Chapter 5

MASS AND ENERGY ANALYSIS OF CONTROL VOLUMES

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Objectives

- Develop the conservation of mass principle.
- Apply the conservation of mass principle to various systems including steady- and unsteady-flow control volumes.
- Apply the first law of thermodynamics as the statement of the conservation of energy principle to control volumes.
- Identify the energy carried by a fluid stream crossing a control surface as the sum of internal energy, flow work, kinetic energy, and potential energy of the fluid and to relate the combination of the internal energy and the flow work to the property enthalpy.
- Solve energy balance problems for common steady-flow devices such as nozzles, compressors, turbines, throttling valves, mixers, heaters, and heat exchangers.
- Apply the energy balance to general unsteady-flow processes with particular emphasis on the uniform-flow process as the model for commonly encountered charging and discharging processes.

CONSERVATION OF MASS

Conservation of mass: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.

Closed systems: The mass of the system remain constant during a process.

Control volumes: Mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.

Mass m and energy E can be converted to each other according to $E = mc^2$

where c is the speed of light in a vacuum, which is $c = 2.9979 \times 10^8$ m/s.

The mass change due to energy change is negligible.

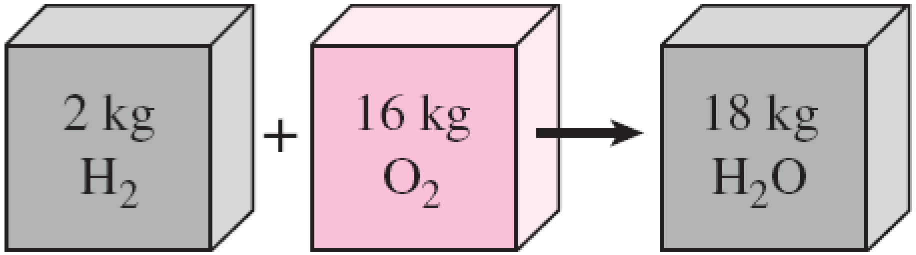


FIGURE 5–1

Mass is conserved even during chemical reactions.

Mass and Volume Flow Rates

$$\delta \dot{m} = \rho V_n \, dA_c$$

$$\dot{m} = \int_{A_c} \delta \dot{m} = \int_{A_c} \rho V_n \, dA_c$$

$$\dot{m} = \rho V_{avg} A_c \qquad (\mathrm{kg/s})$$

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v}$$

Mass flow rate

$$V_{avg} = \frac{1}{A_c} \int_{A_c} V_n \, dA_c$$

Definition of average velocity

Volume flow rate

$$\dot{V} = \int_{A_c} V_n \, dA_c = V_{avg} A_c = V A_c \qquad (\mathrm{m^3/s})$$

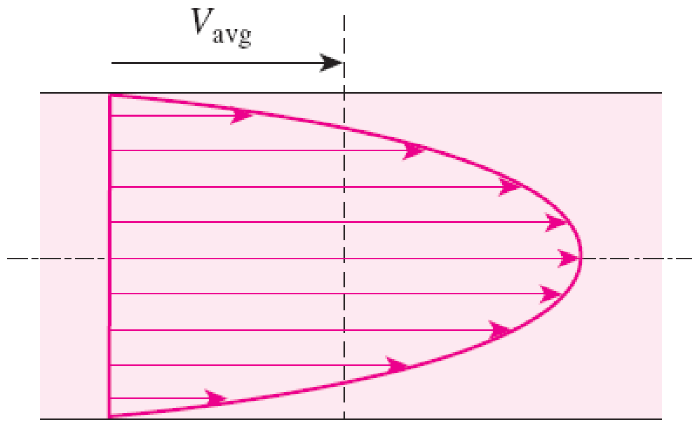


FIGURE 5–3

The average velocity V_{avg} is defined as the average speed through a cross section.

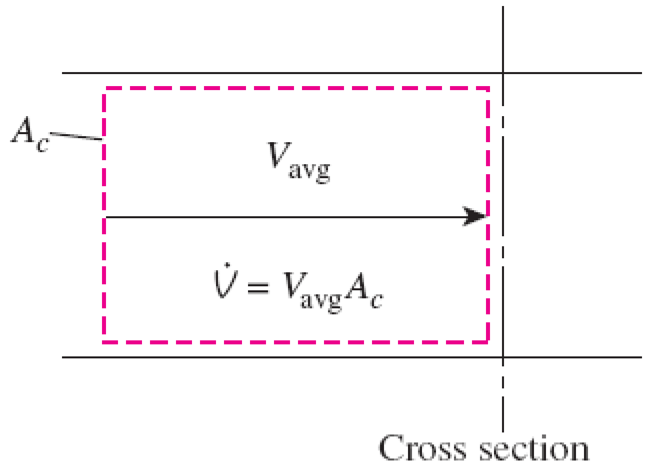


FIGURE 5–4

The volume flow rate is the volume of fluid flowing through a cross section per unit time.

