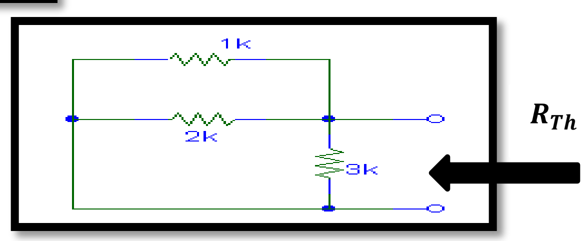


Solution:

Step (1):



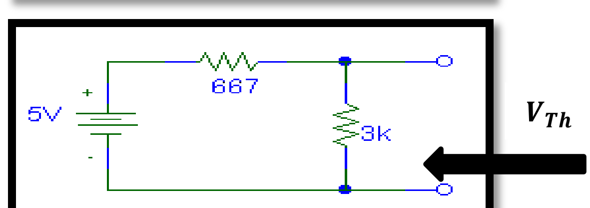
Step (2):



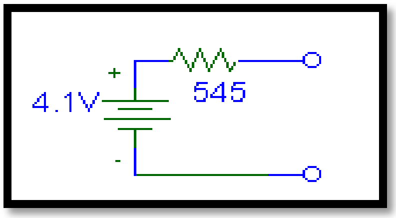
$R_{Th} = 1\|2\|3 = \mathbf{0.545}\boldsymbol{k}\Omega$

Step (3):

$$V_{Th} = \frac{5\times3\times1000}{667+(3\times1000)} = \mathbf{4.1}\boldsymbol{V}$$

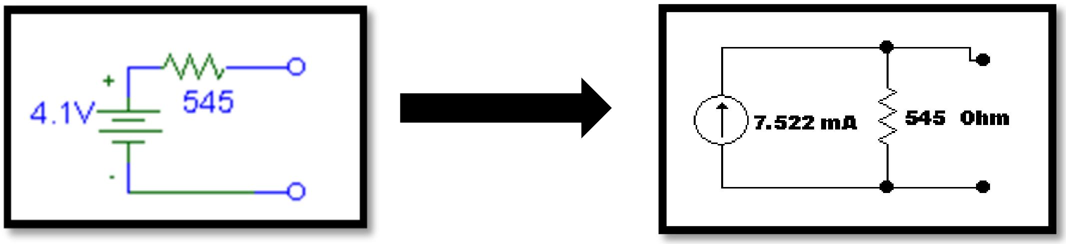


Step (4):



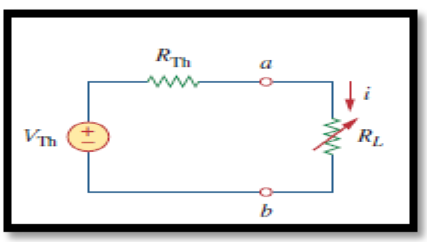
Thevenin's equivalent circuit can be covert to Norton equivalent circuit as shown:

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4.1}{0.545 \times 1000} = \mathbf{7.522}\boldsymbol{mA}, \ \ R_{Th} = R_N = \mathbf{545\Omega}$$



4.4 Maximum Power Transfer

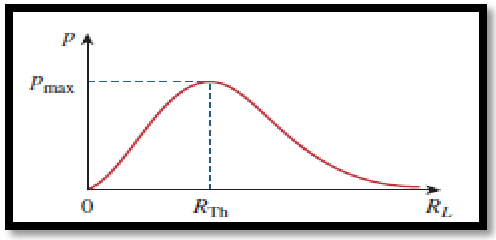
In many practical situations, a circuit is designed to provide power to a load. There are applications in areas such as communications where it is desirable to maximize the power delivered to a load. The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance $\boldsymbol{R_L}$. If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown:



The power delivered to the load can be expressed as:

$$\boxed{\boldsymbol{P = i^2 R_L = (\frac{V_{Th}}{R_{Th} + R_L})^2 \times R_L}}$$

For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as sketched shown:



We notice from this curve, that the power is small for small or large values of $\boldsymbol{R_L}$ but maximum for some value of $\boldsymbol{R_L}$ between 0 and ∞. We now want to show that this maximum power occurs when $\boldsymbol{R_L}$ is equal to $\boldsymbol{R_{Th}}$. This is known as the maximum power theorem.

<u>Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($\boldsymbol{R_L = R_{Th}}$).</u>

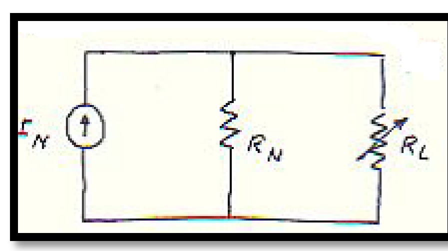
For maximum power transfer: $\boldsymbol{R_{Th} = R_L}$

$$P_{L\,max} = (\frac{V_{Th}}{2R_{Th}})^2 R_{Th} = \frac{{V_{Th}}^2}{4R_{Th}}, \qquad \boldsymbol{P_{L\,max} = \frac{V_{Th}^2}{4R_{Th}}} \qquad \boldsymbol{Eq.(1)}$$

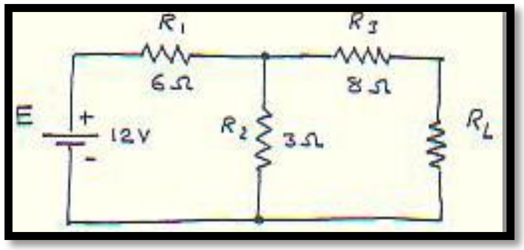
When dealing with Norton equivalent circuit as shown, maximum power transfer takes place when:

$$\boldsymbol{R_N = R_L}, \quad P = i^2 R_L = \left(I_N \times \frac{R_N}{R_N + R_L}\right)^2 \times R_L, \; P_{L\,max} = \left(I_N \times \frac{R_N}{2R_N}\right)^2 R_N$$

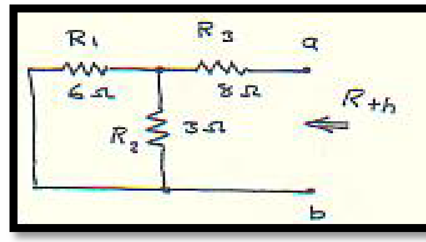
$$\boldsymbol{P_{L\,max} = \frac{I_N^2 R_N}{4}} \qquad \boldsymbol{Eq.(2)}$$



<u>Example</u>: For the network shown, determine the value of $\boldsymbol{R_L}$ for maximum power transfer, and calculate the power delivered under these conditions.

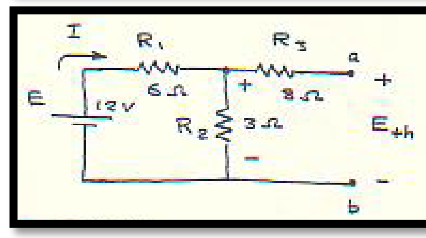


Solution:



$R_{Th} = (6 \| 3) + 8 = \frac{6 \times 3}{6+3} + 8 = 2 + 8 = \mathbf{10}\Omega$

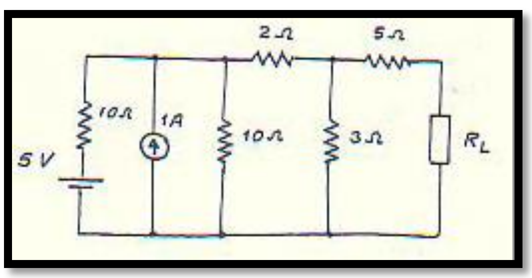
For maximum power transfer $\boldsymbol{R_L = R_{Th} = 10}\Omega$



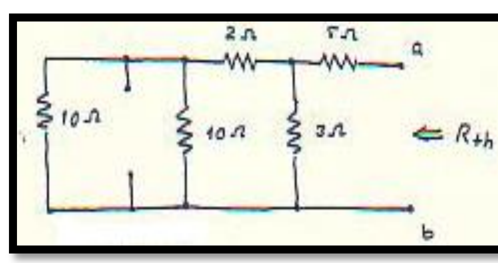
$$E_{Th} = \frac{12 \times 3}{6 + 3} = \boldsymbol{4V},$$

$$P_{L\,max} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(4)^2}{4 \times 10} = \mathbf{0.4}\boldsymbol{W}$$

Example: For the circuit shown, obtain the condition for maximum power transfer to the load $\boldsymbol{R_L}$**.** Hence determine the maximum power transferred.



Solution:



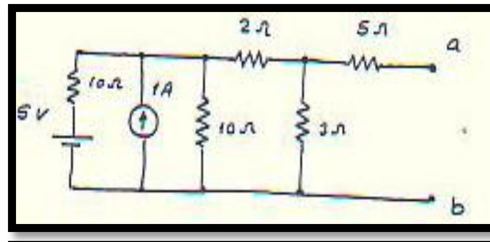
$R_{Th} = ((10 \| 10) + 2) \| 3) + 5 = ((5 + 2) \| 3) + 5$

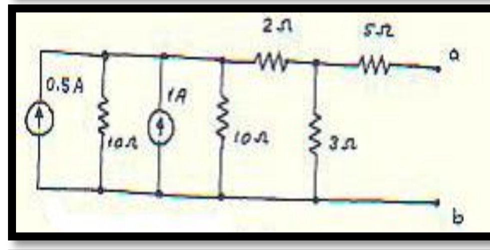
$$R_{Th} = \left(\frac{7 \times 3}{7 + 3}\right) + 5 = \mathbf{7.1}\Omega,$$

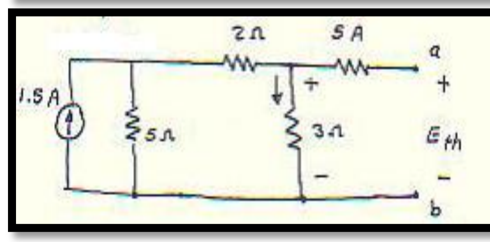
$R_L = R_{Th} = \mathbf{7.1}\Omega \quad for\ maximum\ power\ transfer$

E_{Th} Must be found for maximum power transfer.

We convert the voltage source to a current source as shown.



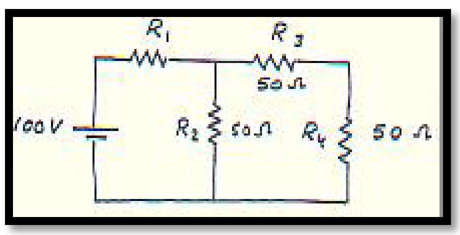




$I = \frac{1.5\times5}{2+3+5} = \mathbf{0.75A}, E_{Th} = 0.75 \times 3 = \mathbf{2.25V}$

$$P_{L\,max} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(2.25)^2}{4 \times 7.1} = \mathbf{0.178W = 178mW}$$

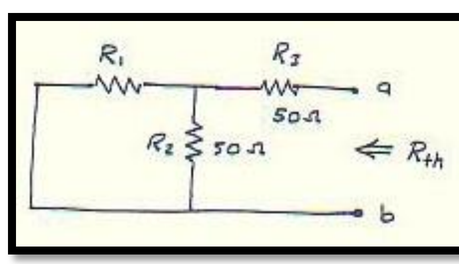
Example: For the circuit shown, find the value of the resistor $\boldsymbol{R_1}$, such that the resistor $\boldsymbol{R_4}$will receive maximum power.



Solution:

Since $\boldsymbol{R_4}$ receive maximum power, then:

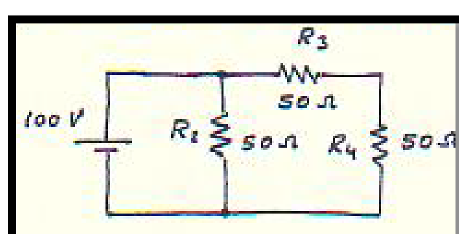
$R_4 = R_{Th},\ \boldsymbol{R_{Th} = 50}\Omega$



$$R_{Th} = (R_1 \| R_2) + 50 = \frac{R_1 \times 50}{R_1 + 50} + 50$$

$$50 = \frac{50R_1}{R_1 + 50} + 50, \qquad \frac{50R_1}{R_1 + 50} = 0, 50R_1 = 0$$

$\boldsymbol{R_1 = 0}$ **,** for maximum power transfer, the circuit will be as:

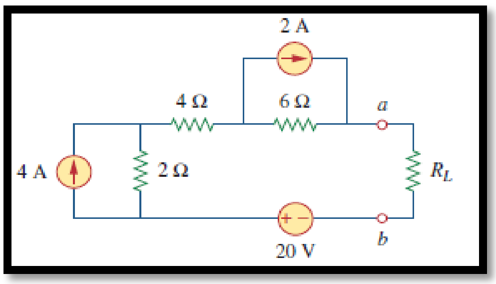


Q: (a) For the circuit shown, obtain the Thevenin equivalent at terminals a-b.

(b) Calculate the current in $\boldsymbol{R_L} = \mathbf{8\Omega}$.

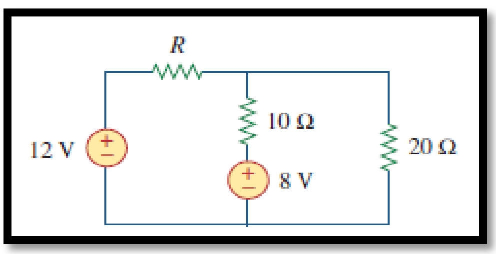
(c) Find $\boldsymbol{R_L}$ for maximum power deliverable to $\boldsymbol{R_L}$.

(d) Determine that maximum power.



Answer: $\boldsymbol{R_{Th} = 12\Omega, V_{Th} = 40V, I_L = 2A, R_L = 12\Omega, P_{L\,max} = 33.33W}$

Q: Compute the value of **R** that results in maximum power transfer to the **10Ω** resistor in circuit shown. Find the maximum power.



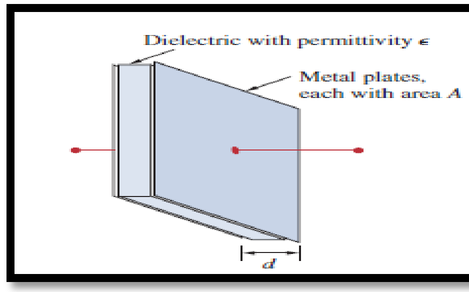
Answer: R=20Ω, $\boldsymbol{P_{L\,max} = 0.1W}$.

R-L-C Circuit

❖ Capacitors

A capacitor is **a passive element** designed to store energy in its electric field. Besides resistors, capacitors are the most common electrical components. Capacitors are used extensively in electronics, communications, computers, and power systems.

<u>A capacitor consists of two conducting plates separated by an insulator (or dielectric).</u>



When a voltage source $\boldsymbol{v}$ is connected to the capacitor, the source deposits a positive charge $\boldsymbol{q}$ on one plate and a negative charge on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by $\boldsymbol{q}$, is directly proportional to the applied voltage so that:

$$\boldsymbol{q = Cv} \qquad \boldsymbol{Eq.(1)}$$

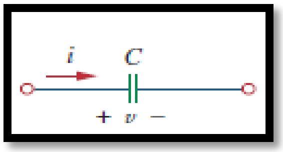
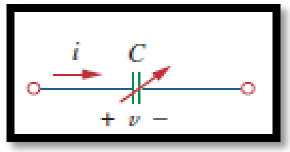
Where **C**, the constant of proportionality, is known as the capacitance of the capacitor.

<u>Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).</u>

Although the capacitance **C** of a capacitor is the ratio of the charge $\boldsymbol{q}$ per plate to the applied voltage $\boldsymbol{v}$ it does not depend on $\boldsymbol{q}$ or $\boldsymbol{v}$. It depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown, the capacitance is given by:

$$\boldsymbol{C = \frac{\epsilon A}{d}} \qquad \boldsymbol{Eq.(2)}$$

Where **A** is the surface area of each plate, **d** is the distance between the plates, and $\boldsymbol{\epsilon}$ is the permittivity of the dielectric material between the plates. The circuit symbols for fixed and variable capacitors are shown in the figure below.

To obtain the current-voltage relationship of the capacitor, we take the derivative of both sides of Eq. (1). Since:

$$\boldsymbol{i = \frac{dq}{dt} \qquad Eq.(3)}$$

Differentiating both sides of Eq. (1) gives:

$$\boxed{\boldsymbol{i = C\frac{dv}{dt} \qquad Eq.(4)}}$$

The voltage-current relation of the capacitor can be obtained by integrating both sides of Eq. (4). We get:

$$\boxed{\boldsymbol{v(t) = \frac{1}{C}\int_{t_1}^{t_2} idt \quad Eq.(5)}}$$

The instantaneous power delivered to the capacitor is:

$$\boldsymbol{P = vi = Cv\frac{dv}{dt} \qquad Eq.(6)}$$

The energy stored in the capacitor is therefore:

$$W = \int_{t_1}^{t_2} Pdt = \int_{t_1}^{t_2} vidt = \int_{t_1}^{t_2} Cv\frac{dv}{dt}dt = C\int_{t_1}^{t_2} vdv = C\frac{v^2}{2}$$

$$\boldsymbol{W = \frac{1}{2}Cv^2 \qquad Eq.(7) \quad or \quad W = \frac{q^2}{2C} \qquad Eq.(8)}$$

We note from Eq. (4) that when the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. Thus,

A capacitor is an open circuit to dc.

Example: (a) Calculate the charge stored on a **3-pF** capacitor with **20 V** across it.

(b) Find the energy stored in the capacitor.

Solution:

$$(a)\ q = Cv = 3 \times 10^{-12} \times 20 = \boldsymbol{60pC}$$

$$(b)\ W = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = \boldsymbol{600pJ}$$

Example: The voltage across a **5-$\boldsymbol{\mu F}$**capacitor is: $\boldsymbol{v(t) = 10cos6000t\ V,}$ calculate the current through it.

