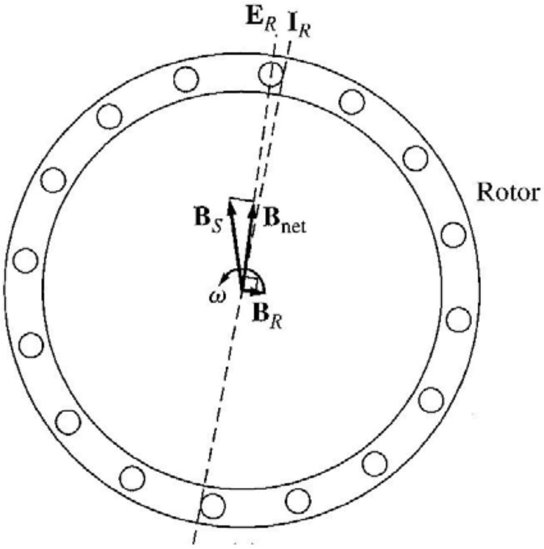
Chapter 7: Induction Motor (Part II)

Having looked at the principles of operation and equivalent circuit of the induction motor, an examination of the torque-speed relationship will be carried out.

7.5. Induction motor torque-speed characteristics

Induced torque from a physical standpoint

No load condition



On the left is a figure of the magnetic fields in an **induction motor at no load**

⇒ rotor speed very nearly at __________________________

Currents in the **stator** will produce a **stator field** $\bar{\boldsymbol{B}}_{\boldsymbol{s}}$.

The **induced rotor currents** will also produce field $\bar{B}_{\text{R}}$.

The **net magnetic field**, $\bar{B}_{\text{net}}$ is produced by the **combination of these two fields**, whereby:

- $\bar{B}_{\text{net}}$ is produced by magnetising current $\bar{I}_{\text{M}}$
- $|\bar{I}_{\text{M}}|$ and hence $\bar{B}_{\text{net}}$ directly proportional to $\bar{E}_1$

(refer to the induction motor equivalent circuit)

If $\bar{E}_1$ is **constan**t ⇒ ____________________________________

In **reality**, $\bar{E}_1$ **varies as load changes** due to the voltage drops across the stator impedances R_1 and X_1.
However, these voltage drops are relatively small
⇒ <u>So $\bar{E}_1$ is approximately constant with changes in load</u>

At no load:

- ***n_m* is near *n_{sync}*** ⇨ slip s is very small
  (relative motion between rotor and $\bar{B}_{\text{net}}$ is small)

- the rotor induced voltage $\bar{E}_{\text{R}}$ ____________________
  (since $e_{ind} \propto v_{rel}$)

- Hence, **small $\bar{I}_{\text{R}}$ will flow** in the rotor and will **almost** be **in phase** with $\bar{E}_{\text{r}}$.
  (Due to rotor frequency $f_r \propto s$ ⇨ $X_R \propto f_r$)

- Small $\bar{I}_{\text{R}}$ produces <u>a small magnetic field $\bar{B}_{\text{R}}$</u> at an angle **slightly greater than 90° behind** $\bar{B}_{\text{net}}$.

- Therefore, the **induced torque** will be ________________
  due to small $\vec{B}_R$ (just enough to overcome the motor's rotational losses) since:

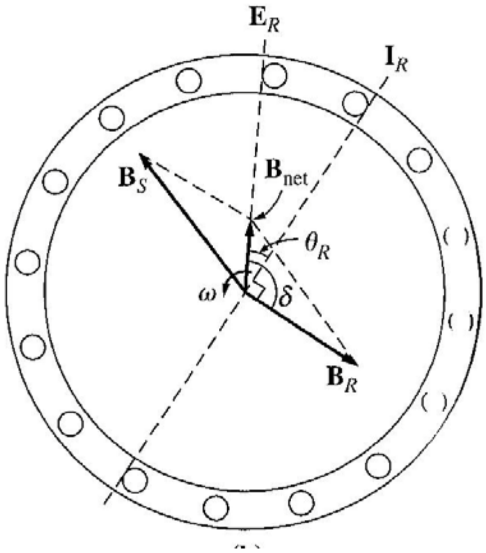
$$\tau_{\text{ind}} = k\bar{B}_{\text{R}} \times \bar{B}_{\text{net}}$$

and the magnitude is given by

$$\tau_{\text{ind}} = kB_{\text{R}}B_{\text{net}} \sin \delta$$

Note: Even though $\bar{I}_{\text{R}}$ is small, $\bar{I}_{\text{S}}$ must be quite large to supply most of $\bar{B}_{\text{net}}$. Hence, large no load currents in IM's compared to other types of machines.