Conservation of Mass Principle

$$\begin{pmatrix}\text{Total mass entering} \\ \text{the CV during } \Delta t\end{pmatrix} - \begin{pmatrix}\text{Total mass leaving} \\ \text{the CV during } \Delta t\end{pmatrix} = \begin{pmatrix}\text{Net change of mass} \\ \text{within the CV during } \Delta t\end{pmatrix}$$

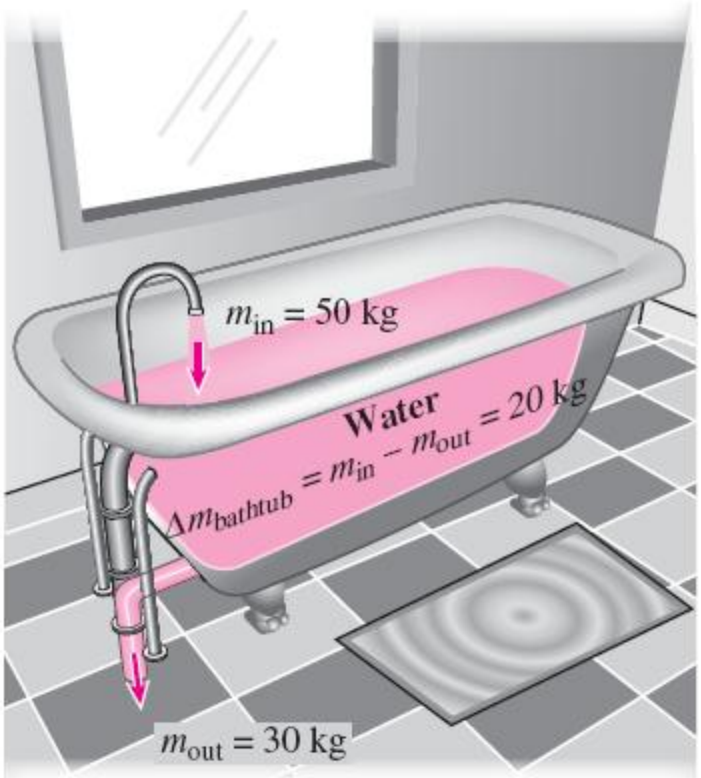


FIGURE 5–5
Conservation of mass principle for an ordinary bathtub.

The conservation of mass principle for a control volume: The net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt.

$$m_{in} - m_{out} = \Delta m_{CV} \qquad \text{(kg)}$$

$$\Delta m_{CV} = m_{final} - m_{initial}$$

$$\dot{m}_{in} - \dot{m}_{out} = dm_{CV}/dt \qquad \text{(kg/s)}$$

These equations are often referred to as the **mass balance** and are applicable to any control volume undergoing any kind of process.

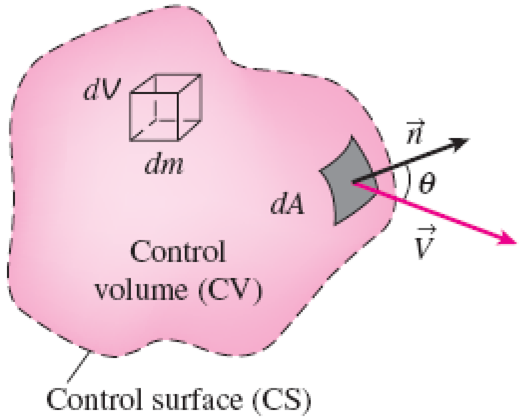


FIGURE 5–6
The differential control volume dV and the differential control surface dA used in the derivation of the conservation of mass relation.

Total mass within the CV: $$m_{CV} = \int_{CV} \rho \, dV$$

Rate of change of mass within the CV: $$\frac{dm_{CV}}{dt} = \frac{d}{dt}\int_{CV} \rho \, dV$$

Normal component of velocity: $$V_n = V\cos\theta = \vec{V}\cdot\vec{n}$$

Differential mass flow rate: $$\delta\dot{m} = \rho V_n \, dA = \rho(V\cos\theta)\, dA = \rho(\vec{V}\cdot\vec{n})\, dA$$

Net mass flow rate: $$\dot{m}_{net} = \int_{CS} \delta\dot{m} = \int_{CS} \rho V_n \, dA = \int_{CS} \rho(\vec{V}\cdot\vec{n})\, dA$$

General conservation of mass: $$\frac{d}{dt}\int_{CV} \rho \, dV + \int_{CS} \rho(\vec{V}\cdot\vec{n})\, dA = 0$$

the time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero.

$$\frac{d}{dt}\int_{CV} \rho \, dV + \sum_{out} \rho \left| V_n \right| dA - +\sum_{in} \rho \left| V_n \right| dA = 0$$

$$\frac{d}{dt}\int_{CV} \rho \, dV = \sum_{in} \dot{m} - \sum_{out} \dot{m} \quad \text{or} \quad \frac{dm_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

General conservation of mass in rate form

Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.

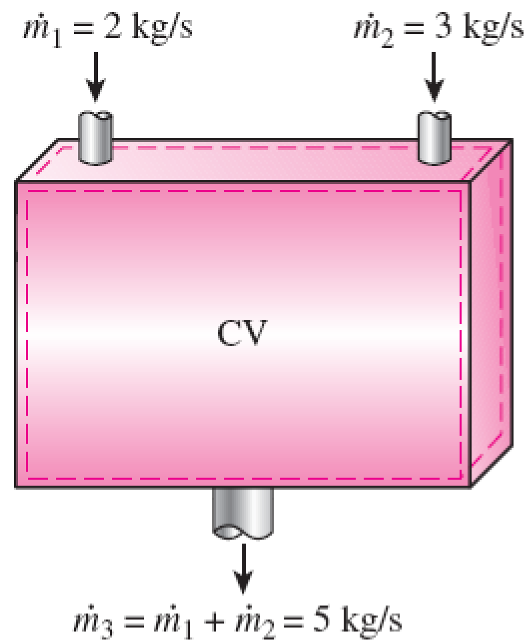


FIGURE 5–7
Conservation of mass principle for a two-inlet–one-outlet steady-flow system.

For steady-flow processes, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate.*

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \qquad (\text{kg/s})$$

Multiple inlets and exits

$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Single stream

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).

Special Case: Incompressible Flow

The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids.

