Load and Stress Analysis

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_1, \tau_2 = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2} \qquad \tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} \qquad \tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2}$$

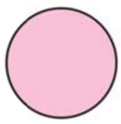
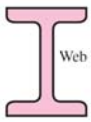
$$E = 2G(1 + \nu)$$

Two-Plane Bending

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

Table 3–2

Formulas for Maximum Shear Stress Due to Bending

Beam Shape	Formula	Beam Shape	Formula
 Rectangular	$\tau_{max} = \frac{3V}{2A}$	 Hollow, thin-walled round	$\tau_{max} = \frac{2V}{A}$
 Circular	$\tau_{max} = \frac{4V}{3A}$	 Structural I beam (thin-walled)	$\tau_{max} = \frac{V}{A_{web}}$

Torsion

$$\theta = \frac{Tl}{GJ} \qquad \tau_{max} = \frac{Tr}{J} \qquad J = \frac{\pi d^4}{32} \qquad J = \frac{\pi}{32}(d_o^4 - d_i^4)$$

Stress Concentration

$$K_t = \frac{\sigma_{max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{max}}{\tau_0}$$

Stresses in Pressurized Cylinders

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2} \qquad (3\text{–}49)$$

The special case of $p_o = 0$ gives

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

$$\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right) \qquad (3\text{–}50)$$

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2} \qquad (3\text{–}51)$$

Thin-Walled Vessels

$$(\sigma_t)_{\text{av}} = \frac{p d_i}{2t} \quad (\sigma_t)_{\max} = \frac{p(d_i + t)}{2t} \qquad \sigma_l = \frac{p d_i}{4t}$$

Stresses in Rotating Rings

$$\sigma_i = \rho\omega^2 \left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1+3\nu}{3+\nu} r^2\right)$$

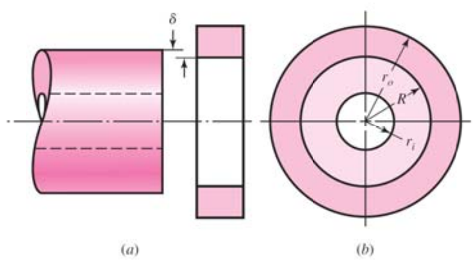
$$\sigma_r = \rho\omega^2 \left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right) \qquad (3\text{–}55)$$

Press and Shrink Fits

$$p = \frac{\delta}{R\left[\dfrac{1}{E_o}\left(\dfrac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \dfrac{1}{E_i}\left(\dfrac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)\right]} \qquad (3\text{–}56)$$

$E_o = E_i = E$, $\nu_o = \nu_i$, the relation simplifies to

$$p = \frac{E\delta}{2R^3}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2}\right] \qquad (3\text{–}57)$$



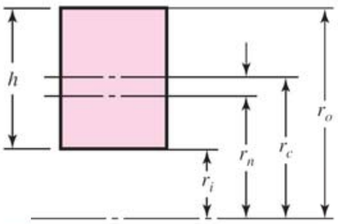
$$(\sigma_t)_i\Big|_{r=R} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} \qquad (3\text{–}58)$$

$$(\sigma_t)_o\Big|_{r=R} = p\frac{r_o^2 + R^2}{r_o^2 - R^2} \qquad (3\text{–}59)$$

Curved Beams in Bending

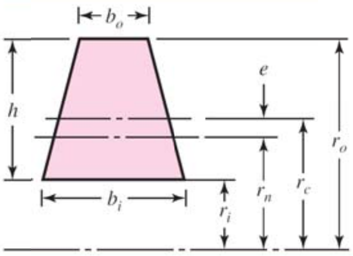
$$\sigma = \frac{My}{Ae(r_n - y)} \qquad (3\text{–}64)$$

$$\sigma_i = \frac{Mc_i}{Aer_i} \qquad \sigma_o = -\frac{Mc_o}{Aer_o} \qquad (3\text{–}65)$$



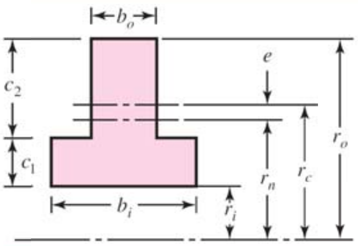
$$r_c = r_i + \frac{h}{2}$$

$$r_n = \frac{h}{\ln(r_o/r_i)}$$



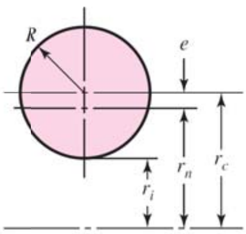
$$r_c = r_i + \frac{h}{3}\frac{b_i + 2b_o}{b_i + b_o}$$

$$r_n = \frac{A}{b_o - b_i + [(b_i r_o - b_o r_i)/h]\ln(r_o/r_i)}$$



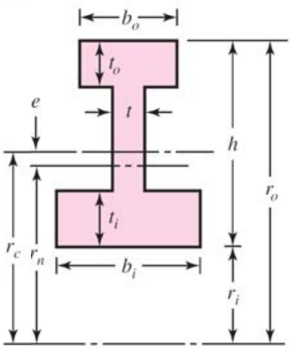
$$r_c = r_i + \frac{b_i c_1^2 + 2b_o c_1 c_2 + b_o c_2^2}{2(b_o c_2 + b_i c_1)}$$

$$r_n = \frac{b_i c_1 + b_o c_2}{b_i \ln[(r_i + c_1)/r_i] + b_o \ln[r_o/(r_i + c_1)]}$$



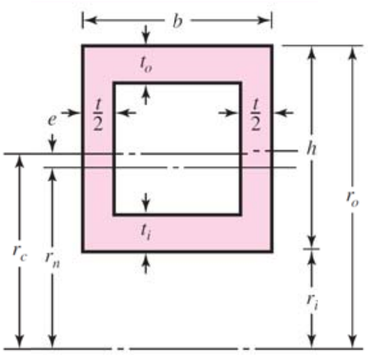
$$r_c = r_i + R$$

$$r_n = \frac{R^2}{2\left(r_c - \sqrt{r_c^2 - R^2}\right)}$$



$$r_c = r_i + \frac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b_i - t) + t_o(b_o - t)(h - t_o/2)}{t_i(b_i - t) + t_o(b_o - t) + ht}$$

$$r_n = \frac{t_i(b_i - t) + t_o(b_o - t) + ht_o}{b_i \ln\dfrac{r_i + t}{r_i} + t\ln\dfrac{r_o - t_o}{r_i + t_i} + b_o \ln\dfrac{r_o}{r_o - t_o}}$$



$$r_c = r_i + \frac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b - t) + t_o(b - t)(h - t_o/2)}{ht + (b - t)(t_i + t_o)}$$

$$r_n = \frac{(b - t)(t_i + t_o) + ht}{b\left(\ln\dfrac{r_i + t_i}{r_i} + \ln\dfrac{r_o}{r_o - t_o}\right) + t\ln\dfrac{r_o - t_o}{r_i + t_i}}$$