On load condition



On the left is a figure of the magnetic fields in a **loaded induction motor**

- As the **load increases**, slip s increases and the **rotor speed** ________________
- A ________________ rotor **induced voltage** $\bar{E}_{\text{R}}$ is produced. Hence, $\bar{I}_{\text{R}}$ flowing will be larger.
- Hence, $\bar{B}_{\text{R}}$ also ______________

- However, the **angle** of $\bar{I}_R$ and $\bar{B}_R$ **changes** since:
  larger slip $\Rightarrow$ rise in f_r $\Rightarrow$ increase in X_R.
- Therefore, $\bar{I}_R$ **lags further** behind $\bar{E}_R$.

- The torque angle δ has also ______________________ .

The increase in $\bar{B}_R$ tends to increase the torque whereas the increase in δ tends to decrease torque (since $\delta > 90°$).

But the **effect of** $\bar{B}_R$ **is larger than the effect of increase** in δ.

Hence, the **overall torque increases** to supply the motor's increased load.

As **load is further increased** (δ increases):
'sin δ' term decreases (the value is going towards the 0 cross over point for a sine wave) at a much **greater rate** than the **increment of** $\bar{B}_R$.

At this point, **any further increase** will **reduce torque** and hence will stop the motor. This effect is known as **pullout torque**.

Modelling the torque-speed characteristics of an induction motor

We know that, $\tau_{\text{ind}} = kB_R B_{\text{net}} \sin\delta$.

Each term can be considered separately to derive the overall torque behaviour:

a) $\bar{B}_R \propto \bar{I}_R$ (provided the rotor **core is unsaturated**). Hence, $\bar{B}_R$ **increases with** $\bar{I}_R$ which in turn **increases with slip** (decrease in speed).

b) $\bar{B}_{\text{net}} \propto \bar{E}_R$ and will remain **approximately constant**.

c) The **angle δ increases with slip**. Hence, **'sin δ' term decreases**. From the figure of the induction motor *on load condition*,

$$\delta = \theta_R + 90°$$

Where θ_R = the rotor power-factor angle

Therefore, $\sin\delta = \sin(\theta_R + 90°) = \cos\theta_R$= **power factor of rotor**

Rotor power factor angle can be calculated since:

$$\theta_{\mathrm{R}} = \tan^{-1}\frac{X_{\mathrm{R}}}{R_{\mathrm{R}}} = \tan^{-1}\frac{sX_{\mathrm{R0}}}{R_{\mathrm{R}}}$$

Hence, the rotor power factor:

The **torque-speed characteristic** can be constructed from the graphical manipulation of the three properties (a)-(c) which is shown on the next page.

The characteristic curve can be divided into three regions:

1. ***Low-slip region*** ($s\uparrow$ linearly, $n_{\mathrm{m}}\downarrow$ linearly) **:**
   - X_{R} negligible $\Rightarrow$ $\mathrm{PF}_{\mathrm{R}} \approx 1$
   - $\bar{I}_{\mathrm{R}}$ increases linearly with s

   Contains the entire steady-state **normal operating range** of an induction motor.

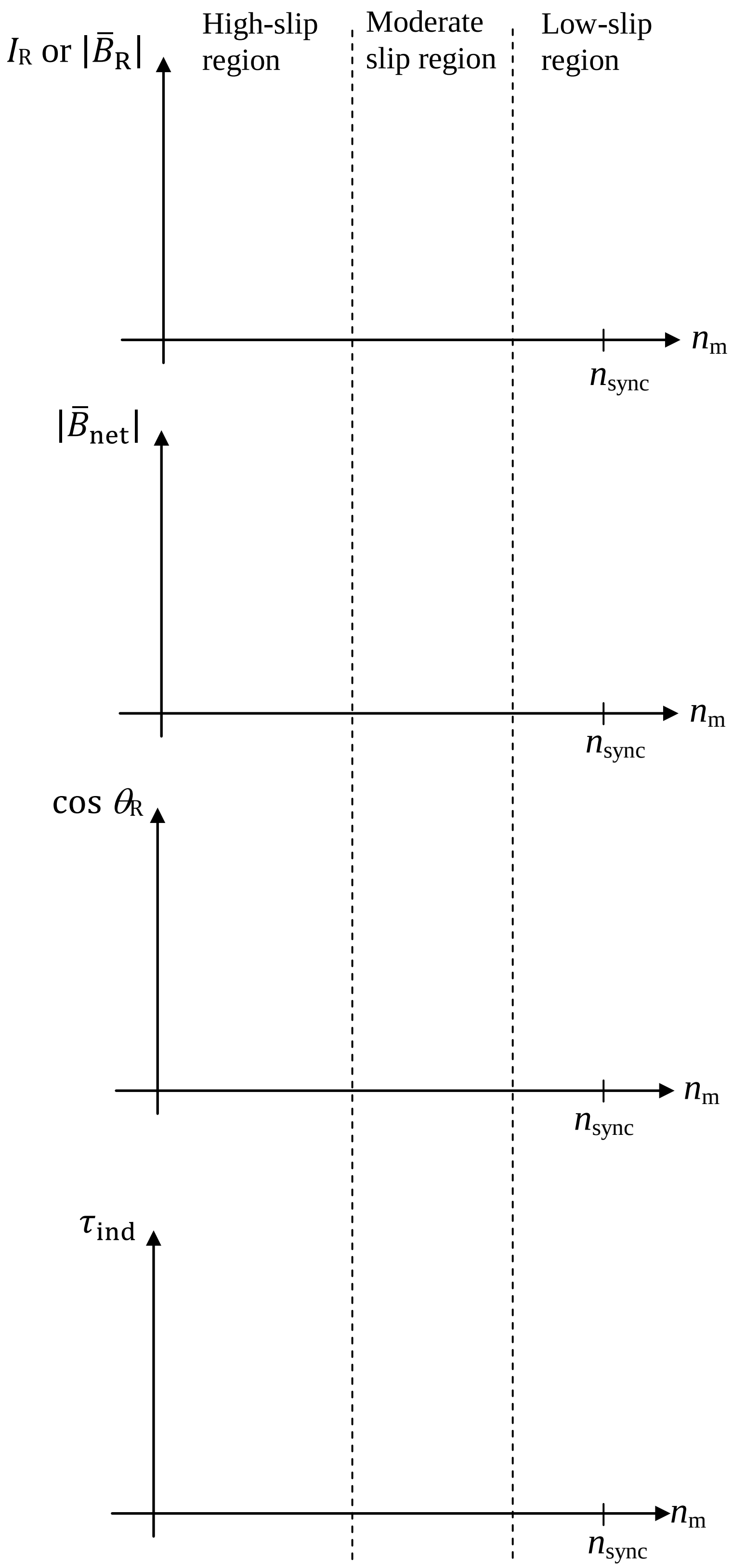
2. ***Moderate-slip region***:
   - X_{R} same order of magnitude as R_{R} $\Rightarrow$ PF_{R} droops
   - $\bar{I}_{\mathrm{R}}$ doesn't increase as rapidly as in low-slip region