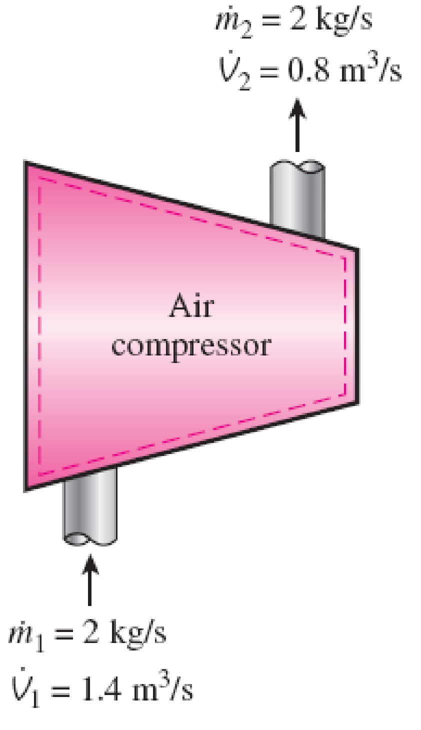


FIGURE 5–8
During a steady-flow process, volume flow rates are not necessarily conserved although mass flow rates are.

$$\sum_{\text{in}} \dot{V} = \sum_{\text{out}} \dot{V} \qquad (\text{m}^3/\text{s})$$

Steady, incompressible

$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$$

Steady, incompressible flow (single stream)

There is no such thing as a "conservation of volume" principle.

For steady flow of liquids, the volume flow rates, as well as the mass flow rates, remain constant since liquids are essentially incompressible substances.

FLOW WORK AND THE ENERGY OF A FLOWING FLUID

Flow work, or flow energy: The work (or energy) required to push the mass into or out of the control volume. This work is necessary for maintaining a continuous flow through a control volume.

$$F = PA$$

$$W_{\text{flow}} = FL = PAL = PV \qquad \text{(kJ)}$$

$$w_{\text{flow}} = Pv \qquad \text{(kJ/kg)}$$

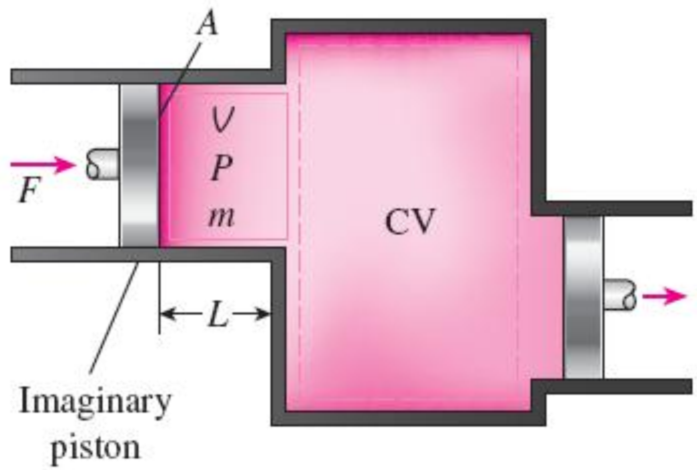


FIGURE 5–11
Schematic for flow work.

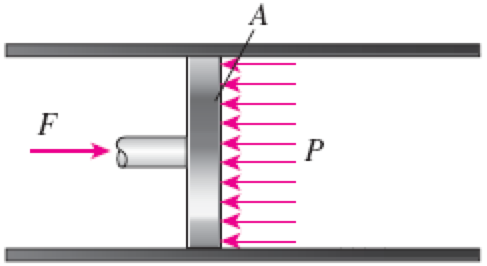


FIGURE 5–12
In the absence of acceleration, the force applied on a fluid by a piston is equal to the force applied on the piston by the fluid.

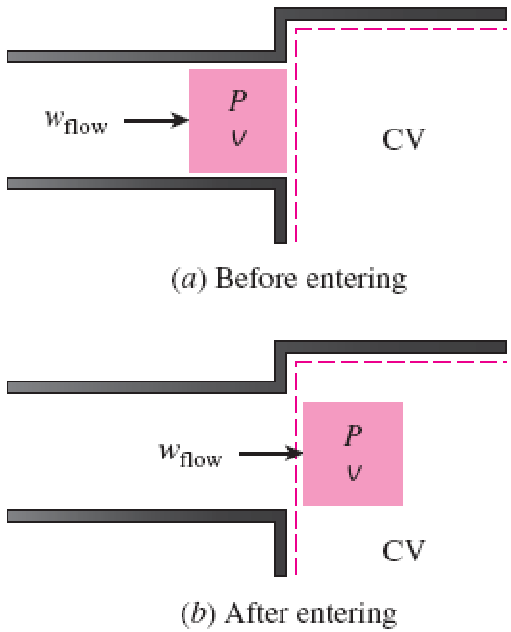


FIGURE 5–13
Flow work is the energy needed to push a fluid into or out of a control volume, and it is equal to Pv.