Solution:

$$i = C\frac{dv}{dt} = 5 \times 10^{-6} \times \frac{d}{dt}(10cos6000t) = -5 \times 10^{-6} \times 6000 \times 10sin6000t$$

$$i = \boldsymbol{-0.3sin6000t\ A}$$

Q: What is the voltage across a $\boldsymbol{4.5-\mu F}$capacitor if the charge on one plate is **0.12 mC**? How much energy is stored?

Answer: 26.67 A, 1.6 mJ.

Q: If a **10-$\boldsymbol{\mu F}$**capacitor is connected to a voltage source with:

$\boldsymbol{v(t) = 75sin2000t\ V,}$ determine the current through the capacitor.

Answer: 1.5cos2000t A.

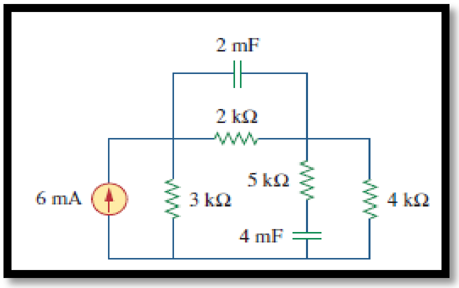
Example: Determine the voltage across a **2-$\boldsymbol{\mu F}$**capacitor if the current through it is: $\boldsymbol{i(t) = 6e^{-3000t}mA,}$ Assume that the initial capacitor voltage is zero.

Solution:

$$v = \frac{1}{C}\int_0^t idt = \frac{1}{2 \times 10^{-6}}\int_0^t 6e^{-3000t} \times 10^{-3}dt = \frac{3 \times 10^3}{-3000}e^{-3000t}\Big|_0^t$$

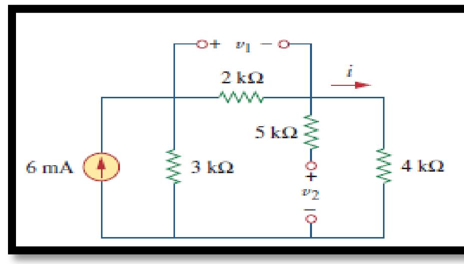
$$\boldsymbol{v = (1 - e^{-3000t})V}$$

<u>Example</u>: Obtain the energy stored in each capacitor in the circuit shown under dc conditions.



<u>Solution</u>:

Under dc conditions, we replace each capacitor with an open circuit, as shown.



The current through the series combination of the **2kΩ** and **4kΩ** resistors is obtained by current division as:

$$i = \frac{6 \times 3}{3 + 2 + 4} = \boldsymbol{2mA}$$

$Hence, the\ voltages\ \boldsymbol{v_1} and\ \boldsymbol{v_2} across\ the\ capacitors\ are$:

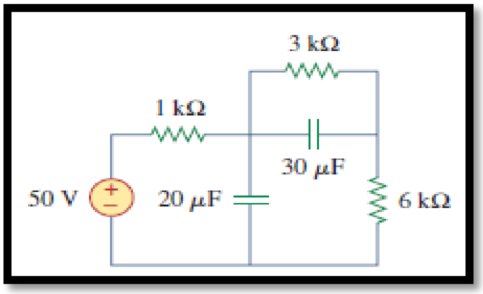
$$v_1 = 2000i = 2000 \times 2 \times 10^{-3} = \boldsymbol{4V}, v_2 = 4000i = \boldsymbol{8V}$$

And the energies stored in them are:

$$W_1 = \frac{1}{2} C_1 v_1^2 = \frac{1}{2}(2 \times 10^{-3}) \times (4)^2 = \boldsymbol{16mJ}$$

$$W_2 = \frac{1}{2} C_2 v_2^2 = \frac{1}{2}(4 \times 10^{-3}) \times (8)^2 = \boldsymbol{128mJ}$$

Q: Under dc conditions, find the energy stored in the capacitors in the circuit shown.

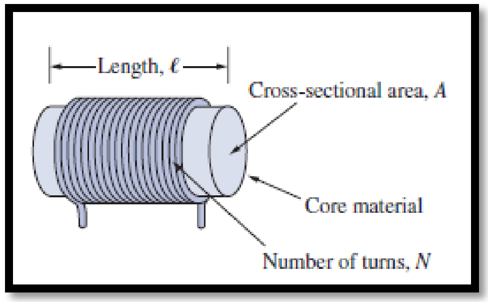


Answer: 20.25 mJ, 3.375 mJ.

❖ Inductors

An inductor is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems.

Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire, as shown.



<u>An inductor consists of a coil of conducting wire.</u>

If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current as shown:

$$\boxed{v = L\frac{di}{dt} \qquad Eq.(1)}$$

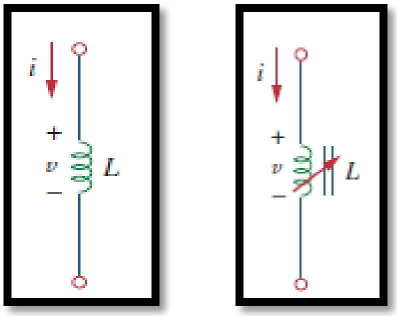
Where **L** is the constant of proportionality called the inductance of the inductor.

<u>Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).</u>

The inductance of an inductor depends on its physical dimension and construction. For example, for the inductor, (solenoid) shown, the inductance is given by:

$$\boldsymbol{L = \frac{N^2 \mu A}{l}} \qquad \boldsymbol{Eq. (2)}$$

Where **N** is the number of turns, $\boldsymbol{l}$ is the length, **A** is the cross-sectional area, and $\boldsymbol{\mu}$ is the permeability of the core. We can see from Eq. (2) that inductance can be increased by increasing the number of turns of coil, using material with higher permeability as the core, increasing the cross-sectional area, or reducing the length of the coil. The circuit symbols for fixed and variables inductors are shown in the figure below.



The current-voltage relationship is obtained from Eq. (1) as:

$$di = \frac{1}{L} v dt$$

Integrating gives:

$$\boxed{\boldsymbol{i = \frac{1}{L} \int_{t_0}^{t} v dt} \qquad \boldsymbol{Eq. (3)}}$$

The inductor is designed to store energy in its magnetic field. The energy stored can be obtained from Eq. (1). The power delivered to the inductor is:

$$\boldsymbol{P = vi = \left(L \frac{di}{dt}\right) i} \qquad \boldsymbol{Eq. (4)}$$

The energy stored is:

$$W = \int_{t_1}^{t_2} P dt = L \int_{t_1}^{t_2} \frac{di}{dt} i dt = L \int_{t_1}^{t_2} i di = L(\frac{i^2}{2})$$

$$\boldsymbol{W = \frac{1}{2}Li^2} \qquad \boldsymbol{Eq.(5)}$$

We note from Eq. (1) that the voltage across an inductor is zero when the current is constant. Thus,

<u>An inductor acts like a short circuit to dc.</u>

<u>Example</u>: The current through a **0.1-H** inductor is $\boldsymbol{i(t) = 10te^{-5t}}$**.** Find the voltage across the inductor and the energy stored in it.

<u>Solution</u>:

$$v = L\frac{di}{dt} = 0.1 \times \frac{d}{dt}(10te^{-5t}) = t \times (-5e^{-5t}) + e^{-5t} \times 1 = -5te^{-5t} + e^{-5t}$$

$$\boldsymbol{v = e^{-5t}(1 - 5t)V}$$

The energy stored:

$$W = \frac{1}{2} \times 0.1 \times (10te^{-5t})^2 = \frac{1}{2} \times 0.1 \times 100t^2e^{-10t} = \boldsymbol{5t^2e^{-10t}J}$$

Q: If the current through a 1-mH inductor is $\boldsymbol{i(t) = 60cos100t\ mA}$, find the terminal voltage and the energy stored.

Answer: -6 sin 100t mV, 1.8 $\boldsymbol{cos^2}$ (100t) $\boldsymbol{\mu}$J.

<u>Example</u>: Find the current through a **5-H** inductor if the voltage across it is:

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0 & t < 0 \end{cases}$$

Also, find the energy stored at t=5s. Assume $i(v) > 0$.

<u>Solution</u>:

$$i = \frac{1}{L}\int_{t_0}^{t} v\,dt = \frac{1}{5}\int_{0}^{t} 30t^2\,dt = 6 \times \frac{t^3}{3}\Big|_0^t = 2(t^3 - 0) = \boldsymbol{2t^3A}$$

$$P = vi = 30t^2 \times 2t^3 = \boldsymbol{60t^5W}$$

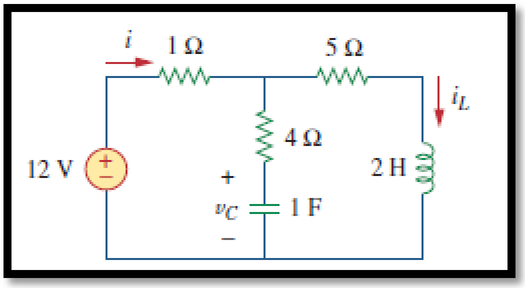
$$W = \int_0^t P dt = \int_0^5 60t^5\, dt = 60\frac{t^6}{6}\Big|_0^5 = 10(5^6 - 0) = \mathbf{156.25}\boldsymbol{kJ}$$

Or $W = \frac{1}{2}Li^2 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li^2(0) = \frac{1}{2}(5)(2\times 5^3)^2 - 0 = \mathbf{156.25}\boldsymbol{kJ}$

Q: The terminal voltage of a **2-H** inductor is $\boldsymbol{v} = \mathbf{10(1} - \boldsymbol{t})\boldsymbol{V}.$ Find the current flowing through it at **t=4s** and the energy stored in it at **t=4s.** Assume $\boldsymbol{i}(\mathbf{0}) = \mathbf{2}\boldsymbol{A}$.

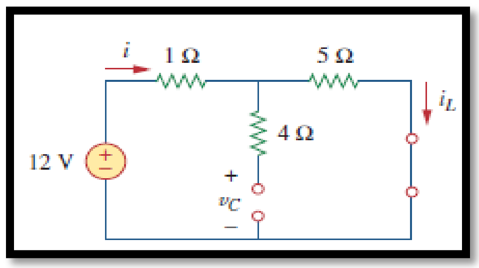
Answer: -18 A, 320 J.

Example: Consider the circuit shown. Under dc conditions, find: (a) $\boldsymbol{i}$, $\boldsymbol{v_c}$ and $\boldsymbol{i_L}$(b) the energy stored in the capacitor and inductor.



Solution:

(a) Under dc conditions, we replace the capacitor with an **open circuit** and the inductor with a **short circuit**, as shown:



$$i = i_L = \frac{12}{1+5} = \mathbf{2}\boldsymbol{A}$$

The voltage $\boldsymbol{v_c}$ is the same as the voltage across the **5Ω** resistor. Hence:

$$v_c = 5i = 5\times 2 = \mathbf{10}\boldsymbol{V}$$

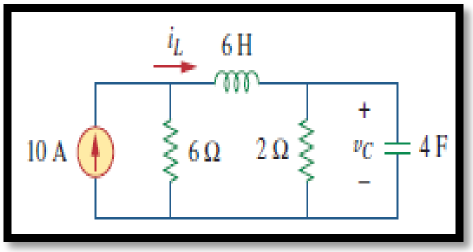
(b) The energy in the capacitor is:

$$W_c = \frac{1}{2}Cv_c^2 = \frac{1}{2}\times 1\times(10)^2 = \mathbf{50}\boldsymbol{J}$$

And that in the inductor is:

$$W_L = \frac{1}{2} L i_L^2 = \frac{1}{2} \times 2 \times (2)^2 = \boldsymbol{4J}$$

Q: Determine $\boldsymbol{v_c}, \boldsymbol{i_L}$ and the energy stored in the capacitor and inductor in the circuit shown under dc conditions.

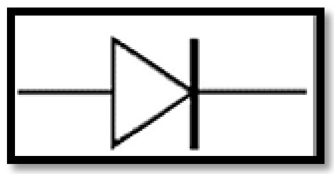


Answer: 15 V, 7.5 A, 450 J, 168.75 J.

Basics and Types of Diodes

❖ Introduction to Diode:

A diode is an electric device that permits the flow of current only in one direction and restricts the flow in the opposite direction. The word "**diode**" is traditionally aloof for tiny signal appliances, **I ≤ 1 A**. When a diode is positioned in a simple battery lamp circuit, then the diode will either permit or stop flow of current through the lamp. There are various sorts of diode but their fundamental role is identical. The most ordinary kind of diode is silicon diode; it is placed in a glass cylinder.



❖ Diode Operation:

A diode starts its operations when a voltage signal applies across its terminals. A DC volt is applied so that diode starts its operation in a circuit and this is known as **Biasing**. Diode is similar to a **switch** which is one way, hence it can be either in conduction more or non-conduction mode. "**ON**" mode of the diode, is attained by **forward biasing**, which simply means that higher or positive potential is applied on the anode and on the cathode, negative or lower potential is applied of a diode. Whereas the "**OFF**" mode of the diode is attained with the aid of **reverse biasing** which simply means that higher or positive potential is applied on the cathode and on the anode, negative or lower potential is applied of a diode.

In the "**ON**" situation the practical diode provides forward resistance. A diode needs forward bias voltage to get in the "ON" mode this is known as **cut-in-voltage**. Whereas the diode initiates conducting in reverse biased manner when reverse bias voltage goes beyond its limit and this is known as **breakdown voltage**. The diode rests in **OFF** mode when no voltage is applicable across it.

❖ Function of Diode:

The key function of a diode is **to obstruct the flow of current in one direction and permit the flow of current in the other direction**. Current passing through the diode is known as **forward current** whereas the current blocked by the diode is known as the **reverse current**.

❖ Diode Equation:

The equation of diode expresses the current flow via diode as a function of voltage. The ideal diode equation is:

$$\boldsymbol{I = I_0(e^{\frac{qV}{KT}} - 1)}$$

Where:

I – Stands for the net current passing through a diode.

$\boldsymbol{I_0}$**-** Stands for dark saturation current, the diode seepage current density in the deficiency of light.

V – Stands for applied voltage across the terminals of the diode.

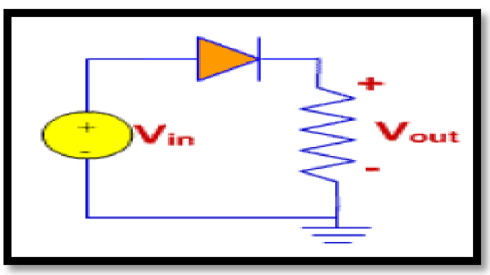
q – Stands for fixed value of electron charge.

k – Stands for Boltzmann’s constant.

T – Stands for fixed temperature (K).

❖ Diode Circuits:

The basic aim behind this study is to show how diodes can be employed in circuits. Now let us analyze a simple diode circuit.



1. When diode is in **ON** mode, no voltage is there across it; hence it acts like a **short circuit**.
2. Whereas when diode is in **OFF** mode, there is zero current, hence it behaves like an **open circuit**.

3. From the above two conditions, either one can take place at a time. This helps us to check out what will happen in any circuit with diodes.

❖ Diode Characteristics:

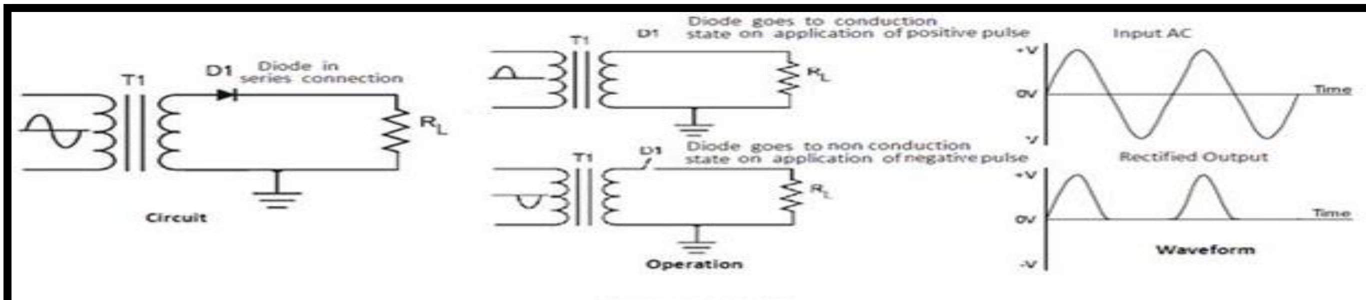
Diodes have attributes that allow them to carry out a number of electronic functions. Three vital characteristics of diodes are as follows:

- Forward Voltage Drop- forward bias about **seven volts**.
- Reverse Voltage Drop- Weakened layer broadens, generally the applied voltage.
- Reverse breakdown voltage- reverse voltage drop that'll force flow of current and in maximum cases demolish the diodes.

❖ Application of Diodes:

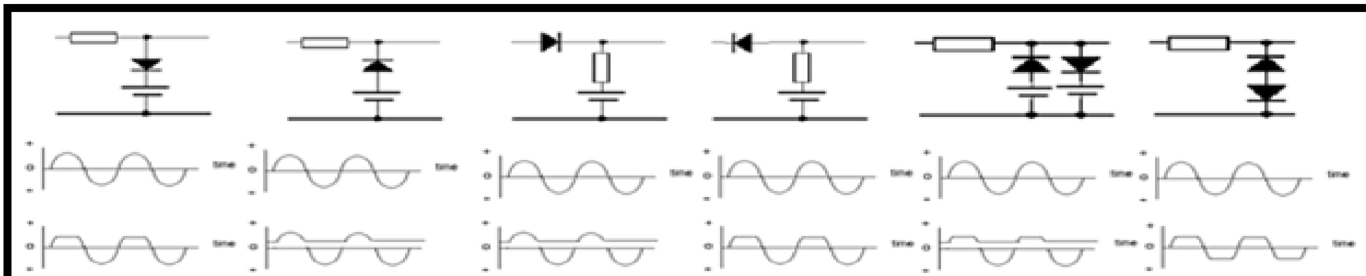
Diodes are employed in a variety of applications such as clipper, rectification, clamper, comparator, voltage multiplier, filters, sampling gates, etc.