   Peak torque (**pullout torque**) occurs **in this region**.

3. ***High-slip region***:
   - Increase in $\bar{I}_{\mathrm{R}}$ completely overshadowed by decrease in PF_{R}.
   - τ_{ind} decreases with increase in load

Note:

- Typical pullout torque ≈ 200% to 250% of τ_{rated}.
- The starting torque ≈150% of the τ_{rated}.
  Hence induction motor may be started at full load.


High-slip region
Moderate slip region
Low-slip region
I_R or $|\bar{B}_R|$
n_m
n_{sync}
$|\bar{B}_{net}|$
n_m
n_{sync}
$\cos\theta_R$
n_m
n_{sync}
τ_{ind}
n_m
n_{sync}

The derivation of the induction motor induced-torque equation

A general expression for induced torque can be derived from the equivalent circuit of the motor as well as the power flow diagram.

It is known that,

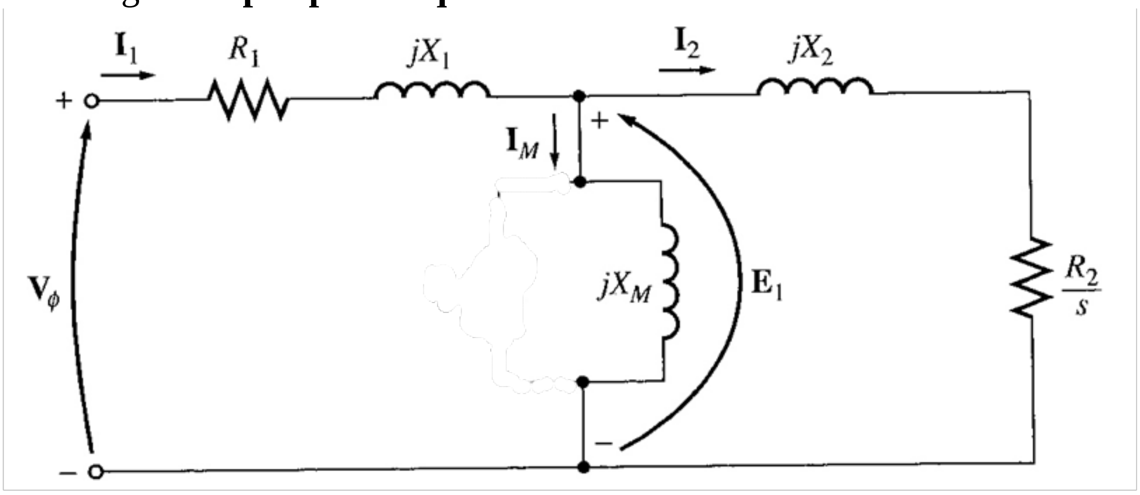
$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_{\text{m}}}$$

or

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}$$

The latter is more useful since ω_{sync} **is always constant**.
Hence, to find an expression for τ_{ind}, we must derive an expression for P_{AG}.

