# Chapter 3
# PROPERTIES OF PURE SUBSTANCES

Mechanical Engineering Dept.
University of Diyala

# PURE SUBSTANCE

- **Pure substance**: A substance that has a fixed chemical composition throughout.
- Air is a mixture of several gases, but it is considered to be a pure substance.

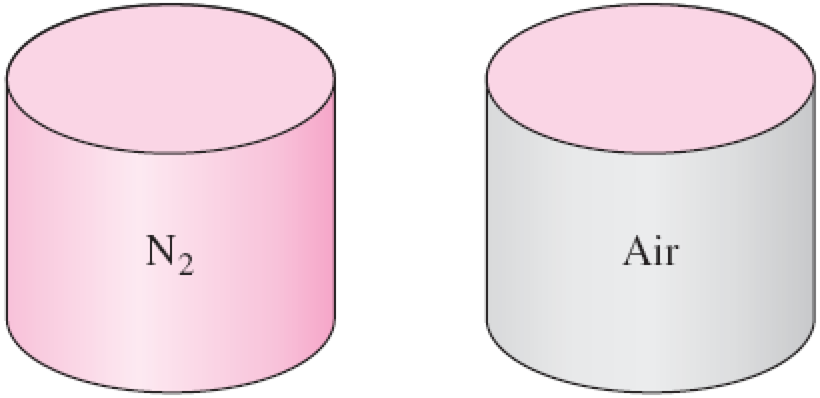
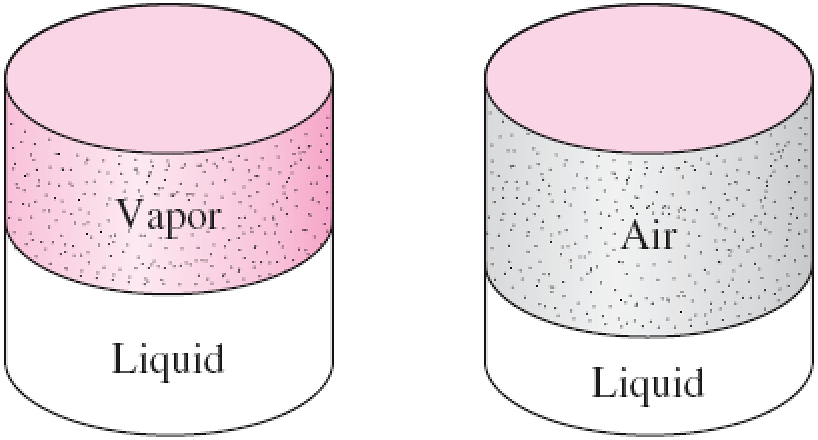


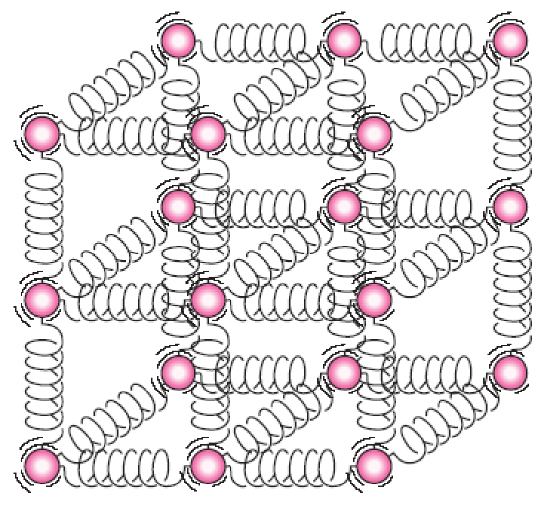
**FIGURE 3–1**
Nitrogen and gaseous air are pure substances.



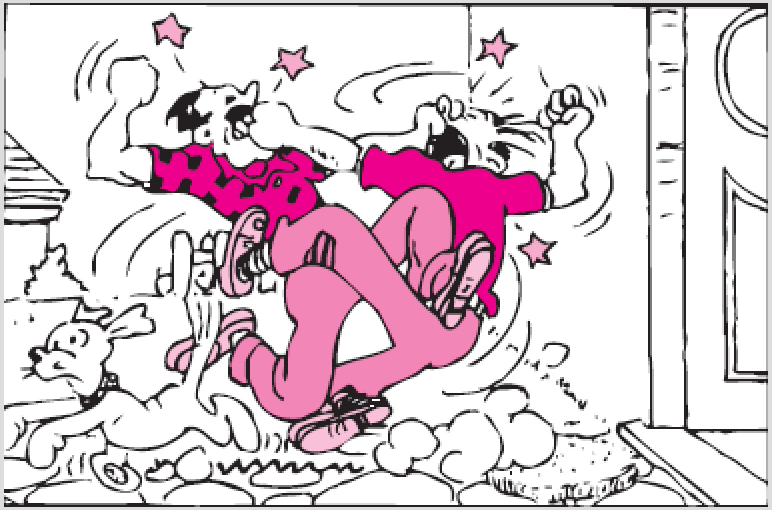
(*a*) $H_2O$ (*b*) Air

**FIGURE 3–2**
A mixture of liquid and gaseous water is a pure substance, but a mixture of liquid and gaseous air is not.

# PHASES OF A PURE SUBSTANCE



The molecules in a solid are kept at their positions by the large springlike inter-molecular forces.



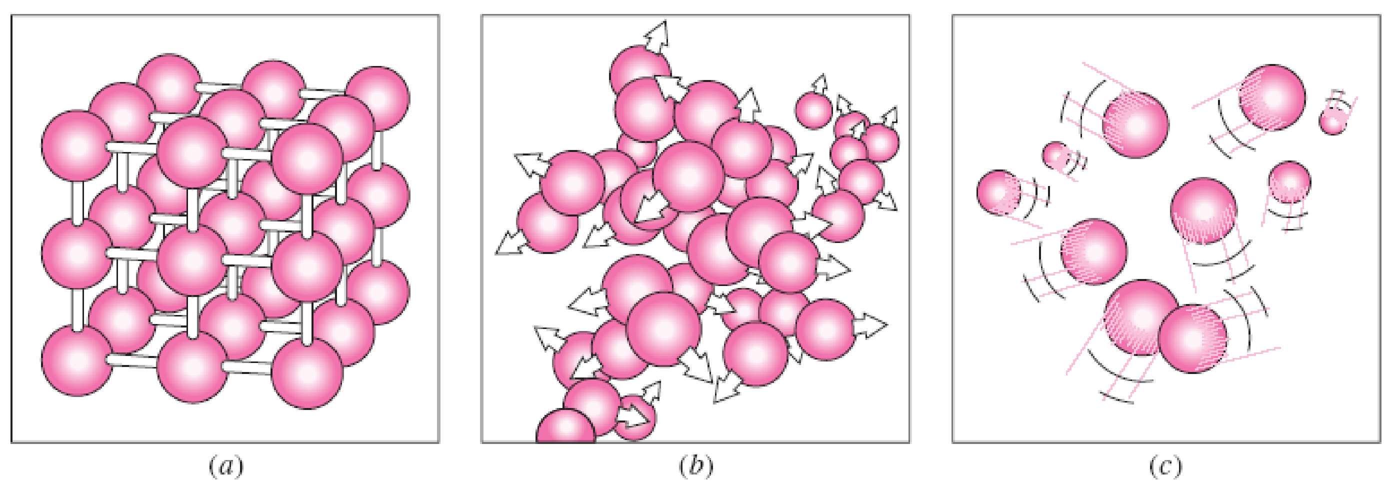


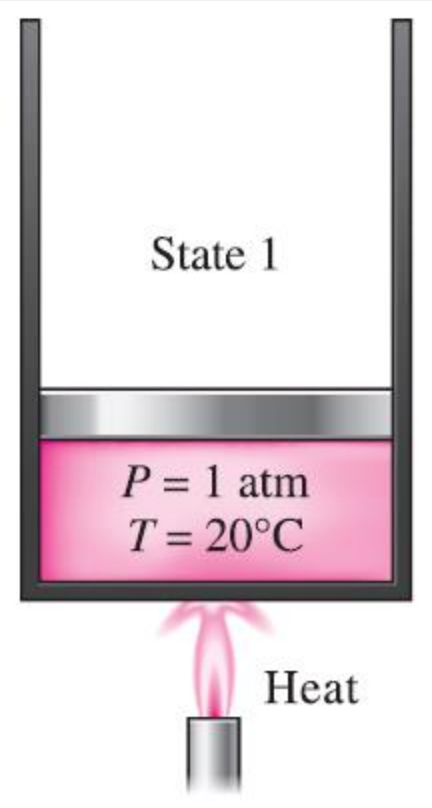
**FIGURE 3–5**

The arrangement of atoms in different phases: (*a*) molecules are at relatively fixed positions in a solid, (*b*) groups of molecules move about each other in the liquid phase, and (*c*) molecules move about at random in the gas phase.

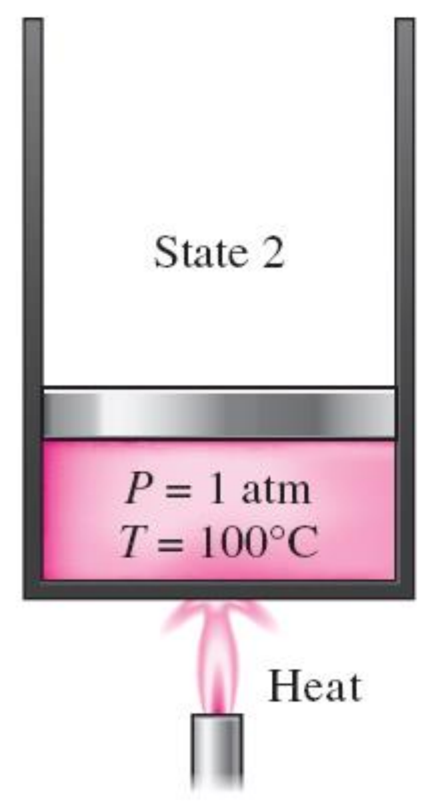
In a solid, the attractive and repulsive forces between the molecules tend to maintain them at relatively constant distances from each other.

# PHASE-CHANGE PROCESSES OF PURE SUBSTANCES

- **Compressed liquid (subcooled liquid)**: A substance that it is *not about to vaporize.*
- **Saturated liquid**: A liquid that is *about to vaporize.*

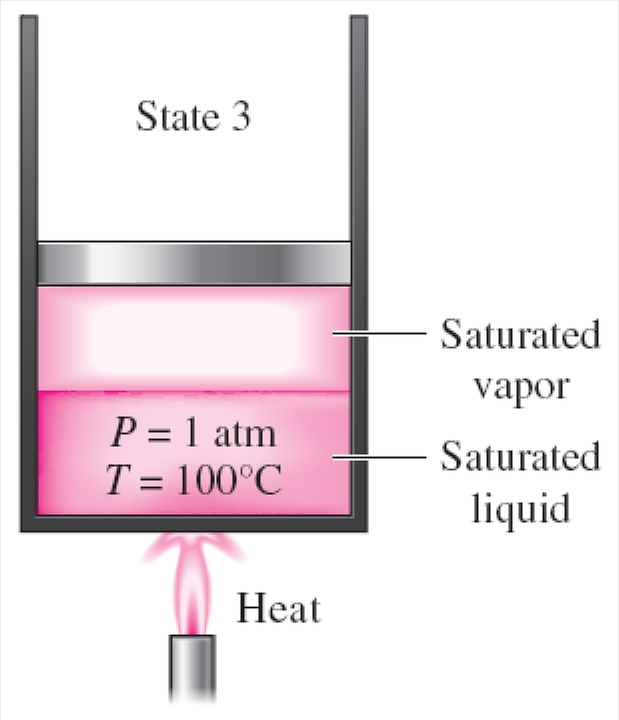


At 1 atm and 20°C, water exists in the liquid phase (***compressed liquid***).



At 1 atm pressure and 100°C, water exists as a liquid that is ready to vaporize (***saturated liquid***).

- **Saturated vapor**: A vapor that is *about to condense.*
- **Saturated liquid–vapor mixture**: The state at which the *liquid and vapor phases coexist* in equilibrium.
- **Superheated vapor**: A vapor that is *not about to condense* (i.e., not a saturated vapor).

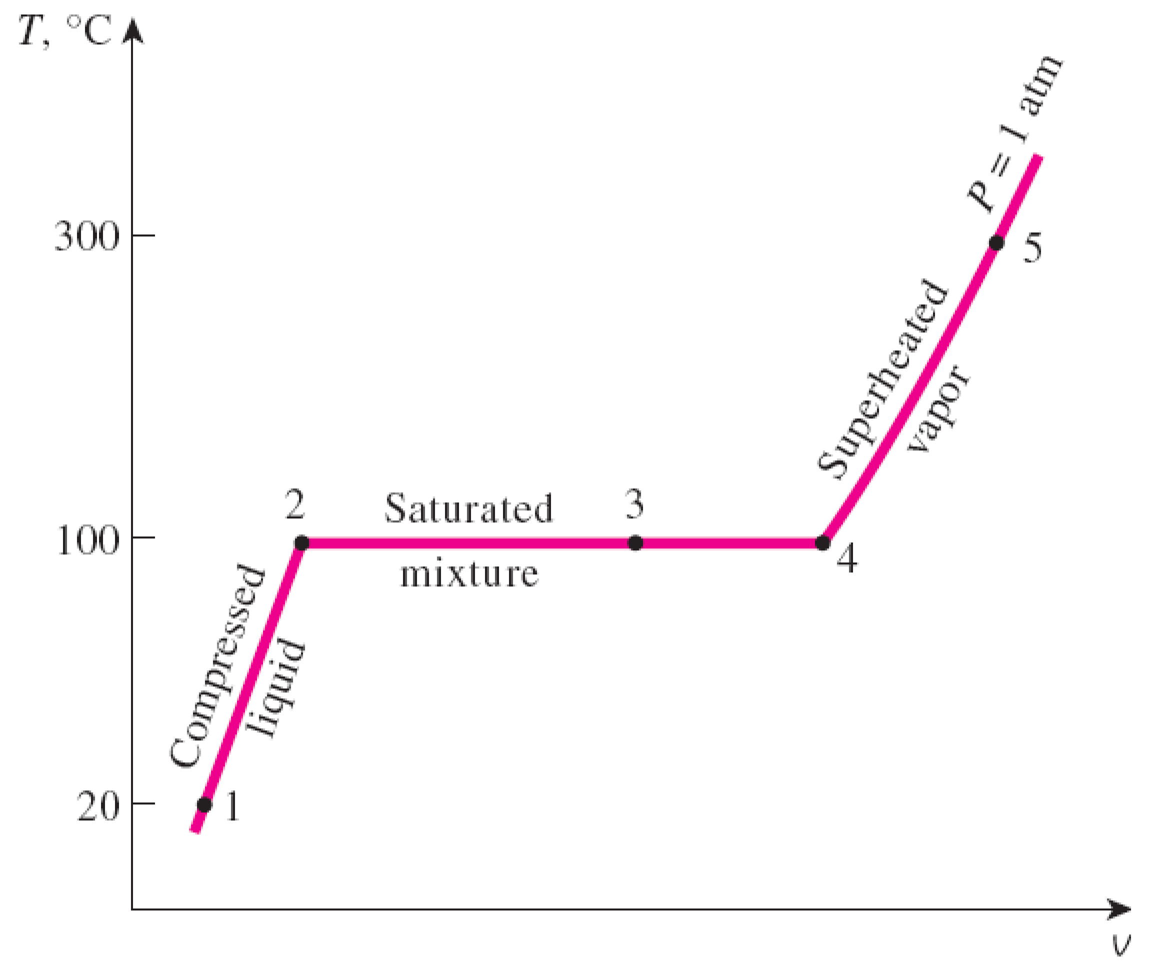


As more heat is transferred, part of the saturated liquid vaporizes (***saturated liquid–vapor mixture***).

At 1 atm pressure, the temperature remains constant at 100°C until the last drop of liquid is vaporized (***saturated vapor***).

As more heat is transferred, the temperature of the vapor starts to rise (***superheated vapor***).

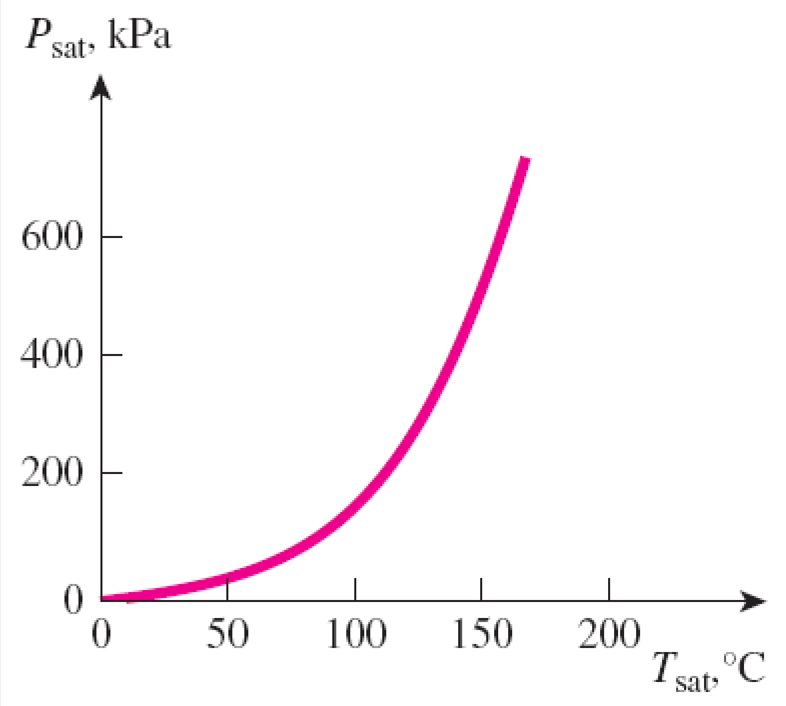
If the entire process between state 1 and 5 described in the figure is reversed by cooling the water while maintaining the pressure at the same value, the water will go back to state 1, retracing the same path, and in so doing, the amount of heat released will exactly match the amount of heat added during the heating process.



*T-v* diagram for the heating process of water at constant pressure.

# Saturation Temperature and Saturation Pressure

- The temperature at which water starts boiling depends on the pressure; therefore, if the pressure is fixed, so is the boiling temperature.
- Water boils at 100°C at 1 atm pressure.
- **Saturation temperature $T_{sat}$**: The temperature at which a pure substance changes phase at a given pressure.
- **Saturation pressure $P_{sat}$**: The pressure at which a pure substance changes phase at a given temperature.



The liquid–vapor saturation curve of a pure substance (numerical values are for water).

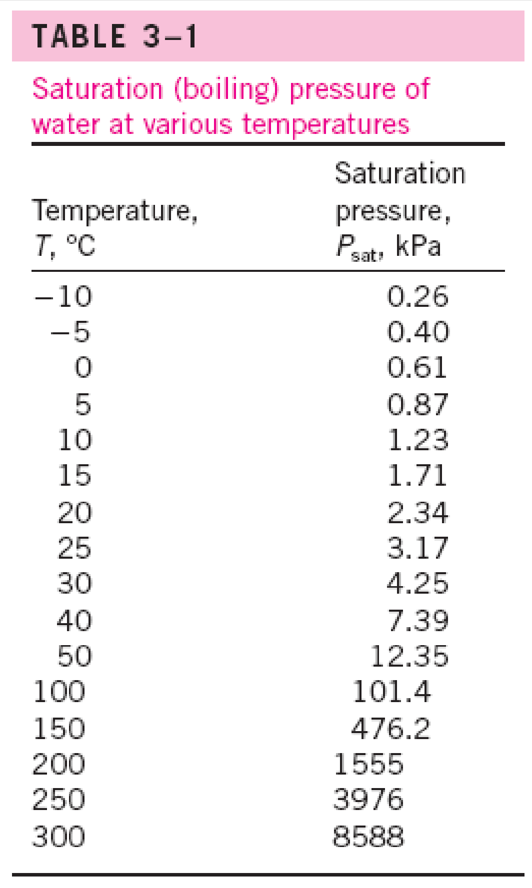


**TABLE 3–1**

Saturation (boiling) pressure of water at various temperatures

| Temperature, $T$, °C | Saturation pressure, $P_{sat}$, kPa |
|---|---|
| −10 | 0.26 |
| −5 | 0.40 |
| 0 | 0.61 |
| 5 | 0.87 |
| 10 | 1.23 |
| 15 | 1.71 |
| 20 | 2.34 |
| 25 | 3.17 |
| 30 | 4.25 |
| 40 | 7.39 |
| 50 | 12.35 |
| 100 | 101.4 |
| 150 | 476.2 |
| 200 | 1555 |
| 250 | 3976 |
| 300 | 8588 |

- **Latent heat**: The amount of energy absorbed or released during a phase-change process.
- **Latent heat of fusion**: The amount of energy absorbed during melting. It is equivalent to the amount of energy released during freezing.
- **Latent heat of vaporization:** The amount of energy absorbed during vaporization and it is equivalent to the energy released during condensation.
- The magnitudes of the latent heats depend on the temperature or pressure at which the phase change occurs.
- At 1 atm pressure, the latent heat of fusion of water is 333.7 kJ/kg and the latent heat of vaporization is 2256.5 kJ/kg.
- The atmospheric pressure, and thus the boiling temperature of water, decreases with elevation.