Contact Stresses

Spherical Contact

$$a = \sqrt[3]{\frac{3F}{8}\frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}} \tag{3–68}$$

$$p_{\max} = \frac{3F}{2\pi a^2} \tag{3–69}$$

$$\sigma_1 = \sigma_2 = \sigma_x = \sigma_y = -p_{\max}\left[\left(1 - \left|\frac{z}{a}\right| \tan^{-1}\frac{1}{|z/a|}\right)(1+\nu) - \frac{1}{2\left(1+\frac{z^2}{a^2}\right)}\right] \tag{3–70}$$

$$\sigma_3 = \sigma_z = \frac{-p_{\max}}{1+\frac{z^2}{a^2}} \tag{3–71}$$

$$\tau_{\max} = \tau_{1/3} = \tau_{2/3} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_2 - \sigma_3}{2} \tag{3–72}$$

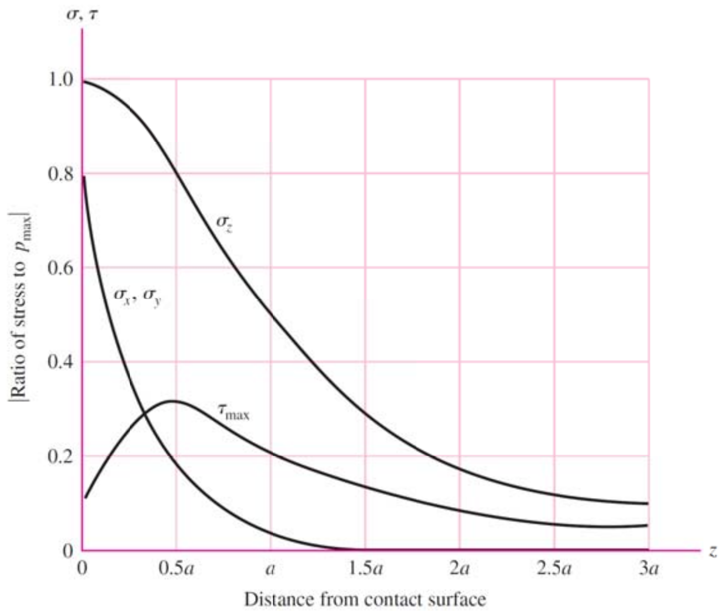


Figure 3–37

Magnitude of the stress components below the surface as a function of the maximum pressure of contacting spheres. Note that the maximum shear stress is slightly below the surface at $z = 0.48a$ and is approximately $0.3p_{max}$. The chart is based on a Poisson ratio of 0.30. Note that the normal stresses are all compressive stresses.

Cylindrical Contact

$$b = \sqrt{\frac{2F}{\pi l}\frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}} \tag{3–73}$$

$$p_{\max} = \frac{2F}{\pi b l} \tag{3–74}$$

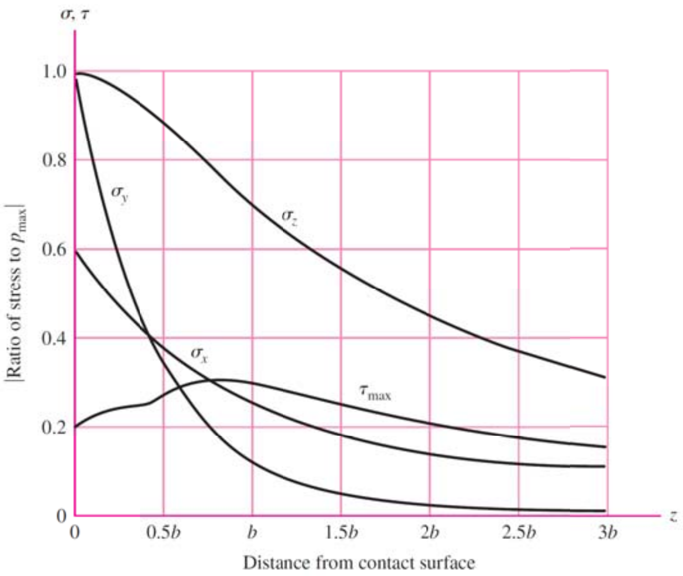
$$\sigma_x = -2\nu p_{\max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right) \tag{3–75}$$

$$\sigma_y = -p_{\max}\left(\frac{1+2\frac{z^2}{b^2}}{\sqrt{1+\frac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right|\right) \tag{3–76}$$

$$\sigma_3 = \sigma_z = \frac{-p_{\max}}{\sqrt{1+z^2/b^2}} \tag{3–77}$$

Figure 3–39

Magnitude of the stress components below the surface as a function of the maximum pressure for contacting cylinders. The largest value of τ_{max} occurs at $z/b = 0.786$. Its maximum value is $0.30p_{max}$. The chart is based on a Poisson ratio of 0.30. Note that all normal stresses are compressive stresses.



Long Columns with Central Loading

$$P_{cr} = \frac{C\pi^2 EI}{l^2} \qquad (4\text{–}40)$$

$$\frac{P_{cr}}{A} = \frac{C\pi^2 E}{(l/k)^2} \qquad (4\text{–}41)$$

Table 4–2

End-Condition Constants for Euler Columns [to Be Used with Eq. (4–40)]

Column End Conditions	End-Condition Constant C: Theoretical Value	Conservative Value	Recommended Value*
Fixed-free	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
Rounded-rounded	1	1	1
Fixed-rounded	2	1	1.2
Fixed-fixed	4	1	1.2

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2 CE}{S_y}\right)^{1/2} \qquad (4\text{–}42)$$

Intermediate-Length Columns with Central Loading

$$\frac{P_{cr}}{A} = S_y - \left(\frac{S_y}{2\pi}\frac{l}{k}\right)^2 \frac{1}{CE} \qquad \frac{l}{k} \le \left(\frac{l}{k}\right)_1 \qquad (4\text{–}43)$$

5 Failures Resulting from Static Loading

Maximum-Shear-Stress Theory for Ductile Materials

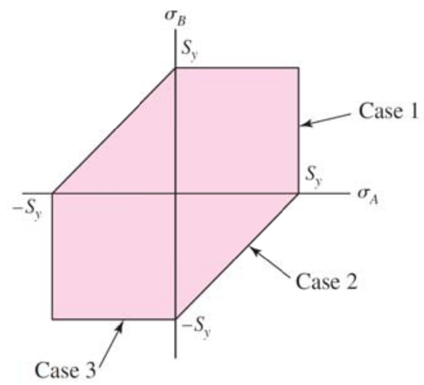
$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \ge \frac{S_y}{2} \quad \text{or} \quad \sigma_1 - \sigma_3 \ge S_y \qquad (5\text{–}1)$$

$$S_{sy} = 0.5 S_y$$

$$\tau_{max} = \frac{S_y}{2n} \quad \text{or} \quad \sigma_1 - \sigma_3 = \frac{S_y}{n} \qquad (5\text{–}3)$$

Figure 5–7

The maximum-shear-stress (MSS) theory for plane stress, where σ_A and σ_B are the two nonzero principal stresses.