Referring to the **per-phase equivalent circuit** of the motor:



$$P_{\text{AG}} = I_2^2 \frac{R_2}{s}$$

Therefore, the total air gap power:

$$P_{\text{AG}} = 3I_2^2 \frac{R_2}{s}$$

Hence, if I_2 can be determined, then P_{AG} and is τ_{ind} known.

This can be easily achieved by constructing a **Thevenin equivalent circuit** to the left of the impedances X_2 and R_2/s.
Thevenin's theorem states that any linear circuit that can be separated by two terminals from the rest of the system can be replaced by a single voltage source in series with an equivalent impedance.

Therefore, the **per-phase equivalent circuit** reduces to the following **Thevenin equivalent circuit**:

Calculation via Thevenin equivalent method:

1) Derive the **Thevenin voltage** (potential divider rule): open-circuit the terminals after the R_c and X_m branch. Hence,

$$\bar{V}_{\mathrm{TH}} = \frac{jX_{\mathrm{M}}}{R_1 + jX_1 + jX_{\mathrm{M}}}\bar{V}_{\phi}$$

Hence, the **magnitude** is:

Since $\boldsymbol{X_{\mathrm{M}}}$ >> $\boldsymbol{X_1}$ and $\boldsymbol{X_1 + X_{\mathrm{M}} \gg R_1}$, the magnitude of the Thevenin voltage is quite accurately approximated by:

2) Find the **Thevenin impedance:** take out the source and replace by a short circuit. Hence,

$$Z_{\mathrm{TH}} = R_{\mathrm{TH}} + jX_{\mathrm{TH}} = \frac{jX_{\mathrm{M}}(R_1 + jX_1)}{R_1 + j(X_1 + X_{\mathrm{M}})}$$

Again, since $\boldsymbol{X_{\mathrm{M}}} >> \boldsymbol{X_1}$ and $\boldsymbol{X_1} + \boldsymbol{X_{\mathrm{M}}} \gg \boldsymbol{R_1}$,

3) Therefore, the current $\bar{I}_2$ flowing in the Thevenin equivalent circuit of the induction motor is given by:

$$\bar{I}_2 = \frac{\bar{V}_{\mathrm{TH}}}{R_{\mathrm{TH}} + \frac{R_2}{s} + j(X_{\mathrm{TH}} + X_2)}$$

And the current magnitude will be:

$$|\bar{I}_2| = I_2 = \frac{V_{\mathrm{TH}}}{\sqrt{\left(R_{\mathrm{TH}} + \frac{R_2}{s}\right)^2 + (X_{\mathrm{TH}} + X_2)^2}}$$

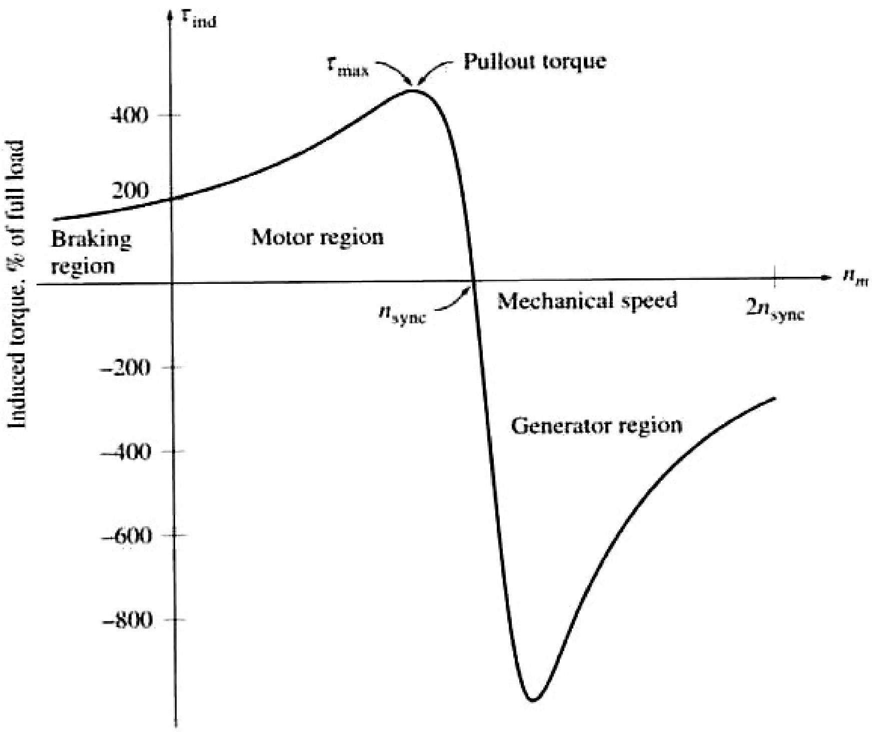
Hence, the **air gap power** is:

$$P_{\mathrm{AG}} = 3I_2^2\frac{R_2}{s} = \frac{3{V_{\mathrm{TH}}}^2\frac{R_2}{s}}{\left(R_{\mathrm{TH}} + \frac{R_2}{s}\right)^2 + (X_{\mathrm{TH}} + X_2)^2}$$

Finally, the **induced torque expression** is:

$$\tau_{\mathrm{ind}} = \frac{P_{\mathrm{AG}}}{\omega_{\mathrm{sync}}}$$

A **plot of the induction motor torque as a function of speed** (and slip) above and below the normal operating range is shown in the next page.