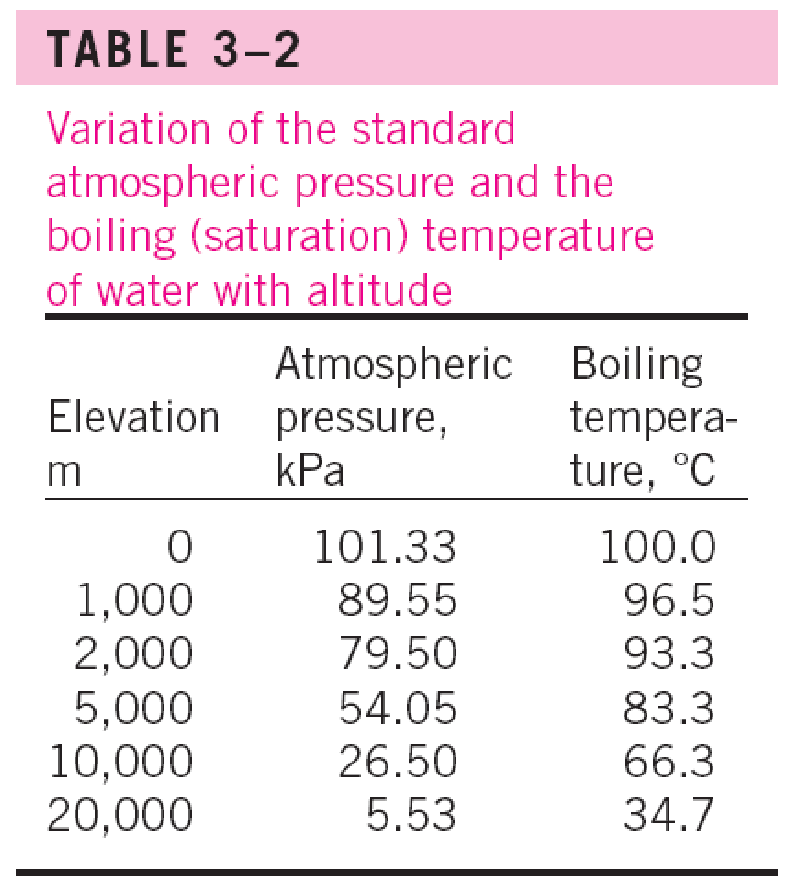


**TABLE 3–2**

Variation of the standard atmospheric pressure and the boiling (saturation) temperature of water with altitude

| Elevation m | Atmospheric pressure, kPa | Boiling temperature, °C |
|---|---|---|
| 0 | 101.33 | 100.0 |
| 1,000 | 89.55 | 96.5 |
| 2,000 | 79.50 | 93.3 |
| 5,000 | 54.05 | 83.3 |
| 10,000 | 26.50 | 66.3 |
| 20,000 | 5.53 | 34.7 |

# Some Consequences of $T_{sat}$ and $P_{sat}$ Dependence

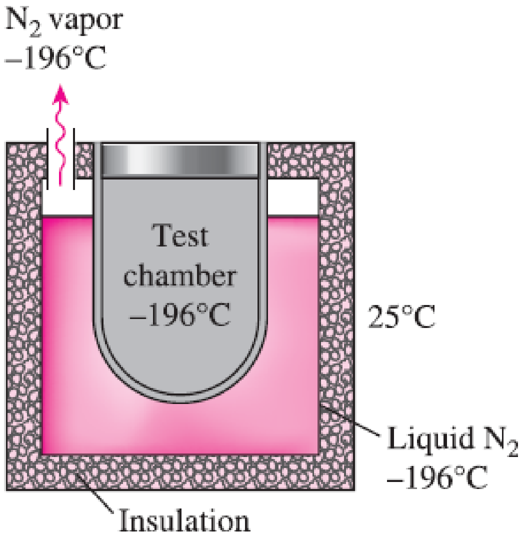
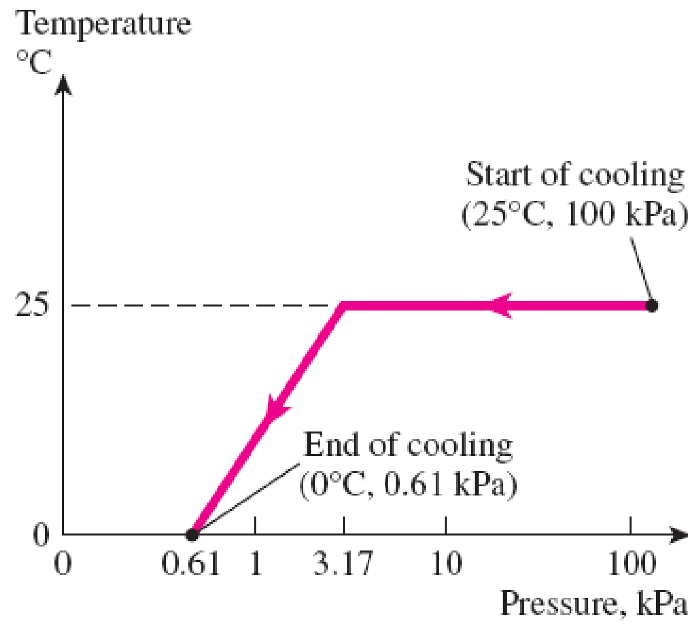


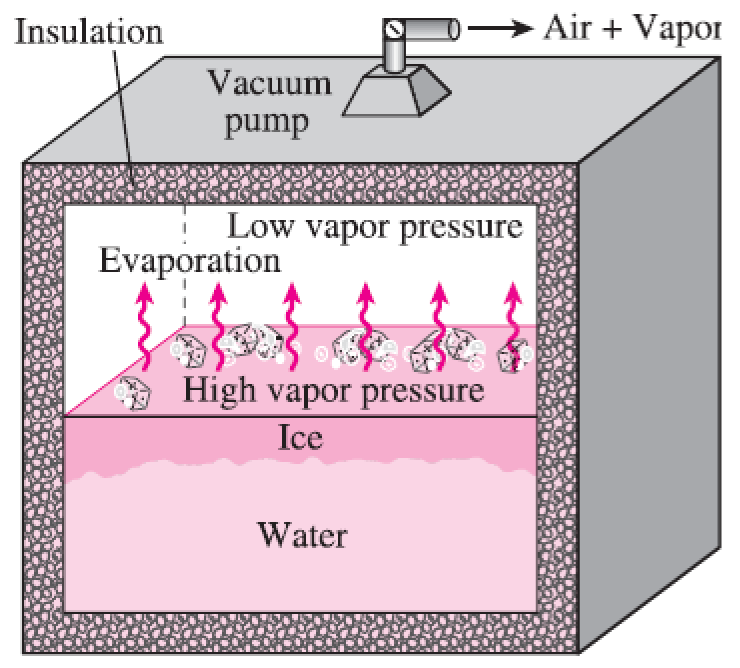
**FIGURE 3–13**

The temperature of liquid nitrogen exposed to the atmosphere remains constant at −196°C, and thus it maintains the test chamber at −196°C.

The variation of the temperature of fruits and vegetables with pressure during vacuum cooling from 25°C to 0°C.

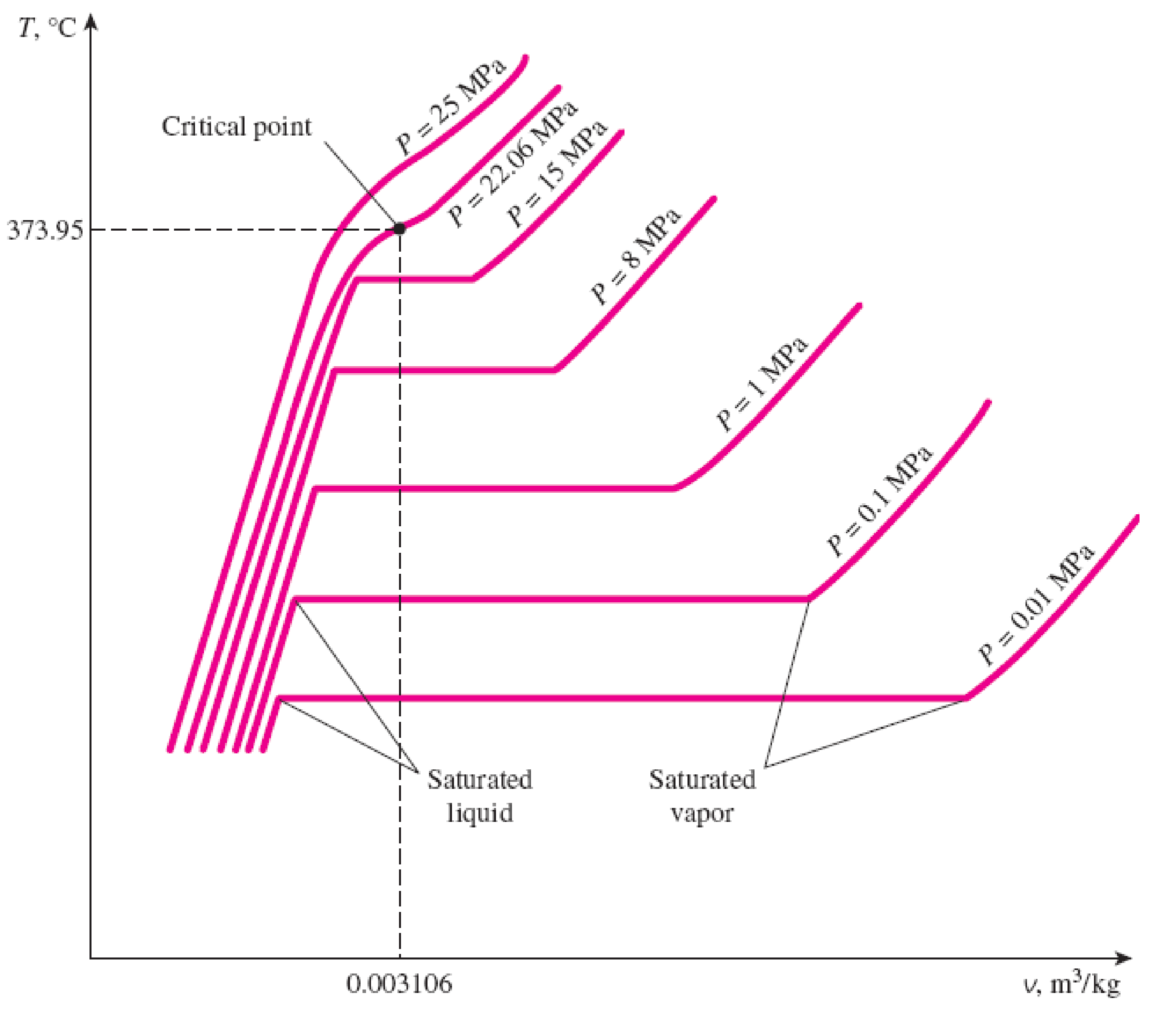


In 1775, ice was made by evacuating the air space in a water tank.



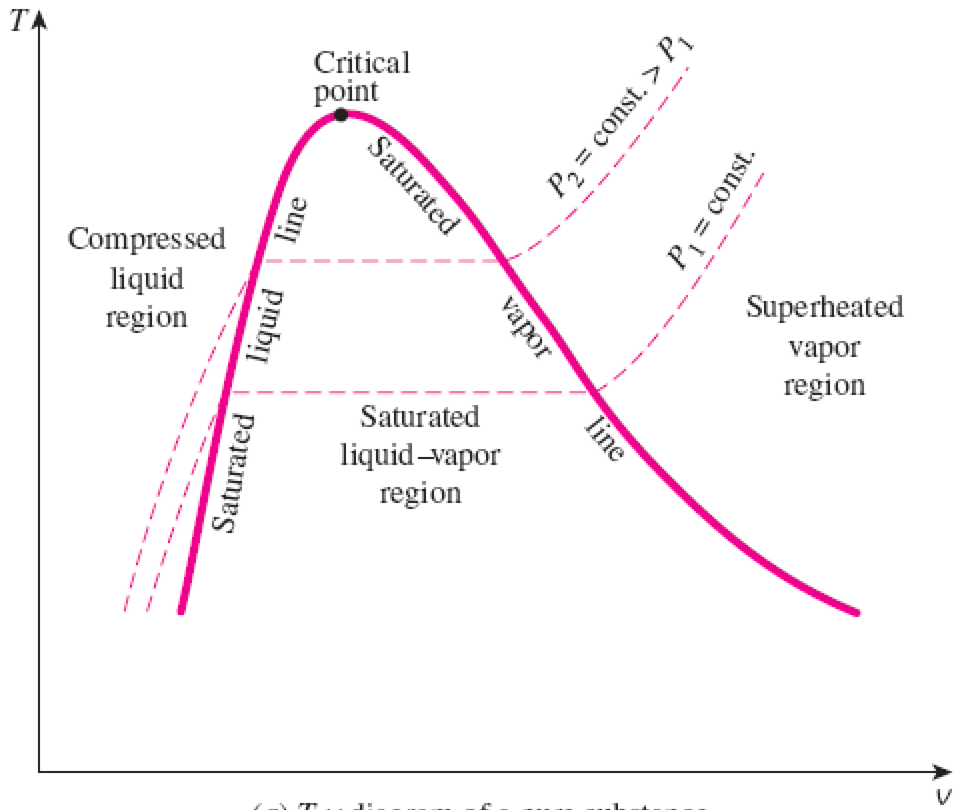
# PROPERTY DIAGRAMS FOR PHASE-CHANGE PROCESSES

The variations of properties during phase-change processes are best studied and understood with the help of property diagrams such as the *T*-*v*, *P*-*v*, and *P*-*T* diagrams for pure substances.



*T*-*v* diagram of constant-pressure phase-change processes of a pure substance at various pressures (numerical values are for water).

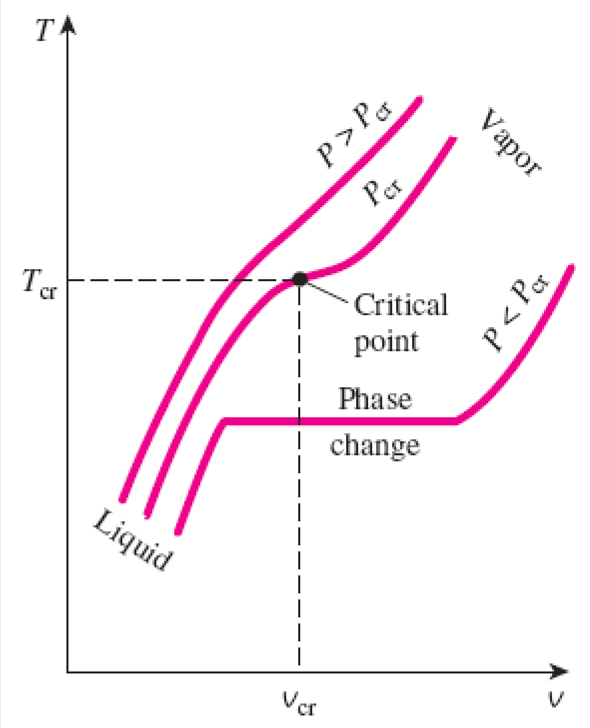
- **saturated liquid line**
- **saturated vapor line**
- **compressed liquid region**
- **superheated vapor region**
- **saturated liquid–vapor mixture region (wet region)**



(*a*) *T*-*v* diagram of a pure substance

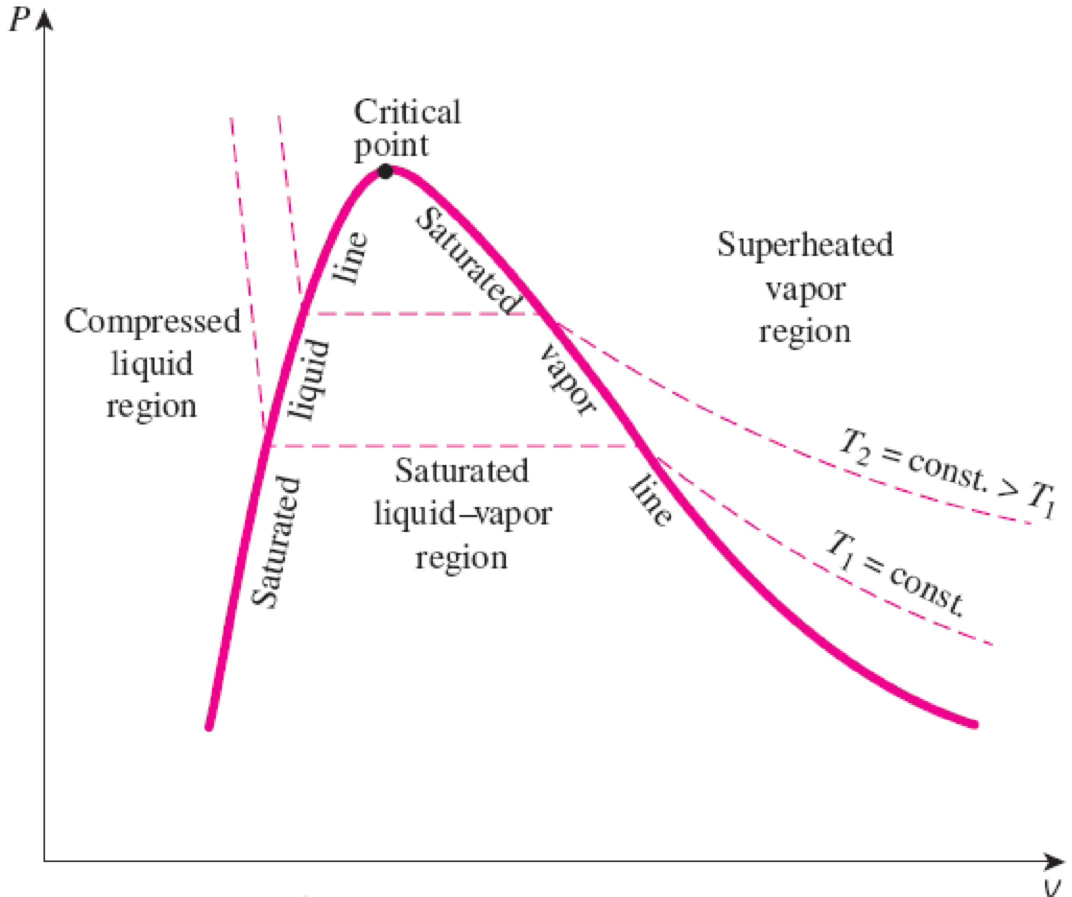
**FIGURE 3–18**

Property diagrams of a pure substance.



At supercritical pressures ($P > P_{cr}$), there is no distinct phase-change (boiling) process.

**Critical point**: The point at which the saturated liquid and saturated vapor states are identical.



(b) P-v diagram of a pure substance

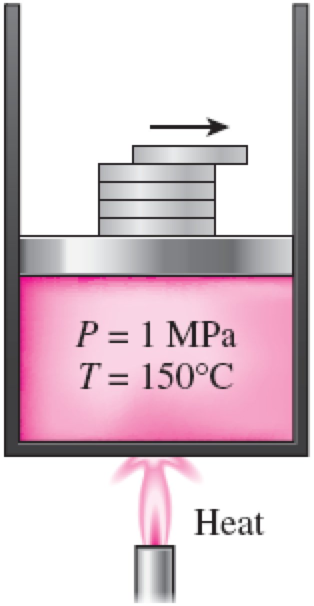


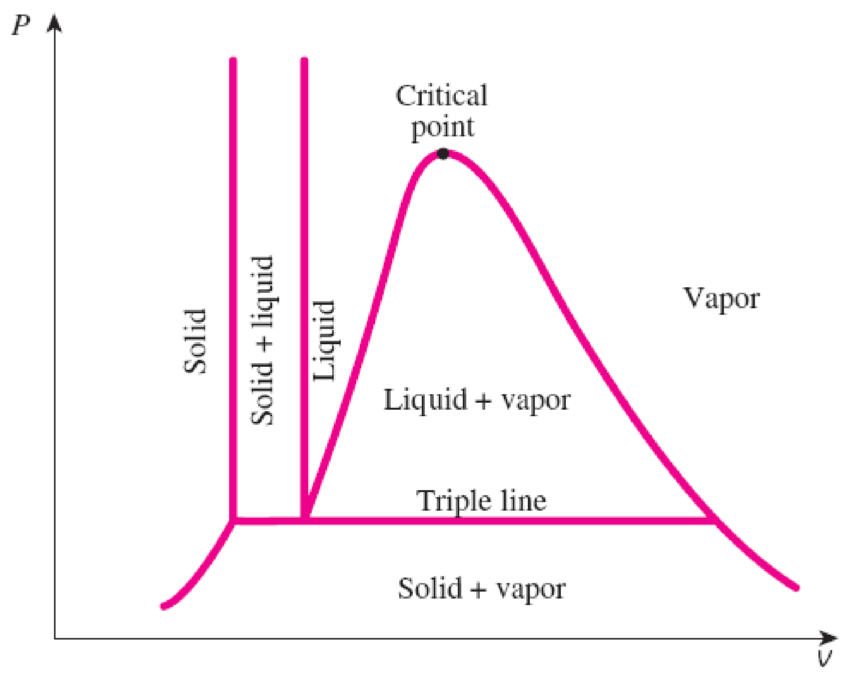
**FIGURE 3–19**

The pressure in a piston–cylinder device can be reduced by reducing the weight of the piston.

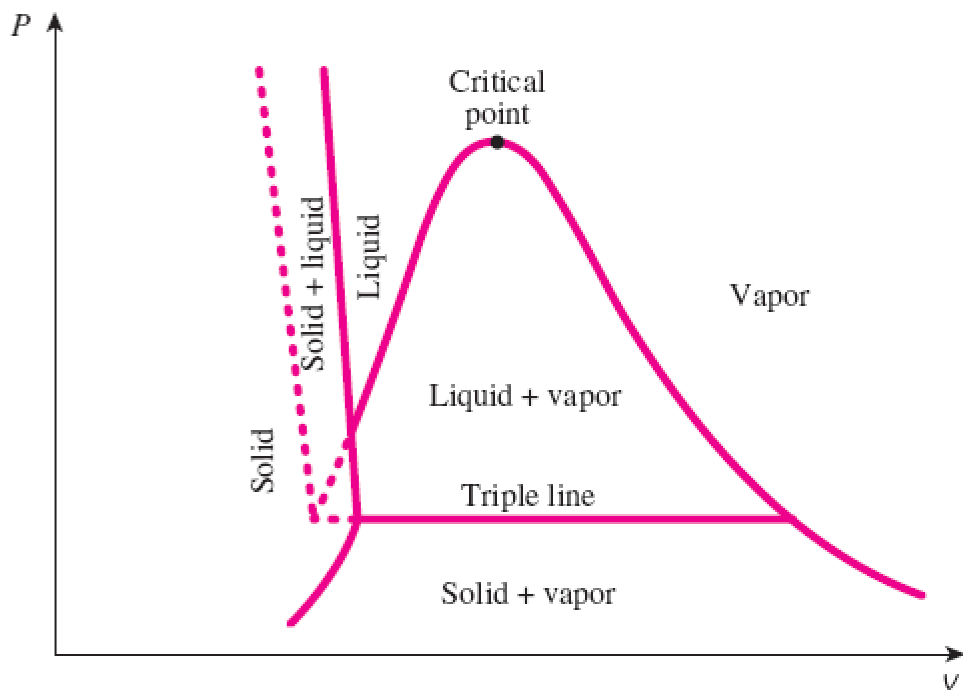
# Extending the Diagrams to Include the Solid Phase

For water,
$T_{tp}$ = 0.01°C
$P_{tp}$ = 0.6117 kPa

At triple-point pressure and temperature, a substance exists in three phases in equilibrium.