Total Energy of a Flowing Fluid

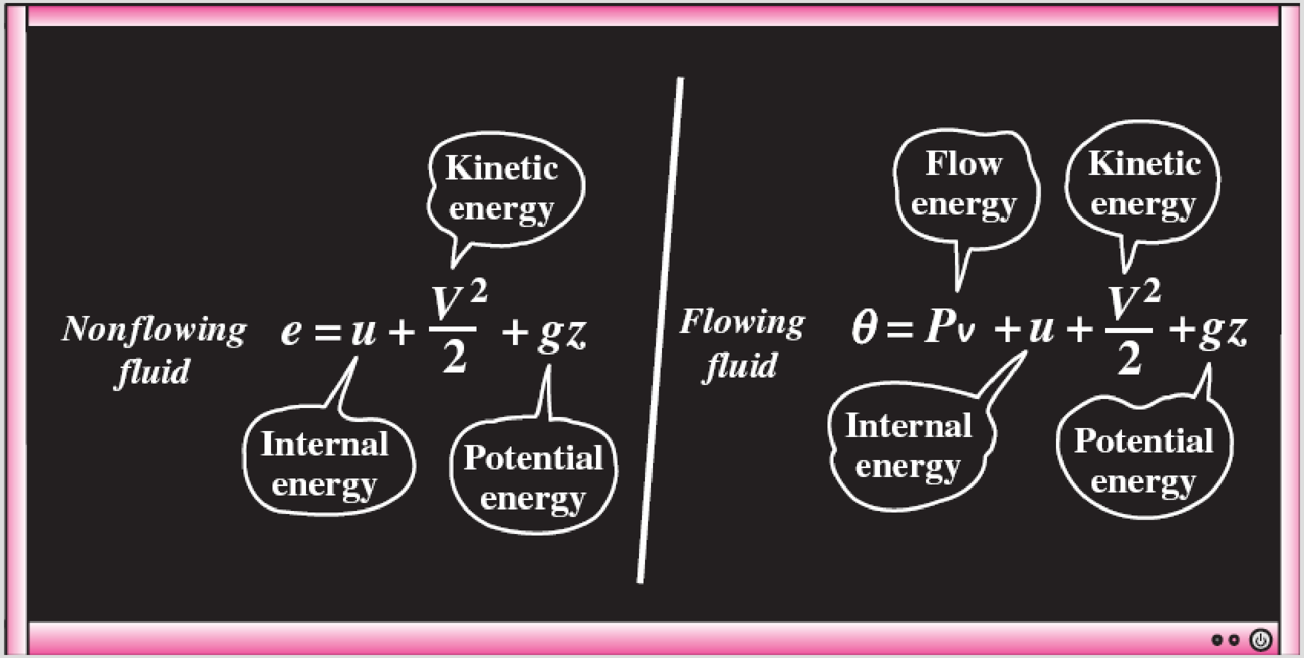
$$e = u + \mathrm{ke} + \mathrm{pe} = u + \frac{V^2}{2} + gz \qquad \mathrm{(kJ/kg)}$$

$$\theta = Pv + e = Pv + (u + \mathrm{ke} + \mathrm{pe})$$

$$h = u + Pv$$

$$\theta = h + \mathrm{ke} + \mathrm{pe} = h + \frac{V^2}{2} + gz \qquad \mathrm{(kJ/kg)}$$

The flow energy is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy.



The total energy consists of three parts for a nonflowing fluid and four parts for a flowing fluid.

Energy Transport by Mass

Amount of energy transport: $$E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right) \qquad \text{(kJ)}$$

Rate of energy transport: $$\dot{E}_{\text{mass}} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right) \qquad \text{(kW)}$$

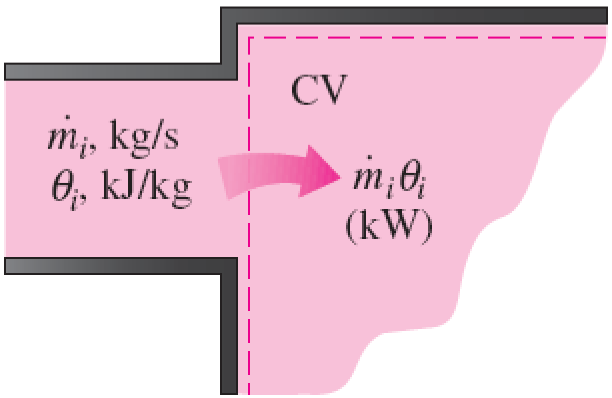


FIGURE 5–15

The product $\dot{m}_i\theta_i$ is the energy transported into control volume by mass per unit time.

When the kinetic and potential energies of a fluid stream are negligible

$$E_{\text{mass}} = mh \qquad \dot{E}_{\text{mass}} = \dot{m}h$$

When the properties of the mass at each inlet or exit change with time as well as over the cross section

$$E_{\text{in,mass}} = \int_{m_i} \theta_i \, \delta m_i = \int_{m_i} \left(h_i + \frac{V_i^2}{2} + gz_i\right) \delta m_i$$

ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS

FIGURE 5–17
Many engineering systems such as power plants operate under steady conditions.

Steady-flow process: *A process during which a fluid flows through a control volume steadily*.

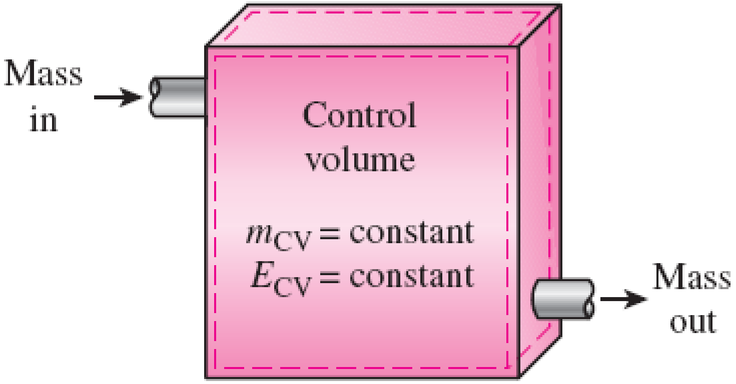


FIGURE 5–18
Under steady-flow conditions, the mass and energy contents of a control volume remain constant.

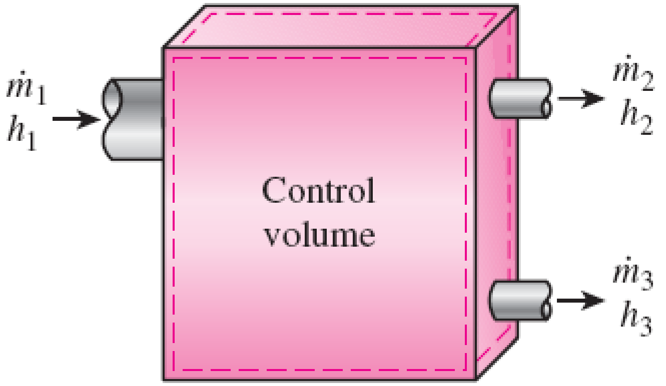


FIGURE 5–19
Under steady-flow conditions, the fluid properties at an inlet or exit remain constant (do not change with time).