Distortion-Energy Theory for Ductile Materials

$$\sigma' \geq S_y \qquad \sigma' = \frac{S_y}{n}$$

$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2} \qquad (5\text{–}12)$$

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} \qquad (5\text{–}13)$$

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\right]^{1/2} \qquad (5\text{–}14)$$

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2} \qquad (5\text{–}15)$$

Coulomb-Mohr Theory for Ductile Materials

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = \frac{1}{n} \qquad (5\text{–}26)$$

$$S_{sy} = \frac{S_{yt}S_{yc}}{S_{yt}+S_{yc}} \qquad (5\text{–}27)$$

Modifications of the Mohr Theory for Brittle Materials

Brittle-Coulomb-Mohr

$$\sigma_A = \frac{S_{ut}}{n} \qquad \sigma_A \geq \sigma_B \geq 0 \qquad (5\text{–}31a)$$

$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \qquad \sigma_A \geq 0 \geq \sigma_B \qquad (5\text{–}31b)$$

$$\sigma_B = -\frac{S_{uc}}{n} \qquad 0 \geq \sigma_A \geq \sigma_B \qquad (5\text{–}31c)$$

Modified Mohr

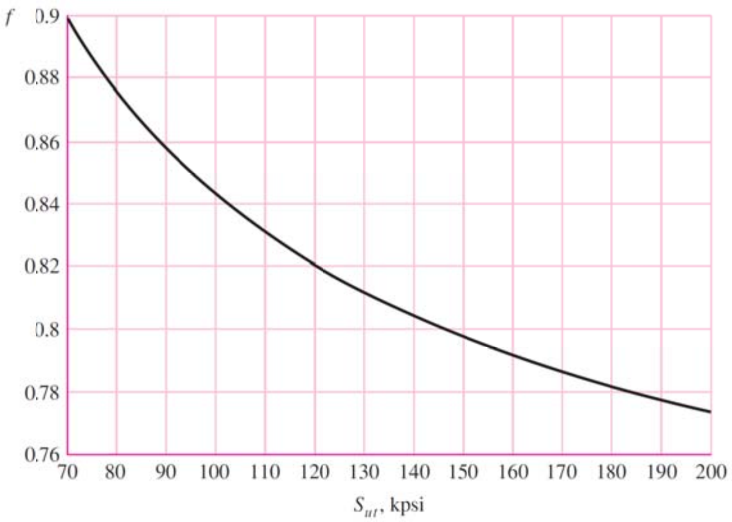
$$\sigma_A = \frac{S_{ut}}{n} \qquad \sigma_A \geq \sigma_B \geq 0$$
$$\sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left|\frac{\sigma_B}{\sigma_A}\right| \leq 1 \qquad (5\text{–}32a)$$

$$\frac{(S_{uc}-S_{ut})\,\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \qquad \sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left|\frac{\sigma_B}{\sigma_A}\right| > 1 \qquad (5\text{–}32b)$$

$$\sigma_B = -\frac{S_{uc}}{n} \qquad 0 \geq \sigma_A \geq \sigma_B \qquad (5\text{–}32c)$$

6 Fatigue Failure Resulting from Variable Loading

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \qquad (6\text{–}8)$$



1 kpsi = 6.895 MPa

$$S_f = a\,N^b \qquad (6\text{–}13)$$

$$a = \frac{(f\,S_{ut})^2}{S_e} \qquad (6\text{–}14)$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) \qquad (6\text{–}15)$$

$$N = \left(\frac{\sigma_a}{a}\right)^{1/b} \qquad (6\text{–}16)$$

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \qquad (6\text{–}18)$$

$$k_a = a S_{ut}^b$$

Table 6–2

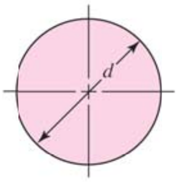
Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

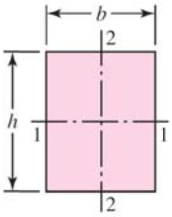
$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \qquad (6\text{–}20)$$

For axial loading there is no size effect, so

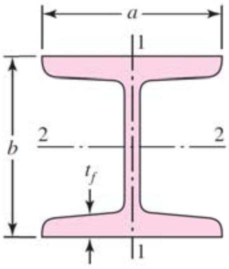
$$k_b = 1$$



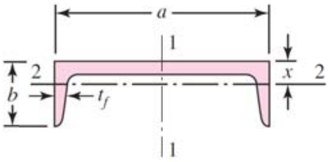
$$A_{0.95\sigma} = 0.01046d^2$$
$$d_e = 0.370d$$



$$A_{0.95\sigma} = 0.05hb$$
$$d_e = 0.808\sqrt{hb}$$



$$A_{0.95\sigma} = \begin{cases} 0.10at_f & & \text{axis 1-1} \\ 0.05ba & t_f > 0.025a & \text{axis 2-2} \end{cases}$$



$$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b - x) & \text{axis 2-2} \end{cases}$$

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases} \tag{6–26}$$

Table 6–4

Effect of Operating Temperature on the Tensile Strength of Steel.* (S_T = tensile strength at operating temperature; S_{RT} = tensile strength at room temperature; $0.099 \le \hat{\sigma} \le 0.110$)

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4 \tag{6–27}$$

$$k_d = \frac{S_T}{S_{RT}} \tag{6–28}$$

$$k_e = 1 - 0.08\, z_a \tag{6–29}$$

Table 6–5

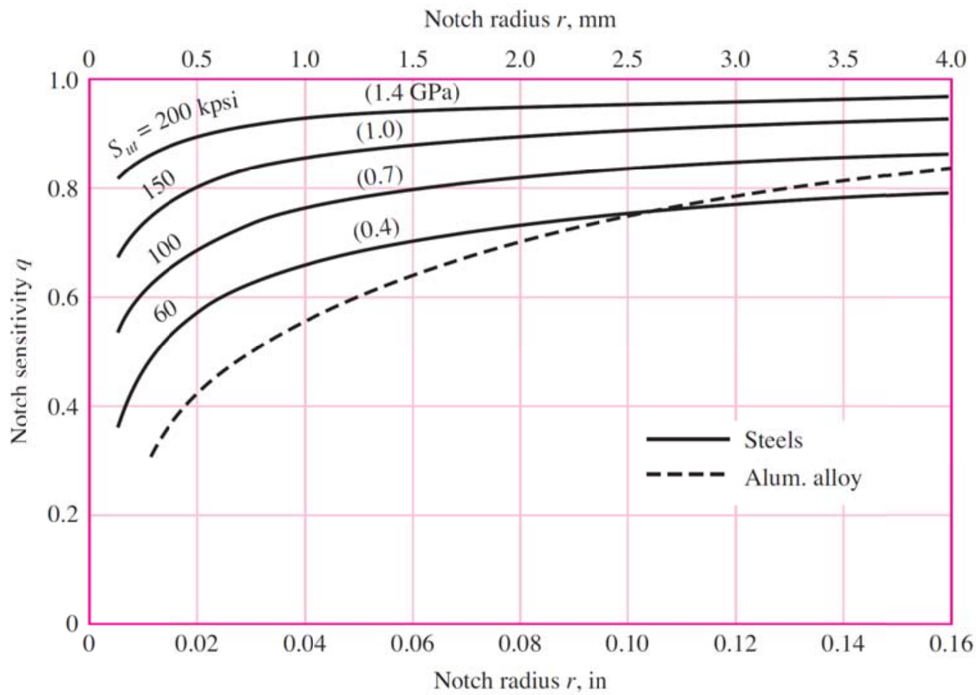
Reliability Factors k_e Corresponding to 8 Percent Standard Deviation of the Endurance Limit