1. **Rectification**: Rectification symbolizes the alteration of AC volt into DC volt. Some of the common examples of rectification circuits are- FWR (full wave rectifier), bridge rectifier & HWR (half wave rectifier).

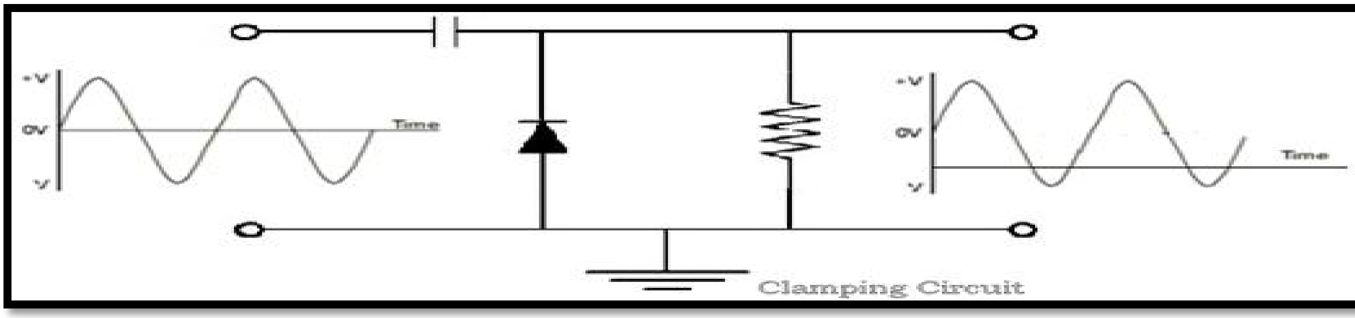


HALF WAVE RECTIFIER

2. **Clipper**: Diode can be employed to trim down some fraction of pulse devoid of deforming the left over fraction of the waveform.


CLIPPERS

3. **Clamper**: A clamping circuit limits the level of voltage to go beyond a limit by changing the DC level. The crest to crest is not influenced by clamping. Capacitors, resistors & diodes all are used to create clamping circuits.



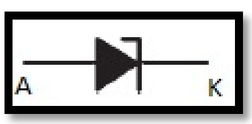
❖ Types of Diodes:

All sort of diodes are dissimilar in means of construction, characteristics & applications. Following are some of the types of diodes:

- **Zener Diodes.**
- **LED (Light Emitting Diodes).**
- **Photodiodes.**
- **Shockley Diode.**
- **Tunnel Diodes.**
- **Varactor Diodes.**

❖ Zener Diode:

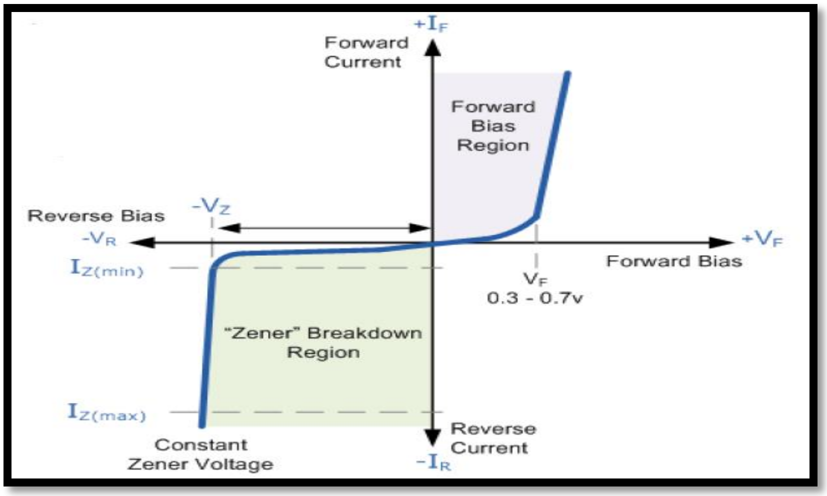
Zener diode works in **reverse bias situation** when the voltage attains the breakdown peak. An even voltage can be attained by insertion of a resistor across it to limit the flow of current. This Zener diode is employed to give reference voltage in power supplying circuits.



- **Zener Diode Characteristics**:

Special diodes such as zener diodes are intended & manufactured to function in the **opposite direction without being broken**.

1. The zener diode acts like a common silicon diode, during the forward bias.



2. Changeable quantity of reverse current can go through the diode devoid of destructing it. The **Vz** (zener voltage) or breakdown voltage across the diode upholds comparatively steady.
3. Producers rate zener diodes as per their zener voltage value and the highest PD (Power Dissipation) i.e. at 25ºC. This provides a signal of the highest IR (reverse current), that a diode securely carries out.

- **Zener Diode Applications:**

Zener diodes have many applications in transistor circuitry. Here we are discussing various vital points in Zener diode applications:

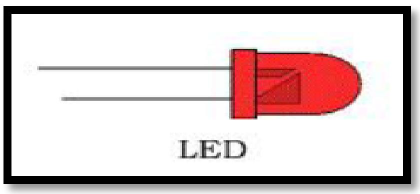
1. **Zener Diode Shunt Regulator**- This diode is commonly employed as a Voltage Regulator or Shunt Regulator.
2. **Meter Protection**- This diode may also come across its functions in meter security.
3. **Zener Diode as Peak Clipper**- This diodes can be employed to cut off the maximum value of incoming waveform.
4. **Switching operation**- This diode can generate an unexpected alteration from low to high current, so it is functional in switching applications. It is relatively speedy in switching processes.

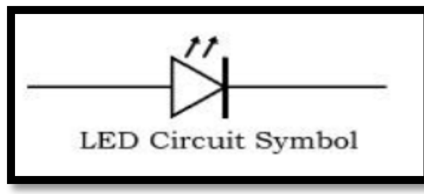
❖ Tunnel Diode:

A tunnel diode is a highly conducting two terminal p-n jnction diode doped heavily approximately **1000 times** upper than a usual junction diode. A tunnel diode is also named as **Esaki diode**, it's named after Esaki diode who is a Nobel prize winner in physics for discovering electron tunneling outcome employed in these diodes. Tunnel diodes are helpful in several circuit purposes like in-microwave oscillation, binary memory & microwave amplification. Tunnel diodes are generally made-up from **gallium or germanium** or **gallium arsenide**. These all comprise tiny prohibited energy breaks and elevated ion motilities.

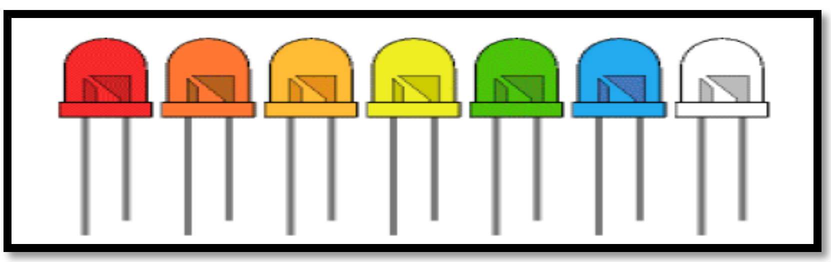
❖ Light Emitting Diode or LED:

LED is a semiconductor appliance that **produces visible light beams or infrared light beams when an electric current is passed through it**. Visible LEDs can be seen in several electronic devices such as microwaves' number display light, brake lights, and even cameras to make use of Infrared LEDs. In LEDs (light emitting diodes) light is created by a solid situation procedure which is named as electroluminescence.



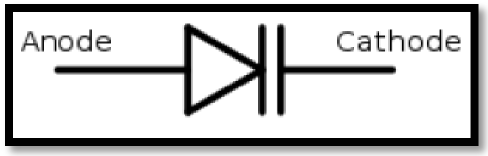


Light emitting diodes are available in various colors like- orange, red, yellow, amber, green, white & blue. Blue & white LEDs are more costly in comparison to other LEDs. The color of a Light emitting diodes is decided by the semi-conductor substance, not by the coloring the plastic of the body.



❖ Varactor diode:

Varicap or Varactor diode is that shows the attributes of a **variable capacitor**. The exhaustion area at the p-n junction behaves as the di-electric and plates of an ordinary capacitor and grounds expansion and contraction by the voltage applied to the varicap diode. This action boosts and reduces the capacitance. The graphic symbol for the varicap diode is shown below.



Varactors are employed in fine-tuning circuits and can be employed as high frequency amplifiers. Even though varactor or varicap diodes can be employed inside several sorts of circuit, they discover applications inside 2 key areas:-

1. RF filters.
2. Voltage controlled oscillators, VCOs.

❖ Photo Diode:

Photo diodes are extensively used in various kinds of electronics such as detectors in compact disc players to optical telecommunications systems. Photo diode technology is popular because its trouble-free, inexpensive yet strong configuration. As photodiodes provide dissimilar properties, various photodiode technologies are utilized in a number of areas. There are 4 types of photo diodes:

1. PN photodiode.
2. Schottky photodiode.
3. Avalanche photodiode.
4. PIN photodiode.

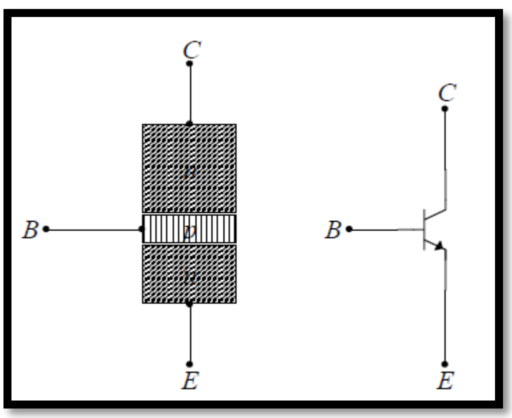
Transistors:

Bipolar Junction Transistors (BJT)

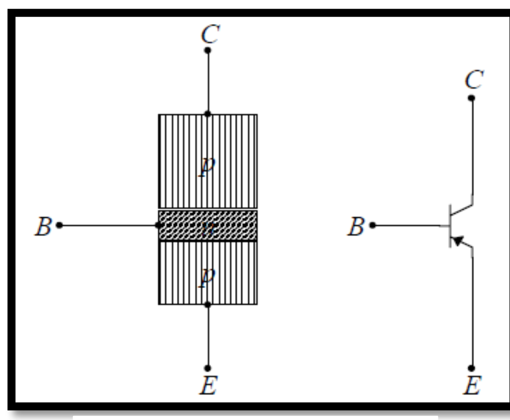
❖ General configuration and definitions

The transistor is the main building block "element" of electronics. It is a semiconductor device and it comes in two general types: the Bipolar Junction Transistor (**BJT**) and the Field Effect Transistor (**FET**). Here we will describe the system characteristics of the BJT configuration and explore its use in fundamental signal shaping and amplifier circuits.

The BJT is a three terminal device and it comes in two different types. **The npn BJT and the pnp BJT**. The BJT symbols and their corresponding block diagrams are shown:



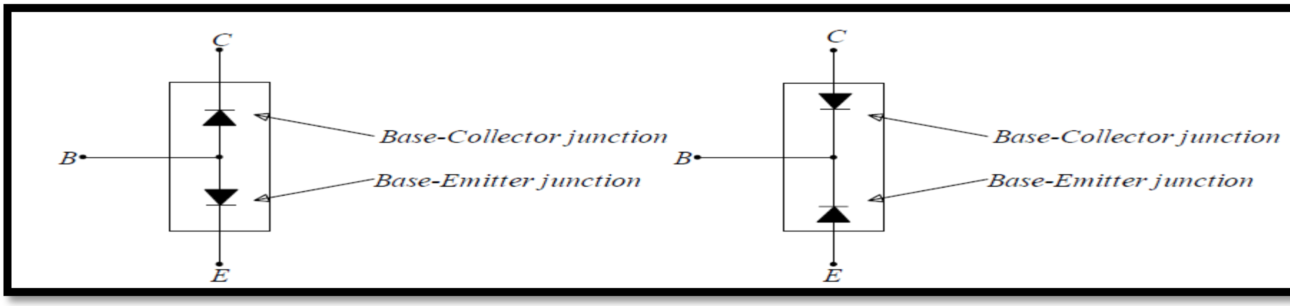
npn transistor



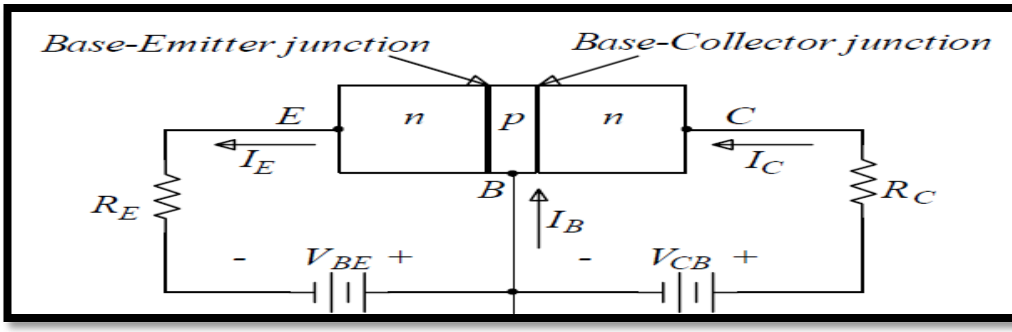
pnp transistor

The BJT is fabricated with three separately doped regions. The npn device has one p region between two n regions and the pnp device has one n region between two p regions. The BJT has two junctions (boundaries between the n and the p regions). These junctions are similar to the junctions we saw in the diodes and thus they may be forward biased or reverse biased. By relating these junctions to a diode model the pnp BJT may be modeled as shown:

The three terminals of the BJT are called the Base (**B**), the Collector (**C**) and the Emitter (**E**). Since each junction has two possible states of operation (forward or reverse bias) the BJT with its two junctions has four possible states of operation.



Before proceeding let's consider the BJT npn structure shown:



With the voltage $\boldsymbol{V_{BE}}$ and $\boldsymbol{V_{CB}}$ as shown, the Base-Emitter (**B-E**) junction is forward biased and the Base-Collector (**B-C**) junction is reverse biased. The current through the B-E junction is related to the B-E voltage as:

$$\boldsymbol{I_E = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)}$$

Due to the large differences in the doping concentrations of the emitter and the base regions the electrons injected into the base region (from the emitter region) results in the emitter current $\boldsymbol{I_E}$. Furthermore the number of electrons injected into the collector region is directly related to the electrons injected into the base region from the emitter region.

Therefore, the collector current is related to the emitter current which is in turn a function of the B-E voltage.

<u>The voltage between two terminals controls the current through the third terminal.</u>

This is the basic principle of the BJT.

The collector current and the base current are related by:

$\boldsymbol{I_C = \beta I_B} \qquad \boldsymbol{Eq.(1)}$

And by applying **KCL** we obtain:

$\boldsymbol{I_E = I_B + I_C} \qquad \boldsymbol{Eq.(2)}$

And thus from equations (1) and (2) the relationship between the emitter and the base currents is:

$I_E = I_B + I_C = I_B + \beta I_B = \boldsymbol{I_B(1+\beta)} \qquad \boldsymbol{Eq.(3)}$

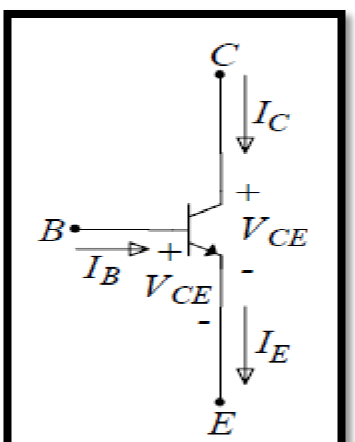
And equivalently:

$$I_B = \frac{I_E}{1+\beta}, \quad I_C = \beta \times \frac{I_E}{1+\beta} = \boldsymbol{\frac{\beta}{1+\beta} I_E} \qquad \boldsymbol{Eq.(4)}$$

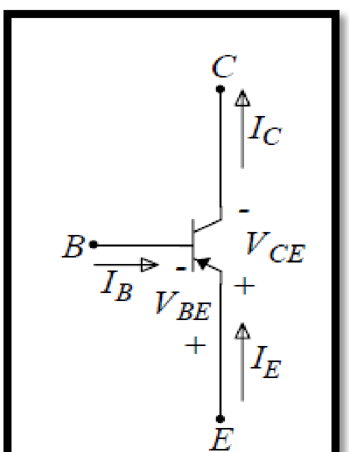
The fraction $\boldsymbol{\frac{\beta}{1+\beta}}$ is called $\boldsymbol{\alpha}$.

For the transistors of interest $\boldsymbol{\beta = 100}$ which corresponds to $\boldsymbol{\alpha = 0.99}$ and $\boldsymbol{I_C \cong I_E}$.

The direction of the currents and the voltage polarities for the **npn** and the **pnp** BJTs are shown.