Mass and Energy balances for a steady-flow process

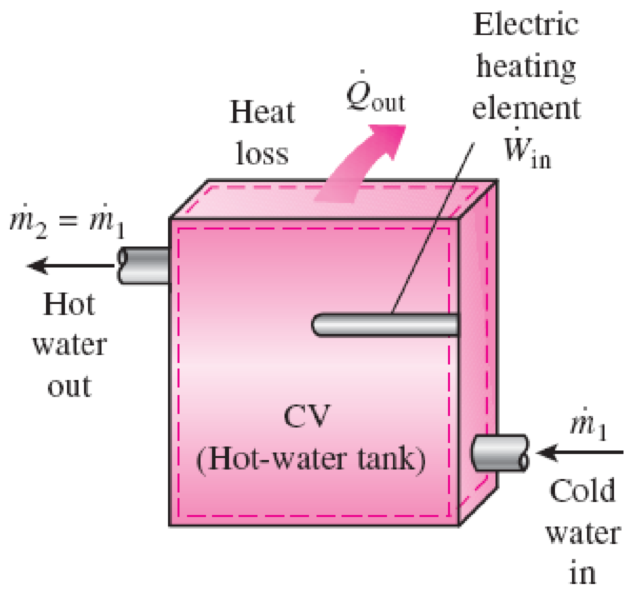
$$\sum_{in} \dot{m} = \sum_{out} \dot{m} \qquad (\text{kg/s})$$

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Mass balance

A water heater in steady operation.



Energy balance

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{dE_{system}/dt}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc., energies}}} \overset{0 \text{ (steady)}}{=} 0$$

$$\underbrace{\dot{E}_{in}}_{\substack{\text{Rate of net energy transfer in} \\ \text{by heat, work, and mass}}} = \underbrace{\dot{E}_{out}}_{\substack{\text{Rate of net energy transfer out} \\ \text{by heat, work, and mass}}} \qquad (\text{kW})$$

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m}\theta = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m}\theta$$

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each inlet}} = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each exit}}$$

Energy balance relations with sign conventions (i.e., heat input and work output are positive)

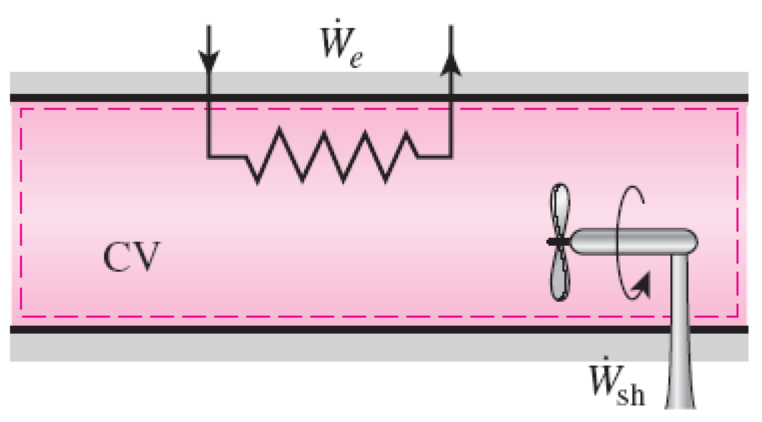
$$\dot{Q} - \dot{W} = \underbrace{\sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each exit}} - \underbrace{\sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each inlet}}$$

$$\dot{Q} - \dot{W} = \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$q - w = h_2 - h_1 \qquad q = \dot{Q}/\dot{m} \qquad w = \dot{W}/\dot{m}$$

when kinetic and potential energy changes are negligible



Under steady operation, shaft work and electrical work are the only forms of work a simple compressible system may involve.

$$\frac{\text{J}}{\text{kg}} \equiv \frac{\text{N}\cdot\text{m}}{\text{kg}} \equiv \left(\text{kg}\frac{\text{m}}{\text{s}^2}\right)\frac{\text{m}}{\text{kg}} \equiv \frac{\text{m}^2}{\text{s}^2}$$

$$\left(\text{Also, } \frac{\text{Btu}}{\text{lbm}} \equiv 25{,}037\ \frac{\text{ft}^2}{\text{s}^2}\right)$$