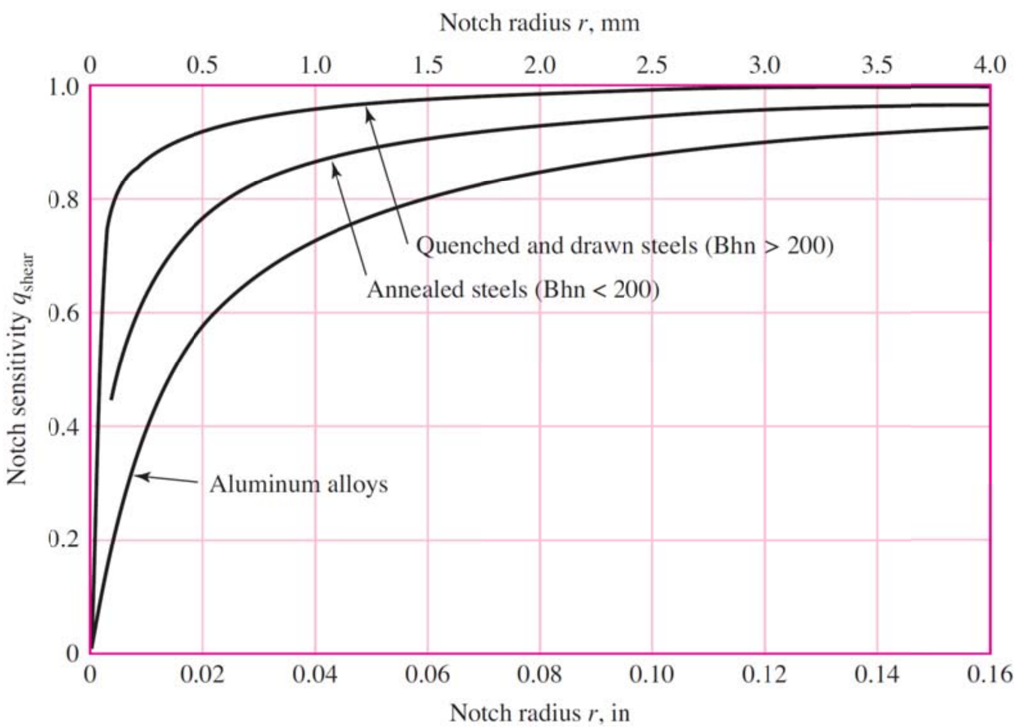
Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Stress Concentration and Notch Sensitivity

$$\sigma_{\max} = K_f \sigma_0 \quad \text{or} \quad \tau_{\max} = K_{fs} \tau_0$$

$$K_f = 1 + q(K_t - 1) \quad \text{or} \quad K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1) \qquad (6\text{–}32)$$





$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2} \qquad R = \frac{\sigma_{\min}}{\sigma_{\max}} \qquad A = \frac{\sigma_a}{\sigma_m}$$

$$\sigma_a = \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right|$$

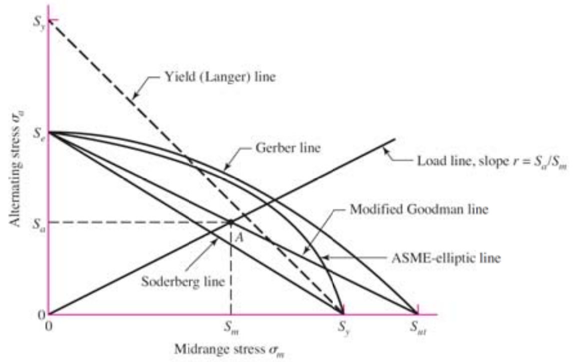


Table 6–6

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Modified Goodman and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$	$S_a = \frac{r S_e S_{ut}}{r S_{ut} + S_e}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$	$S_m = \frac{(S_y - S_e) S_{ut}}{S_{ut} - S_e}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{\text{crit}} = S_a / S_m$

Fatigue factor of safety

$$n_f = \frac{1}{\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}}}$$

Table 6–7

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Gerber and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_a = \frac{r^2 S_{ut}^2}{2 S_e}\left[-1 + \sqrt{1 + \left(\frac{2 S_e}{r S_{ut}}\right)^2}\right]$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_m = \frac{S_{ut}^2}{2 S_e}\left[1 - \sqrt{1 + \left(\frac{2 S_e}{S_{ut}}\right)^2 \left(1 - \frac{S_y}{S_e}\right)}\right]$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{\text{crit}} = S_a / S_m$

Fatigue factor of safety

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \qquad \sigma_m > 0$$

Table 6–8

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for ASME-Elliptic and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$	$S_a = \sqrt{\frac{r^2 S_e^2 S_y^2}{S_e^2 + r^2 S_y^2}}$
Load line $r = S_a/S_m$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
Load line $r = S_a/S_m$	$S_m = \frac{S_y}{1 + r}$
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$	$S_a = 0,\ \frac{2 S_y S_e^2}{S_e^2 + S_y^2}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_m = S_y - S_a,\ r_{crit} = S_a/S_m$

Fatigue factor of safety

$$n_f = \sqrt{\frac{1}{(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2}}$$

For many *brittle* materials, the first quadrant fatigue failure criteria follows a concave upward Smith-Dolan locus represented by

$$\frac{S_a}{S_e} = \frac{1 - S_m/S_{ut}}{1 + S_m/S_{ut}} \qquad (6\text{–}50)$$

or as a design equation,

$$\frac{n\sigma_a}{S_e} = \frac{1 - n\sigma_m/S_{ut}}{1 + n\sigma_m/S_{ut}} \qquad (6\text{–}51)$$

For a radial load line of slope r, we substitute S_a/r for S_m in Eq. (6–50) and solve for S_a, obtaining

$$S_a = \frac{r S_{ut} + S_e}{2}\left[-1 + \sqrt{1 + \frac{4 r S_{ut} S_e}{(r S_{ut} + S_e)^2}}\right] \qquad (6\text{–}52)$$

$$S_a = S_e + \left(\frac{S_e}{S_{ut}} - 1\right) S_m \qquad -S_{ut} \le S_m \le 0 \quad \text{(for cast iron)} \qquad (6\text{–}53)$$

Combinations of Loading Modes

$$\sigma_a' = \left\{\left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}}\frac{(\sigma_a)_{\text{axial}}}{0.85}\right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}}\right]^2\right\}^{1/2} \qquad (6\text{–}55)$$

$$\sigma_m' = \left\{\left[(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}}\right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}}\right]^2\right\}^{1/2} \qquad (6\text{–}56)$$

7 Shafts and Shaft Components

$$\sigma_a = K_f \frac{32 M_a}{\pi d^3} \qquad \sigma_m = K_f \frac{32 M_m}{\pi d^3} \qquad (7\text{–}3)$$

$$\tau_a = K_{fs} \frac{16 T_a}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16 T_m}{\pi d^3} \qquad (7\text{–}4)$$

$$\sigma_a' = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32 K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2} \qquad (7\text{–}5)$$

$$\sigma_m' = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32 K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2} \qquad (7\text{–}6)$$

DE-Goodman

$$\frac{1}{n} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2}\right\} \tag{7–7}$$

$$d = \left(\frac{16n}{\pi}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2}\right\}\right)^{1/3} \tag{7–8}$$