(a) $\boldsymbol{npn}$ transistor



(b) pnp transistor

❖ Transistor Voltages:

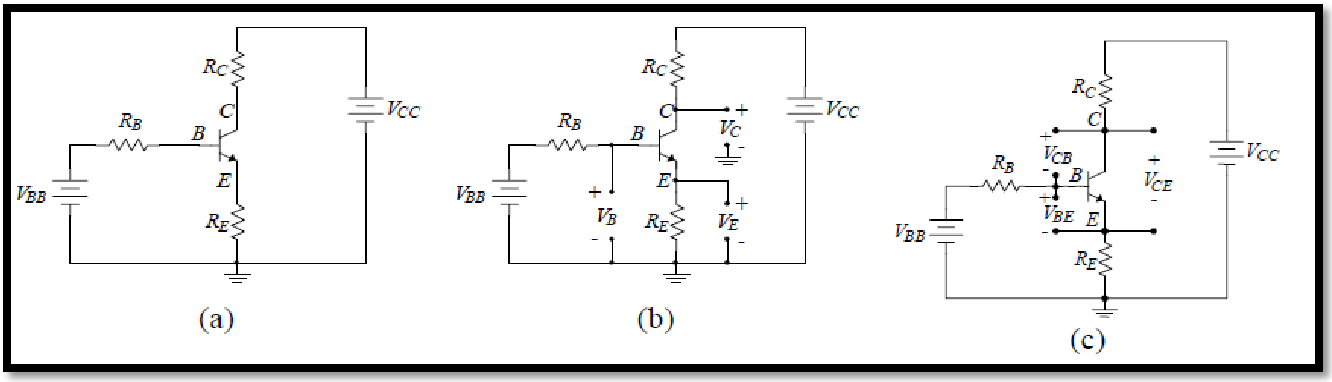
Three different types of voltages are involved in the description of transistors and transistor circuits. They are:

Transistor supply voltages: $\boldsymbol{V_{CC}}, \boldsymbol{V_{BB}}$.

Transistor terminal voltages: $\boldsymbol{V_{C}}, \boldsymbol{V_{B}}, \boldsymbol{V_{E}}$.

Voltages across transistor junctions: $\boldsymbol{V_{BE}}, \boldsymbol{V_{CE}}, \boldsymbol{V_{CB}}$.

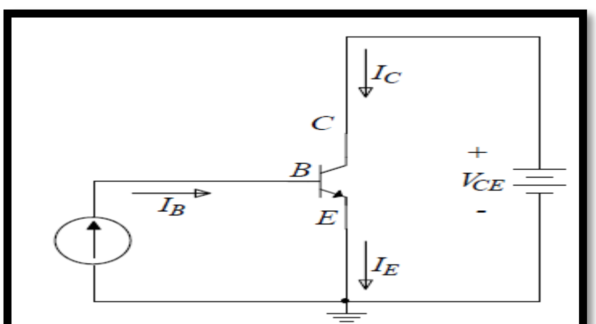
All of these voltages and their polarities are shown for the **npn BJT**.



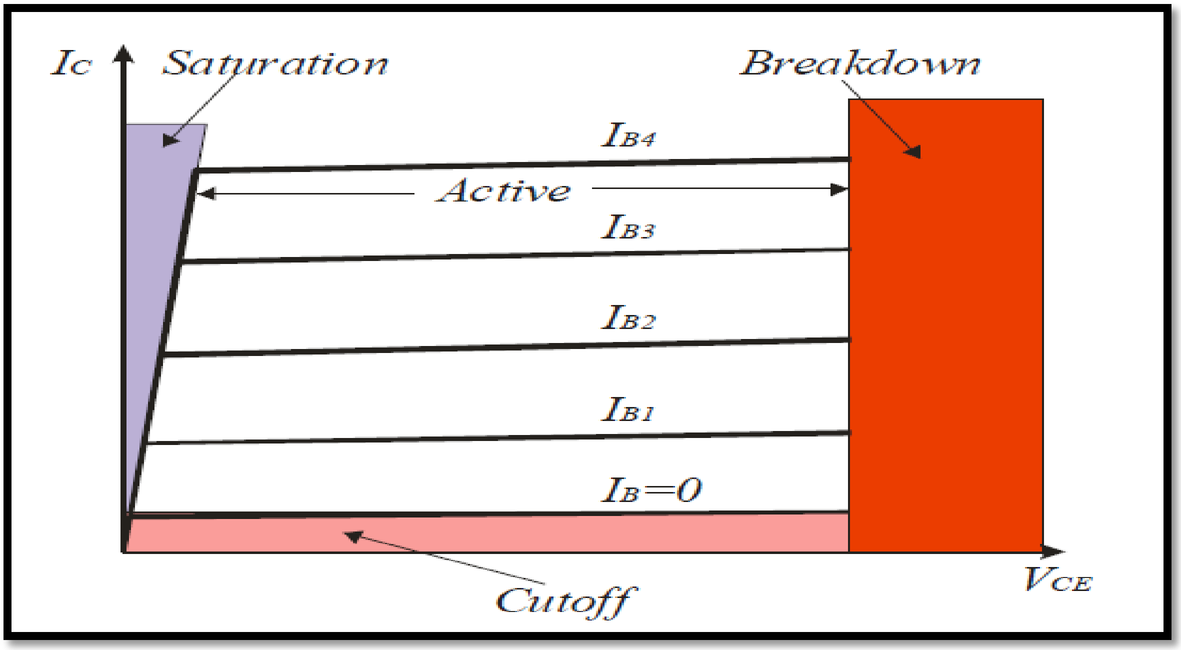
❖ Transistor Operation and Characteristic i-v curves

The three terminals of the transistors and the two junctions, present us with multiple operating regimes. In order to distinguish these regimes we have to look at the **i-v** characteristics of the device.

The most important characteristic of the BJT is the plot of the collector current, $\boldsymbol{I_C}$, versus the collector-emitter voltage, $\boldsymbol{V_{CE}}$, for various values of the base current, $\boldsymbol{I_B}$ as shown on the circuit below:



The figure below shows the qualitative characteristic curves of a BJT. The plot indicates the four regions of operation: **the saturation**, **the cutoff**, **the active** and **the breakdown**. Each family of curves is drawn for a different base current and in this plot $\boldsymbol{I_{B4} > I_{B3} > I_{B2} > I_{B1}}$.



The characteristics of each region of operation are summarized below:

1. Cutoff region:

Base-emitter junction is reverse biased. No current flow.

2. Saturation region:

Base-emitter junction forward biased.

Collector-base junction is forward biased.

$\boldsymbol{I_C}$ Reaches a maximum which is independent of $\boldsymbol{I_B}$ and $\boldsymbol{\beta}$.

No control.

$\boldsymbol{V_{CE} < V_{BE}}$.

3. Active region:

Base-emitter junction forward biased.

Collector-base junction is reverse biased.

Control, $\boldsymbol{I_C = \beta I_B}$ (as can be seen from the fig., there is a small slope of $\boldsymbol{I_C}$ with $\boldsymbol{V_{CE}}$. $\boldsymbol{V_{BE} < V_{CE} < V_{CC}}$.

4. Breakdown region:

$\boldsymbol{I_C}$ and $\boldsymbol{V_{CE}}$ exceed specifications.

Damage to the transistor.

❖ Basic BJT Applications:

1. As switch circuit.
2. Digital Logic.
3. Amplifier Circuit.

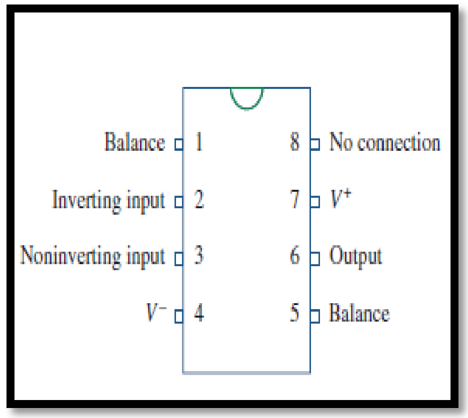
Operational Amplifiers

An operational amplifier is designed so that it performs some mathematical operations when external components, such as resistors and capacitors, are connected to its terminals. Thus,

<u>An op amp is an active circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division, differentiation, and integration.</u>

The op amp is an electronic device consisting of a complex arrangement of resistors, transistors, capacitors, and diodes.

The figure below shows a typical op-amp. It consists of **8 pins,** Pin or terminal 8 is unused, and terminals 1 and 5 are of little concern to us. The five important terminals are:



1. The inverting input, **pin 2**.

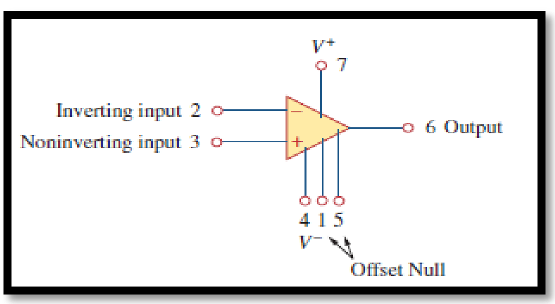
2. The noninverting input, **pin 3**.

3. The output, **pin 6**.

4. The positive power supply $\boldsymbol{V^+}$, **pin 7**.

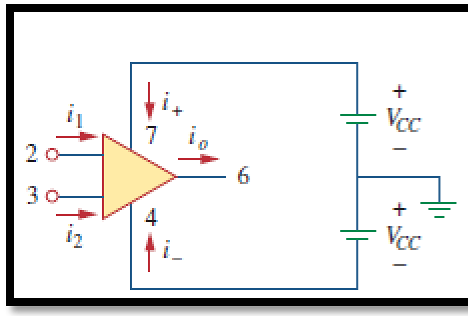
5. The negative power supply $\boldsymbol{V^-}$, **pin 4**.

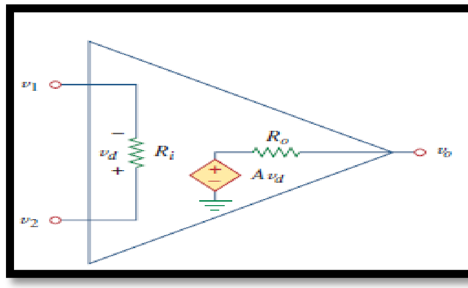
The circuit symbol for the op amp is the triangle as shown; the op amp has two inputs and one output. The inputs are marked with minus (-) and plus (+) to specify inverting and noninverting inputs, respectively. An input applied to the noninverting terminal will appear with the same polarity at the output, while an input applied to the inverting terminal will appear inverted at the output.



As an active element, the op amp must be powered by a voltage supply as typically shown. By **KCL**:

$i_0 = i_1 + i_2 + i_+ + i_-$





The equivalent circuit model of an op amp is shown in fig. below; the output section consists of a voltage-controlled source in series with the output resistance $\boldsymbol{R_0}$. $\boldsymbol{R_i}$ is the input resistance. The differential input voltage $\boldsymbol{v_d}$ is given by:

$\boldsymbol{v_d = v_2 - v_1}$

Where $\boldsymbol{v_1}$ is the voltage between the inverting terminal and ground and $\boldsymbol{v_2}$ is the voltage between the noninverting terminal and ground. The op amp senses the difference between the two inputs, multiplies it by the gain **A**, and causes the resulting voltage to appear at the output.

Thus, the output $\boldsymbol{v_0}$ is given by:

$$\boldsymbol{v_0 = Av_d = A(v_2 - v_1)}$$

A is called the **open-loop voltage gain** because it is the gain of the op amp without any external feedback from output to input.

❖ Ideal Op Amp

To facilitate the understanding of op amp circuits, we will assume ideal op amps. An op amp is ideal if it has the following characteristics:

1. Infinite open-loop gain, **A=∞.**

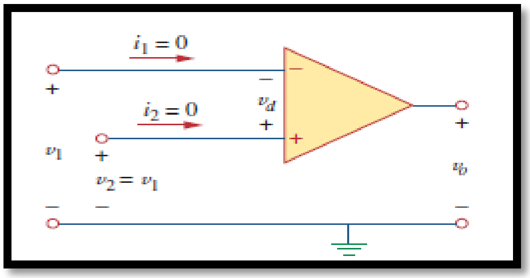
2. Infinite input resistance, $\boldsymbol{R_i} = \infty$.

3. Zero output resistance, $\boldsymbol{R_0} = \boldsymbol{0}$.

<u>An ideal op amp is an amplifier with infinite open-loop gain, infinite input resistance, and zero output resistance.</u>

For circuit analysis, the ideal op amp is illustrated in the figure below:

Two important characteristics of the ideal op amp are:



1. The currents into both input terminals are **zero**:

$$\boldsymbol{i_1 = 0,\qquad i_2 = 0 \quad Eq.(1)}$$

This is due to infinite input resistance. An infinite resistance between the input terminals implies that an open circuit exists there and current cannot enter the op amp. But the output current is not necessarily zero.

2. The voltage across the input terminals is equal to **zero**; i.e.,

$$\boldsymbol{v_d = v_2 - v_1 = 0}$$

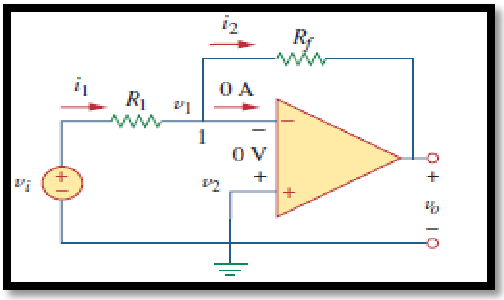
Or $\boldsymbol{v_1 = v_2} \qquad \boldsymbol{Eq.(2)}$

Thus, an ideal op amp has zero current into its two input terminals and the voltage between the two input terminals is equal to zero. **Equations (1) and (2)** are extremely important and should be regarded as the key handles to analyzing op amp circuits.

❖ Inverting Amplifier

We consider some useful op amp circuits that often serve as modules for designing more complex circuits. The first of such op amp circuits is the inverting amplifier shown in the fig. below. In this circuit, the noninverting input is grounded, $\boldsymbol{v_i}$ is connected to the inverting input through $\boldsymbol{R_1}$, and the feedback resistor $\boldsymbol{R_f}$ is connected between the inverting input and output. Our goal is to obtain the relationship between the input voltage $\boldsymbol{v_i}$ and the output voltage $\boldsymbol{v_0}$. Applying **KCL** at node 1:

$$i_1 = i_2, \quad \frac{v_i - v_1}{R_1} = \frac{v_1 - v_0}{R_f}$$



But $\boldsymbol{v_1 = v_2 = 0}$ for an ideal op amp, since the noninverting terminal is grounded. Hence:

$$\frac{v_i}{R_1} = \frac{-v_0}{R_f}, \quad \boldsymbol{v_0 = -\frac{R_f}{R_1} v_i}$$

The voltage gain is:

$$\boldsymbol{A = \frac{v_0}{v_i} = -\frac{R_f}{R_1}}$$

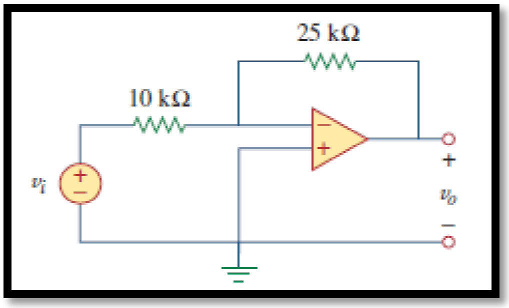
The designation of the circuit as an inverter arises from the **negative sign**.

An inverting amplifier reverses the polarity of the input signal while amplifying it.

Notice that the gain is the feedback resistance divided by the input resistance which means that the gain depends only on the external elements connected to the op amp. The inverting amplifier is used, for example, in a current-to-voltage converter.

Example: Refer to the op amp shown. If $\boldsymbol{v_i = 0.5V}$, calculate: (a) the output voltage $\boldsymbol{v_0}$, and (b) the current in the **10-kΩ** resistor.

Solution:

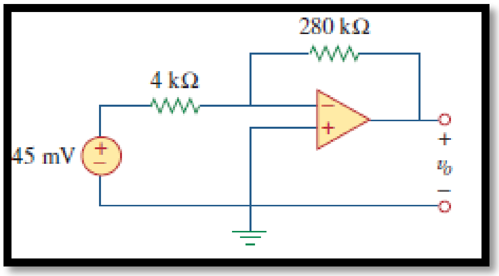


(a) $v_0 = -\frac{R_f}{R_1} v_i = -\frac{25}{10} \times 0.5 = \boldsymbol{-1.25V}$

(b) The current through the **10-kΩ** resistor is:

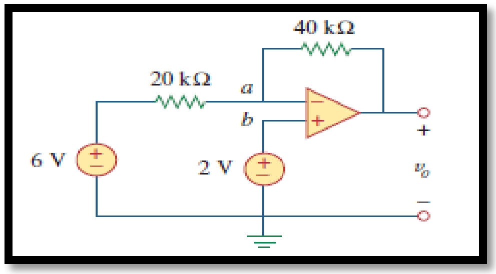
$$i = \frac{v_i - 0}{10 \times 1000} = \frac{0.5}{10000} = \boldsymbol{50 \mu A}$$

Q: Find the output of the op amp circuit shown. Calculate the current through the feedback resistor.



Answer: -3.15 V, 26.25 μA.

Example: Determine $\boldsymbol{v_0}$ in the op amp circuit shown below.