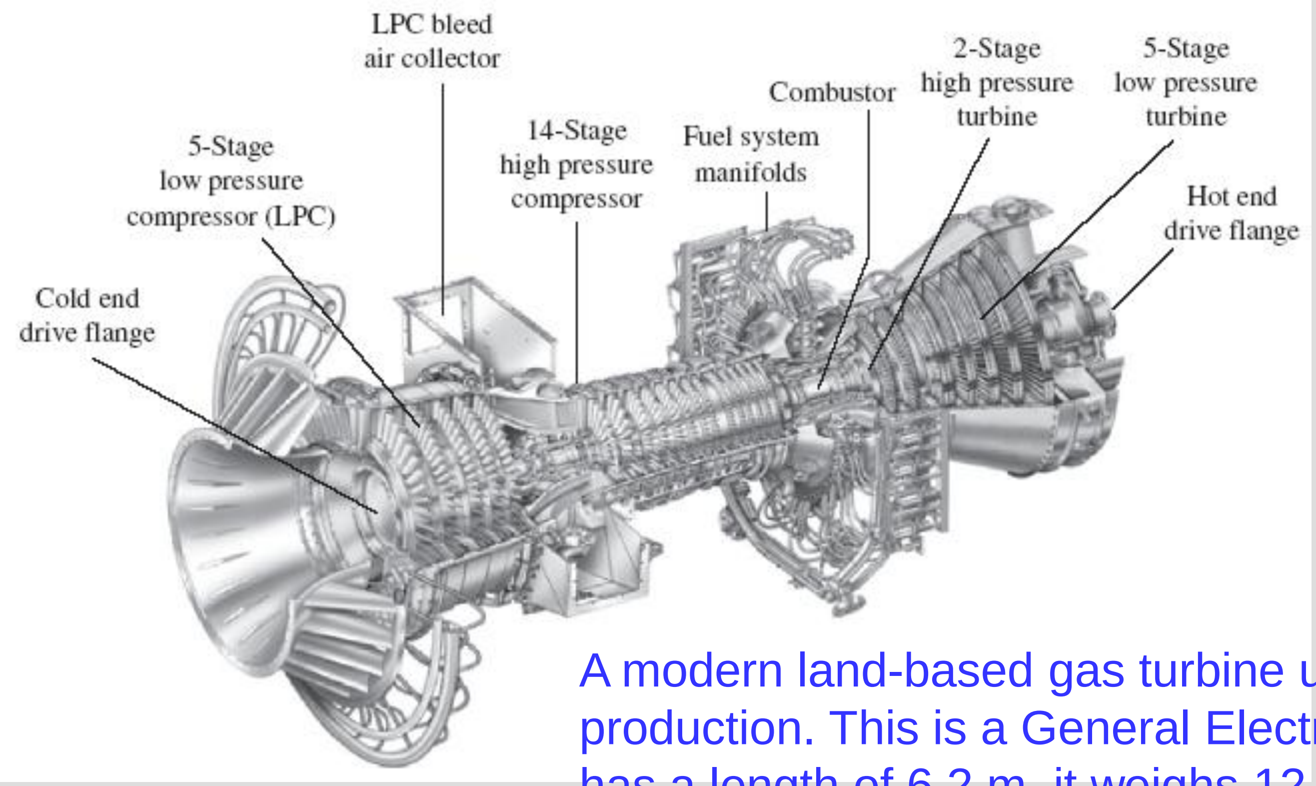
The units m^2/s^2 and J/kg are equivalent.

V_1 m/s	V_2 m/s	Δke kJ/kg
0	45	1
50	67	1
100	110	1
200	205	1
500	502	1

At very high velocities, even small changes in velocities can cause significant changes in the kinetic energy of the fluid.

SOME STEADY-FLOW ENGINEERING DEVICES

Many engineering devices operate essentially under the same conditions for long periods of time. The components of a steam power plant (turbines, compressors, heat exchangers, and pumps), for example, operate nonstop for months before the system is shut down for maintenance. Therefore, these devices can be conveniently analyzed as steady-flow devices.



A modern land-based gas turbine used for electric power production. This is a General Electric LM5000 turbine. It has a length of 6.2 m, it weighs 12.5 tons, and produces 55.2 MW at 3600 rpm with steam injection.

Nozzles and Diffusers

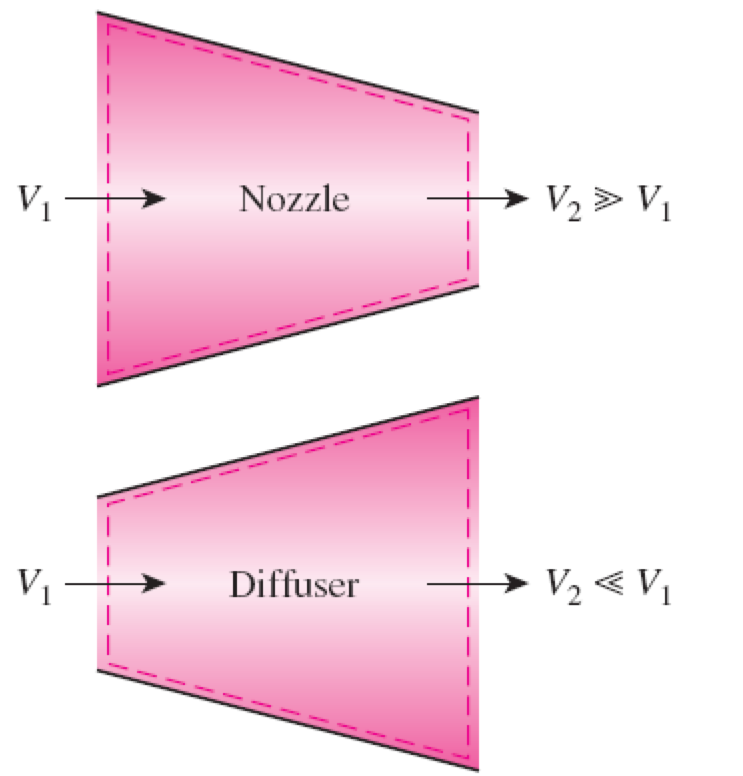


FIGURE 5–25
Nozzles and diffusers are shaped so that they cause large changes in fluid velocities and thus kinetic energies.

Nozzles and diffusers are commonly utilized in jet engines, rockets, spacecraft, and even garden hoses.

A **nozzle** is a device that *increases the velocity of a fluid* at the expense of pressure.

A **diffuser** is a device that *increases the pressure of a fluid* by slowing it down.

The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.