Comments on the induction motor torque-speed curve

1) At **synchronous speed,** $\tau_{\text{ind}} = 0$.

2) The **curve is nearly linear** between **no load and full load**.

3) The maximum torque is known as **pullout torque** or **breakdown torque**. It is approximately 2 to 3 times the rated full-load torque of the motor.

4) The **starting torque** is slightly **larger** than its full-load torque. So, IM will start carrying any load it can supply at full power

5) Torque for a given slip varies as **square** of the **applied voltage**. This is useful as one form of IM speed control.

6) If rotor is driven **faster than synchronous speed**,

τ_{ind} direction _______________ and machine becomes a _______________

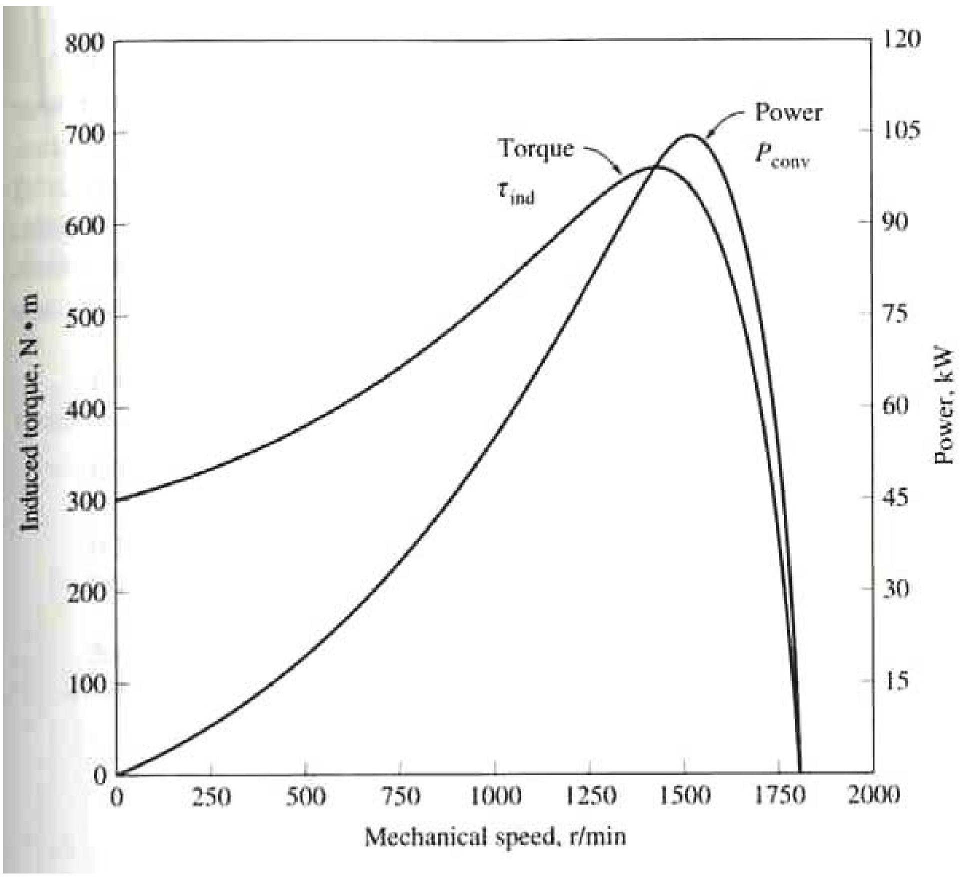
7) If motor is **turning backward relative** to the **direction** of magnetic fields (achieved by reversing the magnetic field rotation direction),

τ_{ind} will **stop the machine** very **rapidly (braking)** and try to rotate in the other direction.
Can be achieved by switching two stator phases, i.e. ***plugging***.

The power converted to mechanical form in an induction motor is:

$$P_{\text{conv}} = \tau_{\text{ind}}\omega_{\text{ind}}$$

Hence, a characteristic to show the variation of converted power with speed (i.e. load) can be obtained.



Note that:

- **Peak power supplied** by the induction motor occurs at **different speed** to **maximum torque**.
- **No power** is converted **when rotor speed = 0**.