Solution:

Applying **KCL** at node a:

$$\frac{v_i - v_a}{20 \times 1000} = \frac{v_a - v_0}{40 \times 1000}, \quad \boldsymbol{v_0 = 3v_a - 12}$$

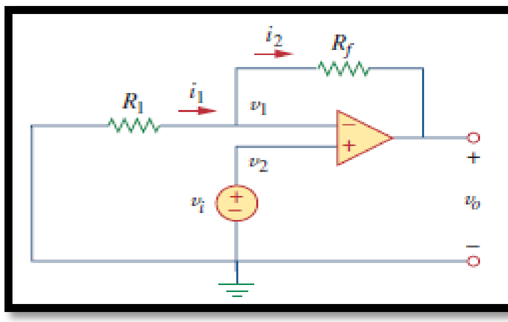
But $v_a = v_b = \boldsymbol{2V}$ for an ideal op amp, because of the zero voltage drop across the input terminals of the op amp. Hence:

$$v_0 = 3 \times 2 - 12 = -\boldsymbol{6V}$$

Notice that if $v_b = 0 = v_a$, then $\boldsymbol{v_0} = -\boldsymbol{12V}$

❖ Noninverting Amplifier

Another important application of the op amp is the noninverting amplifier shown in the fig. below. In this case, the input voltage $\boldsymbol{v_i}$ is applied directly at the noninverting input terminal, and resistor $\boldsymbol{R_1}$ is connected between the ground and the inverting terminal. We are interested in the output voltage and the voltage gain. Application of **KCL** at the inverting terminal gives:



$$i_1 = i_2, \qquad \frac{0 - v_1}{R_1} = \frac{v_1 - v_0}{R_f}$$

But:

$$v_1 = v_2 = v_i, \qquad \frac{-v_i}{R_1} = \frac{v_i - v_0}{R_f} \ or \ \boldsymbol{v_0} = \left(\boldsymbol{1} + \frac{\boldsymbol{R_f}}{\boldsymbol{R_1}}\right)\boldsymbol{v_i}$$

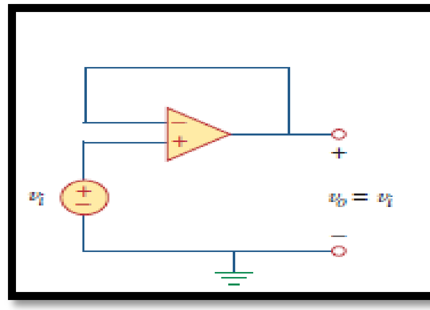
The voltage gain is:

$$\boldsymbol{A_v} = \frac{\boldsymbol{V_0}}{\boldsymbol{V_i}} = \boldsymbol{1} + \frac{\boldsymbol{R_f}}{\boldsymbol{R_1}}$$

Which does not have a negative sign. Thus, the output has the same polarity as the input.

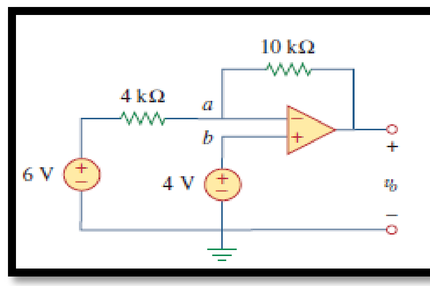
<u>A noninverting amplifier is an op amp circuit designed to provide a positive voltage gain.</u>

Again we notice that the gain depends only on the external resistors. Notice that if feedback resistor $\boldsymbol{R_f = 0}$ (short circuit) or $\boldsymbol{R_1} = \infty$ (open circuit) or both, the gain becomes **1**. Under these conditions ($\boldsymbol{R_f} =$ $\boldsymbol{0}$ and $\boldsymbol{R_1} = \infty$), the circuit becomes as shown, which is called **a voltage follower (or unity gain amplifier)** because the output follows the input. Thus, for a voltage follower:



$$\boldsymbol{v_0 = v_i}$$

<u>Example</u>: For the op amp circuit shown, calculate the output voltage $\boldsymbol{v_0}$.



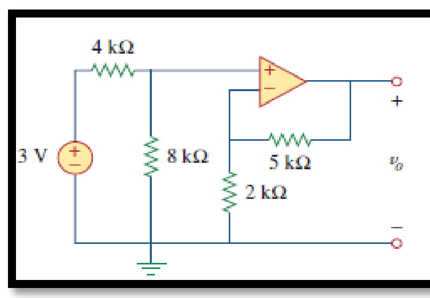
<u>Solution</u>:

Applying **KCL** at node a:

$$\frac{6 - v_a}{4} = \frac{v_a - v_0}{10}, \quad but\ \boldsymbol{v_a = v_b = 4V}$$

$$\frac{6 - 4}{4} = \frac{4 - v_0}{10}, \quad v_0 = -\boldsymbol{1V}$$

Q: Calculate $\boldsymbol{v_0}$ in the circuit shown.



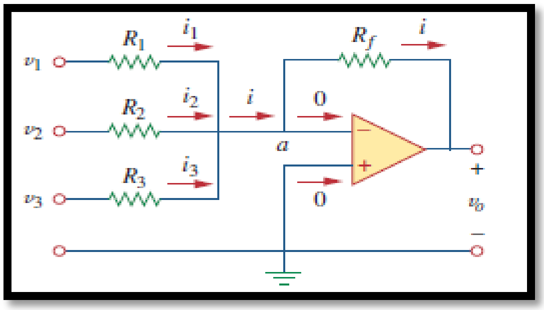
Answer: 7V.

❖ Summing Amplifier

Besides amplification, the op amp can perform addition. The addition is performed by the summing amplifier.

<u>A summing amplifier is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.</u>

The summing amplifier is shown in the fig., is a variation of the inverting amplifier. It takes advantage of the fact that the inverting configuration can handle many inputs at the same time. We keep in mind that the current entering each op amp input is zero. Applying **KCL** at node a gives:



$$i = i_1 + i_2 + i_3 \qquad Eq. (1)$$

But:

$$i_1 = \frac{v_1 - v_a}{R_1}, i_2 = \frac{v_2 - v_a}{R_2}, i_3 = \frac{v_3 - v_a}{R_3}, i = \frac{v_a - v_0}{R_f} \qquad Eq. (2)$$

We note that $v_a = 0$ and substitute Eq. (2) into Eq. (1). We get:

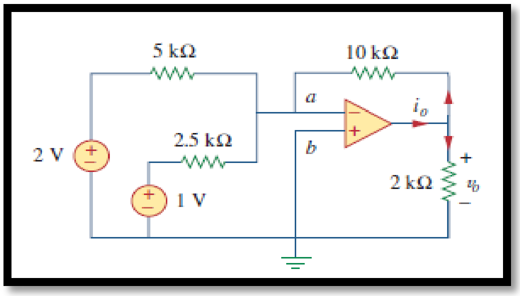
$$\boldsymbol{v_0 = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right) \qquad Eq. (3)}$$

Indicating that the output voltage is a weighted sum of the inputs. For this reason, the circuit shown is called **a summer**. Needless to say, the summer can have **more than three inputs**.

<u>Example</u>: Calculate $\boldsymbol{v_0}$ and $\boldsymbol{i_0}$ in the op amp circuit shown below.

<u>Solution</u>:

This is a summer with two inputs. Using Eq. (3) gives:

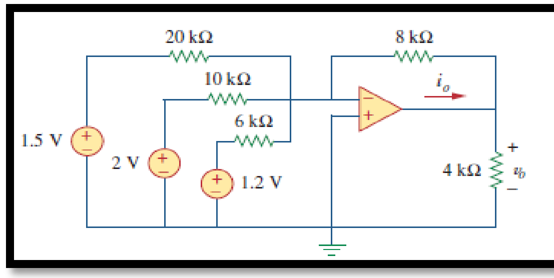


$$v_0 = -\left(\frac{10}{5} \times 2 + \frac{10}{2.5} \times 1\right) = \boldsymbol{-8V}$$

The current i_0 is the sum of the currents through the **10-kΩ** and **2-kΩ** resistors. Both of these resistors have voltage $\boldsymbol{v_0 = -8V}$ across them, since $\boldsymbol{v_a = v_b = 0}$. Hence:

$$i_0 = \frac{v_0 - 0}{10} + \frac{v_0 - 0}{2} = -0.8 - 4 = -\mathbf{4.8}\boldsymbol{mA}$$

Q: Find $\boldsymbol{v_0}$ and $\boldsymbol{i_0}$ in the op amp circuit shown below.



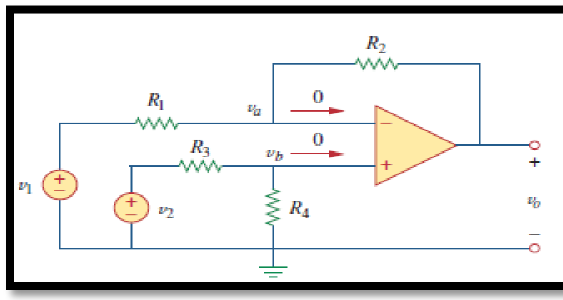
Answer: -3.8 V, -1.425 mA.

❖ Difference Amplifier

Difference (or differential) amplifiers are used in various applications where there is a need to amplify the difference between two input signals.

A difference amplifier is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.

Consider the op amp circuit shown. Keep in mind that zero currents enter the op amp terminals. Applying **KCL** to node a:



$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_0}{R_2}$$

Or

$$\boldsymbol{v_0 = \left(\frac{R_2}{R_1} + 1\right) v_a - \frac{R_2}{R_1} v_1 \qquad Eq.(1)}$$

Applying **KCL** to node b:

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4}$$

Or

$$\boldsymbol{v_b = \left(\frac{R_4}{R_3 + R_4}\right) v_2 \qquad Eq.(2)}$$

But $\boldsymbol{v_a = v_b}$. Substituting Eq. (2) into Eq. (1) yields:

$$v_0 = \left(\frac{R_2}{R_1} + 1\right)\left(\frac{R_4}{R_3 + R_4}\right) v_2 - \frac{R_2}{R_1} v_1$$

Or

$$\boldsymbol{v_0 = \frac{R_2\left(1 + \frac{R_1}{R_2}\right)}{R_1\left(1 + \frac{R_3}{R_4}\right)} v_2 - \frac{R_2}{R_1} v_1 \qquad Eq. (3)}$$

Since a difference amplifier must reject a signal common to the two inputs, the amplifier must have the property that $\boldsymbol{v_0 = 0}$ when $\boldsymbol{v_1 = v_2}$. This property exists when:

$$\boldsymbol{\frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad Eq. (4)}$$

Thus, when the op amp circuit is a difference amplifier, Eq. (3) becomes:

$$\boldsymbol{v_0 = \frac{R_2}{R_1}(v_2 - v_1) \qquad Eq. (5)}$$

If $\boldsymbol{R_2 = R_1}$ and $\boldsymbol{R_3 = R_4}$, the difference amplifier becomes a subtractor, with the output:

$$\boldsymbol{v_0 = v_2 - v_1 \qquad Eq. (6)}$$

Example: Design an op amp circuit with inputs $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ such that:

$\boldsymbol{v_0 = -5v_1 + 3v_2}$.

Solution:

The circuit requires that:

$$\boldsymbol{v_0 = 3v_2 - 5v_1}$$

If we desire to use only one op amp, we can use the difference op amp circuit.

Comparing the previous eq. with Eq. (3), we see:

$$\frac{R_2}{R_1} = 5, \qquad \boldsymbol{R_2 = 5R_1}$$

Also:

$$5\frac{\left(1+\frac{R_1}{R_2}\right)}{\left(1+\frac{R_3}{R_4}\right)} = 3, \quad \frac{\frac{6}{5}}{1+\frac{R_3}{R_4}} = \frac{3}{5}, \qquad 2 = 1 + \frac{R_3}{R_4}, \qquad \boldsymbol{R_3 = R_4}$$

If we choose $\boldsymbol{R_1 = 10k}\Omega$ and $\boldsymbol{R_2 = 50k}\Omega$, then $\boldsymbol{R_3 = 20k}\Omega$ and $\boldsymbol{R_4 = 20k}\Omega$**.**

Q: Design a difference amplifier with gain **7.5**.

Answer: Typical: $\boldsymbol{R_1 = R_3 = 20k}\Omega$**,** $\boldsymbol{R_2 = R_4 = 150k}\Omega$**.**

Rectifiers

❖ Introduction

A rectifier is an electrical device that converts alternating current (AC) which periodically reverses direction, to direct current (DC), which flows in only one direction. The process is known as rectification. Rectifiers have many uses, but are often found serving as components of DC power supplies and high-voltage direct current power transmission systems. Rectification may serve in roles other than to generate direct current for use as a source of power. As noted, detectors of radio signals serve as rectifiers. In gas heating systems flame rectification is used to detect presence of a flame. Because of the alternating nature of the input AC sine wave, the process of rectification alone produces a DC current that, though unidirectional, consists of pulses of current. Many applications of rectifiers, such as power supplies for radio, television and computer equipment, require a steady constant DC current (as would be produced by a battery). In these applications the output of the rectifier is smoothed by an electronic filter (usually a capacitor) to produce a steady current.

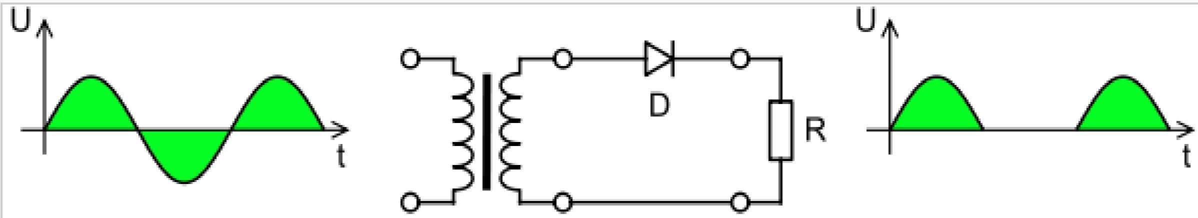
❖ Rectifier circuits

Rectifier circuits may be **single-phase or multi-phase** (three being the most common number of phases). Most low power rectifiers for domestic equipment are single-phase, but three-phase rectification is very important for industrial applications and for the transmission of energy as DC.

❖ Single-phase rectifiers

1. Half-wave rectification

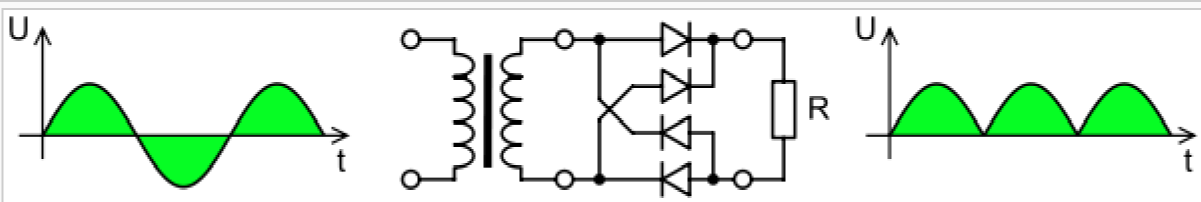
In half-wave rectification of a single-phase supply, either the positive or negative half of the AC wave is passed, while the other half is blocked. Because only one half of the input waveform reaches the output, mean voltage is lower. Half-wave rectification requires a single diode in a single-phase supply, or three in a three-phase supply. Rectifiers yield a unidirectional but pulsating direct current; half-wave rectifiers produce far more ripple than full-wave rectifiers, and much more filtering is needed to eliminate harmonics of the AC frequency from the output.



Half-wave rectifier

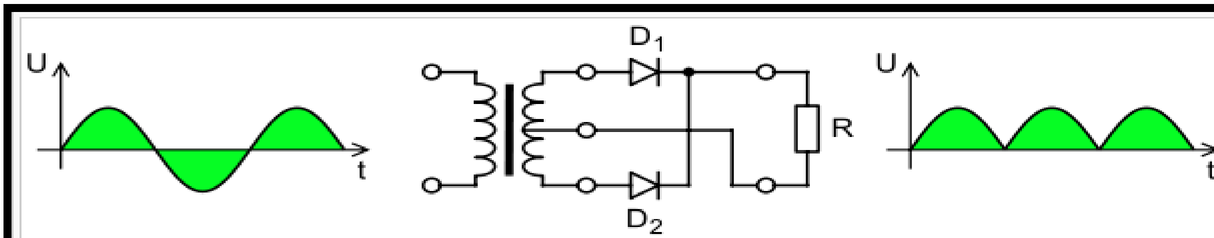
2. Full-wave rectification

A full-wave rectifier converts the whole of the input waveform to one of constant polarity (positive or negative) at its output. Full-wave rectification converts both polarities of the input waveform to pulsating DC (direct current), and yields a higher average output voltage. Two diodes and a center tapped transformer, or four diodes in a bridge configuration and any AC source (including a transformer without center tap), are needed. Single semiconductor diodes, double diodes with common cathode or common anode, and four-diode bridges, are manufactured as single components.



Graetz bridge rectifier: a full-wave rectifier using four diodes.

For single-phase AC, if the transformer is center-tapped, then two diodes back-to-back (cathode-to-cathode or anode-to-anode, depending upon output polarity required) can form a full-wave rectifier. Twice as many turns are required on the transformer secondary to obtain the same output voltage than for a bridge rectifier, but the power rating is unchanged.



Full-wave rectifier using a center tap transformer and 2 diodes.

The average output voltage of an ideal single-phase half-wave rectifier is:

$$\boldsymbol{V_{dc} = \frac{V_{peak}}{\pi}}$$

The average output voltage of an ideal single-phase full-wave rectifier is:

$$\boldsymbol{V_{dc} = \frac{2V_{peak}}{\pi}}$$

Where:

$V_{dc} = V_{av}, \textit{the DC or average output voltage.}$

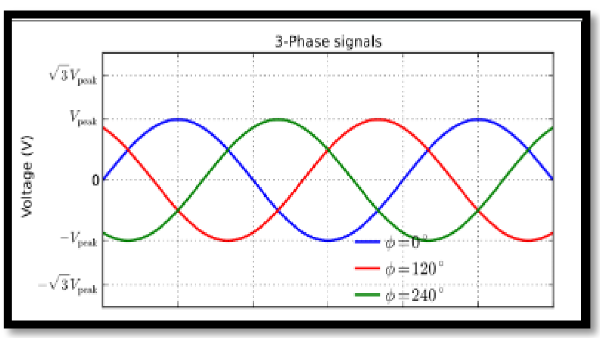
$V_{peak}, \textit{the peak value of the phase input voltages.}$

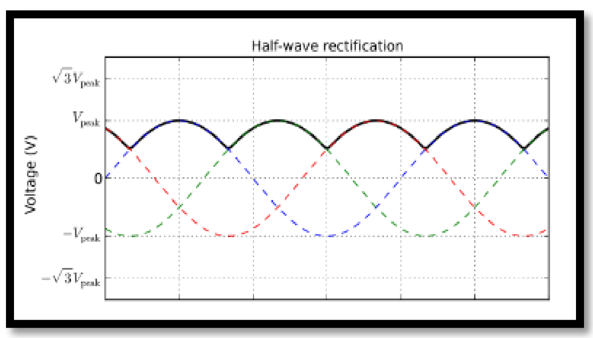
❖ Three-phase rectifiers

Single-phase rectifiers are commonly used for power supplies for domestic equipment. However, for most industrial and high-power applications, three-phase rectifier circuits are the norm. As with single-phase rectifiers, three-phase rectifiers can take the form of a half-wave circuit, a full-wave circuit using a center-tapped transformer, or a full-wave bridge circuit.