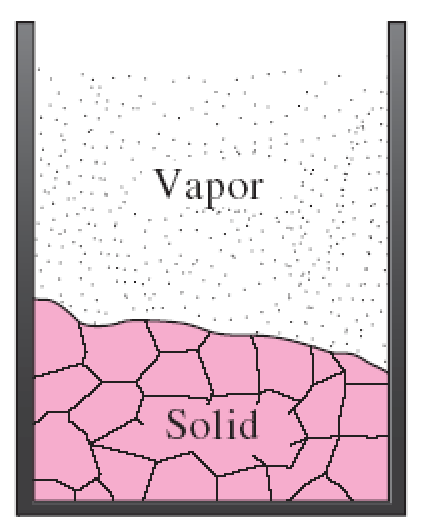
(*a*) *P*-*v* diagram of a substance that contracts on freezing



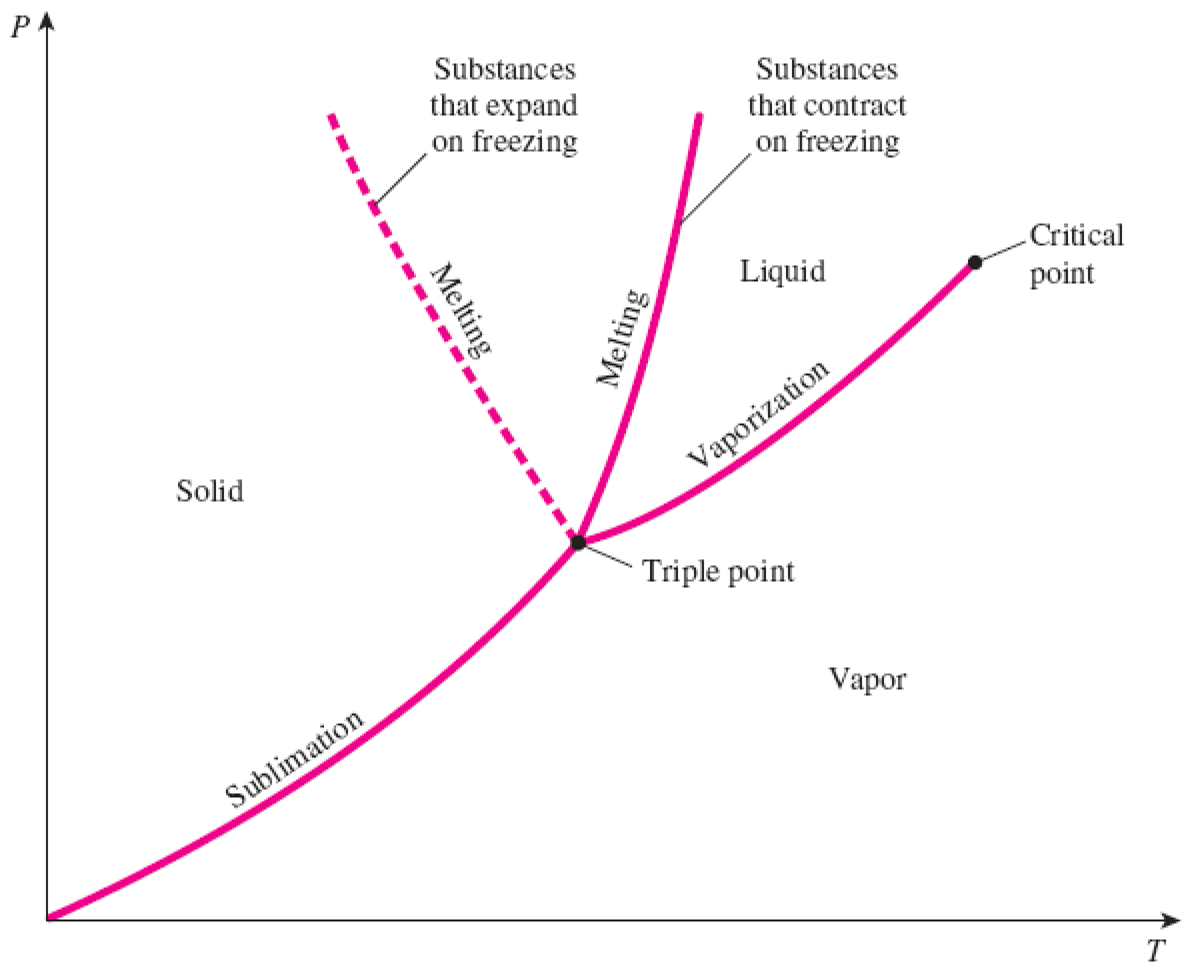
(*b*) *P*-*v* diagram of a substance that expands on freezing (such as water)

**Sublimation**: Passing from the solid phase directly into the vapor phase.

# Phase Diagram



At low pressures (below the triple-point value), solids evaporate without melting first (*sublimation*).



*P-T* diagram of pure substances.

The *P*-*v*-*T* surfaces present a great deal of information at once, but in a thermodynamic analysis it is more convenient to work with two-dimensional diagrams, such as the *P*-*v* and *T*-*v* diagrams.

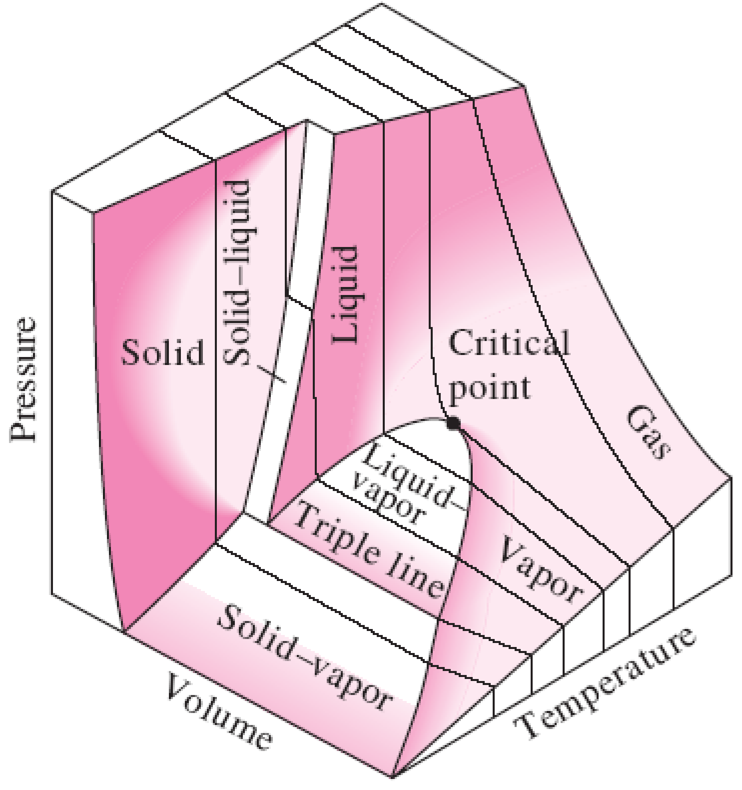


**FIGURE 3–24**

*P*-*v*-*T* surface of a substance that *contracts* on freezing.

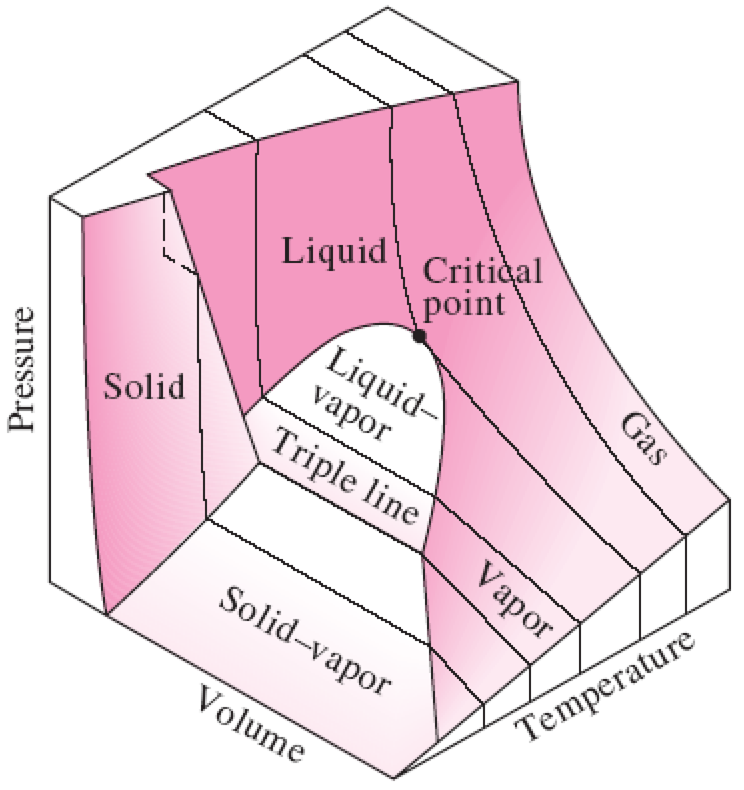


**FIGURE 3–25**

*P*-*v*-*T* surface of a substance that *expands* on freezing (like water).

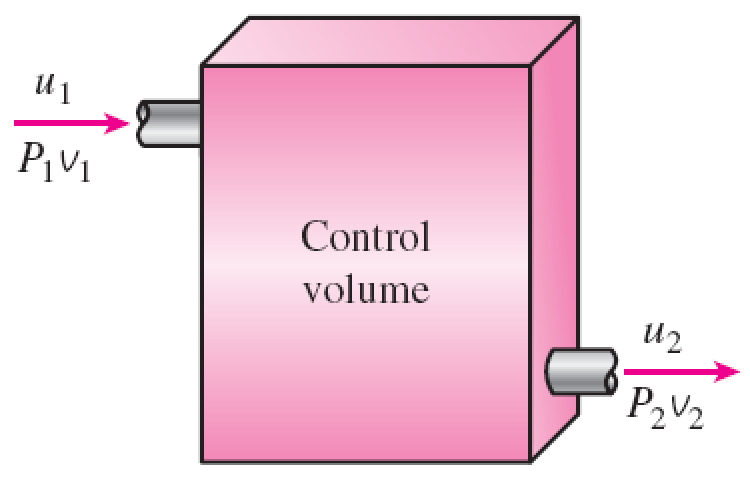
# PROPERTY TABLES

- For most substances, the relationships among thermodynamic properties are too complex to be expressed by simple equations.
- Therefore, properties are frequently presented in the form of tables.
- Some thermodynamic properties can be measured easily, but others cannot and are calculated by using the relations between them and measurable properties.
- The results of these measurements and calculations are presented in tables in a convenient format.

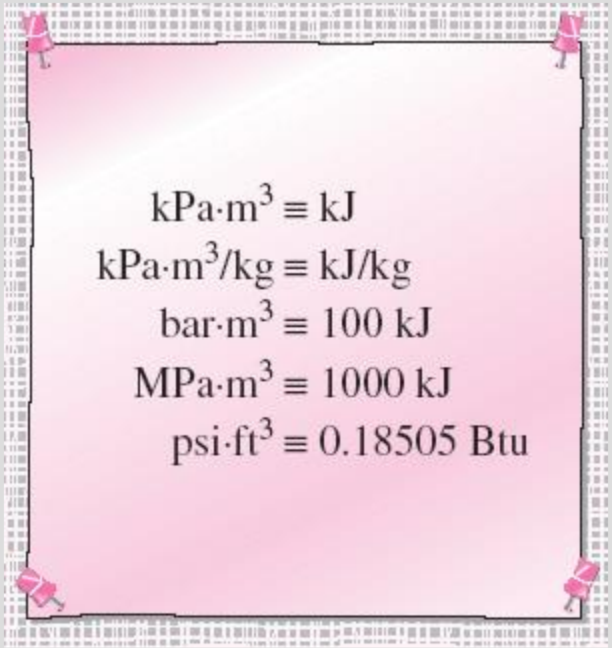
## Enthalpy—A Combination Property

$$h = u + Pv \qquad (\text{kJ/kg})$$

$$H = U + PV \qquad (\text{kJ})$$



The combination *u* + *Pv* is frequently encountered in the analysis of control volumes.



$$\text{kPa}\cdot\text{m}^3 \equiv \text{kJ}$$

$$\text{kPa}\cdot\text{m}^3/\text{kg} \equiv \text{kJ/kg}$$

$$\text{bar}\cdot\text{m}^3 \equiv 100\ \text{kJ}$$

$$\text{MPa}\cdot\text{m}^3 \equiv 1000\ \text{kJ}$$

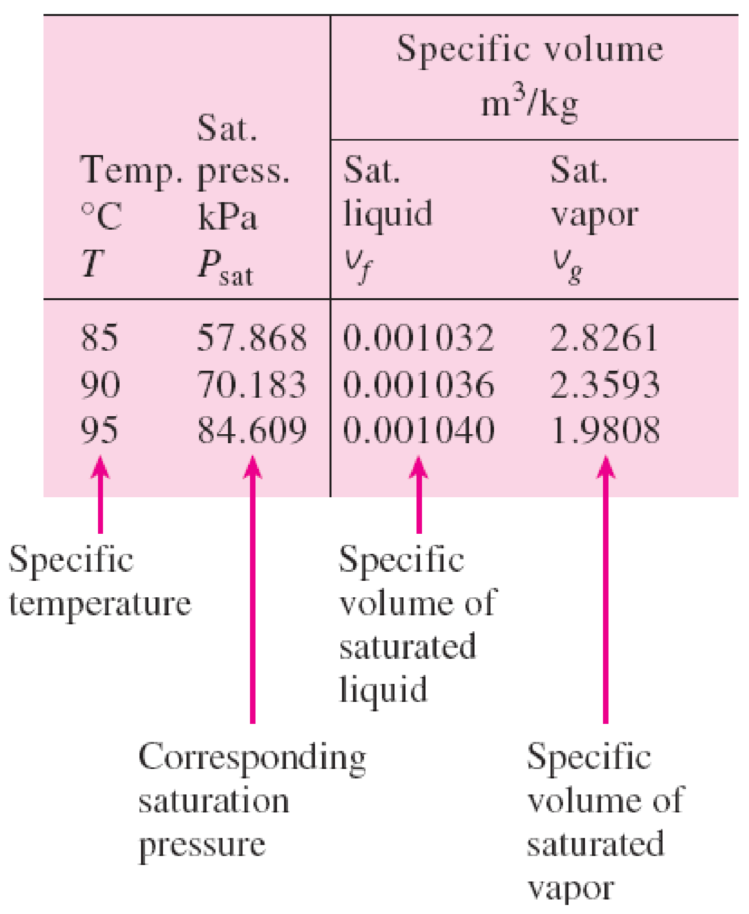
$$\text{psi}\cdot\text{ft}^3 \equiv 0.18505\ \text{Btu}$$

The product *pressure* × *volume* has energy units.

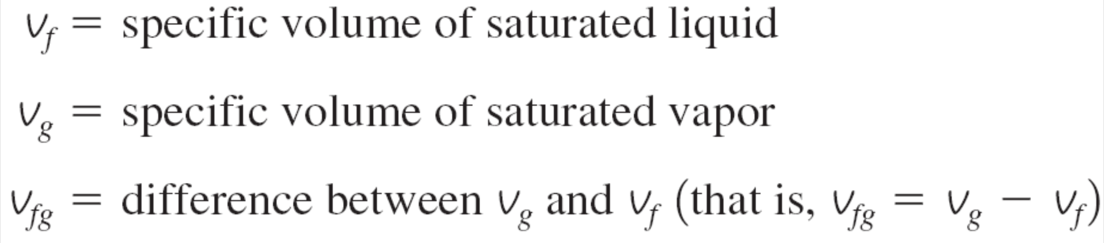
# Saturated Liquid and Saturated Vapor States

- **Table A–4**: Saturation properties of water under temperature.
- **Table A–5**: Saturation properties of water under pressure.

A partial list of Table A–4.



| | | Specific volume $m^3/kg$ | |
|---|---|---|---|
| Temp. °C $T$ | Sat. press. kPa $P_{sat}$ | Sat. liquid $v_f$ | Sat. vapor $v_g$ |
| 85 | 57.868 | 0.001032 | 2.8261 |
| 90 | 70.183 | 0.001036 | 2.3593 |
| 95 | 84.609 | 0.001040 | 1.9808 |

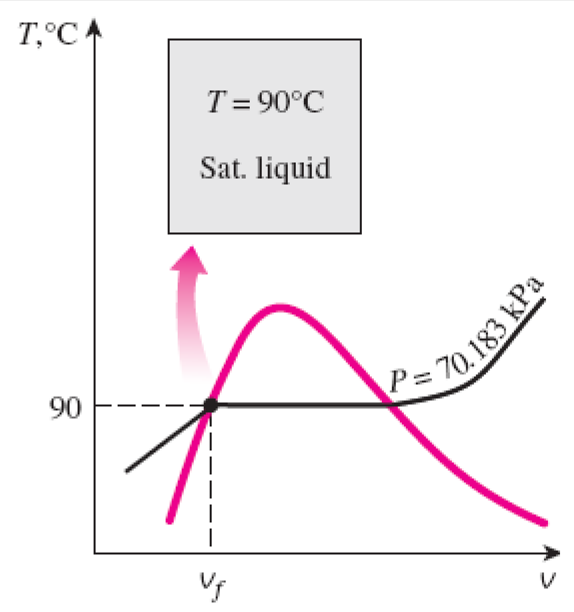


$v_f$ = specific volume of saturated liquid

$v_g$ = specific volume of saturated vapor

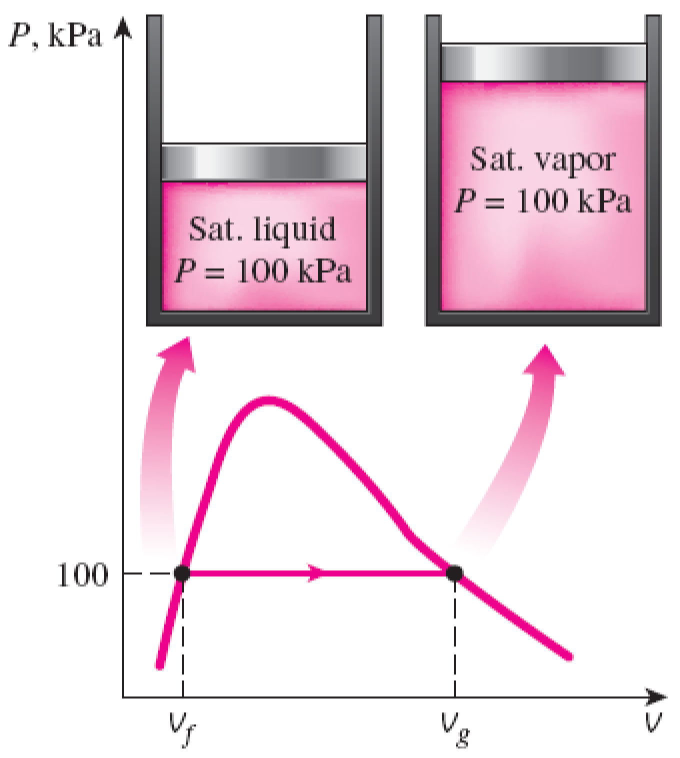
$v_{fg}$ = difference between $v_g$ and $v_f$ (that is, $v_{fg} = v_g - v_f$)

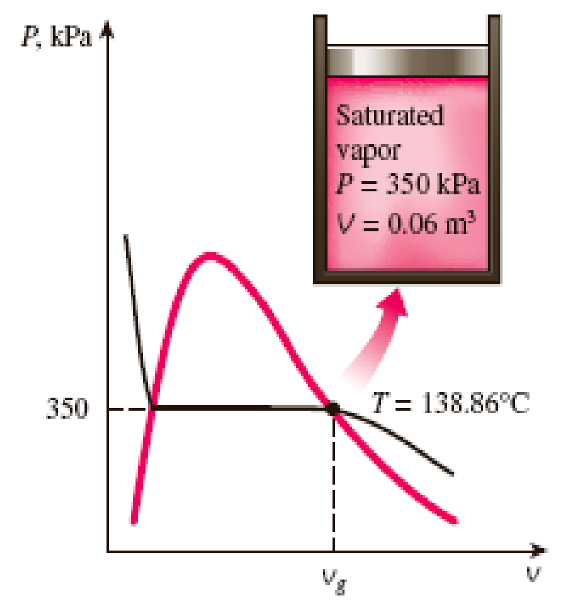
**Enthalpy of vaporization**, $h_{fg}$ **(Latent heat of vaporization):** The amount of energy needed to vaporize a unit mass of saturated liquid at a given temperature or pressure.



**Examples**:

Saturated liquid and saturated vapor states of water on *T-v* and *P-v* diagrams.





# Saturated Liquid–Vapor Mixture

**Quality, *x*** : The ratio of the mass of vapor to the total mass of the mixture.

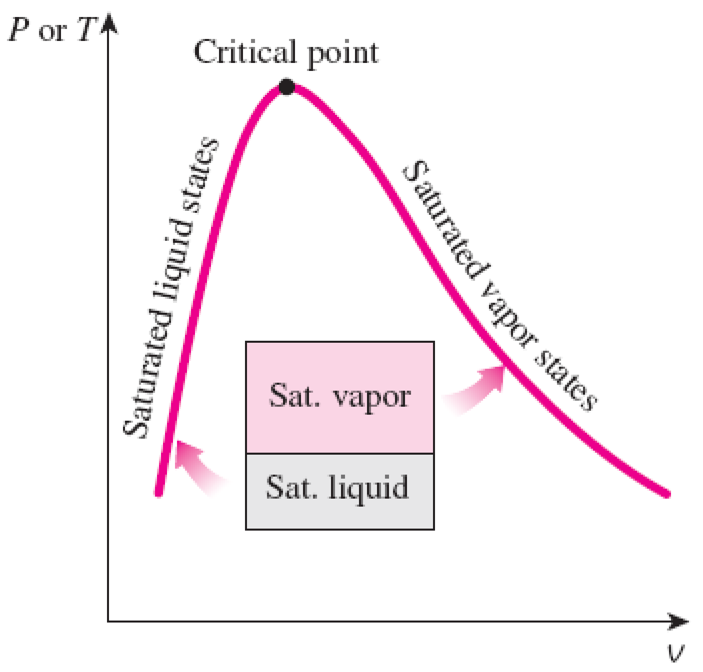
**Quality is between 0 and 1 → 0: sat. liquid, 1: sat. vapor.**

*The properties of the saturated liquid are the same whether it exists alone or in a mixture with saturated vapor.*

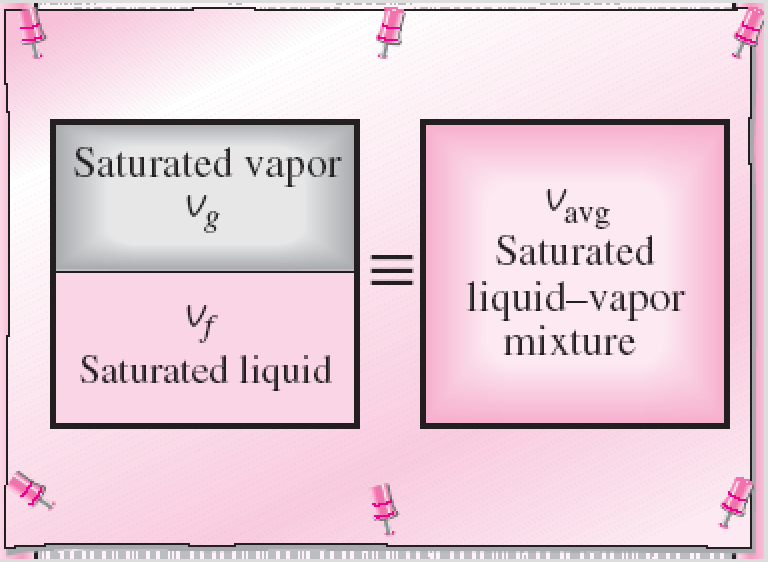
Temperature and pressure are dependent properties for a mixture.

$$x = \frac{m_{\text{vapor}}}{m_{\text{total}}}$$

$$m_{\text{total}} = m_{\text{liquid}} + m_{\text{vapor}} = m_f + m_g$$



The relative amounts of liquid and vapor phases in a saturated mixture are specified by the ***quality x***.