DE-Gerber

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\} \tag{7–9}$$

$$d = \left(\frac{8nA}{\pi S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}\right)^{1/3} \tag{7–10}$$

where

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}$$

$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2}$$

DE-ASME Elliptic

$$\frac{1}{n} = \frac{16}{\pi d^3}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2} \tag{7–11}$$

$$d = \left\{\frac{16n}{\pi}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}\right\}^{1/3} \tag{7–12}$$

DE-Soderberg

$$\frac{1}{n} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_{yt}}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2}\right\} \tag{7–13}$$

$$d = \left(\frac{16n}{\pi}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_{yt}}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2}\right\}\right)^{1/3} \tag{7–14}$$

$$\sigma'_{\max} = \left[(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2\right]^{1/2} = \left[\left(\frac{32K_f(M_m + M_a)}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}(T_m + T_a)}{\pi d^3}\right)^2\right]^{1/2} \tag{7–15}$$

$$n_y = \frac{S_y}{\sigma'_{\max}}$$

Figure A–15–1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.

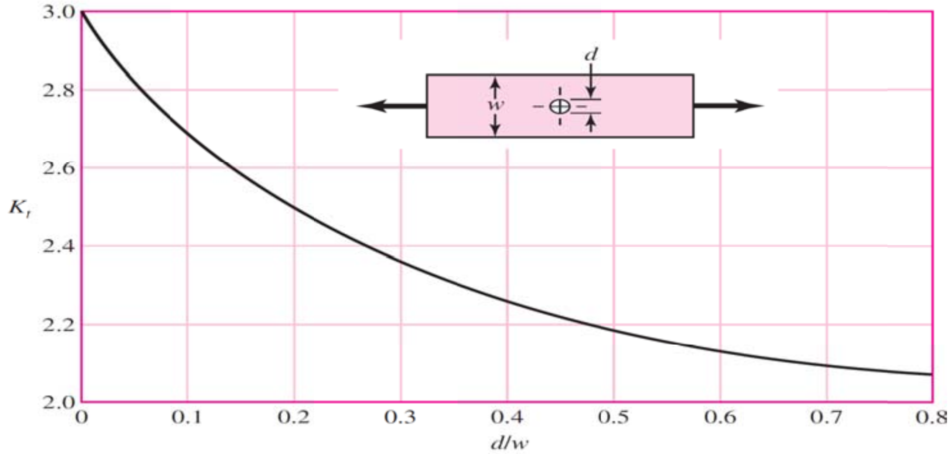


Figure A–15–2

Rectangular bar with a transverse hole in bending. $\sigma_0 = Mc/I$, where $I = (w - d)h^3/12$.

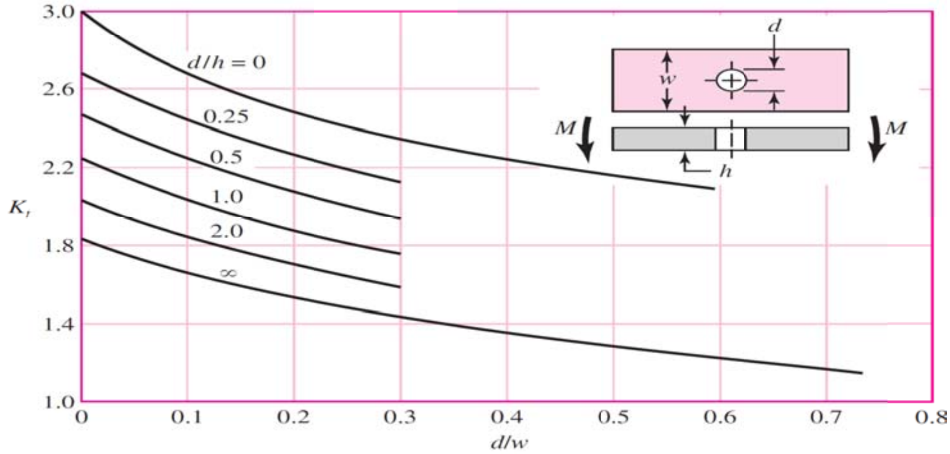


Figure A–15–3

Notched rectangular bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

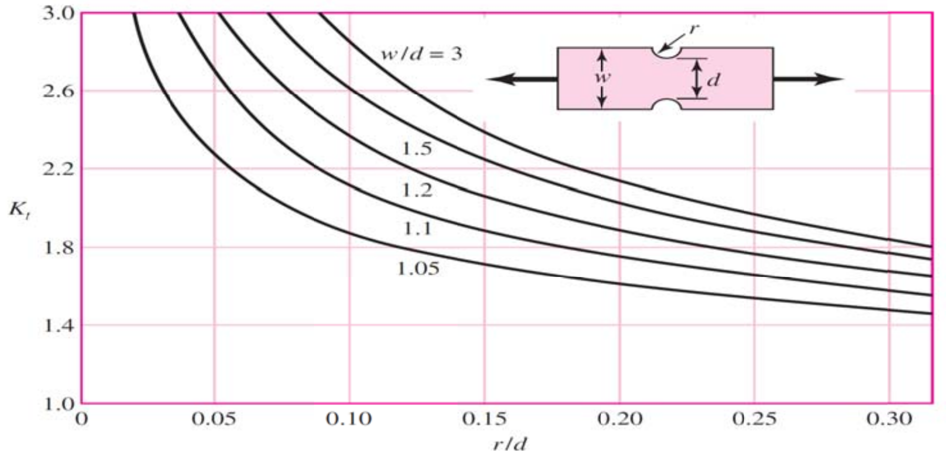


Figure A–15–4

Notched rectangular bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, and t is the thickness.