1. Three-phase, half-wave circuit

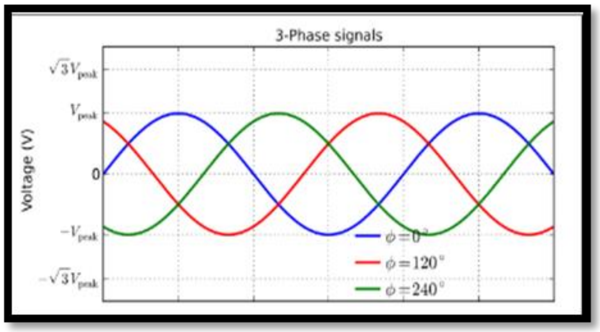
An uncontrolled three-phase, half-wave circuit requires three diodes, one connected to each phase. This is the simplest type of three-phase rectifier but suffers from relatively high harmonic distortion on both the AC and DC connections.

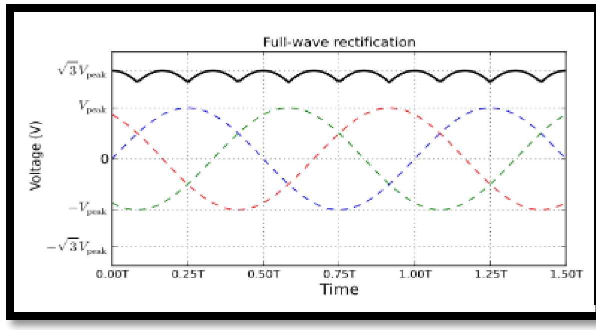




2. Three-phase, full-wave circuit using center-tapped transformer

If the AC supply is fed via a transformer with a center tap, a rectifier circuit with improved harmonic performance can be obtained. This rectifier now requires six diodes, one connected to each end of each transformer secondary winding. This circuit has a pulse-number of six, and in effect, can be thought of as a six-phase, half-wave circuit.

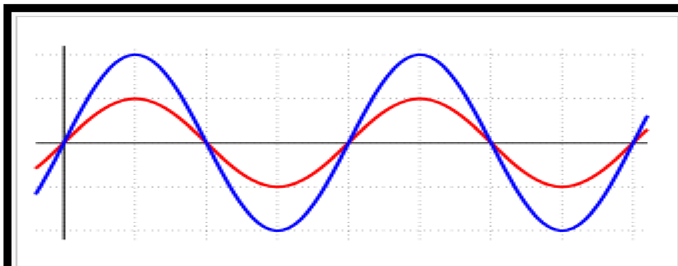




Amplifiers

❖ Introduction

An amplifier, electronic amplifier is an electronic component that can increase the power of a signal. An amplifier functions by taking power from a power supply and controlling the output to match the input signal shape but with a larger amplitude. In this sense, an amplifier modulates the output of the power supply based upon the properties of the input signal. An amplifier is effectively the opposite of an attenuator: while an amplifier provides gain, an attenuator provides loss.



The blue signal is an amplification of the red signal with gain 2

An amplifier can either be a discrete piece of equipment or an electrical circuit contained within another device. Amplification is fundamental to modern electronics, and amplifiers are widely used in almost all electronic equipment. Amplifiers can be categorized in different ways. One is by the frequency of the electronic signal being amplified; audio amplifiers amplify signals in the audio (sound) range of less than 20 kHz, RF amplifiers amplify frequencies in the radio frequency range between 20 kHz and 300 GHz. Another is which quantity, voltage or current is being amplified; amplifiers can be divided into voltage amplifiers, current amplifiers, transconductance amplifiers, and transresistance amplifiers.

❖ Amplifier categorization

Amplifiers are described according to the properties of their inputs, their outputs, and how they relate. All amplifiers have gain, a multiplication factor that relates the magnitude of some property of the output signal to a property the input signal. The gain may be specified as the ratio of output voltage to input voltage (voltage gain), output power to input power (power gain), or some combination of current, voltage, and power. In many cases the property of the output that varies is dependent on the same property of the input, making the gain unitless (though often expressed in decibels (**dB**)). Amplifiers are usually designed to function well in a specific application, for example: radio and television transmitters and receivers, stereo equipment, microcomputers and other digital equipment.

❖ Amplifier architectures

Amplifiers can be categorized by the way they amplify the input signal.

1. Power amplifier

A power amplifier is an amplifier designed primarily to increase the power available to a load.

2. Operational amplifiers (op-amps)

An operational amplifier is an amplifier circuit which typically has very high open loop gain and differential inputs. Feedback via an external circuit can be used to control the transfer function, or gain.

3. Fully differential amplifiers

A fully differential amplifier is similar to the operational amplifier, but also has differential outputs. These are usually constructed using BJTs or FETs.

4. Distributed amplifiers

These use a balanced transmission lines to separate individual single stage amplifiers the outputs of which are summed by the same transmission line.

5. Switched mode amplifiers

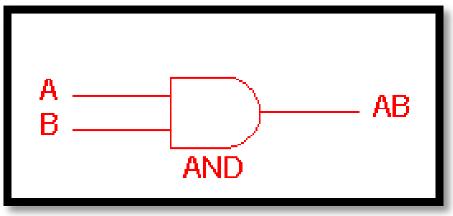
These nonlinear amplifiers have much higher efficiencies than linear amps, and are used where the power saving justifies the extra complexity.

Logic Gates

Digital systems are said to be constructed by using logic gates. These gates are the AND, OR, NOT, NAND, NOR, EXOR and EXNOR gates. The basic operations are described below with the aid of truth tables. Truth tables are used to show logic gate functions.

➢ AND Gate

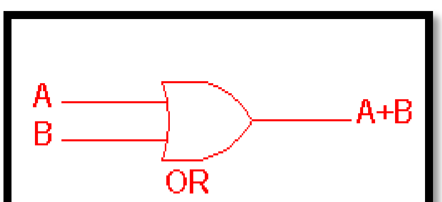
The AND gate is an electronic circuit that gives a high output (**1**) only if all its inputs are high. A dot (**.**) is used to show the AND operation i.e. A.B. Bear in mind that this dot is sometimes omitted i.e. AB.



2 Input AND gate		
A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

➢ OR Gate

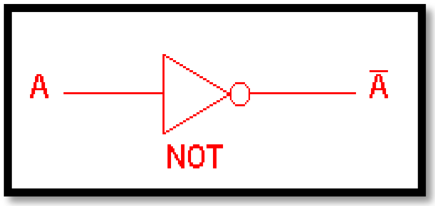
The OR gate is an electronic circuit that gives a high output (**1**) if one or more of its inputs are high. A plus (+) is used to show the OR operation.



2 Input OR gate		
A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

➢ NOT Gate

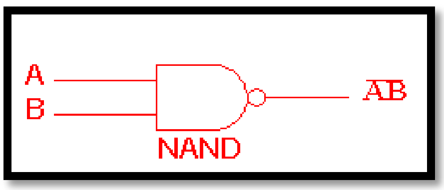
The NOT gate is an electronic circuit that produces an inverted version of the input at its output. It is also known as an **inverter**. If the input variable is A, the inverted output is known as NOT A. This is also shown as A', or A with a bar over the top, as shown at the outputs.



NOT gate	
A	$\overline{A}$
0	1
1	0

➢ NAND Gate

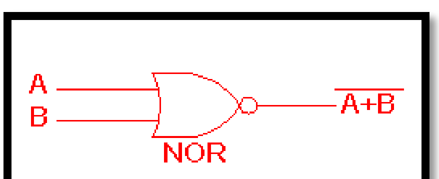
This is **a NOT-AND** gate which is equal to an AND gate followed by a NOT gate. The outputs of all NAND gates are high if any of the inputs are low. The symbol is an AND gate with a small circle on the output. The small circle represents inversion.



2 Input NAND gate		
A	B	$\overline{A.B}$
0	0	1
0	1	1
1	0	1
1	1	0

➢ NOR Gate

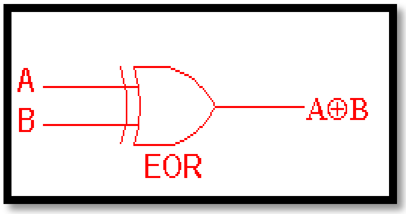
This is a **NOT-OR** gate which is equal to an OR gate followed by a NOT gate. The outputs of all NOR gates are low if any of the inputs are high. The symbol is an OR gate with a small circle on the output. The small circle represents inversion.



2 Input NOR gate		
A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

➢ EXOR Gate

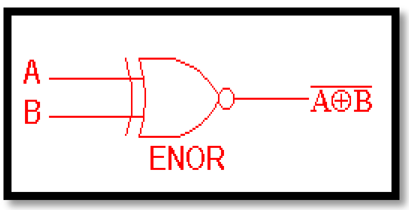
The **'Exclusive-OR'** gate is a circuit which will give a high output if either, but not both, of its two inputs are high. An encircled plus sign ($\oplus$) is used to show the EOR operation.



2 Input EXOR gate		
A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

➢ EXNOR Gate

The 'Exclusive-NOR' gate circuit does the **opposite to the EOR gate**. It will give a low output if either, but not both, of its two inputs are high. The symbol is an EXOR gate with a small circle on the output. The small circle represents inversion.



2 Input EXNOR gate		
A	B	$\overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

Table below is a summary truth table of the input/output combinations for the NOT gate together with all possible input/output combinations for the other gate functions.

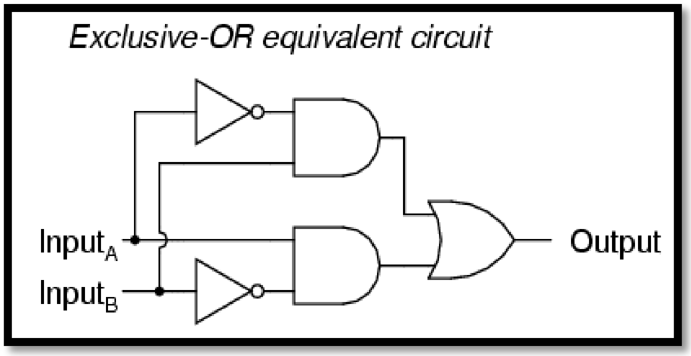
NOT gate	
A	$\overline{A}$
0	1
1	0

INPUTS		OUTPUTS					
A	B	AND	NAND	OR	NOR	EXOR	EXNOR
0	0	0	1	0	1	0	1
0	1	0	1	1	0	1	0
1	0	0	1	1	0	1	0
1	1	1	0	1	0	0	1

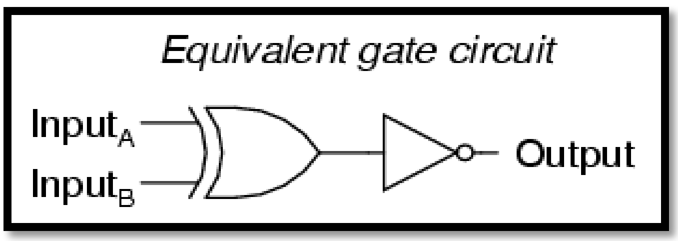
<u>Example</u>: Obtain the **EXOR** gate by using AND and OR gates.

<u>Solution</u>:

$$A \oplus B = \overline{A}B + A\overline{B}$$

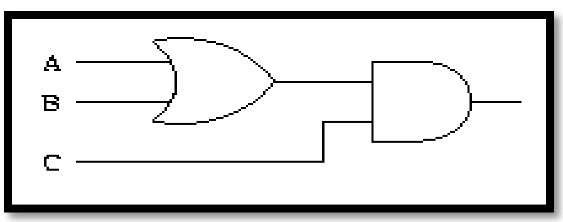


<u>Example</u>: Obtain the equivalent circuit to **EXNOR** gate.



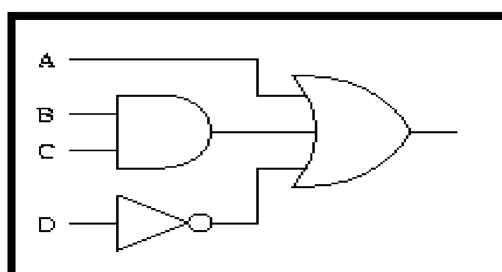
<u>Example</u>: Draw a logic circuit for the function: **F= (A + B) C**.

<u>Solution</u>:



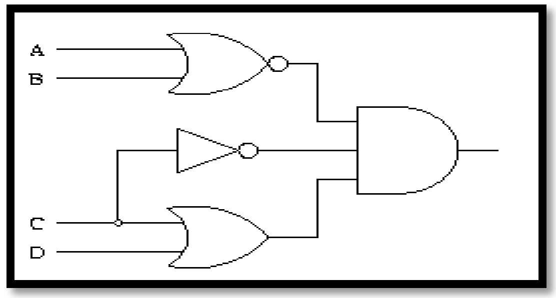
<u>Example</u>: Draw a logic circuit for the function: **F=A + BC + D**.

<u>Solution</u>:



Example: Draw a logic circuit for: **F=** $\boldsymbol{(\overline{A+B})\,(C+D)\,\overline{C}}$.

Solution:



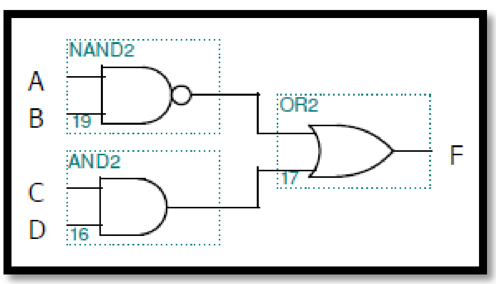
Example: Draw a logic circuit for the following functions:

(a) F=$\boldsymbol{\overline{AB}+CD}$

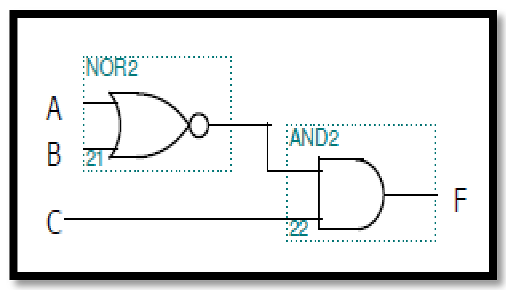
(b) $\boldsymbol{F=C(\overline{A+B})}$

Solution:

(a)



(b)



Q: Draw the logic circuit for the following functions:

(a) F=$\overline{A}B+AC$

(b) F= AB(C+D)

(c) F=$(\overline{A}+\overline{B})(C+D)$

(d) F=A$(\overline{B}+\overline{C}+\overline{D})$

❖ BOOLEAN OPERATIONS AND EXPRESSIONS

Variable, complement, and literal are terms used in Boolean algebra. A variable is a symbol used to represent a logical quantity. Any single variable can have a **1** or a **0** value. The complement is the inverse of a variable and is indicated by a bar over variable (overbar). For example, the complement of the variable A is

$\overline{A}$. If A = 1, then $\overline{A}$ = 0. If A = 0, then $\overline{A}$ = 1. The complement of the variable A is read as "**not A**" or "A bar." Sometimes a prime symbol rather than an overbar is used to denote the complement of a variable; for example, B' indicates the complement of B. A literal is a variable or the complement of a variable.

- ## Laws of Boolean Algebra

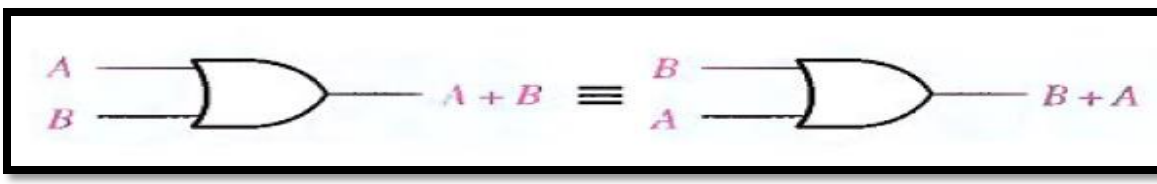
The basic laws of Boolean algebra-the commutative laws for addition and multiplication, the associative laws for addition and multiplication, and the distributive law-are the same as in ordinary algebra.

- **Commutative Laws**

1. The commutative law of addition for two variables is written as:

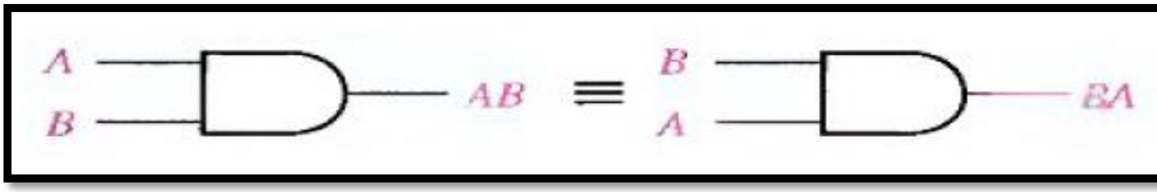
A+B = B+A

This law states that the order in which the variables are ORed makes no difference. Remember, in Boolean algebra as applied to logic circuits, addition and the OR operation are the same. The fig. below illustrates the commutative law as applied to the OR gate.