A two-phase system can be treated as a homogeneous mixture for convenience.

$$v_{\text{avg}} = v_f + x v_{fg} \qquad (\text{m}^3/\text{kg})$$

$$x = m_g / m_t$$

$$x = \frac{v_{\text{avg}} - v_f}{v_{fg}}$$

$$u_{\text{avg}} = u_f + x u_{fg} \qquad (\text{kJ/kg})$$

$$h_{\text{avg}} = h_f + x h_{fg} \qquad (\text{kJ/kg})$$

***y*** → *v*, *u*, or *h*.

$$y_{\text{avg}} = y_f + x y_{fg}$$

$$y_f \leq y_{\text{avg}} \leq y_g$$

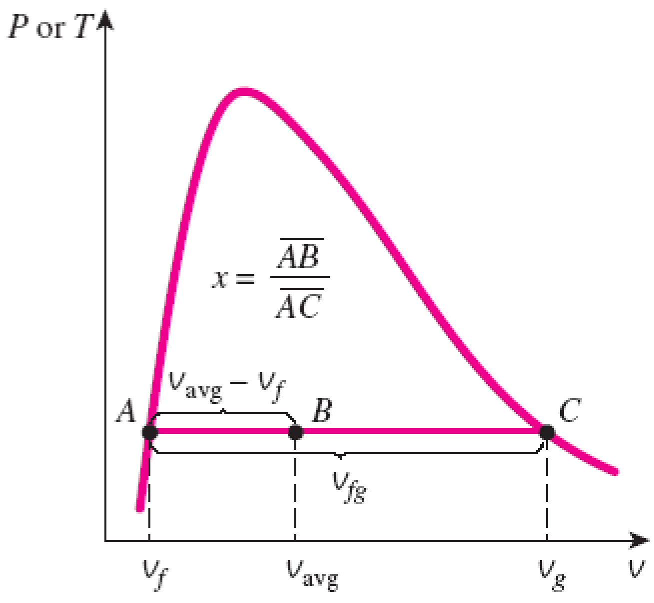


**FIGURE 3–34**
Quality is related to the horizontal distances on *P*-*v* and *T*-*v* diagrams.

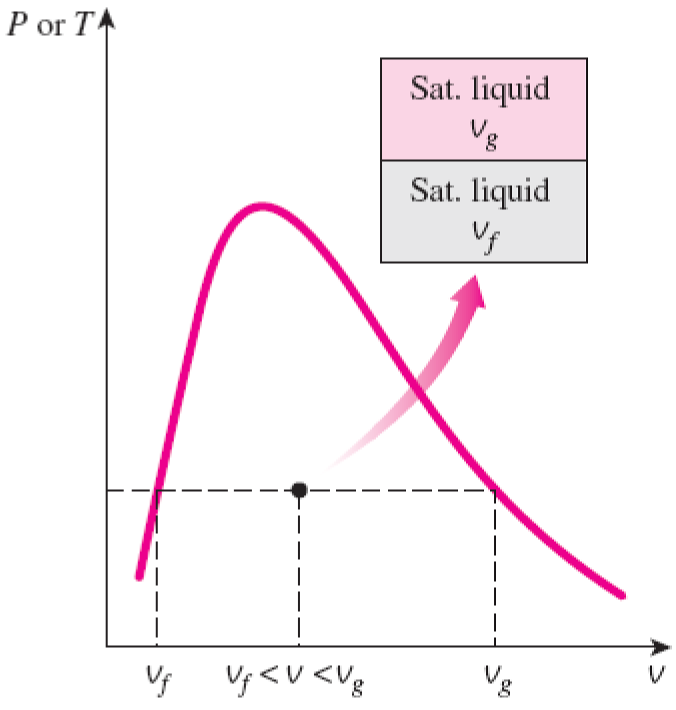
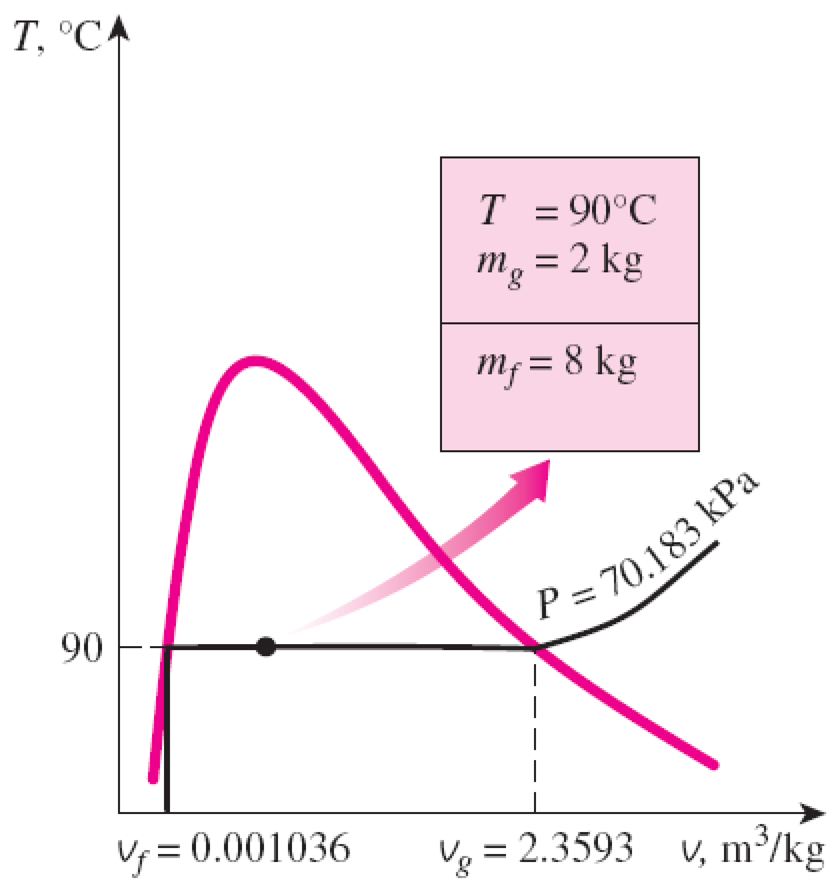
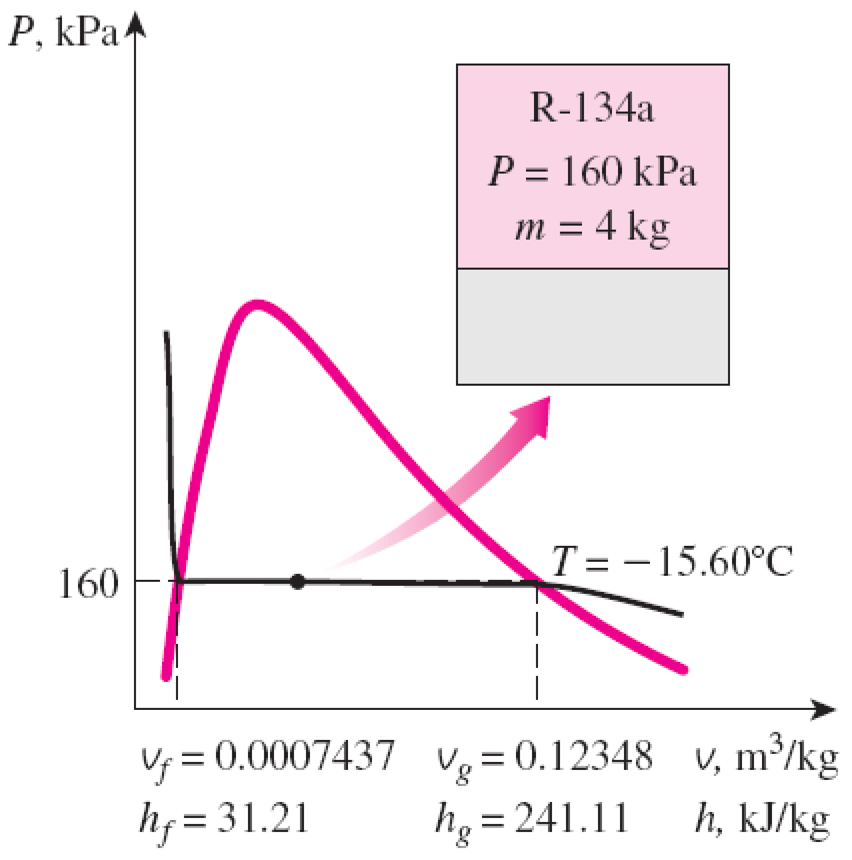


**FIGURE 3–35**
The *v* value of a saturated liquid–vapor mixture lies between the $v_f$ and $v_g$ values at the specified *T* or *P*.

# **Examples**: Saturated liquid-vapor mixture states on *T-v* and *P-v* diagrams.



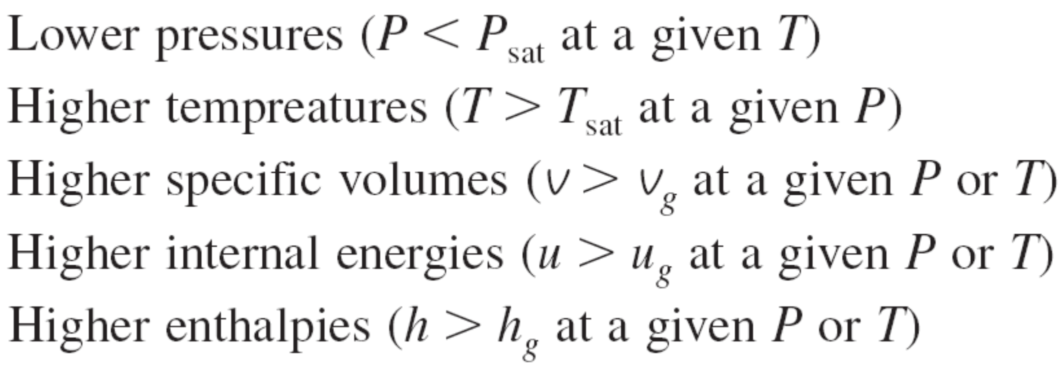


# Superheated Vapor

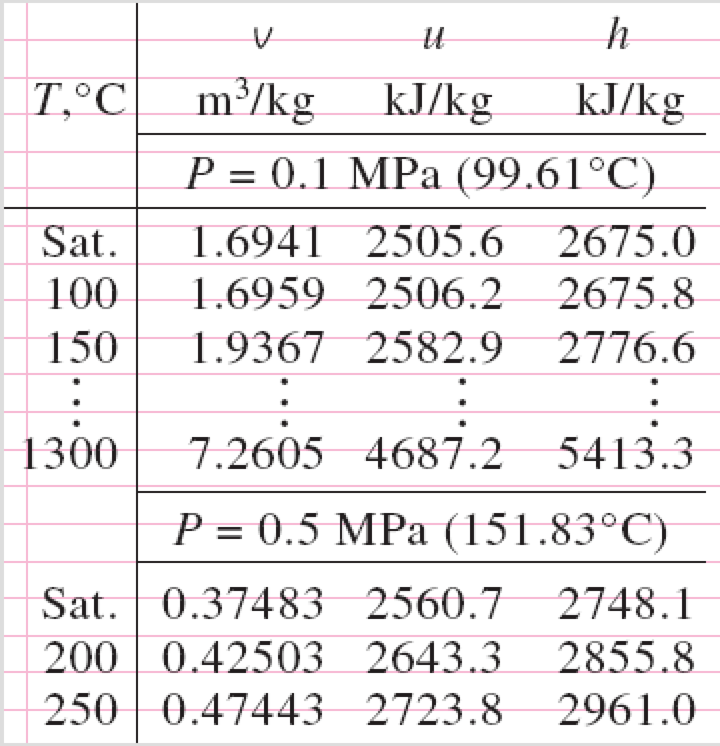
In the region to the right of the saturated vapor line and at temperatures above the critical point temperature, a substance exists as superheated vapor.

In this region, temperature and pressure are independent properties.

Compared to saturated vapor, superheated vapor is characterized by



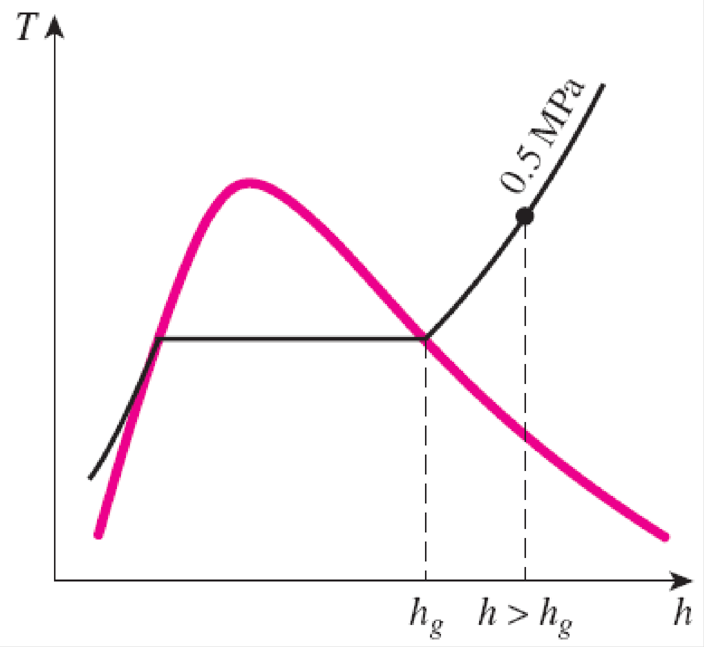
Lower pressures ($P < P_{\text{sat}}$ at a given $T$)
Higher tempreatures ($T > T_{\text{sat}}$ at a given $P$)
Higher specific volumes ($v > v_g$ at a given $P$ or $T$)
Higher internal energies ($u > u_g$ at a given $P$ or $T$)
Higher enthalpies ($h > h_g$ at a given $P$ or $T$)



| $T$,°C | $v$ m³/kg | $u$ kJ/kg | $h$ kJ/kg |
|---|---|---|---|
| | $P$ = 0.1 MPa (99.61°C) | | |
| Sat. | 1.6941 | 2505.6 | 2675.0 |
| 100 | 1.6959 | 2506.2 | 2675.8 |
| 150 | 1.9367 | 2582.9 | 2776.6 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 1300 | 7.2605 | 4687.2 | 5413.3 |
| | $P$ = 0.5 MPa (151.83°C) | | |
| Sat. | 0.37483 | 2560.7 | 2748.1 |
| 200 | 0.42503 | 2643.3 | 2855.8 |
| 250 | 0.47443 | 2723.8 | 2961.0 |

A partial listing of Table A–6.

At a specified $P$, superheated vapor exists at a higher $h$ than the saturated vapor.



# Compressed Liquid

Compressed liquid is characterized by

Higher pressures ($P > P_{\mathrm{sat}}$ at a given $T$)
Lower tempreatures ($T < T_{\mathrm{sat}}$ at a given $P$)
Lower specific volumes ($v < v_f$ at a given $P$ or $T$)
Lower internal energies ($u < u_f$ at a given $P$ or $T$)
Lower enthalpies ($h < h_f$ at a given $P$ or $T$)

The compressed liquid properties depend on temperature much more strongly than they do on pressure.

$y \cong y_{f\,@\,T}$    $y \rightarrow v$, $u$, or $h$

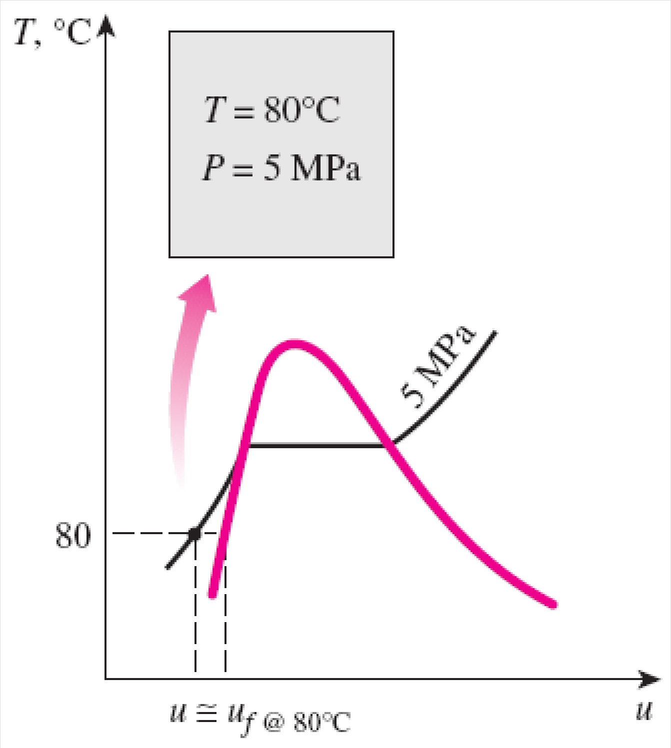
A more accurate relation for *h*

$$h \cong h_{f\,@\,T} + v_{f\,@\,T}\,(P - P_{\mathrm{sat}\,@\,T})$$

**Given: *P* and *T***

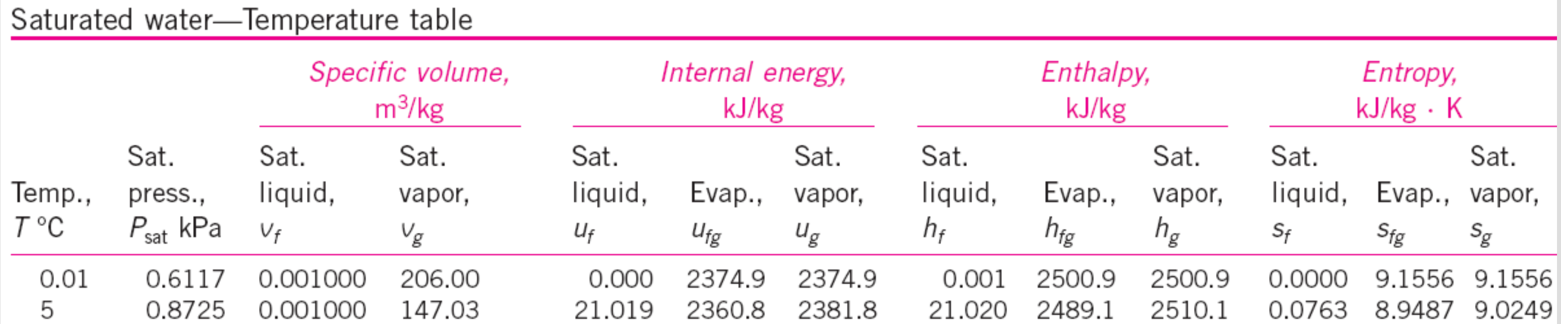
$$v \cong v_{f\,@T}$$
$$u \cong u_{f\,@T}$$
$$h \cong h_{f\,@T}$$

A compressed liquid may be approximated as a saturated liquid at the given temperature.



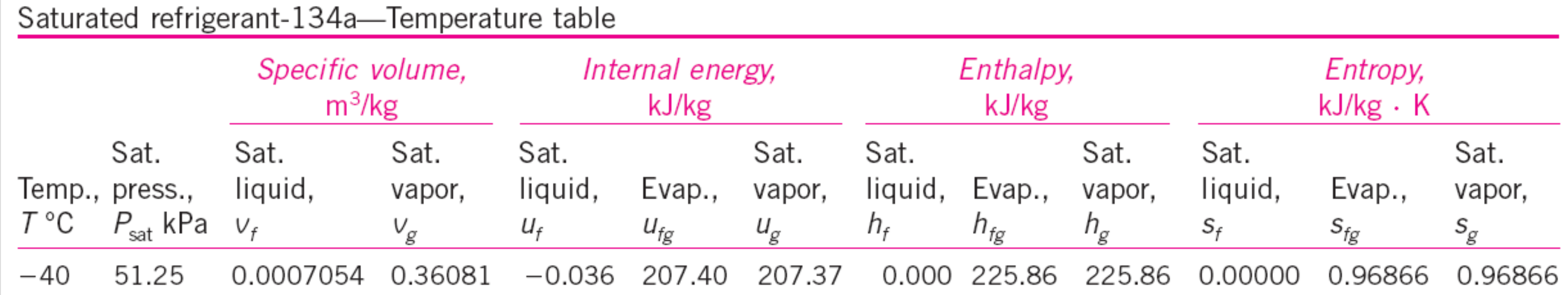
# Reference State and Reference Values

- The values of *u*, *h*, and *s* cannot be measured directly, and they are calculated from measurable properties using the relations between properties.
- However, those relations give the *changes* in properties, not the values of properties at specified states.
- Therefore, we need to choose a convenient *reference state* and assign a value of *zero* for a convenient property or properties at that state.
- The reference state for water is 0.01°C and for R-134a is -40°C in tables.
- Some properties may have negative values as a result of the reference state chosen.
- Sometimes different tables list different values for some properties at the same state as a result of using a different reference state.
- However, In thermodynamics we are concerned with the *changes* in properties, and the reference state chosen is of no consequence in calculations.