Maximum (Pullout) torque in an induction motor

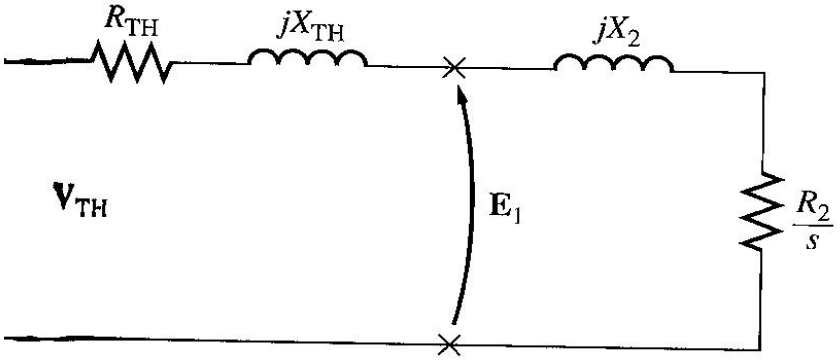
Maximum τ_{ind} occurs when $\boldsymbol{P}_{\mathbf{AG}}$ **is maximum**.

P_{AG} is maximum **when power consumed** by **resistor** $\boldsymbol{R_2/s}$ is **maximum**.

According to the maximum power transfer theorem:

Maximum power transfer is achieved when the magnitude of the load impedance is equal to the source impedance.

Hence, referring to the Thevenin equivalent circuit of the induction motor:



Source impedance = $Z_{\text{source}} = R_{\text{TH}} + jX_{\text{TH}} + jX_2$

Load impedance = $\frac{R_2}{s}$

Hence, **maximum power transfer occurs when**:

$$\frac{R_2}{s} = \sqrt{R_{\text{TH}}^2 + j(X_{\text{TH}} + X_2)^2}$$

Solving equation above for slip, we see that the **slip at pullout torque** is given by:

$$s_{\text{max}} = \frac{R_2}{\sqrt{R_{\text{TH}}^2 + j(X_{\text{TH}} + X_2)^2}}$$

Hence, the resulting equation for the **maximum or pullout torque** is:

$$\tau_{\max} = \frac{3V_{\mathrm{TH}}^2}{2\omega_{\mathrm{sync}}\left[R_{\mathrm{TH}} + \sqrt{R_{\mathrm{TH}}^2 + (X_{\mathrm{TH}} + X_2)^2}\right]}$$

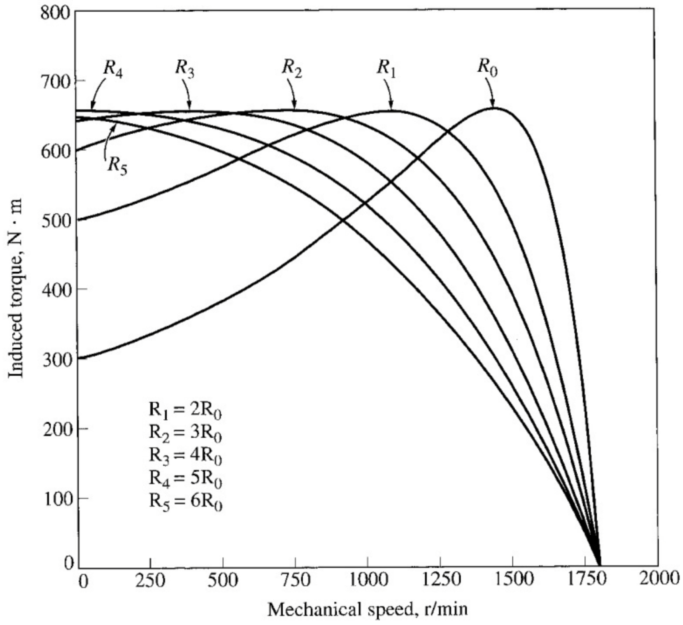
From this we see that:

- Torque is related to the square of supplied voltage.
- Torque is inversely proportional to stator impedances and rotor reactance.
- **$s_{\max}$ is directly proportional to R_2.**
- **$\tau_{\max}$ is independent of R_2.**

As **increase R_2** (i.e. increase $s_{\max}$):

- **pullout speed** of motor **decreases**
- **maximum torque** remains **constant**
- **starting torque increases**

This is **an advantage of** a **wound rotor** induction motor.



Example 7.4

A 2-pole, 50-Hz induction motor supplies 15 kW to a load at a speed of 2950 r/min.

(a) What is the motor's slip?

(b) What is the induced torque in the motor in Nm under these conditions?
(c) What will the operating speed of the motor be if its torque is doubled?
(d) How much power will be supplied by the motor when the torque is doubled?

Example 7.5

A 460-V, 18.65-kW, 60-Hz, 4-pole, Y-connected wound rotor induction motor has the following impedances in ohms per-phase referred to the stator circuit:

$R_1 = 0.641\ \Omega$ $R_2 = 0.332\ \Omega$
$X_1 = 1.106\ \Omega$ $X_2 = 0.464\ \Omega$ $X_m = 26.3\ \Omega$

(a) What is the max torque of this motor? At what speed and slip does it occur?
(b) What is the starting torque?
(c) When the rotor resistance is doubled, what is the speed at which the max torque now occurs? What is the new starting torque?