2. The commutative law of multiplication for two variables is:
   A.B = B.A

This law states that the order in which the variables are ANDed makes no difference. The fig. below il1ustrates this law as applied to the AND gate.

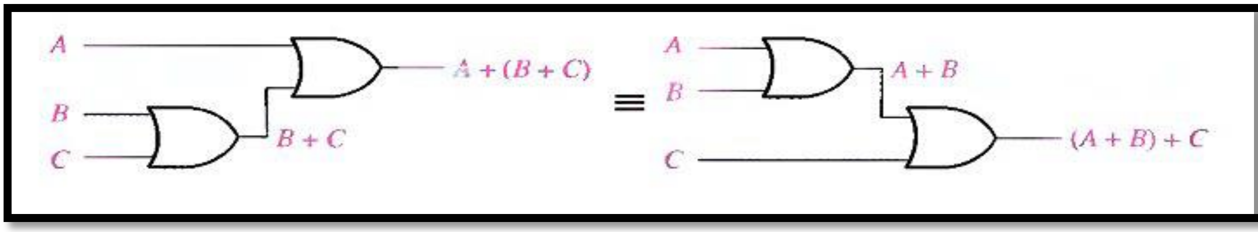


➢ **Associative Laws**

1. The associative law of addition is written as follows for three variables:

A + (B + C) = (A + B) + C

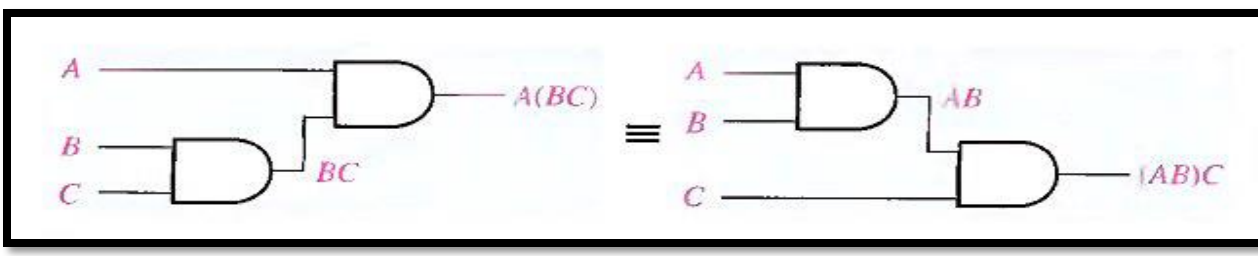
This law states that when ORing more than two variables, the result is the same regardless of the grouping of the variables. The fig. below illustrates this law as applied to 2-input OR gates.



2. The associative law of multiplication is written as follows for three variables:

A (BC) = (AB) C

This law states that it makes no difference in what order the variables are grouped when ANDing more than two variables. The fig. below illustrates this law as applied to 2-input AND gates.



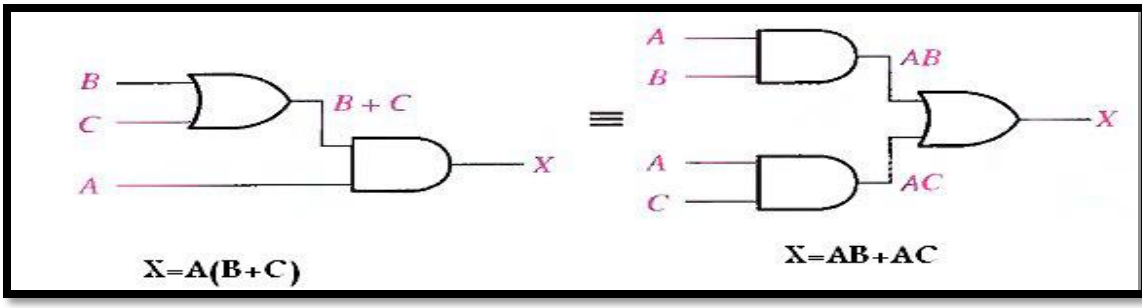
➢ **Distributive Law**

1. The distributive law is written for three variables as follows:

A (B + C) = AB + AC

This law states that ORing two or more variables and then ANDing the result with a single variable is equivalent to ANDing the single variable with each of the two or more variables and then ORing the products. The distributive law also expresses the process of factoring in which the common variable **A** is factored out of the product terms, for example, **AB + AC = A (B + C).**

The fig. below illustrates the distributive law in terms of gate implementation.



- ## Rules of Boolean Algebra

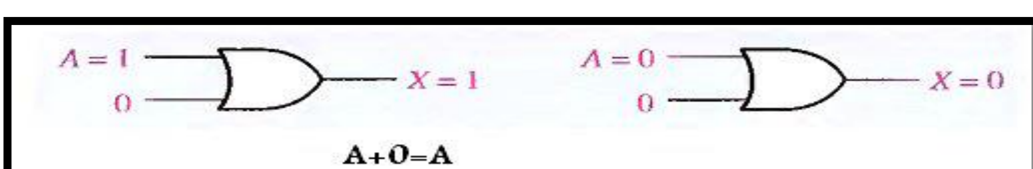
The table below lists **12** basic rules that are useful in manipulating and simplifying Boolean expressions. Rules 1 through 9 will be viewed in terms of their application to logic gates. Rules 10 through 12 will be derived in terms of the simpler rules and the laws previously discussed.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

A, B, or *C* can represent a single variable or a combination of variables.

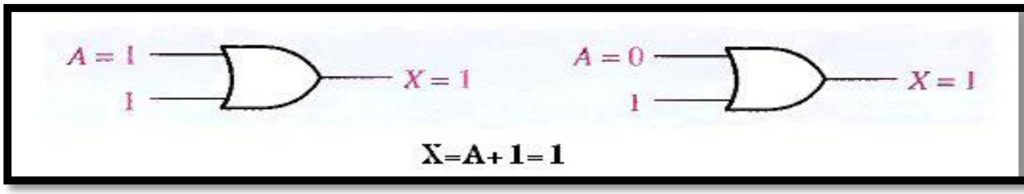
Rule 1. A + 0 = A

A variable ORed with **0** is always equal to the variable. If the input variable A is **1**, the output variable X is **1**, which is equal to A. If A is **0**, the output is **0**, which is also equal to A. This rule is illustrated in fig. below, where the lower input is fixed at **0**.



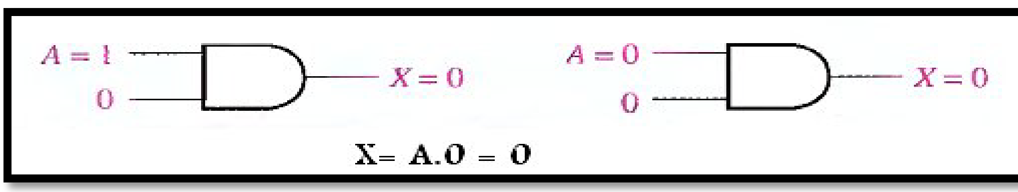
Rule 2. A + 1 = 1

A variable ORed with **1** is always equal to **1**. A 1 on an input to an OR gate produces a **1** on the output, regardless of the value of the variable on the other input. This rule is illustrated in fig. below, where the lower input is fixed at **1**.



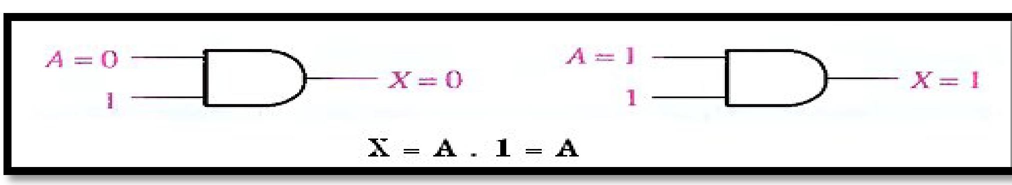
Rule 3. A. 0 = 0

A variable ANDed with 0 is always equal to **0**. Any time one input to an AND gate is **0**, the output is **0**, regardless of the value of the variable on the other input. This rule is illustrated in fig. below, where the lower input is fixed at **0**.



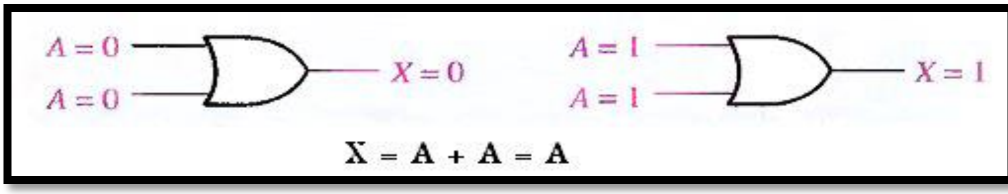
Rule 4. A. 1 = A

A variable ANDed with 1 is always equal to the variable. If A is **0** the output of the AND gate is **0**. If A is 1, the output of the AND gate is **1** because both inputs are now 1s. This rule is shown in fig. below, where the lower input is fixed at **1**.



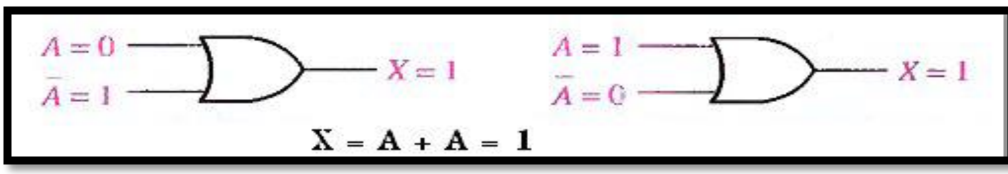
Rule 5. A + A = A

A variable ORed with itself is always equal to the variable. If A is **0**, then **0** + **0** = **0**; and if A is **1**, then 1 + 1 = **1**. This is shown in fig. below, where both inputs are the same variable.



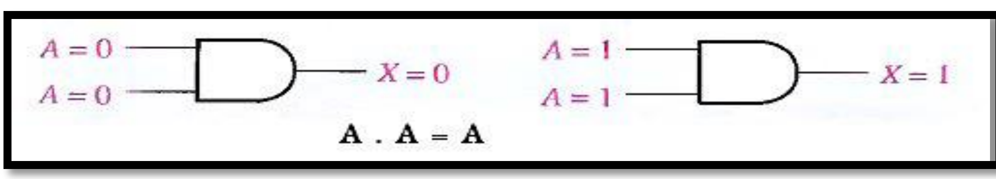
Rule 6. A + A = 1

A variable ORed with its complement is always equal to 1. If A is **0**, then 0 + 0 = 0 + 1 = **1**. If A is l, then 1 + 1 = 1+ 0 = **1**. See fig. below, where one input is the complement of the other.



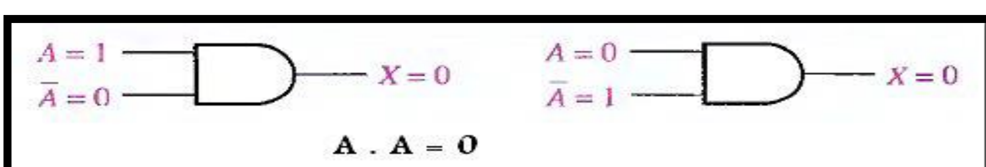
Rule 7. A. A = A

A variable ANDed with itself is always equal to the variable. If A = 0, then 0.0 = **0**; and if A = 1. Then 1.1 = **1**. Fig. below illustrates this rule.



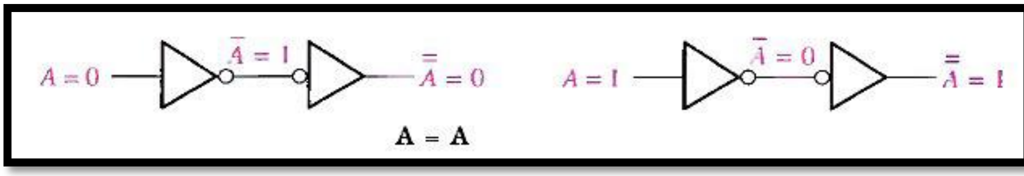
Rule 8. A. $\overline{A}$ = 0

A variable ANDed with its complement is always equal to **0**. Either A or A will always be 0: and when a 0 is applied to the input of an AND gate. The output will be **0** also. Fig. below illustrates this rule.



Rule 9 A = $\overline{\overline{A}}$

The double complement of a variable is always equal to the variable. If you start with the variable A and complement (invert) it once, you get A. If you then take A and complement (invert) it, you get A, which is the original variable. This rule is shown in fig. below using inverters.



Rule 10. A + AB = A

This rule can be proved by applying the distributive law, rule 2, and rule 4 as follows:

A + AB = A (1 + B) Factoring (distributive law)

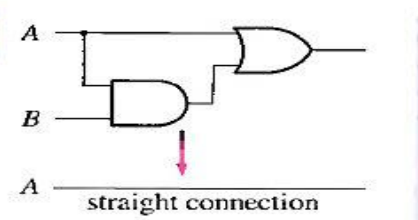
= A. l Rule 2: (1 + B) = 1

= A Rule 4: A. 1 = A

The proof is shown in table below, which shows the truth table and the resulting logic circuit simplification.

A	B	AB	A + AB
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

equal



Rule 11. A + $\overline{A}$B = A + B

This rule can be proved as follows:

A + AB = (A + AB) + $\overline{A}$B Rule 10: A = A + AB

= (AA + AB) + AB Rule 7: A = AA

=AA +AB +AA +AB Rule 8: adding AA = 0

= (A + A)(A + B) Factoring

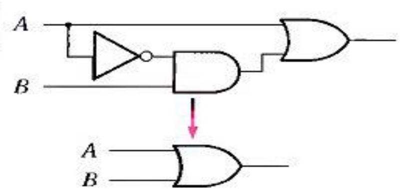
= 1. (A + B) Rule 6: A + A = 1

=**A + B** Rule 4: drop the 1

The proof is shown in table below, which shows the truth table and the resulting logic circuit simplification.

A	B	$\overline{A}B$	$A+\overline{A}B$	$A+B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



Rule 12. (A + B)(A + C) = A + BC

This rule can be proved as follows:

(A + B)(A + C) = AA + AC + AB + BC Distributive law

= A + AC + AB + BC Rule 7: AA = A

= A (1 + C) + AB + BC Rule 2: 1 + C = 1

= A. 1 + AB + BC Factoring (distributive law)

= A (1 + B) + BC Rule 2: 1 + B = 1

= A. 1 + BC Rule 4: A. 1 = A

= **A + BC**

<u>**Example**</u>: Using Boolean algebra techniques, simplify this expression:

AB + A (B + C) + B (B + C)

<u>**Solution**</u>:

Step 1: Apply the distributive law to the second and third terms in the expression, as follows:

AB + AB + AC + BB + BC

Step 2: Apply rule 7 (BB = B) to the fourth term.

AB + AB + AC + B + BC

Step 3: Apply rule 5 (AB + AB = AB) to the first two terms.

AB + AC + B + BC

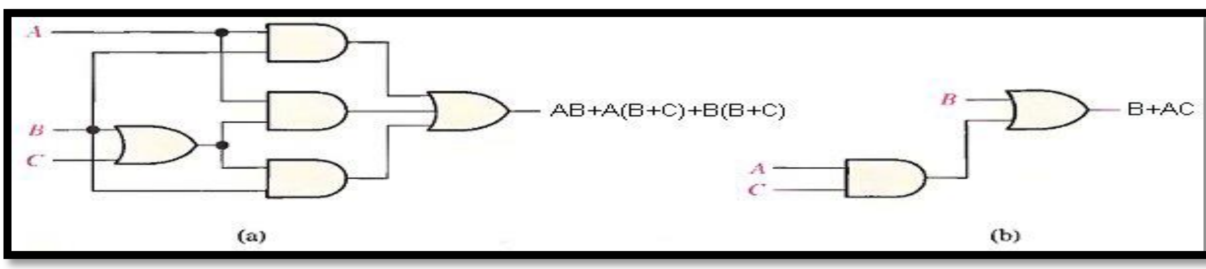
Step 4: Apply rule 10 (B + BC = B) to the last two terms.

AB + AC + B

Step 5: Apply rule 10 (AB + B = B) to the first and third terms.

B+AC

At this point the expression is simplified as much as possible.



❖ DEMORGAN'S THEOREMS

DeMorgan, a mathematician who knew Boole, proposed two theorems that are an important part of Boolean algebra. In practical terms. DeMorgan's theorems provide mathematical verification of the equivalency of the NAND and negative-OR gates and the equivalency of the NOR and negative-AND gates.

One of DeMorgan's theorems is stated as follows:

The complement of a product of variables is equal to the sum of the complements of the variables,

The formula for expressing this theorem for two variables is:

$\overline{\boldsymbol{XY}} = \overline{\boldsymbol{X}} + \overline{\boldsymbol{Y}}$

DeMorgan's second theorem is stated as follows:

The complement of a sum of variables is equal to the product of the complements of the variables.

The formula for expressing this theorem for two variables is

$\overline{\boldsymbol{X} + \boldsymbol{Y}} = \overline{\boldsymbol{X}\,\boldsymbol{Y}}$

<u>Example</u>: Apply DeMorgan's theorems to the expressions $\overline{\boldsymbol{XYZ}}$ and $\overline{\boldsymbol{X} + \boldsymbol{Y} + \boldsymbol{Z}}$.