Saturated water—Temperature table

| | | *Specific volume,* m$^3$/kg | | *Internal energy,* kJ/kg | | | *Enthalpy,* kJ/kg | | | *Entropy,* kJ/kg · K | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temp., $T$ °C | Sat. press., $P_{sat}$ kPa | Sat. liquid, $v_f$ | Sat. vapor, $v_g$ | Sat. liquid, $u_f$ | Evap., $u_{fg}$ | Sat. vapor, $u_g$ | Sat. liquid, $h_f$ | Evap., $h_{fg}$ | Sat. vapor, $h_g$ | Sat. liquid, $s_f$ | Evap., $s_{fg}$ | Sat. vapor, $s_g$ |
| 0.01 | 0.6117 | 0.001000 | 206.00 | 0.000 | 2374.9 | 2374.9 | 0.001 | 2500.9 | 2500.9 | 0.0000 | 9.1556 | 9.1556 |
| 5 | 0.8725 | 0.001000 | 147.03 | 21.019 | 2360.8 | 2381.8 | 21.020 | 2489.1 | 2510.1 | 0.0763 | 8.9487 | 9.0249 |



Saturated refrigerant-134a—Temperature table

| | | *Specific volume,* m$^3$/kg | | *Internal energy,* kJ/kg | | | *Enthalpy,* kJ/kg | | | *Entropy,* kJ/kg · K | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temp., $T$ °C | Sat. press., $P_{sat}$ kPa | Sat. liquid, $v_f$ | Sat. vapor, $v_g$ | Sat. liquid, $u_f$ | Evap., $u_{fg}$ | Sat. vapor, $u_g$ | Sat. liquid, $h_f$ | Evap., $h_{fg}$ | Sat. vapor, $h_g$ | Sat. liquid, $s_f$ | Evap., $s_{fg}$ | Sat. vapor, $s_g$ |
| −40 | 51.25 | 0.0007054 | 0.36081 | −0.036 | 207.40 | 207.37 | 0.000 | 225.86 | 225.86 | 0.00000 | 0.96866 | 0.96866 |

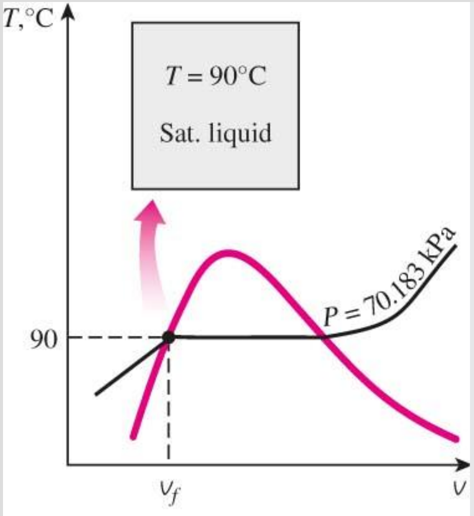
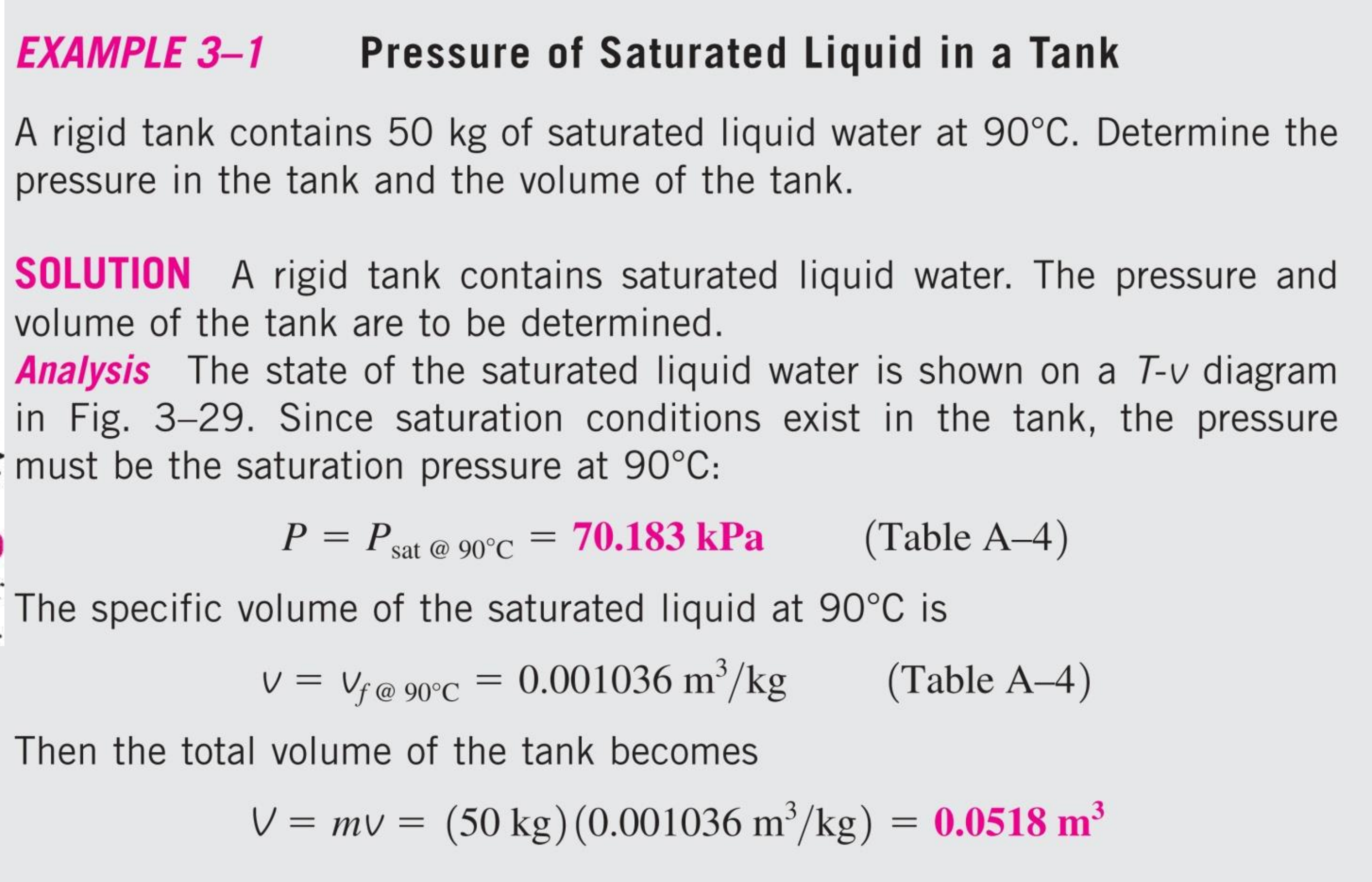


**FIGURE 3–29**
Schematic and $T$-$v$ diagram for Example 3–1.



## EXAMPLE 3–1 Pressure of Saturated Liquid in a Tank

A rigid tank contains 50 kg of saturated liquid water at 90°C. Determine the pressure in the tank and the volume of the tank.

**SOLUTION** A rigid tank contains saturated liquid water. The pressure and volume of the tank are to be determined.

***Analysis*** The state of the saturated liquid water is shown on a $T$-$v$ diagram in Fig. 3–29. Since saturation conditions exist in the tank, the pressure must be the saturation pressure at 90°C:

$$P = P_{\text{sat @ } 90^\circ\text{C}} = \mathbf{70.183\ kPa} \qquad \text{(Table A–4)}$$

The specific volume of the saturated liquid at 90°C is

$$v = v_{f\text{ @ }90^\circ\text{C}} = 0.001036\ \text{m}^3/\text{kg} \qquad \text{(Table A–4)}$$

Then the total volume of the tank becomes

$$V = mv = (50\ \text{kg})(0.001036\ \text{m}^3/\text{kg}) = \mathbf{0.0518\ m^3}$$

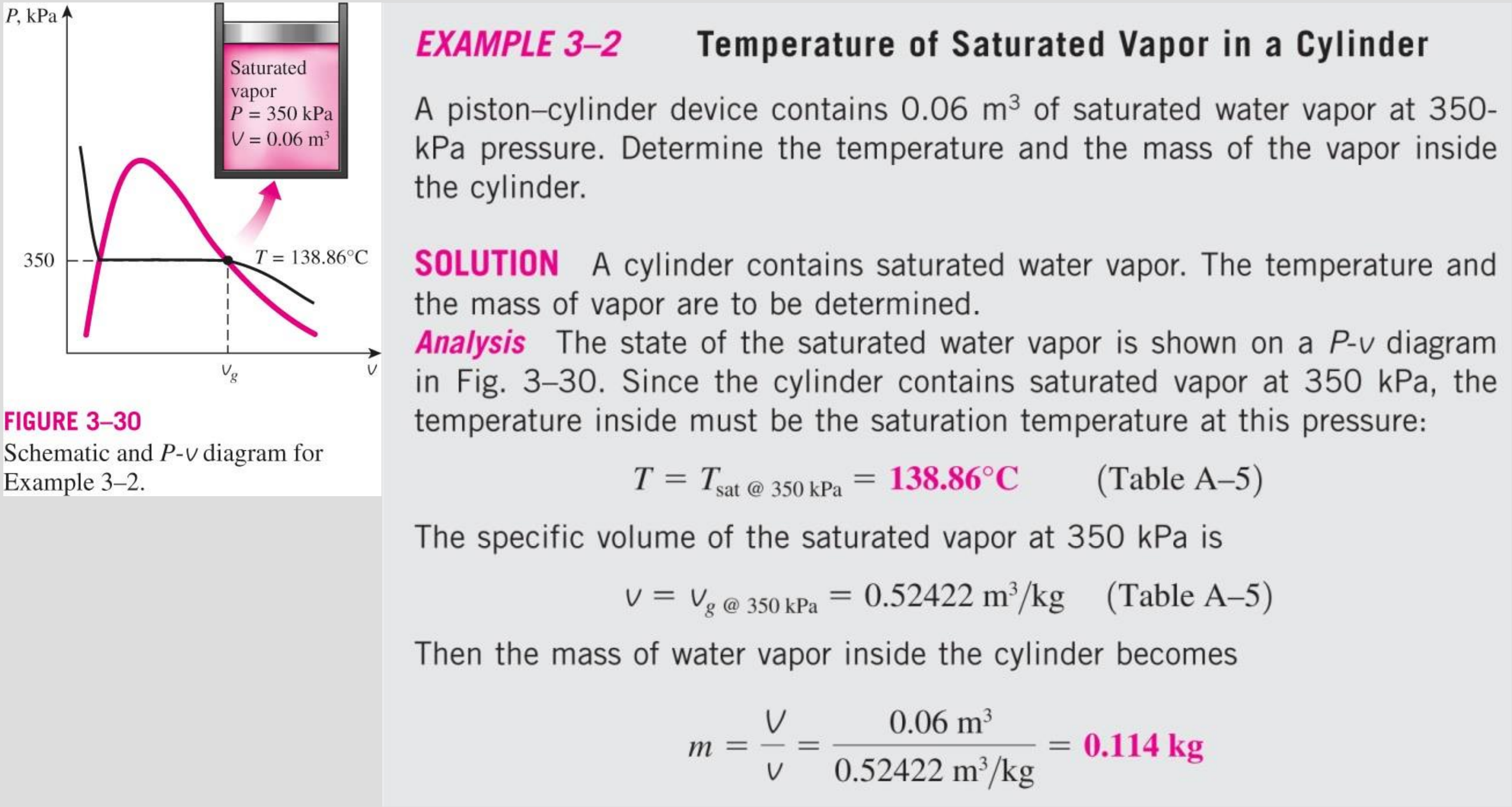


**FIGURE 3–30**
Schematic and $P$-$v$ diagram for Example 3–2.

## EXAMPLE 3–2 Temperature of Saturated Vapor in a Cylinder

A piston–cylinder device contains 0.06 m$^3$ of saturated water vapor at 350-kPa pressure. Determine the temperature and the mass of the vapor inside the cylinder.

**SOLUTION** A cylinder contains saturated water vapor. The temperature and the mass of vapor are to be determined.

***Analysis*** The state of the saturated water vapor is shown on a $P$-$v$ diagram in Fig. 3–30. Since the cylinder contains saturated vapor at 350 kPa, the temperature inside must be the saturation temperature at this pressure:

$$T = T_{\text{sat @ 350 kPa}} = \mathbf{138.86^\circ C} \qquad \text{(Table A–5)}$$

The specific volume of the saturated vapor at 350 kPa is

$$v = v_{g \text{ @ 350 kPa}} = 0.52422 \text{ m}^3/\text{kg} \qquad \text{(Table A–5)}$$

Then the mass of water vapor inside the cylinder becomes

$$m = \frac{V}{v} = \frac{0.06 \text{ m}^3}{0.52422 \text{ m}^3/\text{kg}} = \mathbf{0.114 \text{ kg}}$$

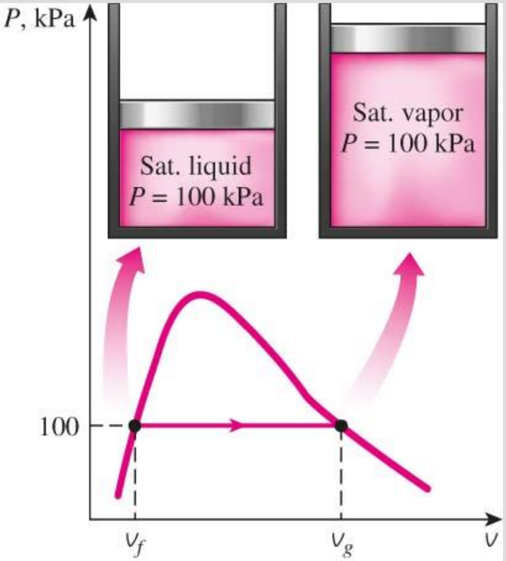


**FIGURE 3–31**
Schematic and *P*-*v* diagram for Example 3–3.

## *EXAMPLE 3–3* Volume and Energy Change during Evaporation

A mass of 200 g of saturated liquid water is completely vaporized at a constant pressure of 100 kPa. Determine (*a*) the volume change and (*b*) the amount of energy transferred to the water.

**SOLUTION** Saturated liquid water is vaporized at constant pressure. The volume change and the energy transferred are to be determined.

***Analysis*** (*a*) The process described is illustrated on a *P*-*v* diagram in Fig. 3–31. The volume change per unit mass during a vaporization process is $v_{fg}$, which is the difference between $v_g$ and $v_f$. Reading these values from Table A–5 at 100 kPa and substituting yield

$$v_{fg} = v_g - v_f = 1.6941 - 0.001043 = 1.6931 \text{ m}^3/\text{kg}$$

Thus,

$$\Delta V = m v_{fg} = (0.2 \text{ kg})(1.6931 \text{ m}^3/\text{kg}) = \mathbf{0.3386 \text{ m}^3}$$

(*b*) The amount of energy needed to vaporize a unit mass of a substance at a given pressure is the enthalpy of vaporization at that pressure, which is $h_{fg}$ = 2257.5 kJ/kg for water at 100 kPa. Thus, the amount of energy transferred is

$$m h_{fg} = (0.2 \text{ kg})(2257.5 \text{ kJ/kg}) = \mathbf{451.5 \text{ kJ}}$$

***Discussion*** Note that we have considered the first four decimal digits of $v_{fg}$ and disregarded the rest. This is because $v_g$ has significant numbers to the first four decimal places only, and we do not know the numbers in the other decimal places. Copying all the digits from the calculator would mean that we are assuming $v_g = 1.694100$, which is not necessarily the case. It could very well be that $v_g = 1.694138$ since this number, too, would truncate to 1.6941. All the digits in our result (1.6931) are significant. But if we did not truncate the result, we would obtain $v_{fg} = 1.693057$, which falsely implies that our result is accurate to the sixth decimal place.

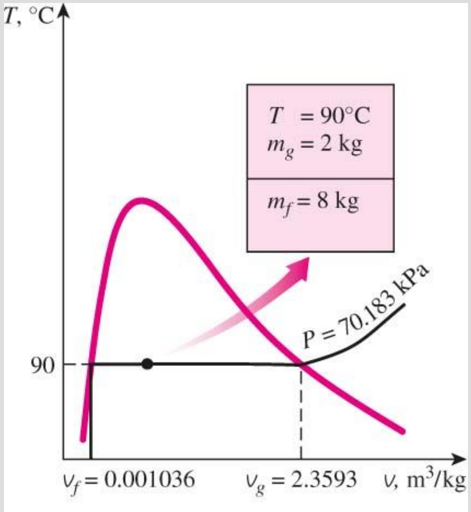


**FIGURE 3–36**
Schematic and *T*-*v* diagram for Example 3–4.

**EXAMPLE 3–4** **Pressure and Volume of a Saturated Mixture**

A rigid tank contains 10 kg of water at 90°C. If 8 kg of the water is in the liquid form and the rest is in the vapor form, determine (*a*) the pressure in the tank and (*b*) the volume of the tank.

**SOLUTION** A rigid tank contains saturated mixture. The pressure and the volume of the tank are to be determined.

***Analysis*** (*a*) The state of the saturated liquid–vapor mixture is shown in Fig. 3–36. Since the two phases coexist in equilibrium, we have a saturated mixture, and the pressure must be the saturation pressure at the given temperature:

$$P = P_{\text{sat @ 90°C}} = \mathbf{70.183\ kPa} \qquad \text{(Table A–4)}$$

(*b*) At 90°C, we have $v_f = 0.001036\ \text{m}^3/\text{kg}$ and $v_g = 2.3593\ \text{m}^3/\text{kg}$ (Table A–4). One way of finding the volume of the tank is to determine the volume occupied by each phase and then add them:

$$\begin{aligned} V &= V_f + V_g = m_f v_f + m_g v_g \\ &= (8\ \text{kg})(0.001036\ \text{m}^3/\text{kg}) + (2\ \text{kg})(2.3593\ \text{m}^3/\text{kg}) \\ &= \mathbf{4.73\ m^3} \end{aligned}$$

Another way is to first determine the quality *x*, then the average specific volume *v*, and finally the total volume:

$$\begin{aligned} x &= \frac{m_g}{m_t} = \frac{2\ \text{kg}}{10\ \text{kg}} = 0.2 \\ v &= v_f + x v_{fg} \\ &= 0.001036\ \text{m}^3/\text{kg} + (0.2)[(2.3593 - 0.001036)\ \text{m}^3/\text{kg}] \\ &= 0.473\ \text{m}^3/\text{kg} \end{aligned}$$

and

$$V = mv = (10\ \text{kg})(0.473\ \text{m}^3/\text{kg}) = 4.73\ \text{m}^3$$

***Discussion*** The first method appears to be easier in this case since the masses of each phase are given. In most cases, however, the masses of each phase are not available, and the second method becomes more convenient.

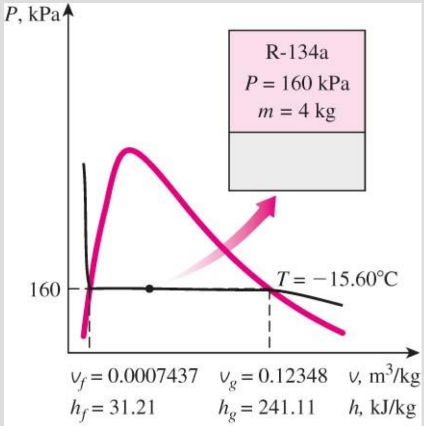


**FIGURE 3–37**
Schematic and $P$-$v$ diagram for Example 3–5.

## EXAMPLE 3–5 Properties of Saturated Liquid–Vapor Mixture

An 80-L vessel contains 4 kg of refrigerant-134a at a pressure of 160 kPa. Determine (*a*) the temperature, (*b*) the quality, (*c*) the enthalpy of the refrigerant, and (*d*) the volume occupied by the vapor phase.

**SOLUTION** A vessel is filled with refrigerant-134a. Some properties of the refrigerant are to be determined.

***Analysis*** (*a*) The state of the saturated liquid–vapor mixture is shown in Fig. 3–37. At this point we do not know whether the refrigerant is in the compressed liquid, superheated vapor, or saturated mixture region. This can be determined by comparing a suitable property to the saturated liquid and saturated vapor values. From the information given, we can determine the specific volume:

$$v = \frac{V}{m} = \frac{0.080 \text{ m}^3}{4 \text{ kg}} = 0.02 \text{ m}^3/\text{kg}$$

At 160 kPa, we read

$$\begin{aligned} v_f &= 0.0007437 \text{ m}^3/\text{kg} \\ v_g &= 0.12348 \text{ m}^3/\text{kg} \end{aligned} \qquad \text{(Table A–12)}$$

Obviously, $v_f < v < v_g$, and, the refrigerant is in the saturated mixture region. Thus, the temperature must be the saturation temperature at the specified pressure:

$$T = T_{\text{sat @ 160 kPa}} = \mathbf{-15.60°C}$$

(*b*) Quality can be determined from

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.02 - 0.0007437}{0.12348 - 0.0007437} = \mathbf{0.157}$$

(*c*) At 160 kPa, we also read from Table A–12 that $h_f$ = 31.21 kJ/kg and $h_{fg}$ = 209.90 kJ/kg. Then,

$$\begin{aligned} h &= h_f + xh_{fg} \\ &= 31.21 \text{ kJ/kg} + (0.157)(209.90 \text{ kJ/kg}) \\ &= \mathbf{64.2 \text{ kJ/kg}} \end{aligned}$$

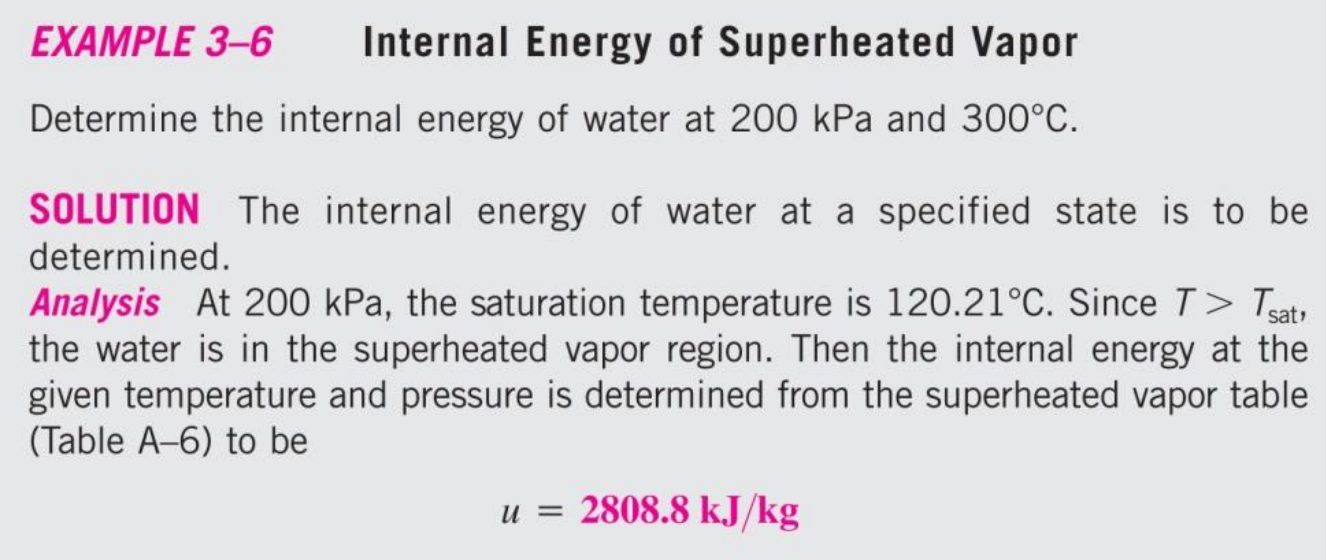
(*d*) The mass of the vapor is

$$m_g = xm_t = (0.157)(4 \text{ kg}) = 0.628 \text{ kg}$$

and the volume occupied by the vapor phase is

$$V_g = m_g v_g = (0.628 \text{ kg})(0.12348 \text{ m}^3/\text{kg}) = \mathbf{0.0775 \text{ m}^3} \text{ (or 77.5 L)}$$

The rest of the volume (2.5 L) is occupied by the liquid.