7.6. Variations in induction motor torque-speed characteristics

Based on the properties of the induction motor torque-speed characteristics, machine designers are faced with a dilemma – **high stating torque or high efficiency?**

Design Option 1:
High R_2 (or R_R)
⇩
- **High starting torque**
- Slip **high** at normal operating conditions
- **Decreased efficiency**
  $P_{conv} = (1-s)\,P_{AG}$
  $\Rightarrow s\uparrow, P_{conv}\downarrow, \eta\downarrow$

Design Option 2:
Low R_2 (or R_R)
⇩
- **Low starting torque**, **high** starting current
- Slip **low** at normal operating conditions
- **High efficiency**
  $P_{conv} = (1-s)\,P_{AG}$
  $\Rightarrow s\downarrow, P_{conv}\uparrow, \eta\uparrow$

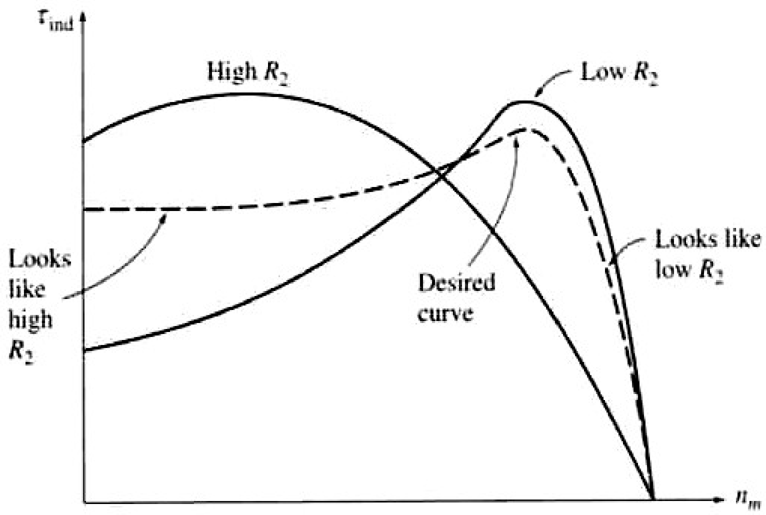
Solution 1:
Use a **wound rotor induction motor** with:

- **extra resistance added** to rotor **during starting**
- **then removed** for better efficiency **during normal operations**

But wound rotor motors are**:**

- **more expensive**
- need **more maintenance**
- more complex automatic control circuit

Better solution:
Utilise leakage reactance in induction motor design **to achieve** the **desired torque-speed curve** shown below**.**



Torque-speed characteristics curve combining high-resistance effects at low speeds (high slip) with low resistance effects at high speed (low slip).

Control of motor characteristics by cage rotor design

Leakage reactance, X_2 (referred rotor leakage reactance) is due to

⇨ rotor flux lines that do not couple with stator windings

If **rotor bar** (or part of a bar) is:

far away from stator

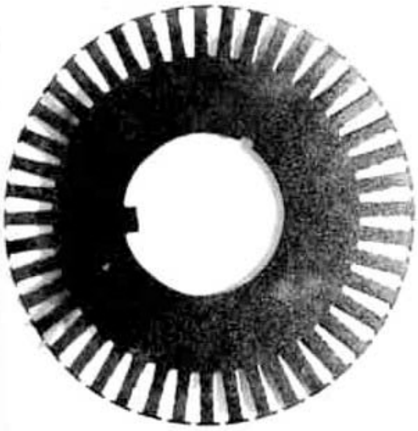
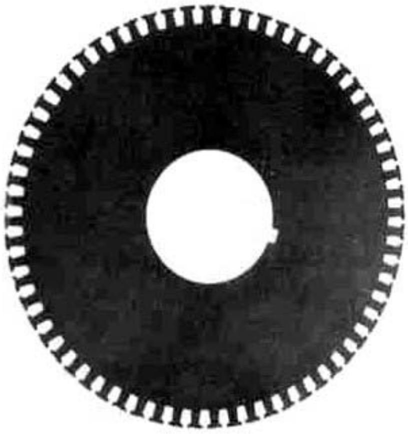
more rotor flux leakage
⇨ **value of X_2** ________

nearer to stator

less rotor flux leakage
⇨ **value of X_2** ________

Generally, the farther away the rotor bar is from the stator, the greater is X_2, since only a small percentage of the bar's flux will reach the stator.

Typical rotor designs:

National Electrical Manufacturers Association (NEMA) **design**	**Class A** 	**Class D** 
Rotor bars	Quite large cross section, placed near surface	Small cross section, placed near surface
R_2 or R_R		
X_2		
Pullout torque occurs at	Near n_{sync} (low slip)	Far from n_{sync} (high slip)
Starting torque		
Starting current	High	Low
Efficiency		
Typical applications	• driving fans • pumps • other machine tools	Extremely high-inertia type loads

NEMA Class A = typical induction motor design
NEMA Class D = like **wound rotor induction motor** with **extra resistance added** to rotor.