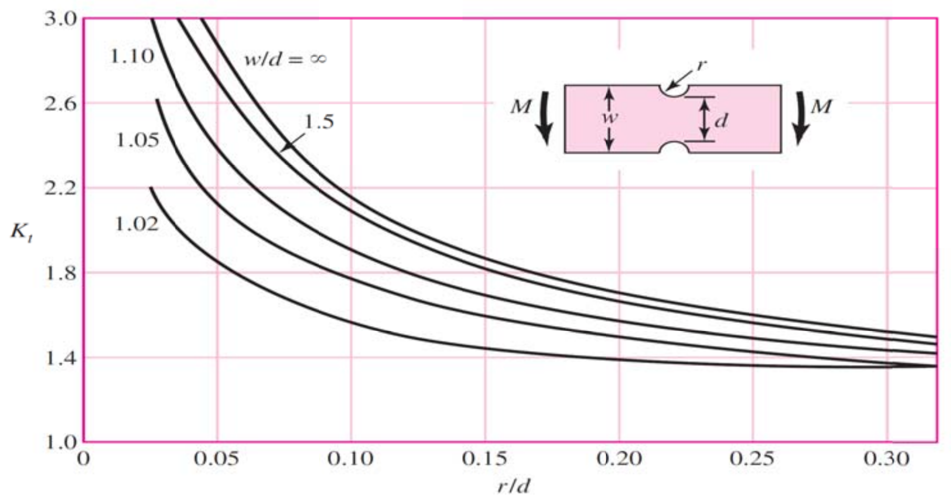


Figure A–15–5

Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

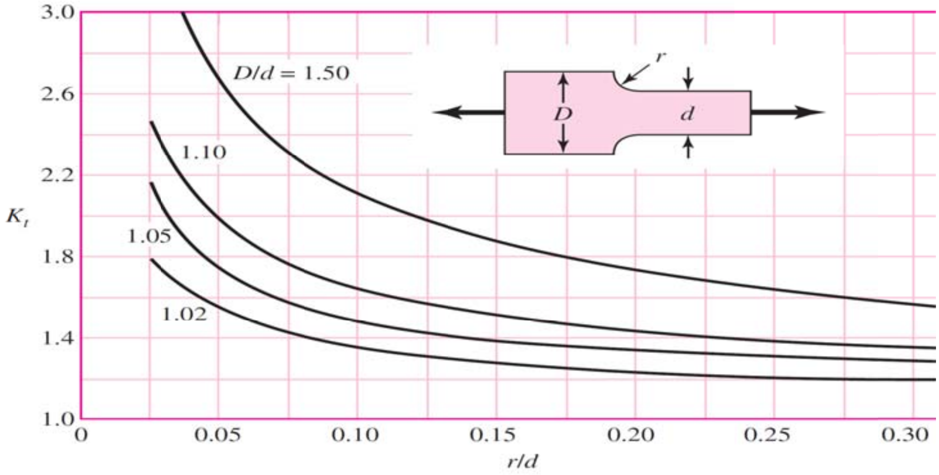


Figure A–15–6

Rectangular filleted bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, t is the thickness.

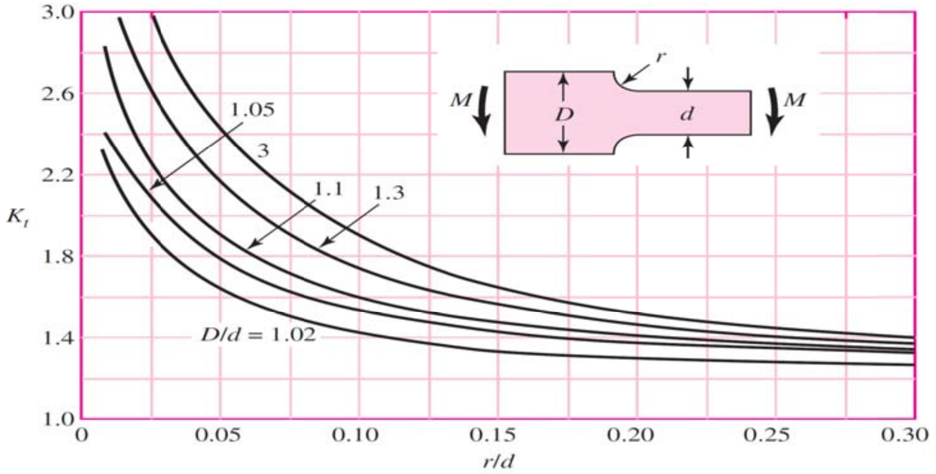


Figure A–15–7

Round shaft with shoulder fillet in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.

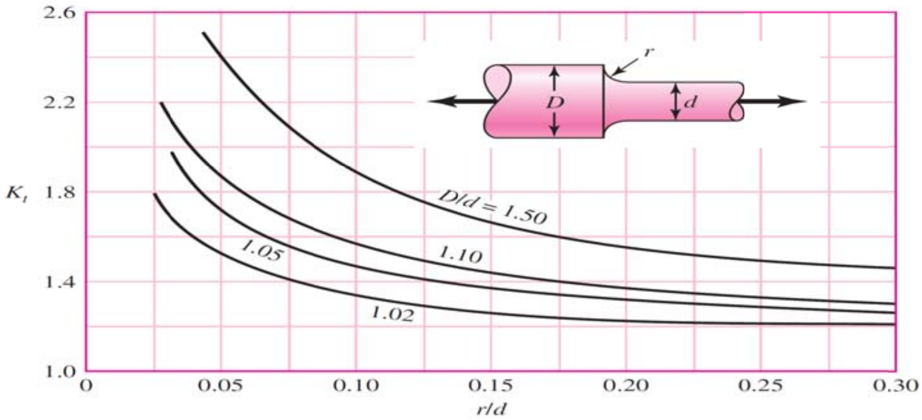


Figure A–15–8

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.

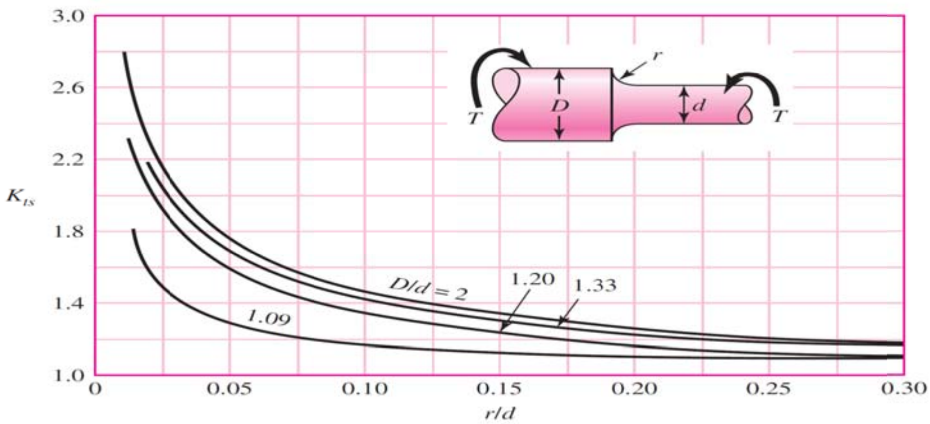


Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

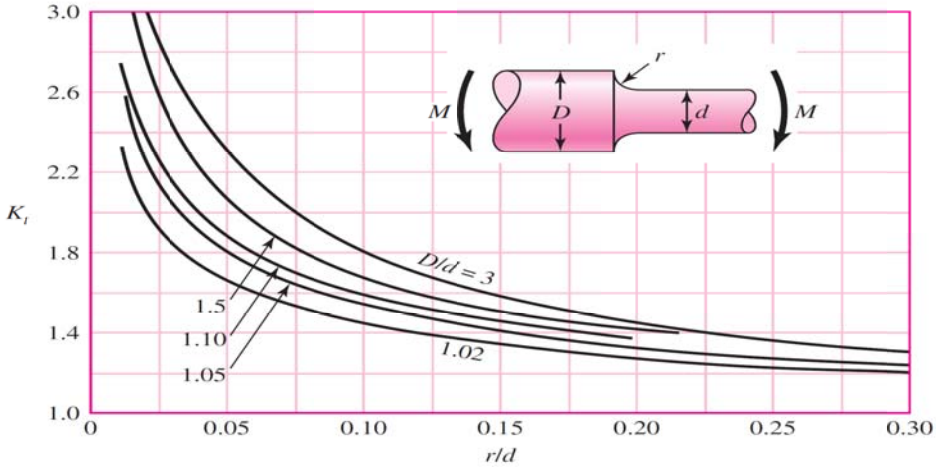


Figure A–15–10

Round shaft in torsion with transverse hole.