<u>Solution</u>:

$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$

$\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$

<u>Example</u>: Apply DeMorgan's theorems to the expressions $\overline{\boldsymbol{WXYZ}}$ and $\overline{\boldsymbol{W}} + \overline{\boldsymbol{X}} + \overline{\boldsymbol{Y}} + \overline{\boldsymbol{Z}}$.

<u>Solution</u>:

$\overline{WXYZ} = \overline{W} + \overline{X} + \overline{Y} + \overline{Z}$

$\overline{W + X + Y + Z} = \overline{W} + \overline{X} + \overline{Y} + \overline{Z}$

Q: Simplify the Boolean expressions, and then draw their logic circuits.

1- A$\overline{B}$ + A ($\overline{B + C}$) + B ($\overline{B + C}$).

2- [A$\overline{B}$(C + BD) +$\overline{A}\,\overline{B}$] C.

3- $\overline{A}$BC + A$\overline{B}\,\overline{C}$+ $\overline{A}\,\overline{B}\,\overline{C}$ + A$\overline{B}$C + ABC.

<u>Example</u>: Apply DeMorgan's theorems to the following expression:

$$\overline{\overline{A + B\overline{C}} + D(\overline{E + \overline{F}})}$$

Solution:

Step l. Identify the terms to which you can apply DeMorgan's theorems, and think of each term as a single variable. Let $\overline{A + B\overline{C}}$= X and D $\overline{(E\ +\ \overline{F})}$ = Y.

Step 2. Since $\overline{X + Y}$= $\overline{X}\ \overline{Y}$

$$\overline{\overline{A + B\overline{C}} + D\overline{(E + \overline{F})}} = (\overline{\overline{A + B\overline{C}}})(\overline{D\overline{(E + \overline{F})}})$$

Step 3. Use rule 9 (A = $\overline{\overline{A}}$) to cancel the double bars over the left term (this is not part of DeMorgan's theorem).

$$\left(\overline{\overline{A + B\overline{C}}}\right)\left(\overline{D\left(\overline{E + \overline{F}}\right)}\right) = \left(A + B\overline{C}\right)\left(\overline{D\left(\overline{E + \overline{F}}\right)}\right)$$

Step 4. Applying DeMorgan's theorem to the second term,

(A + B$\overline{C}$)($\overline{D(\overline{E + \overline{F}})}$) = (A + B$\overline{C}$) ($\overline{D}$ + ($\overline{\overline{E\ +\ \overline{F}}}$))

Step 5. Use rule 9 (A =$\overline{\overline{A}}$) to cancel the double bars over the E + $\overline{F}$ part of the term.

(A + B$\overline{C}$)($\overline{D}$ + $\overline{\overline{E + \overline{F}}}$) = (**A + B$\boldsymbol{\overline{C}}$) ($\boldsymbol{\overline{D}}$ + E +$\boldsymbol{\overline{F}}$**).

Q: Apply DeMorgan's theorems to each of the following expressions, and then draw their logic circuits:

(a) $\overline{(A + B + C)D}$ (b) $\overline{ABC + DEF}$ (c) $\overline{A\overline{B} + C\overline{D} + EF}$

Flip Flops

❖ Introduction

In electronics, a flip-flop or latch is a circuit that has two stable states and can be used to store state information. The circuit can be made to change state by signals applied to one or more control inputs and will have one or two outputs. It is the basic storage element in sequential logic. Flip-flops and latches are fundamental building blocks of digital electronics systems used in computers, communications, and many other types of systems. Flip-flops and latches are used as data storage elements. A flip-flop stores a single bit (binary digit) of data; one of its two states represents a "one" and the other represents a "zero". Such data storage can be used for storage of state, and such a circuit is described as sequential logic.

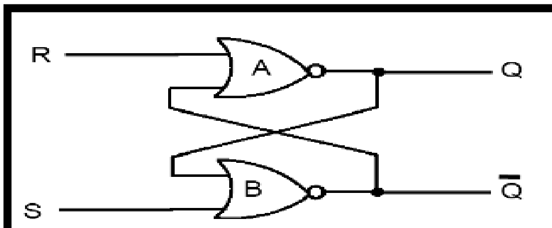
❖ Implementation

Flip-flops can be either simple (transparent or **asynchronous**) or clocked (**synchronous**); the transparent ones are commonly called latches. The word latch is mainly used for storage elements, while clocked devices are described as flip-flops. Simple flip-flops can be built using logic gates.

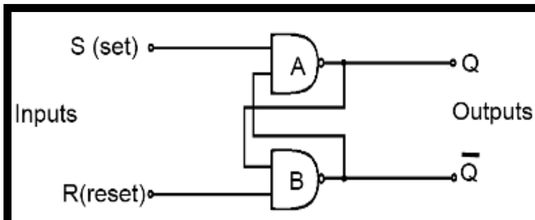
❖ Flip-Flop Types

1. Set-Reset Flip Flop (S-R)

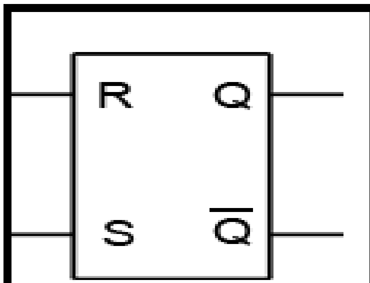
When using static gates as building blocks, the most fundamental latch is the simple SR latch, where S and R stand for set and reset. It can be constructed from a pair of cross-coupled **NOR** or **NAND** logic gates. The stored bit is present on the output marked Q. While the R and S inputs are both low, feedback maintains the Q and $\overline{Q}$ outputs in a constant state, with $\overline{Q}$ the complement of Q. If S (Set) is pulsed high while R (Reset) is held low, then the Q output is forced high, and stays high when S returns to low; similarly, if R is pulsed high while S is held low, then the Q output is forced low, and stays low when R returns to low.



(b) RS Latch Flip Flop NOR Gate



(a) Latch Flip Flop NAND Gate



RS Latch

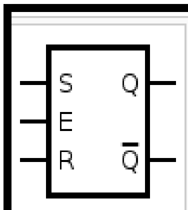
SR latch operation[13]							
Characteristic table				Excitation table			
S	R	Q_{next}	Action	Q	Q_{next}	S	R
0	0	Q	hold state	0	0	0	X
0	1	0	reset	0	1	1	0
1	0	1	set	1	0	0	1
1	1	X	not allowed	1	1	X	0

Note: **X** means don't care, that is, either 0 or 1 is a valid value.

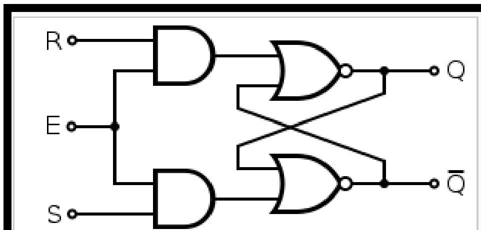
2. Gated SR Flip Flop

A synchronous SR latch (sometimes clocked SR flip-flop) can be made by adding a second level of NAND gates to the inverted SR latch (or a second level of AND gates to the direct SR latch). The extra NAND gates further invert the inputs so the simple SR latch becomes a gated SR latch (and a simple SR latch would transform into a gated SR latch with inverted enable). With E high (enable true), the signals can pass through the input gates to the encapsulated latch; all signal combinations except for (0, 0) = hold then immediately reproduce on the (Q,$\overline{Q}$) output, i.e. the latch is transparent. With E low (enable false) the latch is closed (opaque) and remains in the state it was left the last time E was high. The enable input is sometimes a **clock signal**, but more often a read or write strobe.

Gated SR latch operation	
E/C	Action
0	No action (keep state)
1	The same as non-clocked SR latch



Symbol for a gated SR latch



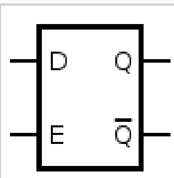
A gated SR latch circuit diagram constructed from NOR gates.

3. D Flip Flop

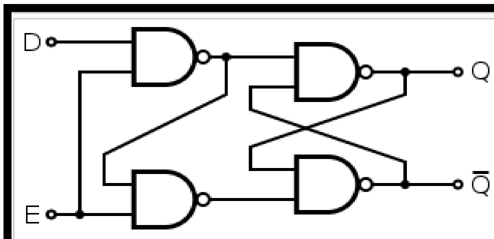
This latch exploits the fact that, in the two active input combinations (**01** and **10**) of a gated SR latch, R is the complement of S. The input NAND stage converts the two D input states (0 and 1) to these two input combinations for the next SR latch by inverting the data input signal. The low state of the enable signal produces the inactive "11" combination. Thus a gated D-latch may be considered as a one-input synchronous SR latch. This configuration prevents application of the restricted input combination. It is also known as transparent latch, data latch, or simply gated latch. It has a data input and an enable signal (sometimes named clock, or control). The word transparent comes from the fact that, when the enable input is on, the signal propagates directly through the circuit, from the input D to the output Q.

Gated D latch truth table

E/C	D	Q	$\overline{Q}$	Comment
0	X	Q_{prev}	$\overline{Q}_{prev}$	No change
1	0	0	1	Reset
1	1	1	0	Set



Symbol for a gated D latch



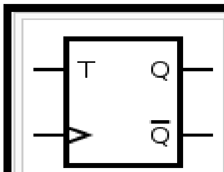
A gated D latch based on an $\overline{SR}$ NAND latch

4. T Flip Flop

If the **T** input is high, the T flip-flop changes state ("toggles") whenever the clock input is strobed. If the T input is low, the flip-flop holds the previous value. This behavior is described by the characteristic equation:

$$Q_{next} = T \oplus Q = T\overline{Q} + \overline{T}Q$$

And can be described in a truth table:



A circuit symbol for a T-type flip-flop

T flip-flop operation[26]

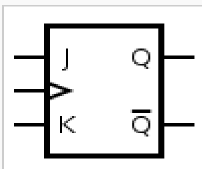
Characteristic table				Excitation table			
T	Q	Q_{next}	Comment	Q	Q_{next}	T	Comment
0	0	0	hold state (no clk)	0	0	0	No change
0	1	1	hold state (no clk)	1	1	0	No change
1	0	1	toggle	0	1	1	Complement
1	1	0	toggle	1	0	1	Complement

5. JK Flip Flop

The JK flip-flop augments the behavior of the SR flip-flop (J=Set, K=Reset) by interpreting the J = K = 1 condition as a "flip" or toggle command. Specifically, the combination J = 1, K = 0 is a command to set the flip-flop; the combination J = 0, K = 1 is a command to reset the flip-flop; and the combination J = K = 1 is a command to toggle the flip-flop, i.e., change its output to the logical complement of its current value. Setting J = K = 0 maintains the current state. To synthesize a D flip-flop, simply set K equal to the complement of J. Similarly, to synthesize a T flip-flop, set K equal to J. The JK flip-flop is therefore a universal flip-flop, because it can be configured to work as an SR flip-flop, a D flip-flop, or a T flip-flop. The characteristic equation of the JK flip-flop is:

$$Q_{next} = J\overline{Q} + \bar{J}Q$$

And the corresponding truth table is:



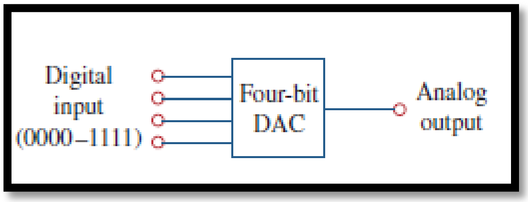
A circuit symbol for a positive-edge-triggered JK flip-flop

JK flip-flop operation[26]

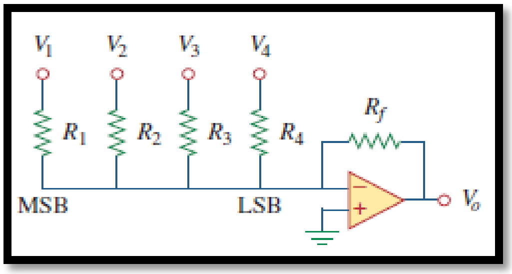
Characteristic table				Excitation table				
J	K	Comment	Q_{next}	Q	Q_{next}	Comment	J	K
0	0	hold state	Q	0	0	No Change	0	X
0	1	reset	0	0	1	Set	1	X
1	0	set	1	1	0	Reset	X	1
1	1	toggle	$\overline{Q}$	1	1	No Change	X	0

Digital-to-Analog Converter

The digital-to-analog converter (DAC) transforms digital signals into analog form. A typical example of a four-bit DAC is illustrated in fig. shown below.



The four-bit DAC can be realized in many ways. A simple realization is the binary weighted ladder, shown below.



The bits are weights according to the magnitude of their place value, by descending value of $\boldsymbol{R_f/R_n}$so that each lesser bit has half the weight of the next higher. This is obviously an inverting summing amplifier. The output is related to the inputs as shown in the equation below:

$$-\boldsymbol{V_0} = \frac{\boldsymbol{R_f}}{\boldsymbol{R_1}}\boldsymbol{V_1} + \frac{\boldsymbol{R_f}}{\boldsymbol{R_2}}\boldsymbol{V_2} + \frac{\boldsymbol{R_f}}{\boldsymbol{R_3}}\boldsymbol{V_3} + \frac{\boldsymbol{R_f}}{\boldsymbol{R_4}}\boldsymbol{V_4} \quad \boldsymbol{Eq.(1)}$$

Input $\boldsymbol{V_1}$ is called the most significant bit (**MSB**), while input $\boldsymbol{V_4}$ is the least significant bit (**LSB**). Each of the four binary inputs $V_1, \ldots, V_4$ can assume only two voltage levels: **0 or 1 V**. By using the proper input and feedback resistor values, the **DAC** provides a single output that is proportional to the inputs.

<u>Example</u>: In the op amp circuit of the previous fig. , let $\boldsymbol{R_f = 10k\Omega}$, $\boldsymbol{R_1 = 10k\Omega}$, $\boldsymbol{R_2 = 20k\Omega}$, $\boldsymbol{R_3 = 40k\Omega}$, $\boldsymbol{R_4 = 80k\Omega}$. Obtain the analog output for binary inputs [**0000**], [**0001**], [**0010**], . . . , [**1111**].

Solution:

Substituting the given values of the input and feedback resistors in **Eq. (1)** gives:

$$-V_0 = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$

$$= V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4$$

Using this equation, a digital input $[V_1V_2V_3V_4]$ =[0000] produces an analog output of $-V_0 = \mathbf{0V}, [V_1V_2V_3V_4] = [0001] gives - V_0 = \mathbf{0.125V}$.

Similarly,

$[V_1V_2V_3V_4] = [0010], \qquad -V_0 = \mathbf{0.25V}$

$[V_1V_2V_3V_4] = [0011], \qquad -V_0 = 0.25 + 0.125 = \mathbf{0.375V}$

$[V_1V_2V_3V_4] = [0100] \qquad -V_0 = \mathbf{0.5V}$

$[V_1V_2V_3V_4] = [1111] \qquad -V_0 = 1 + 0.5 + 0.25 + 0.125 = \mathbf{1.875V}$

Table below summarizes the result of the digital-to-analog conversion. Note that we have assumed that each bit has a value of 0.125 V.

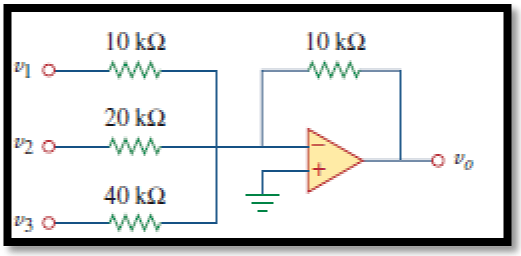
Input and output values of the four-bit DAC.

Binary input $[V_1V_2V_3V_4]$	Decimal value	Output $-V_o$
0000	0	0
0001	1	0.125
0010	2	0.25
0011	3	0.375
0100	4	0.5
0101	5	0.625
0110	6	0.75
0111	7	0.875
1000	8	1.0
1001	9	1.125
1010	10	1.25
1011	11	1.375
1100	12	1.5
1101	13	1.625
1110	14	1.75
1111	15	1.875

Example: A three-bit DAC is shown in the fig. below, determine:

(a) $\boldsymbol{V_0}$ for $[V_1V_2V_3] = [010]$.

(b) Find $\boldsymbol{V_0}$ if $[V_1 V_2 V_3] = [110]$.
(c) If $V_0 = \mathbf{1.25V}$ is desired, what should be $[V_1 V_2 V_3]$?
(d) To get $V_0 = \mathbf{1.75V}$**,** what should $[V_1 V_2 V_3]$?

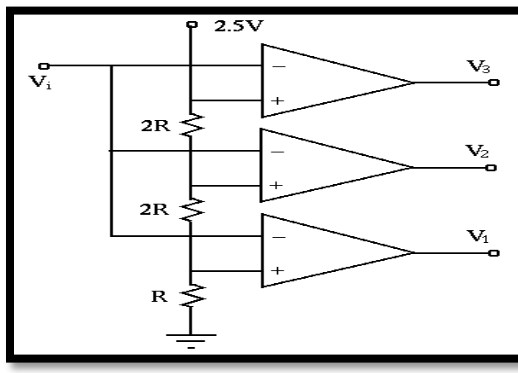


Analog-to-Digital Converter

An Analog-to-Digital (A/D) converter converts an analog voltage into a digital number. The three-bit ADC can be realized in many ways. A simple realization is shown below. The figure shows an A/D converter built by three op-amps to measure voltage $\boldsymbol{V_{in}}$ from 0 to 3 volts with resolution 1 V.

Due to the voltage divider, the input voltages to the three op-amps are, respectively, 2.5V, 1.5V and 0.5V. The outputs of these op-amps are listed below for each of the input voltage levels.

Input voltage	0	1	2	3
Op-amps Outputs	000	001	011	111



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