**EXAMPLE 3–6** **Internal Energy of Superheated Vapor**

Determine the internal energy of water at 200 kPa and 300°C.

**SOLUTION** The internal energy of water at a specified state is to be determined.

***Analysis*** At 200 kPa, the saturation temperature is 120.21°C. Since $T > T_{\text{sat}}$, the water is in the superheated vapor region. Then the internal energy at the given temperature and pressure is determined from the superheated vapor table (Table A–6) to be

$$u = \mathbf{2808.8\ kJ/kg}$$

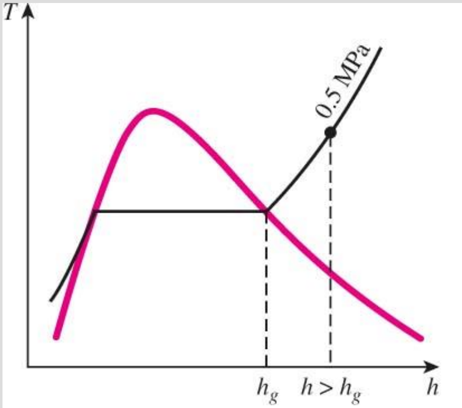
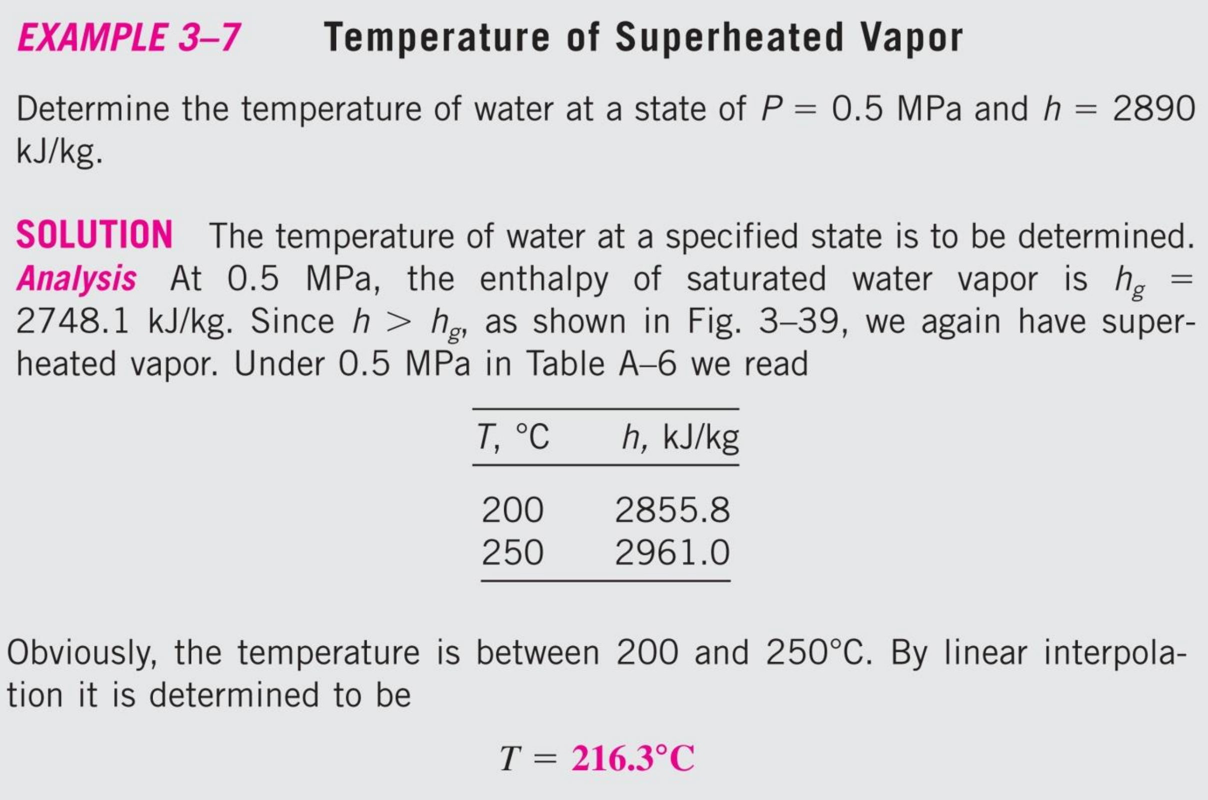


**FIGURE 3–39**
At a specified $P$, superheated vapor exists at a higher $h$ than the saturated vapor (Example 3–7).



### *EXAMPLE 3–7* **Temperature of Superheated Vapor**

Determine the temperature of water at a state of $P = 0.5$ MPa and $h = 2890$ kJ/kg.

**SOLUTION** The temperature of water at a specified state is to be determined.
***Analysis*** At 0.5 MPa, the enthalpy of saturated water vapor is $h_g = 2748.1$ kJ/kg. Since $h > h_g$, as shown in Fig. 3–39, we again have superheated vapor. Under 0.5 MPa in Table A–6 we read

| $T$, °C | $h$, kJ/kg |
|---|---|
| 200 | 2855.8 |
| 250 | 2961.0 |

Obviously, the temperature is between 200 and 250°C. By linear interpolation it is determined to be

$$T = \mathbf{216.3°C}$$

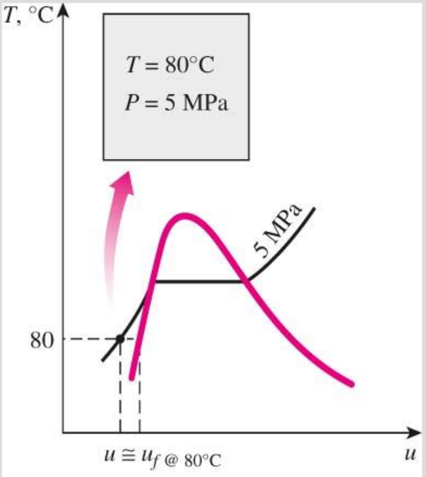
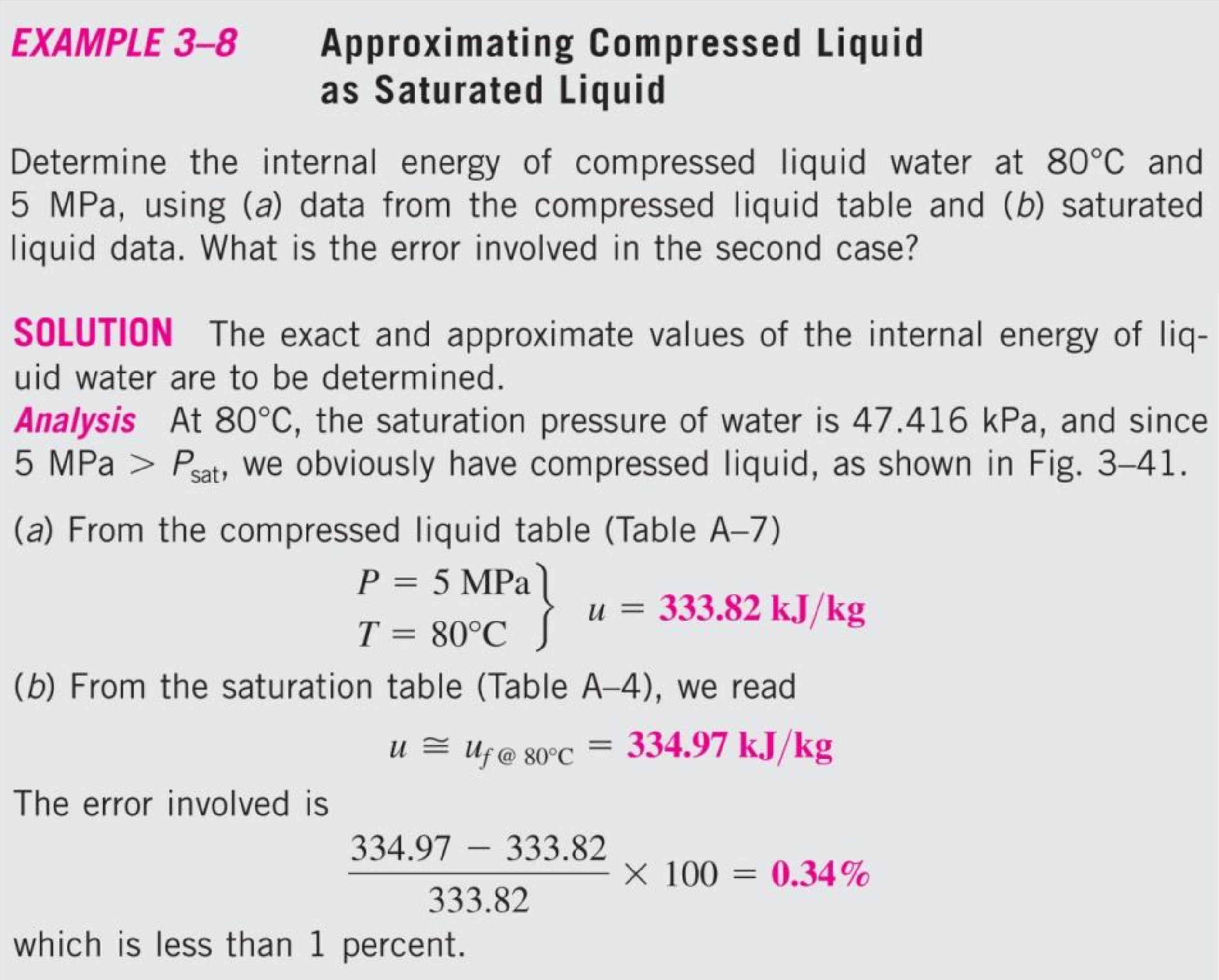


**FIGURE 3–41**
Schematic and *T*-*u* diagram for Example 3–8.



## **EXAMPLE 3–8** Approximating Compressed Liquid as Saturated Liquid

Determine the internal energy of compressed liquid water at 80°C and 5 MPa, using (*a*) data from the compressed liquid table and (*b*) saturated liquid data. What is the error involved in the second case?

**SOLUTION** The exact and approximate values of the internal energy of liquid water are to be determined.

***Analysis*** At 80°C, the saturation pressure of water is 47.416 kPa, and since 5 MPa $> P_{sat}$, we obviously have compressed liquid, as shown in Fig. 3–41.

(*a*) From the compressed liquid table (Table A–7)

$$\left.\begin{array}{l} P = 5\text{ MPa} \\ T = 80^\circ\text{C} \end{array}\right\} \quad u = \mathbf{333.82\ kJ/kg}$$

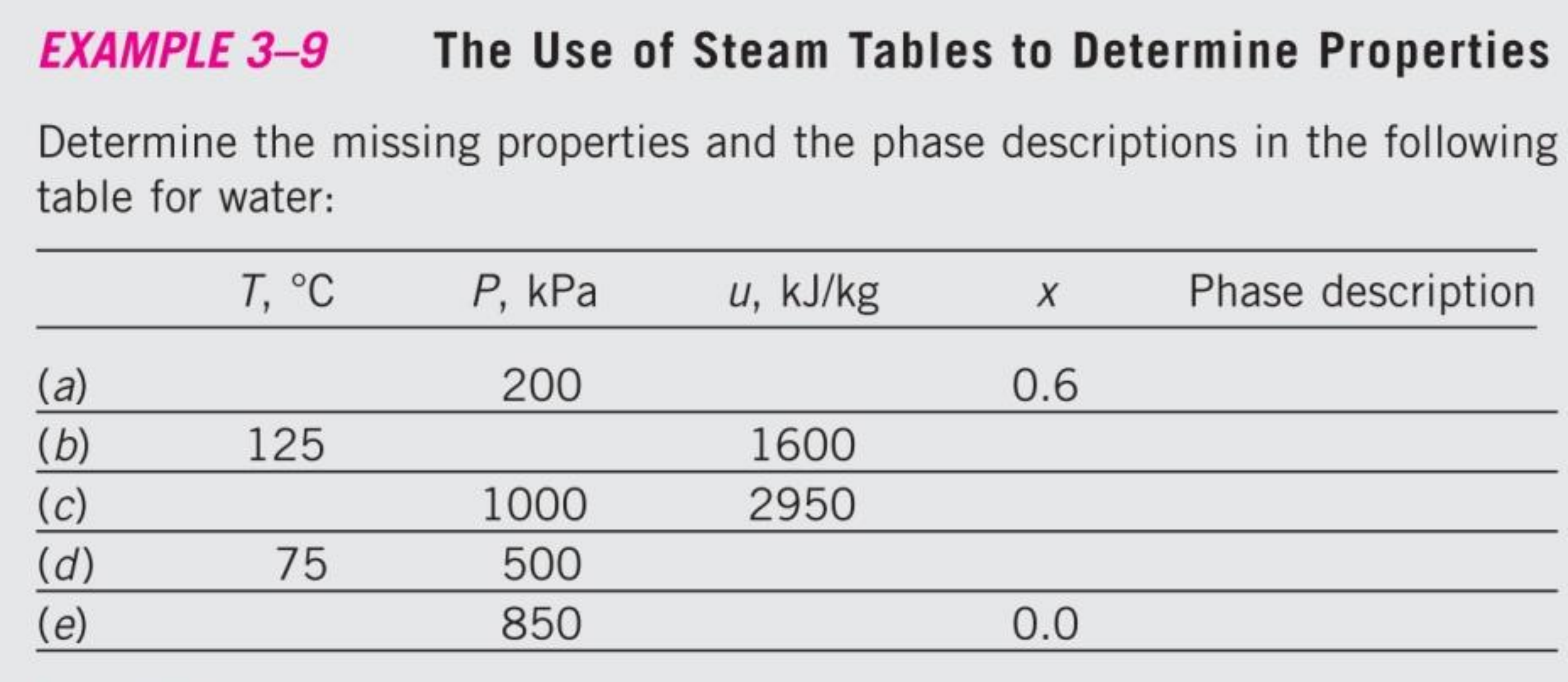
(*b*) From the saturation table (Table A–4), we read

$$u \cong u_{f\,@\,80^\circ\text{C}} = \mathbf{334.97\ kJ/kg}$$

The error involved is

$$\frac{334.97 - 333.82}{333.82} \times 100 = \mathbf{0.34\%}$$

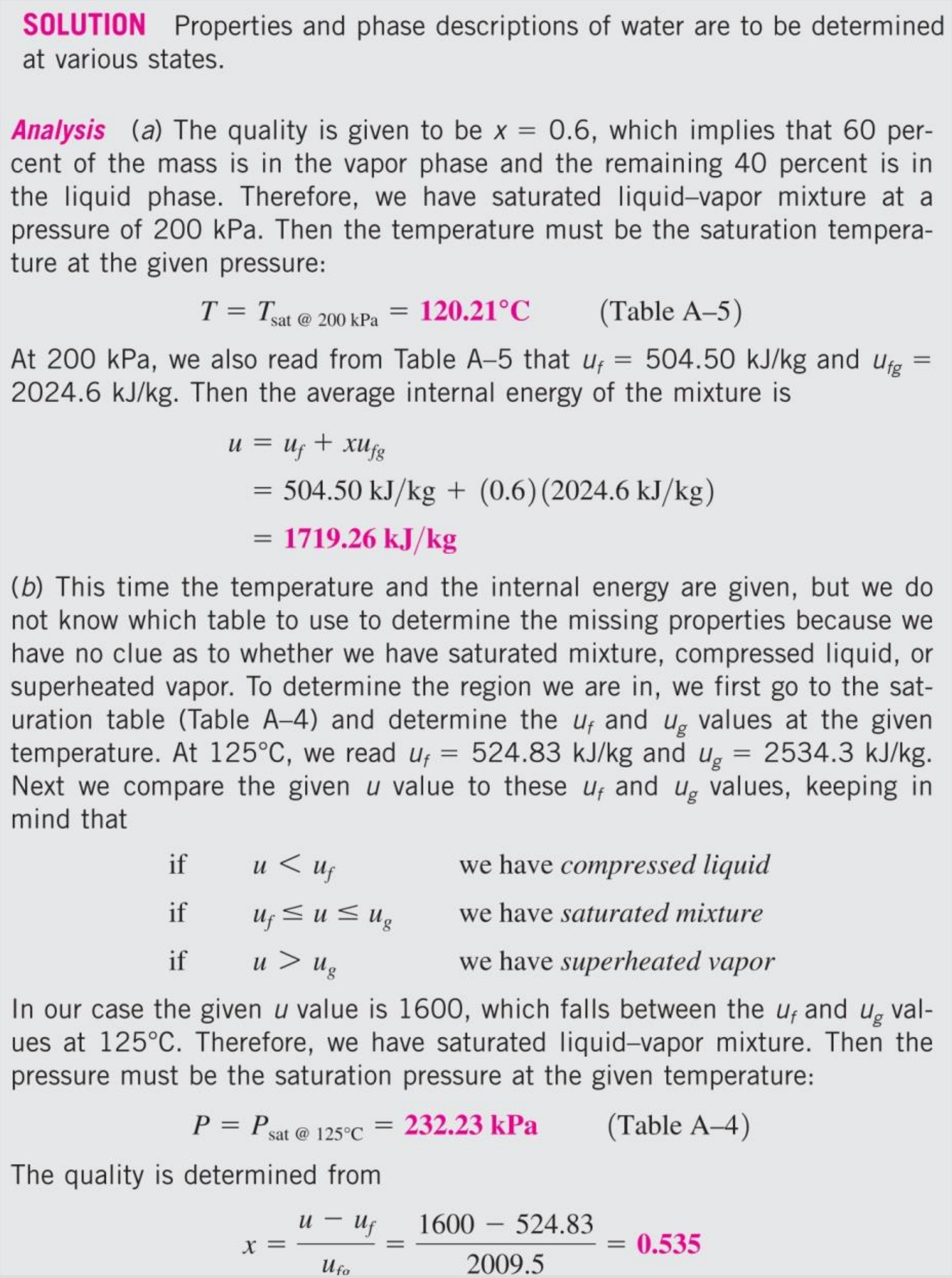
which is less than 1 percent.



**EXAMPLE 3–9** **The Use of Steam Tables to Determine Properties**

Determine the missing properties and the phase descriptions in the following table for water:

| | $T$, °C | $P$, kPa | $u$, kJ/kg | $x$ | Phase description |
|---|---|---|---|---|---|
| (*a*) | | 200 | | 0.6 | |
| (*b*) | 125 | | 1600 | | |
| (*c*) | | 1000 | 2950 | | |
| (*d*) | 75 | 500 | | | |
| (*e*) | | 850 | | 0.0 | |



**SOLUTION** Properties and phase descriptions of water are to be determined at various states.

***Analysis*** (*a*) The quality is given to be $x = 0.6$, which implies that 60 percent of the mass is in the vapor phase and the remaining 40 percent is in the liquid phase. Therefore, we have saturated liquid–vapor mixture at a pressure of 200 kPa. Then the temperature must be the saturation temperature at the given pressure:

$$T = T_{\text{sat @ 200 kPa}} = \mathbf{120.21^\circ C} \qquad \text{(Table A–5)}$$

At 200 kPa, we also read from Table A–5 that $u_f = 504.50$ kJ/kg and $u_{fg} = 2024.6$ kJ/kg. Then the average internal energy of the mixture is

$$\begin{aligned} u &= u_f + xu_{fg} \\ &= 504.50 \text{ kJ/kg} + (0.6)(2024.6 \text{ kJ/kg}) \\ &= \mathbf{1719.26\ kJ/kg} \end{aligned}$$

(*b*) This time the temperature and the internal energy are given, but we do not know which table to use to determine the missing properties because we have no clue as to whether we have saturated mixture, compressed liquid, or superheated vapor. To determine the region we are in, we first go to the saturation table (Table A–4) and determine the $u_f$ and $u_g$ values at the given temperature. At 125°C, we read $u_f = 524.83$ kJ/kg and $u_g = 2534.3$ kJ/kg. Next we compare the given $u$ value to these $u_f$ and $u_g$ values, keeping in mind that

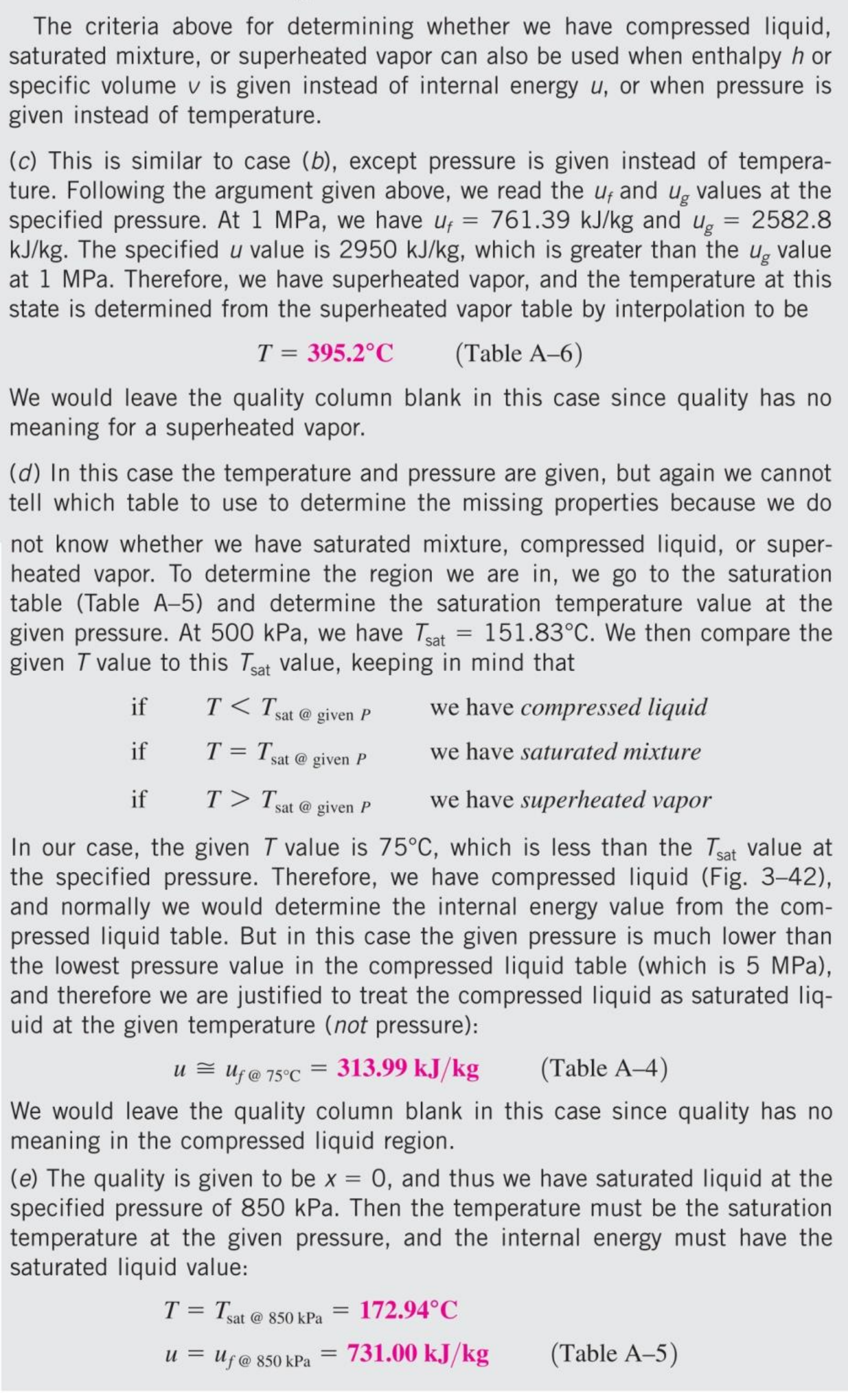
if $u < u_f$ we have *compressed liquid*
if $u_f \le u \le u_g$ we have *saturated mixture*
if $u > u_g$ we have *superheated vapor*

In our case the given $u$ value is 1600, which falls between the $u_f$ and $u_g$ values at 125°C. Therefore, we have saturated liquid–vapor mixture. Then the pressure must be the saturation pressure at the given temperature:

$$P = P_{\text{sat @ 125°C}} = \mathbf{232.23\ kPa} \qquad \text{(Table A–4)}$$

The quality is determined from

$$x = \frac{u - u_f}{u_{fg}} = \frac{1600 - 524.83}{2009.5} = \mathbf{0.535}$$



The criteria above for determining whether we have compressed liquid, saturated mixture, or superheated vapor can also be used when enthalpy $h$ or specific volume $v$ is given instead of internal energy $u$, or when pressure is given instead of temperature.