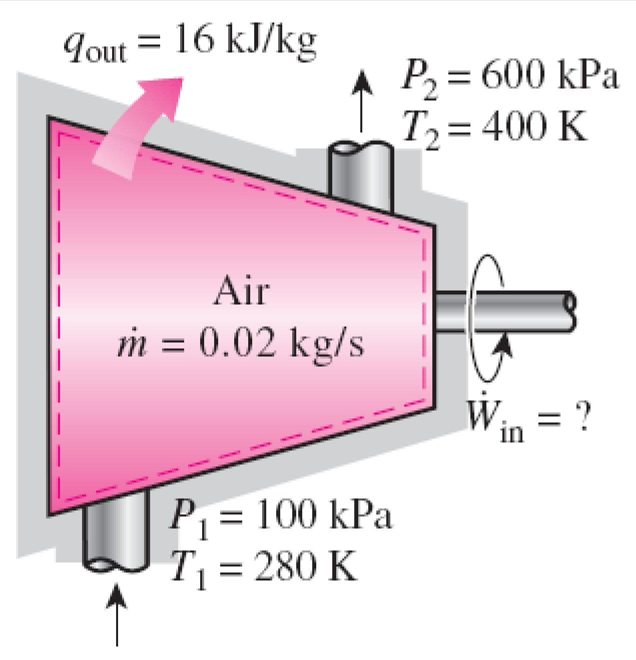
Energy balance for a nozzle or diffuser:

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$

$$(\text{since } \dot{Q} \cong 0,\ \dot{W} = 0, \text{ and } \Delta\text{pe} \cong 0)$$

Turbines and Compressors



Energy balance for the compressor in this figure:

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2$$

$$(\text{since } \Delta\text{ke} = \Delta\text{pe} \cong 0)$$

Turbine drives the electric generator in steam, gas, or hydroelectric power plants.

As the fluid passes through the turbine, work is done against the blades, which are attached to the shaft. As a result, the shaft rotates, and the turbine produces work.

Compressors, as well as **pumps** and **fans**, are devices used to increase the pressure of a fluid. Work is supplied to these devices from an external source through a rotating shaft.

A ***fan*** increases the pressure of a gas slightly and is mainly used to mobilize a gas.

A ***compressor*** is capable of compressing the gas to very high pressures.

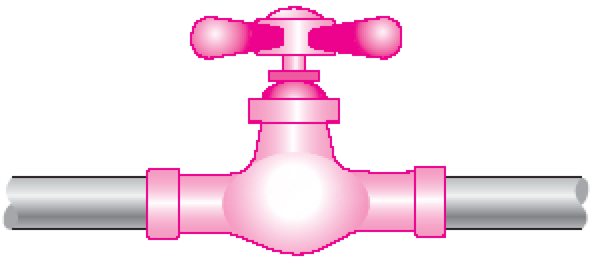
Pumps work very much like compressors except that they handle liquids instead of gases.

Throttling valves

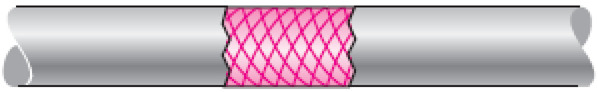
Throttling valves are *any kind of flow-restricting devices* that cause a significant pressure drop in the fluid.

What is the difference between a turbine and a throttling valve?

The pressure drop in the fluid is often accompanied by a *large drop in temperature*, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications.



(*a*) An adjustable valve



(*b*) A porous plug

(*c*) A capillary tube

Energy balance

$$h_2 \cong h_1$$

$$u_1 + P_1 v_1 = u_2 + P_2 v_2$$

$$\text{Internal energy} + \text{Flow energy} = \text{Constant}$$

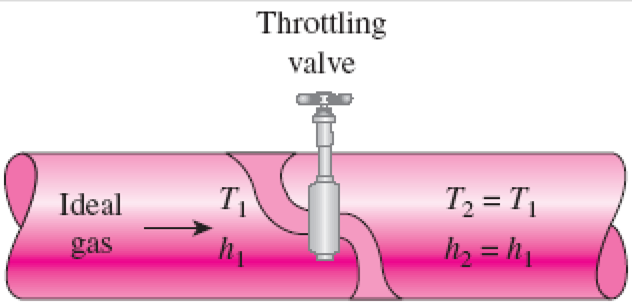


FIGURE 5–30
The temperature of an ideal gas does not change during a throttling (h = constant) process since $h = h(T)$.

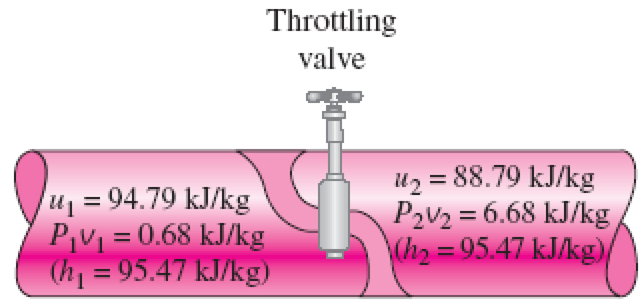


FIGURE 5–31
During a throttling process, the enthalpy (flow energy + internal energy) of a fluid remains constant. But internal and flow energies may be converted to each other.

Mixing chambers

In engineering applications, the section where the mixing process takes place is commonly referred to as a **mixing chamber**.