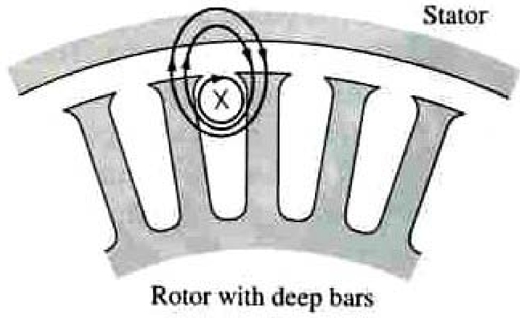
*How can a **variable rotor resistance** be produced to **combine** the **high starting torque and low starting current of Class D** with the **low normal operating slip and high efficiency of Class A**?*

Deep-bar and double-cage rotor designs

The basic concept is illustrated below:

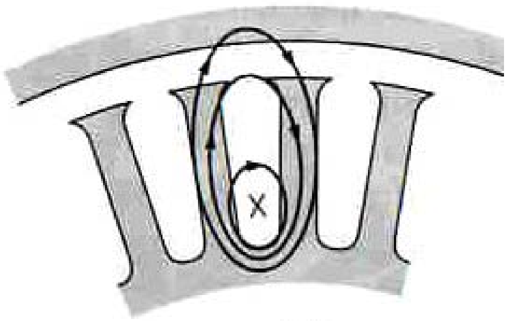
- current flowing in **upper portion of the bar**

  ⇒ flux leakage is low, <u>leakage inductance is small</u>



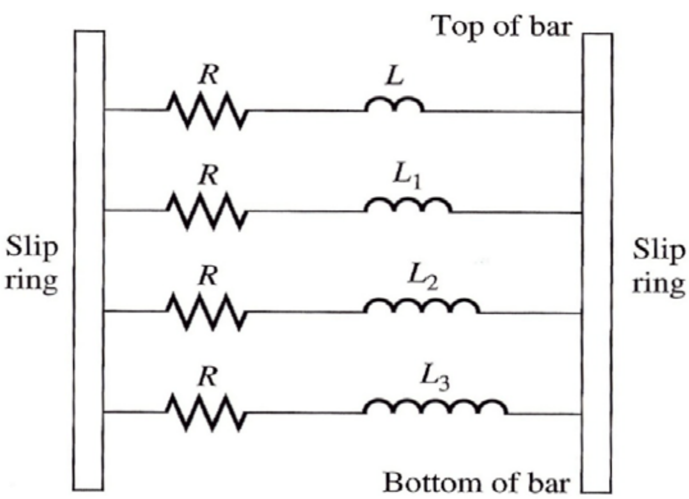
-
-
- current flowing in **lower portion of the bar**

  ⇒ flux leakage is high, ______________________



Since **all parts of the rotor bar are parallel electrically**, the bar essentially represents

⇒ a **series of parallel *R*-*L* circuits**



NEMA design Class B (deep-bar rotor)

Description: **Wide** cross-sectional **bars in deep slots**.

Upper part of a deep rotor bar: the current flowing is tightly coupled to the stator, and hence the **leakage inductance is small** in this region.

Deeper in the bar: the **leakage inductance is higher**.

At **low slips**:

- low rotor frequency
- ***X*** **lower** in all parallel paths (compared to *R*)
- impedance of all parts of bar approx. equal to *R*
- **equal current flows** through **all parts of bar**
- $\boldsymbol{R_R}$ **small** (due to large effective cross-sectional area), hence **good efficiency and higher normal operation speed.**

At **high slips (starting conditions)**:

- higher rotor frequency
- ***X*** **higher** in all parallel paths (compared to *R*)
- **current flow concentrated at upper-part of bar** (low-reactance part)
- $\boldsymbol{R_R}$ **high** (due to lower effective cross-sectional area), hence **high starting torque and lower starting current** (compared to Class A).

Application: similar to **class A**.

NEMA design Class C (double-cage rotor)

Description:
Large, low resistance set of **bars** buried deeply **in the rotor AND small, high resistance** set of **bars** at rotor **surface**.

It is similar to the deep-bar rotor, except that the difference between low-slip and high-slip operation is even **more exaggerated**.

At **high slips (starting conditions)**:

- only **small bars** are effective
- $\boldsymbol{R_R}$ **high**, hence **high starting torque**

At **low slips (normal operating speeds)**:

- **both bars** are effective
- $\boldsymbol{R_R}$ almost as **low** as in deep-bar rotor
- **Good efficiency**

Application: for **high starting torque load** such as loaded pumps, compressors and conveyors.

Typical torque-speed curves for different rotor designs