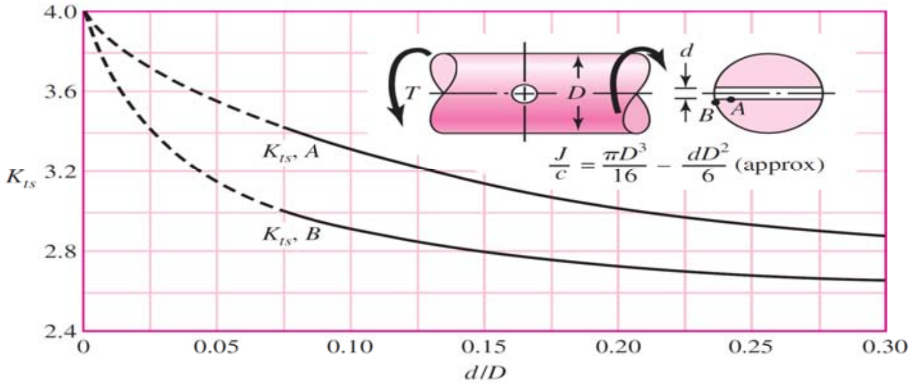


Figure A–15–11

Round shaft in bending with a transverse hole. $\sigma_0 = M/[(\pi D^3/32) - (dD^2/6)]$, approximately.

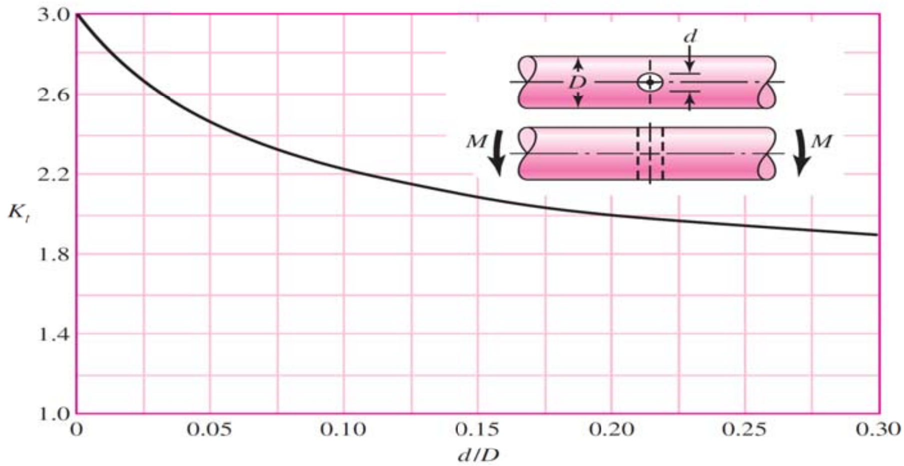


Figure A–15–12

Plate loaded in tension by a pin through a hole. $\sigma_0 = F/A$, where $A = (w - d)t$. When clearance exists, increase K_t 35 to 50 percent. *(M. M. Frocht and H. N. Hill, "Stress Concentration Factors around a Central Circular Hole in a Plate Loaded through a Pin in Hole," J. Appl. Mechanics, vol. 7, no. 1, March 1940, p. A-5.)*

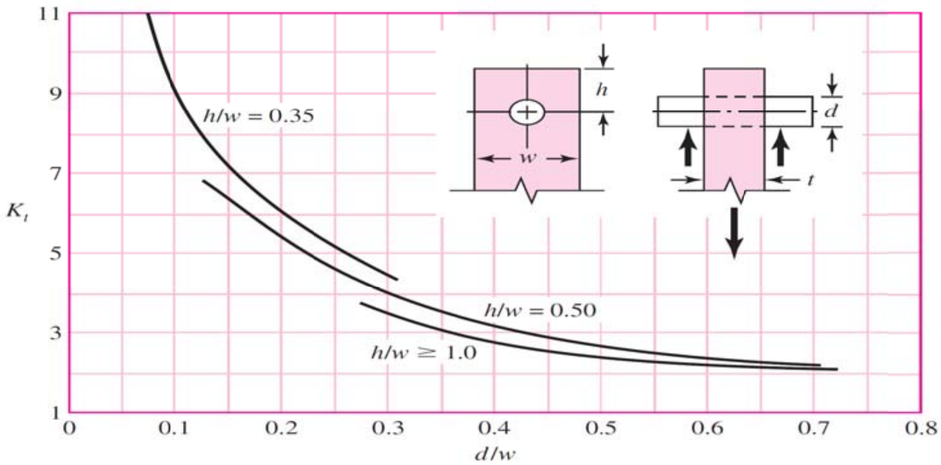


Figure A–15–13

Grooved round bar in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.

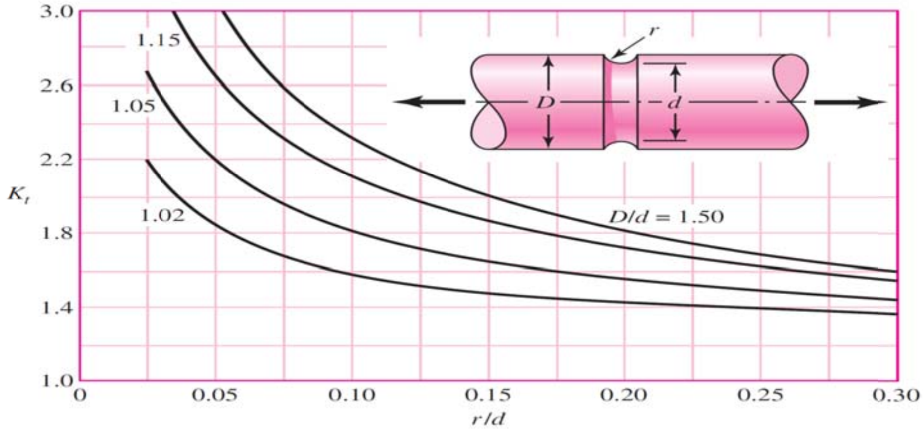


Figure A–15–14

Grooved round bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

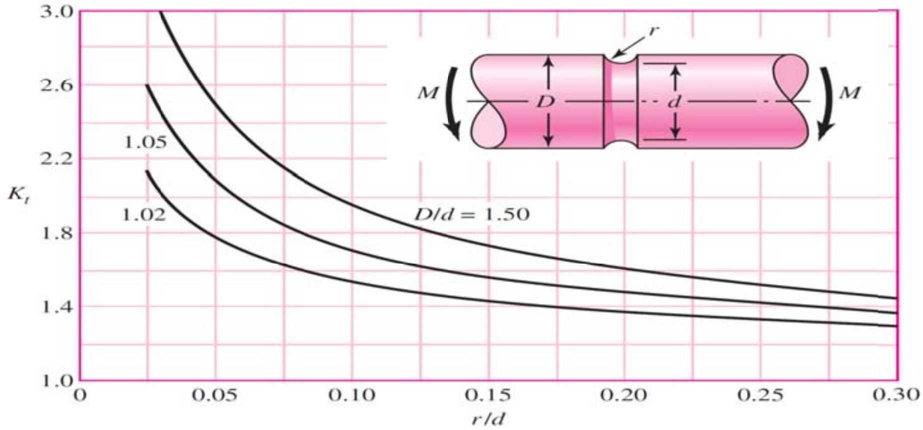


Figure A–15–15

Grooved round bar in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.