(*c*) This is similar to case (*b*), except pressure is given instead of temperature. Following the argument given above, we read the $u_f$ and $u_g$ values at the specified pressure. At 1 MPa, we have $u_f = 761.39$ kJ/kg and $u_g = 2582.8$ kJ/kg. The specified $u$ value is 2950 kJ/kg, which is greater than the $u_g$ value at 1 MPa. Therefore, we have superheated vapor, and the temperature at this state is determined from the superheated vapor table by interpolation to be

$$T = \mathbf{395.2°C} \qquad \text{(Table A–6)}$$

We would leave the quality column blank in this case since quality has no meaning for a superheated vapor.

(*d*) In this case the temperature and pressure are given, but again we cannot tell which table to use to determine the missing properties because we do not know whether we have saturated mixture, compressed liquid, or superheated vapor. To determine the region we are in, we go to the saturation table (Table A–5) and determine the saturation temperature value at the given pressure. At 500 kPa, we have $T_{sat} = 151.83°C$. We then compare the given $T$ value to this $T_{sat}$ value, keeping in mind that

if $T < T_{\text{sat @ given } P}$ we have *compressed liquid*

if $T = T_{\text{sat @ given } P}$ we have *saturated mixture*

if $T > T_{\text{sat @ given } P}$ we have *superheated vapor*

In our case, the given $T$ value is 75°C, which is less than the $T_{sat}$ value at the specified pressure. Therefore, we have compressed liquid (Fig. 3–42), and normally we would determine the internal energy value from the compressed liquid table. But in this case the given pressure is much lower than the lowest pressure value in the compressed liquid table (which is 5 MPa), and therefore we are justified to treat the compressed liquid as saturated liquid at the given temperature (*not* pressure):

$$u \cong u_{f\,@\,75°C} = \mathbf{313.99\ kJ/kg} \qquad \text{(Table A–4)}$$

We would leave the quality column blank in this case since quality has no meaning in the compressed liquid region.

(*e*) The quality is given to be $x = 0$, and thus we have saturated liquid at the specified pressure of 850 kPa. Then the temperature must be the saturation temperature at the given pressure, and the internal energy must have the saturated liquid value:

$$T = T_{\text{sat @ 850 kPa}} = \mathbf{172.94°C}$$

$$u = u_{f\,@\,850\text{ kPa}} = \mathbf{731.00\ kJ/kg} \qquad \text{(Table A–5)}$$

# THE IDEAL-GAS EQUATION OF STATE

- **Equation of state**: Any equation that relates the pressure, temperature, and specific volume of a substance.
- The simplest and best-known equation of state for substances in the gas phase is the ideal-gas equation of state. This equation predicts the *P*-*v*-*T* behavior of a gas quite accurately within some properly selected region.

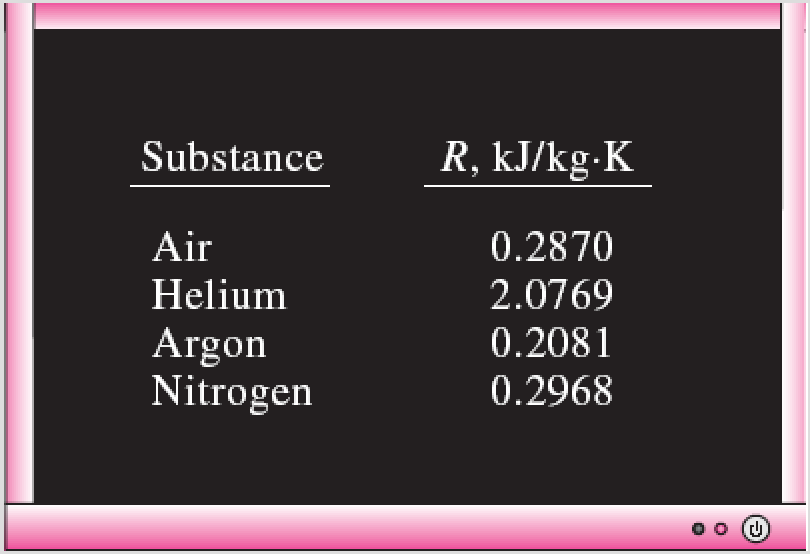
$$P = R\left(\frac{T}{v}\right) \qquad Pv = RT$$

**Ideal gas equation of state**

$$R = \frac{R_u}{M} \qquad (\mathrm{kJ/kg \cdot K \ or \ kPa \cdot m^3/kg \cdot K})$$

*R*: gas constant
*M*: molar mass (kg/kmol)
$R_u$: universal gas constant

$$R_u = \begin{cases} 8.31447 \ \mathrm{kJ/kmol \cdot K} \\ 8.31447 \ \mathrm{kPa \cdot m^3/kmol \cdot K} \\ 0.0831447 \ \mathrm{bar \cdot m^3/kmol \cdot K} \\ 1.98588 \ \mathrm{Btu/lbmol \cdot R} \\ 10.7316 \ \mathrm{psia \cdot ft^3/lbmol \cdot R} \\ 1545.37 \ \mathrm{ft \cdot lbf/lbmol \cdot R} \end{cases}$$



| Substance | *R*, kJ/kg·K |
|---|---|
| Air | 0.2870 |
| Helium | 2.0769 |
| Argon | 0.2081 |
| Nitrogen | 0.2968 |

Different substances have different gas constants.

Mass = Molar mass × Mole number

$$m = MN \qquad (\mathrm{kg})$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

**Ideal gas equation at two states for a fixed mass**

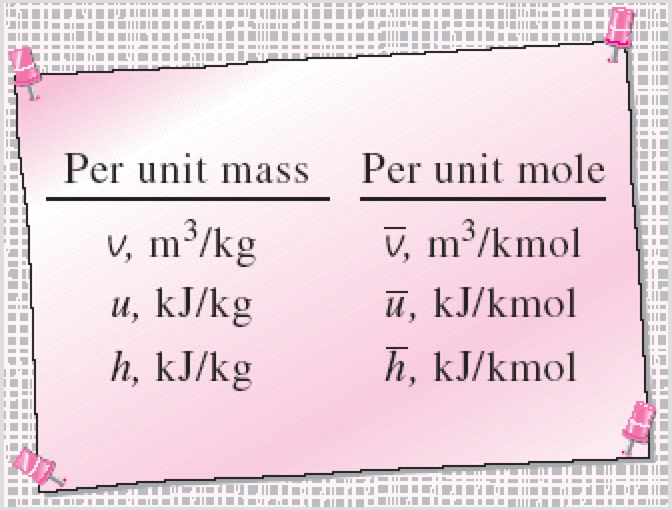
$$V = m\mathrm{v} \longrightarrow PV = mRT$$

$$mR = (MN)R = NR_u \longrightarrow PV = NR_uT$$

$$V = N\overline{\mathrm{v}} \longrightarrow P\overline{\mathrm{v}} = R_uT$$

Various expressions of ideal gas equation

**Real gases behave as an ideal gas at low densities (i.e., low pressure, high temperature).**



| Per unit mass | Per unit mole |
|---|---|
| $\mathrm{v}$, m$^3$/kg | $\overline{\mathrm{v}}$, m$^3$/kmol |
| $u$, kJ/kg | $\overline{u}$, kJ/kmol |
| $h$, kJ/kg | $\overline{h}$, kJ/kmol |

Properties per unit mole are denoted with a bar on the top.

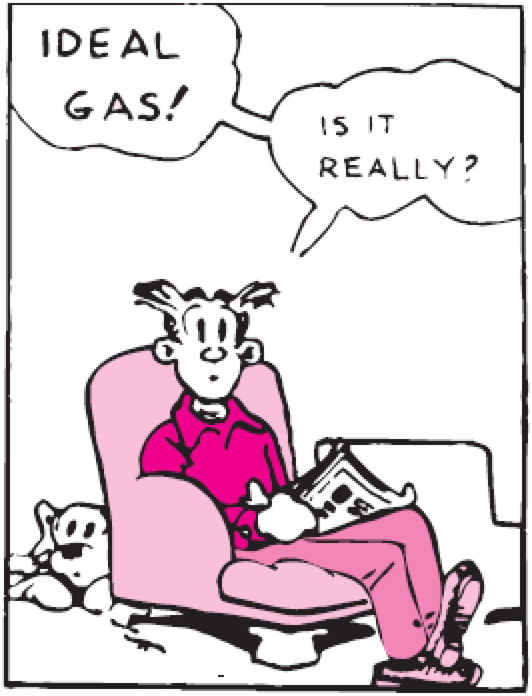
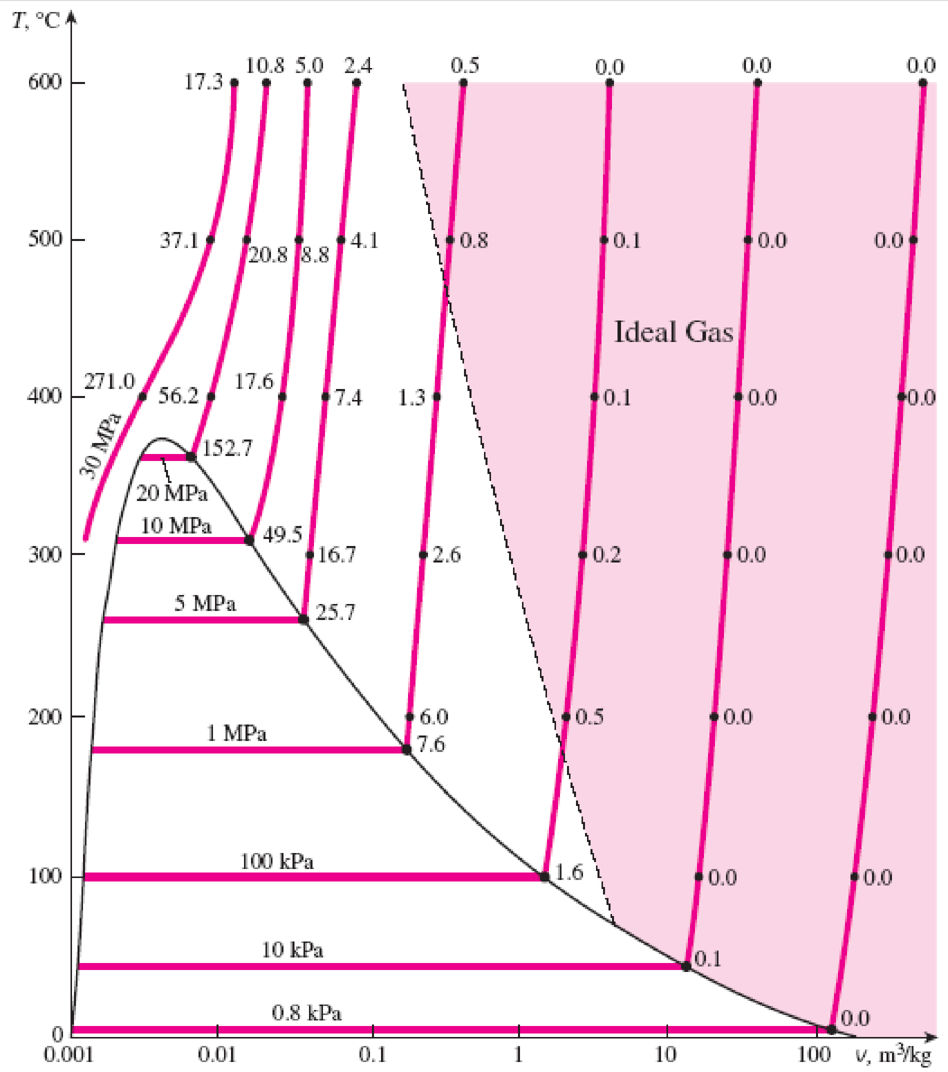


**FIGURE 3–45**
The ideal-gas relation often is not applicable to real gases; thus, care should be exercised when using it.

# Is Water Vapor an Ideal Gas?



- At pressures below 10 kPa, water vapor can be treated as an ideal gas, regardless of its temperature, with negligible error (less than 0.1 percent).
- At higher pressures, however, the ideal gas assumption yields unacceptable errors, particularly in the vicinity of the critical point and the saturated vapor line.
- In air-conditioning applications, the water vapor in the air can be treated as an ideal gas. Why?
- In steam power plant applications, however, the pressures involved are usually very high; therefore, ideal-gas relations should not be used.

Percentage of error ($[|v_{\text{table}} - v_{\text{ideal}}|/v_{\text{table}}] \times 100$) involved in assuming steam to be an ideal gas, and the region where steam can be treated as an ideal gas with less than 1 percent error.

# COMPRESSIBILITY FACTOR—A MEASURE OF DEVIATION FROM IDEAL-GAS BEHAVIOR

**Compressibility factor *Z***
A factor that accounts for the deviation of real gases from ideal-gas behavior at a given temperature and pressure.

$$Pv = ZRT$$

$$Z = \frac{Pv}{RT}$$

$$Z = \frac{v_{\text{actual}}}{v_{\text{ideal}}}$$

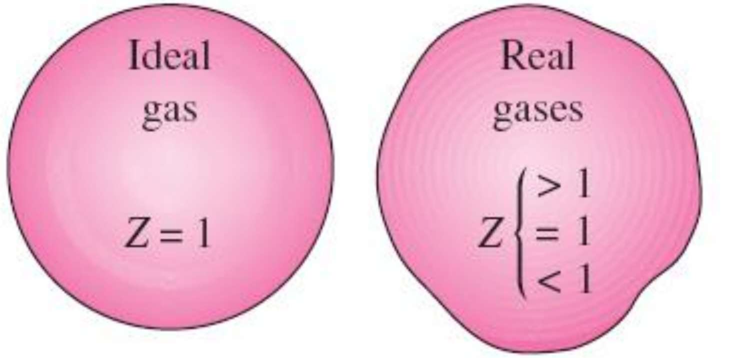


**FIGURE 3–48**
The compressibility factor is unity for ideal gases.

The farther away *Z* is from unity, the more the gas deviates from ideal-gas behavior.

*Gases behave as an ideal gas at low densities (i.e., low pressure, high temperature).*

**Question**: What is the criteria for low pressure and high temperature?

**Answer**: The pressure or temperature of a gas is high or low relative to its critical temperature or pressure.

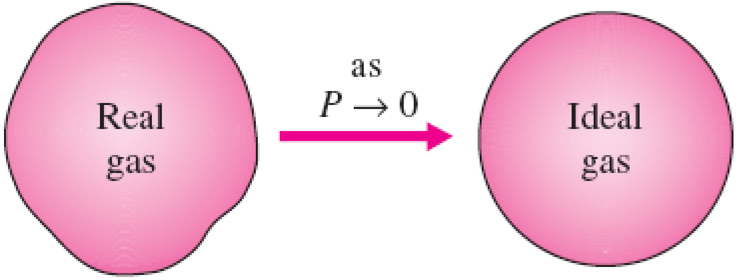


**FIGURE 3–50**
At very low pressures, all gases approach ideal-gas behavior (regardless of their temperature).

$$P_R = \frac{P}{P_{cr}}$$

Reduced pressure

$$T_R = \frac{T}{T_{cr}}$$

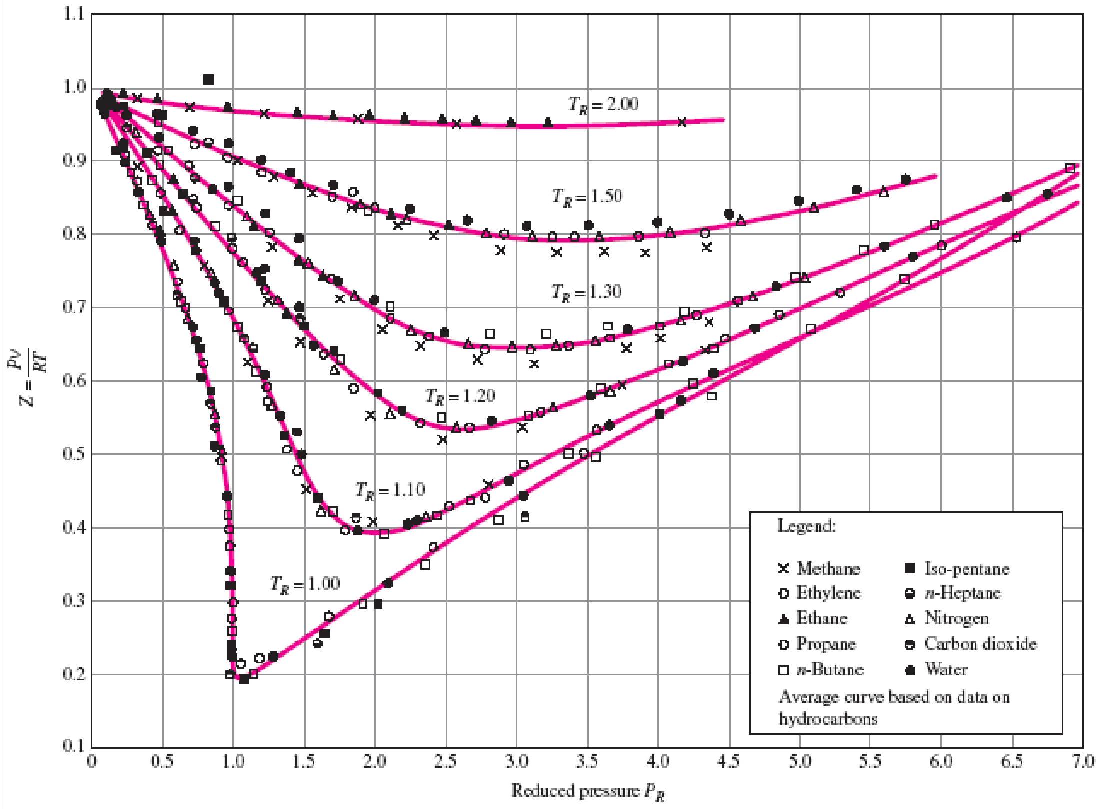
Reduced temperature

$$v_R = \frac{v_{actual}}{RT_{cr}/P_{cr}}$$

Pseudo-reduced specific volume

$$\left.\begin{aligned} P_R &= \frac{P}{P_{cr}} \\ v_R &= \frac{v}{RT_{cr}/P_{cr}} \end{aligned}\right\} \begin{array}{l} Z = \ldots \\ \text{(Fig. A–15)} \end{array}$$

*Z* can also be determined from a knowledge of $P_R$ and $v_R$.



Comparison of *Z* factors for various gases.

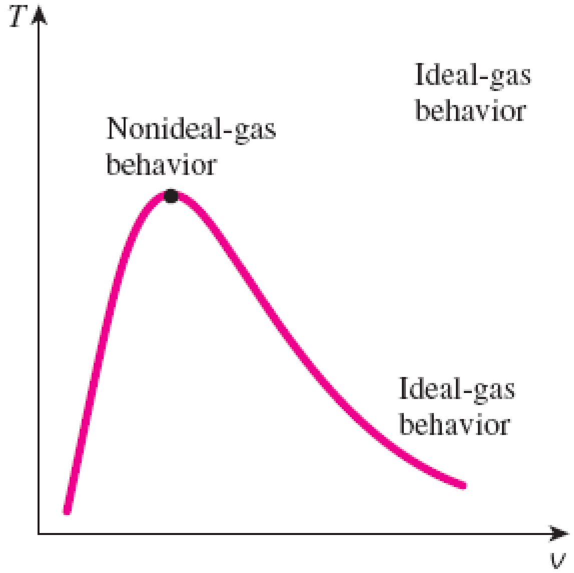


**FIGURE 3–51**

Gases deviate from the ideal-gas behavior the most in the neighborhood of the critical point.

# OTHER EQUATIONS OF STATE

Several equations have been proposed to represent the *P*-*v*-*T* behavior of substances accurately over a larger region with no limitations.

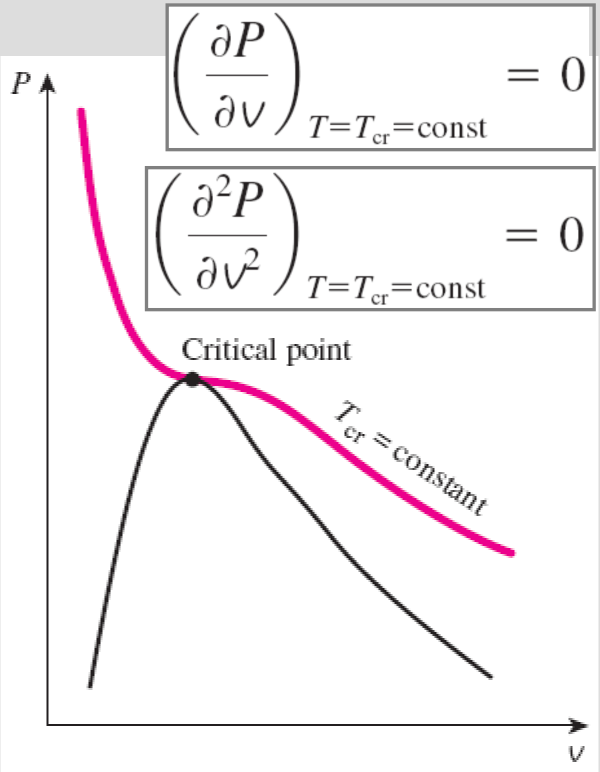
van der Waals
Berthelet
Redlich-Kwang
Beattie-Bridgeman
Benedict-Webb-Rubin
Strobridge
Virial

## Van der Waals Equation of State

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

$$a = \frac{27R^2T_{\mathrm{cr}}^2}{64P_{\mathrm{cr}}} \qquad b = \frac{RT_{\mathrm{cr}}}{8P_{\mathrm{cr}}}$$

Critical isotherm of a pure substance has an inflection point at the critical state.



$$\left(\frac{\partial P}{\partial v}\right)_{T=T_{\mathrm{cr}}=\mathrm{const}} = 0$$

$$\left(\frac{\partial^2 P}{\partial v^2}\right)_{T=T_{\mathrm{cr}}=\mathrm{const}} = 0$$

This model includes two effects not considered in the ideal-gas model: the *intermolecular attraction forces* and the *volume occupied by the molecules themselves*. The accuracy of the van der Waals equation of state is often inadequate.