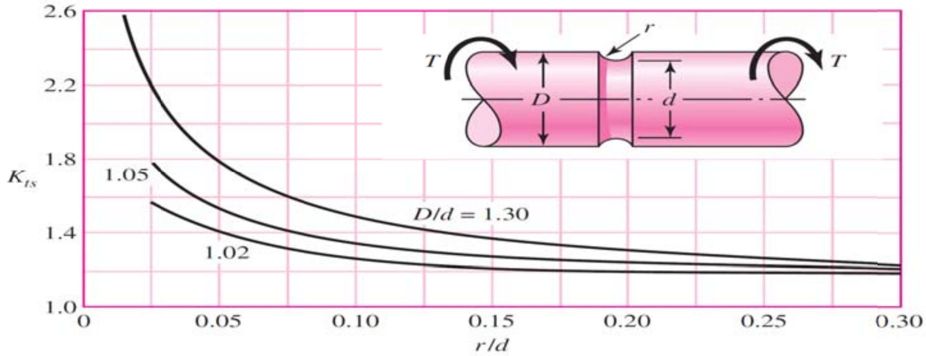


Figure A–15–16

Round shaft with flat-bottom groove in bending and/or tension.

$$\sigma_0 = \frac{4P}{\pi d^2} + \frac{32M}{\pi d^3}$$

Source: W. D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed. John Wiley & Sons, New York, 1997, p. 115

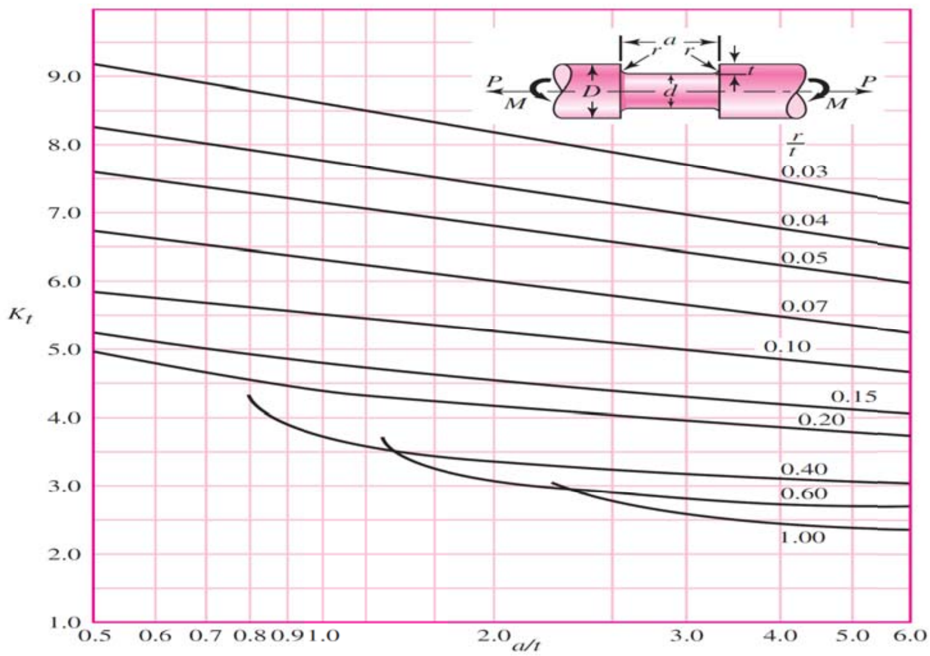
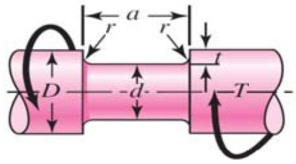


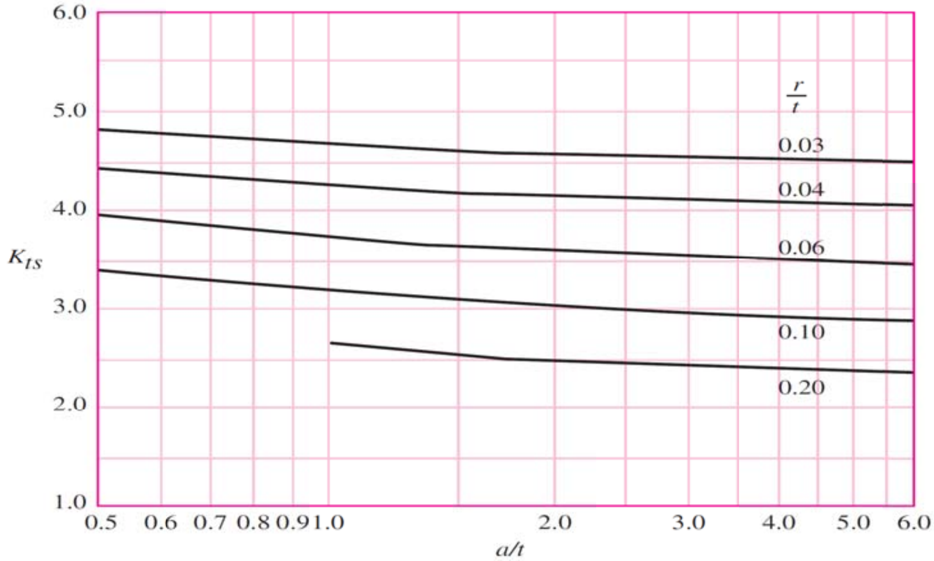
Figure A–15–17

Round shaft with flat-bottom groove in torsion.

$\tau_0 = \dfrac{16T}{\pi d^3}$

Source: W. D. Pilkey, *Peterson's Stress Concentration Factors,* 2nd ed. John Wiley & Sons, New York, 1997, p. 133





Millimeters

0.05, 0.06, 0.08, 0.10, 0.12, 0.16, 0.20, 0.25, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 1.0, 1.1, 1.2, 1.4, 1.5, 1.6, 1.8, 2.0, 2.2, 2.5, 2.8, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 8.0, 9.0, 10, 11, 12, 14, 16, 18, 20, 22, 25, 28, 30, 32, 35, 40, 45, 50, 60, 80, 100, 120, 140, 160, 180, 200, 250, 300

Renard Numbers*

1st choice, R5: 1, 1.6, 2.5, 4, 6.3, 10

2d choice, R10: 1.25, 2, 3.15, 5, 8

3d choice, R20: 1.12, 1.4, 1.8, 2.24, 2.8, 3.55, 4.5, 5.6, 7.1, 9

4th choice, R40: 1.06, 1.18, 1.32, 1.5, 1.7, 1.9, 2.12, 2.36, 2.65, 3, 3.35, 3.75, 4.25, 4.75, 5.3, 6, 6.7, 7.5, 8.5, 9.5

1 UNS No.	2 SAE and/or AISI No.	3 Processing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
G10450	1045	HR	570 (82)	310 (45)	16	40	163
		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197
G10600	1060	HR	680 (98)	370 (54)	12	30	201
G10800	1080	HR	770 (112)	420 (61.5)	10	25	229
G10950	1095	HR	830 (120)	460 (66)	10	25	248