# Beattie-Bridgeman Equation of State

$$P = \frac{R_u T}{\bar{v}^2}\left(1 - \frac{c}{\bar{v} T^3}\right)(\bar{v} + B) - \frac{A}{\bar{v}^2}$$

$$A = A_0\left(1 - \frac{a}{\bar{v}}\right) \qquad B = B_0\left(1 - \frac{b}{\bar{v}}\right)$$

The constants are given in Table 3–4 for various substances. It is known to be reasonably accurate for densities up to about $0.8\rho_{cr}$.

# Benedict-Webb-Rubin Equation of State

$$P = \frac{R_u T}{\bar{v}} + \left(B_0 R_u T - A_0 - \frac{C_0}{T^2}\right)\frac{1}{\bar{v}^2} + \frac{b R_u T - a}{\bar{v}^3} + \frac{a\alpha}{\bar{v}^6} + \frac{c}{\bar{v}^3 T^2}\left(1 + \frac{\gamma}{\bar{v}^2}\right)e^{-\gamma/\bar{v}^2}$$

The constants are given in Table 3–4. This equation can handle substances at densities up to about 2.5 $\rho_{cr}$.

# Virial Equation of State

$$P = \frac{RT}{v} + \frac{a(T)}{v^2} + \frac{b(T)}{v^3} + \frac{c(T)}{v^4} + \frac{d(T)}{v^5} + \ldots$$

The coefficients $a(T)$, $b(T)$, $c(T)$, and so on, that are functions of temperature alone are called *virial coefficients*.

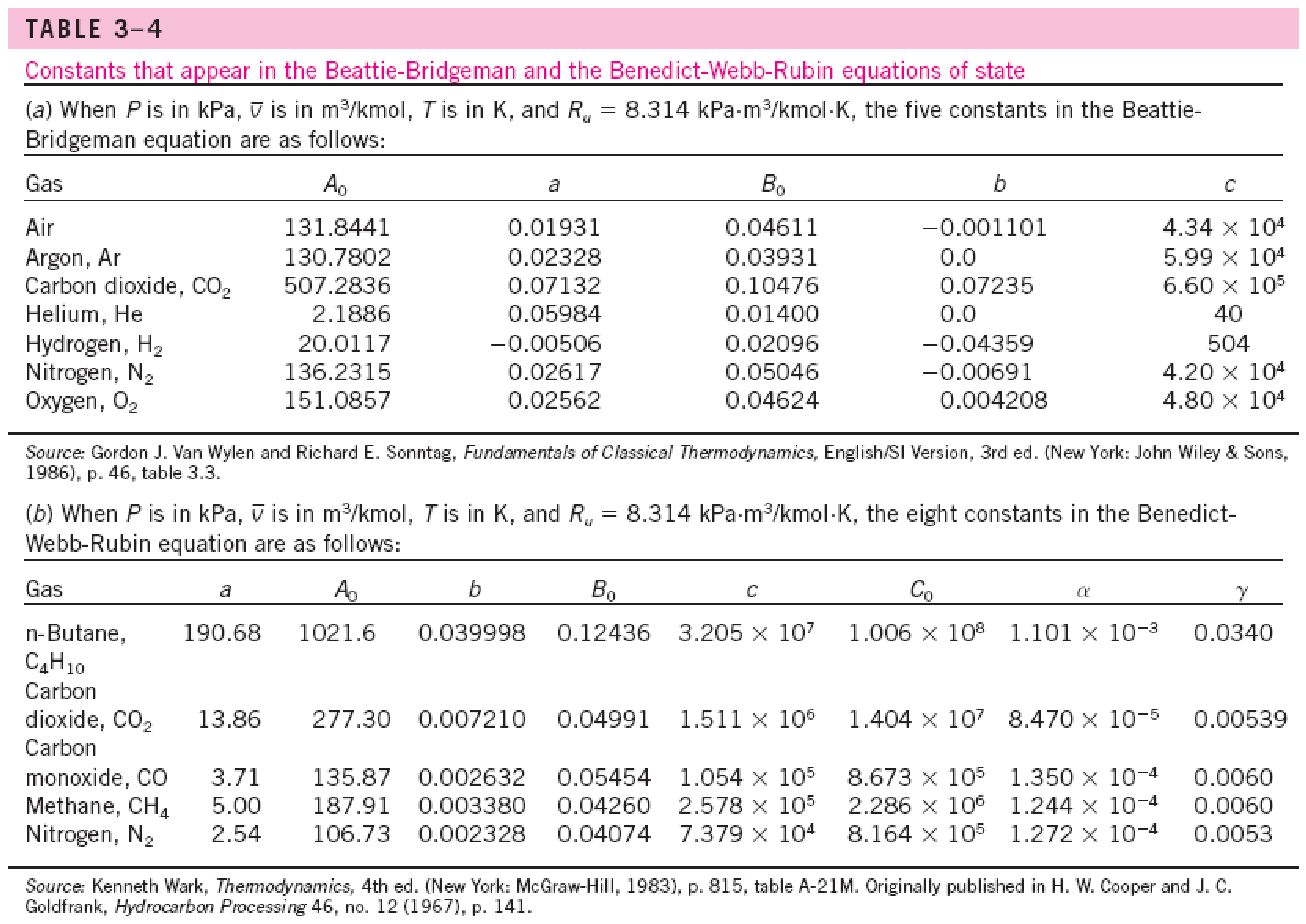


**TABLE 3–4**

Constants that appear in the Beattie-Bridgeman and the Benedict-Webb-Rubin equations of state

(*a*) When $P$ is in kPa, $\bar{v}$ is in m³/kmol, $T$ is in K, and $R_u$ = 8.314 kPa·m³/kmol·K, the five constants in the Beattie-Bridgeman equation are as follows:

| Gas | $A_0$ | $a$ | $B_0$ | $b$ | $c$ |
|---|---|---|---|---|---|
| Air | 131.8441 | 0.01931 | 0.04611 | −0.001101 | $4.34 \times 10^4$ |
| Argon, Ar | 130.7802 | 0.02328 | 0.03931 | 0.0 | $5.99 \times 10^4$ |
| Carbon dioxide, $CO_2$ | 507.2836 | 0.07132 | 0.10476 | 0.07235 | $6.60 \times 10^5$ |
| Helium, He | 2.1886 | 0.05984 | 0.01400 | 0.0 | 40 |
| Hydrogen, $H_2$ | 20.0117 | −0.00506 | 0.02096 | −0.04359 | 504 |
| Nitrogen, $N_2$ | 136.2315 | 0.02617 | 0.05046 | −0.00691 | $4.20 \times 10^4$ |
| Oxygen, $O_2$ | 151.0857 | 0.02562 | 0.04624 | 0.004208 | $4.80 \times 10^4$ |

*Source:* Gordon J. Van Wylen and Richard E. Sonntag, *Fundamentals of Classical Thermodynamics,* English/SI Version, 3rd ed. (New York: John Wiley & Sons, 1986), p. 46, table 3.3.

(*b*) When $P$ is in kPa, $\bar{v}$ is in m³/kmol, $T$ is in K, and $R_u$ = 8.314 kPa·m³/kmol·K, the eight constants in the Benedict-Webb-Rubin equation are as follows:

| Gas | $a$ | $A_0$ | $b$ | $B_0$ | $c$ | $C_0$ | $\alpha$ | $\gamma$ |
|---|---|---|---|---|---|---|---|---|
| n-Butane, $C_4H_{10}$ | 190.68 | 1021.6 | 0.039998 | 0.12436 | $3.205 \times 10^7$ | $1.006 \times 10^8$ | $1.101 \times 10^{-3}$ | 0.0340 |
| Carbon dioxide, $CO_2$ | 13.86 | 277.30 | 0.007210 | 0.04991 | $1.511 \times 10^6$ | $1.404 \times 10^7$ | $8.470 \times 10^{-5}$ | 0.00539 |
| Carbon monoxide, CO | 3.71 | 135.87 | 0.002632 | 0.05454 | $1.054 \times 10^5$ | $8.673 \times 10^5$ | $1.350 \times 10^{-4}$ | 0.0060 |
| Methane, $CH_4$ | 5.00 | 187.91 | 0.003380 | 0.04260 | $2.578 \times 10^5$ | $2.286 \times 10^6$ | $1.244 \times 10^{-4}$ | 0.0060 |
| Nitrogen, $N_2$ | 2.54 | 106.73 | 0.002328 | 0.04074 | $7.379 \times 10^4$ | $8.164 \times 10^5$ | $1.272 \times 10^{-4}$ | 0.0053 |

*Source:* Kenneth Wark, *Thermodynamics,* 4th ed. (New York: McGraw-Hill, 1983), p. 815, table A-21M. Originally published in H. W. Cooper and J. C. Goldfrank, *Hydrocarbon Processing* 46, no. 12 (1967), p. 141.

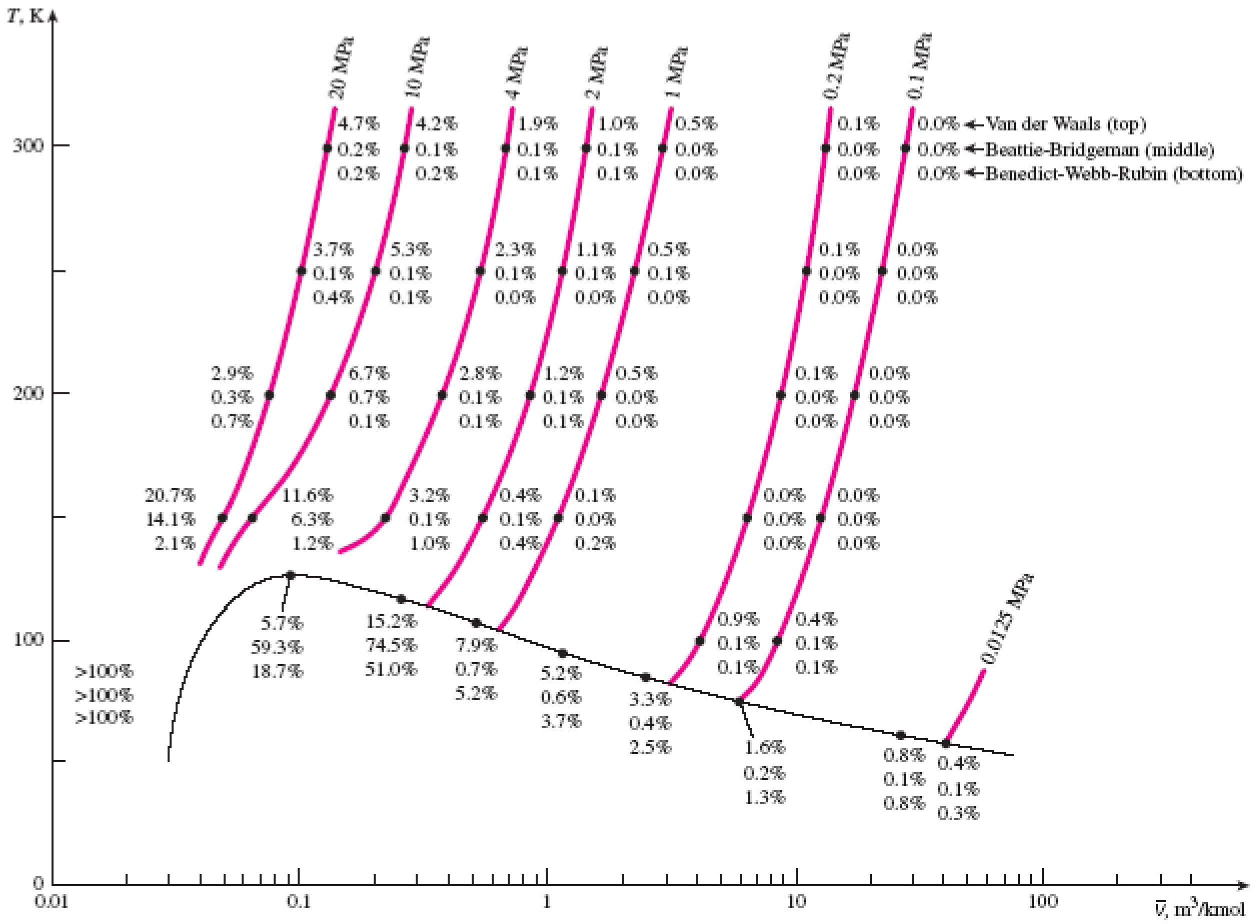
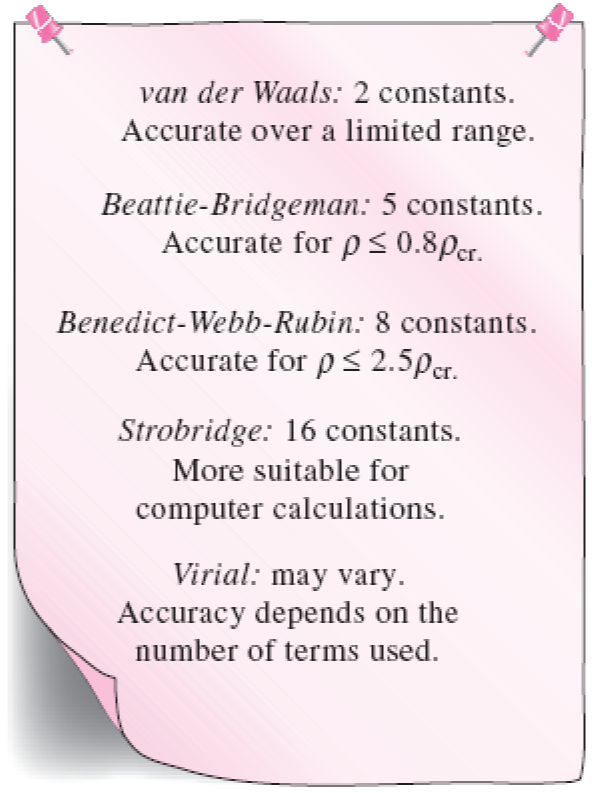


**FIGURE 3–58**

Percentage of error involved in various equations of state for nitrogen (% error = $[(|v_{\text{table}} - v_{\text{equation}}|)/v_{\text{table}}] \times 100$).

Percentage of error involved in various equations of state for nitrogen
(% error = $[(|v_{\text{table}} - v_{\text{equation}}|)/v_{\text{table}}] \times 100$).



*van der Waals:* 2 constants.
Accurate over a limited range.

*Beattie-Bridgeman:* 5 constants.
Accurate for $\rho \leq 0.8\rho_{\text{cr}}$.

*Benedict-Webb-Rubin:* 8 constants.
Accurate for $\rho \leq 2.5\rho_{\text{cr}}$.

*Strobridge:* 16 constants.
More suitable for
computer calculations.

*Virial:* may vary.
Accuracy depends on the
number of terms used.

Complex equations of state represent the *P*-*v*-*T* behavior of gases more accurately over a wider range.

## *EXAMPLE 3-10* **Temperature Rise of Air in a Tire During a Trip**

The gage pressure of an automobile tire is measured to be 210 kPa before a trip and 220 kPa after the trip at a location where the atmospheric pressure is 95 kPa (Fig. 3–46). Assuming the volume of the tire remains constant and the air temperature before the trip is 25°C, determine air temperature in the tire after the trip.

**SOLUTION** The pressure in an automobile tire is measured before and after a trip. The temperature of air in the tire after the trip is to be determined.

***Assumptions*** **1** The volume of the tire remains constant. **2** Air is an ideal gas.

***Properties*** The local atmospheric pressure is 95 kPa.

***Analysis*** The absolute pressures in the tire before and after the trip are

$$P_1 = P_{\text{gage.1}} + P_{\text{atm}} = 210 + 95 = 305\text{ kPa}$$
$$P_2 = P_{\text{gage.2}} + P_{\text{atm}} = 220 + 90 = 315\text{ kPa}$$

Note that air is an ideal gas and the volume is constant, the air temperatures after the trip is determined to be

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \longrightarrow T_2 = \frac{P_2}{P_1}T_1 = \frac{315\text{ kPa}}{305\text{ kPa}}(25 + 273\text{ K}) = 307.8\text{ K} = \mathbf{34.8^\circ C}$$

Therefore, the absolute temperature of air in the tire will increase by 6.9% during this trip.

***Discussion*** Note that the air temperature has risen nearly 10°C during this trip. This shows the importance of measuring the tire pressures before long trips to avoid errors due to temperature rise of air in tire. Also note that the unit Kelvin is used for temperature in the ideal gas relation.

## *EXAMPLE 3–11* **The Use of Generalized Charts**

Determine the specific volume of refrigerant-134a at 1 MPa and 50°C, using (*a*) the ideal-gas equation of state and (*b*) the generalized compressibility chart. Compare the values obtained to the actual value of 0.021796 $m^3/kg$ and determine the error involved in each case.

**SOLUTION** The specific volume of refrigerant-134a is to be determined assuming ideal- and nonideal-gas behavior.

***Analysis*** The gas constant, the critical pressure, and the critical temperature of refrigerant-134a are determined from Table A–1 to be

$$R = 0.0815\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$$
$$P_{\text{cr}} = 4.059\ \text{MPa}$$
$$T_{\text{cr}} = 374.2\ \text{K}$$

(*a*) The specific volume of refrigerant-134a under the ideal-gas assumption is

$$v = \frac{RT}{P} = \frac{(0.0815\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(323\ \text{K})}{1000\ \text{kPa}} = \mathbf{0.026325\ m^3/kg}$$

Therefore, treating the refrigerant-134a vapor as an ideal gas would result in an error of (0.026325 − 0.021796)/0.021796 = **0.208**, or 20.8 percent in this case.

(*b*) To determine the correction factor *Z* from the compressibility chart, we first need to calculate the reduced pressure and temperature:

$$\left.\begin{aligned} P_R &= \frac{P}{P_{\text{cr}}} = \frac{1\ \text{MPa}}{4.059\ \text{MPa}} = 0.246 \\ T_R &= \frac{T}{T_{\text{cr}}} = \frac{323\ \text{K}}{374.2\ \text{K}} = 0.863 \end{aligned}\right\} \quad Z = 0.84$$

Thus

$$v = Zv_{\text{ideal}} = (0.84)(0.026325\ \text{m}^3/\text{kg}) = \mathbf{0.022113\ m^3/kg}$$

***Discussion*** The error in this result is less than **2 percent**. Therefore, in the absence of tabulated data, the generalized compressibility chart can be used with confidence.

### EXAMPLE 3–12 Using Generalized Charts to Determine Pressure

Determine the pressure of water vapor at 350°C and 0.035262 $m^3/kg$, using (*a*) the steam tables, (*b*) the ideal-gas equation, and (*c*) the generalized compressibility chart.

**SOLUTION** The pressure of water vapor is to be determined in three different ways.

***Analysis*** A sketch of the system is given in Fig. 3–53. The gas constant, the critical pressure, and the critical temperature of steam are determined from Table A–1 to be

$$R = 0.4615 \text{ kPa·m}^3\text{/kg·K}$$
$$P_{cr} = 22.06 \text{ MPa}$$
$$T_{cr} = 647.1 \text{ K}$$

(*a*) The pressure at the specified state is determined from Table A–6 to be

$$\left.\begin{aligned} v &= 0.035262 \text{ m}^3\text{/kg} \\ T &= 350°\text{C} \end{aligned}\right\} \quad P = \mathbf{7.0\ MPa}$$

This is the experimentally determined value, and thus it is the most accurate.

(*b*) The pressure of steam under the ideal-gas assumption is determined from the ideal-gas relation to be

$$P = \frac{RT}{v} = \frac{(0.4615 \text{ kPa·m}^3\text{/kg·K})(623 \text{ K})}{0.035262 \text{ m}^3\text{/kg}} = \mathbf{8.15\ MPa}$$

Therefore, treating the steam as an ideal gas would result in an error of (8.15 − 7.0)/7.0 = 0.164, or 16.4 percent in this case.

(*c*) To determine the correction factor *Z* from the compressibility chart (Fig. A–15), we first need to calculate the pseudo-reduced specific volume and the reduced temperature:

$$\left.\begin{aligned} v_R &= \frac{v_{\text{actual}}}{RT_{cr}/P_{cr}} = \frac{(0.035262 \text{ m}^3\text{/kg})(22{,}060 \text{ kPa})}{(0.4615 \text{ kPa·m}^3\text{/kg·K})(647.1 \text{ K})} = 2.605 \\ T_R &= \frac{T}{T_{cr}} = \frac{623 \text{ K}}{647.1 \text{ K}} = 0.96 \end{aligned}\right\} \quad P_R = 0.31$$

Thus,

$$P = P_R P_{cr} = (0.31)(22.06 \text{ MPa}) = \mathbf{6.84\ MPa}$$

***Discussion*** Using the compressibility chart reduced the error from 16.4 to 2.3 percent, which is acceptable for most engineering purposes (Fig. 3–54). A bigger chart, of course, would give better resolution and reduce the reading errors. Notice that we did not have to determine *Z* in this problem since we could read $P_R$ directly from the chart.

# Summary

- Pure substance
- Phases of a pure substance
- Phase-change processes of pure substances
  - ✓ Compressed liquid, Saturated liquid, Saturated vapor, Superheated vapor
  - ✓ Saturation temperature and Saturation pressure
- Property diagrams for phase change processes
  - ✓ The *T-v* diagram, The *P-v* diagram, The *P-T* diagram, *The P-v-T* surface
- Property tables
  - ✓ Enthalpy
  - ✓ Saturated liquid, saturated vapor, Saturated liquid vapor mixture, Superheated vapor, compressed liquid
  - ✓ Reference state and reference values
- The ideal gas equation of state
  - ✓ Is water vapor an ideal gas?
- Compressibility factor
- Other equations of state
  - ✓ van der Waals Equation of State, Beattie-Bridgeman Equation of State
  - ✓ Benedict-Webb-Rubin Equation of State, Virial Equation of State