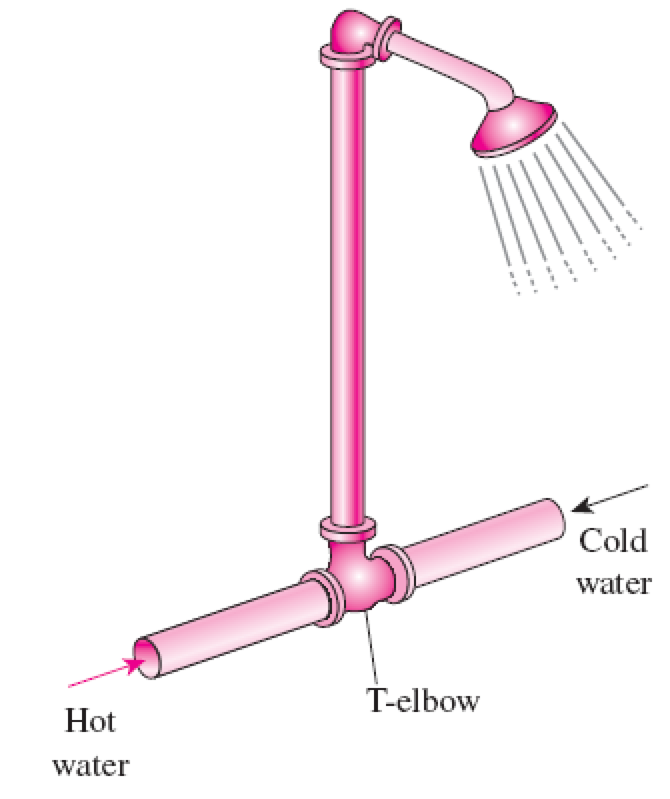
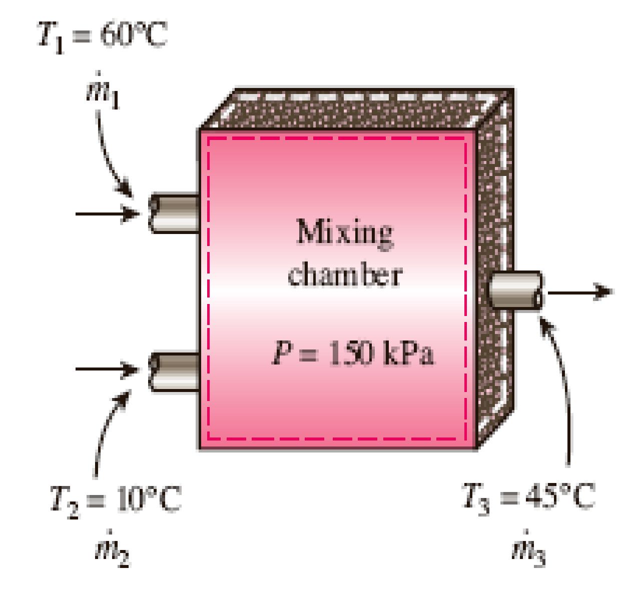


FIGURE 5–32

The T-elbow of an ordinary shower serves as the mixing chamber for the hot- and the cold-water streams.



Energy balance for the adiabatic mixing chamber in the figure is:

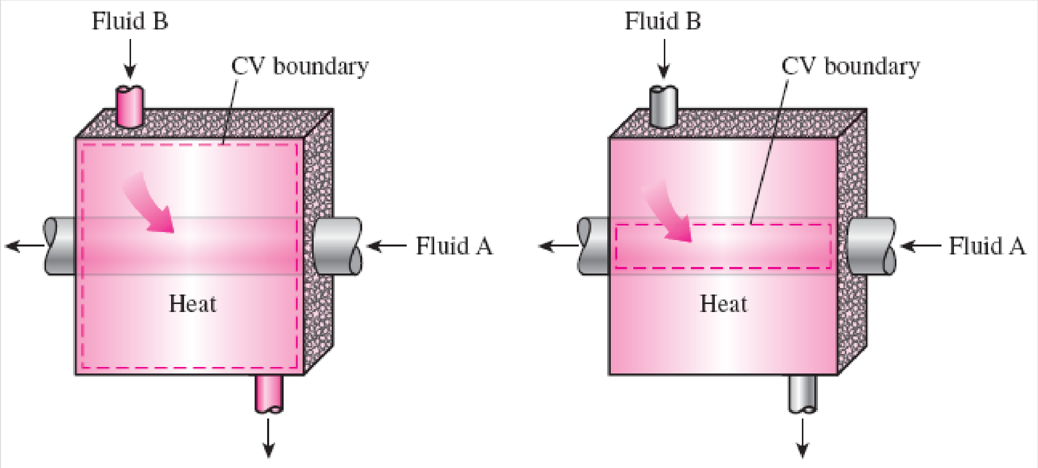
$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

$$(\text{since } \dot{Q} \cong 0,\ \dot{W} = 0,\ \text{ke} \cong \text{pe} \cong 0)$$

Heat exchangers

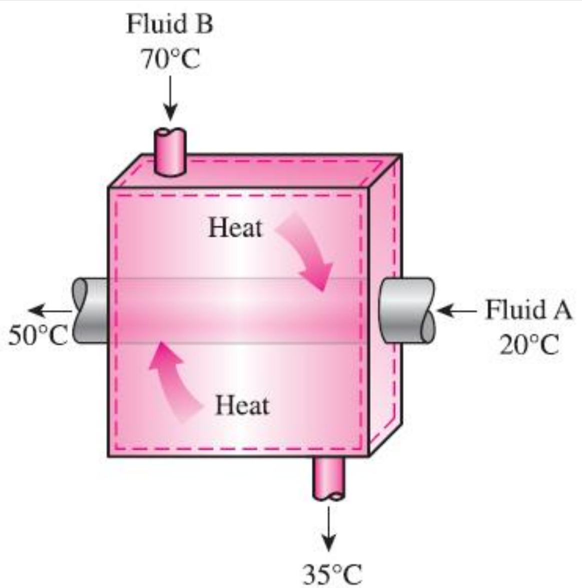
Heat exchangers are devices where two moving fluid streams exchange heat without mixing. Heat exchangers are widely used in various industries, and they come in various designs.



(*a*) System: Entire heat exchanger ($Q_{CV} = 0$)

(*b*) System: Fluid A ($Q_{CV} \neq 0$)

The heat transfer associated with a heat exchanger may be zero or nonzero depending on how the control volume is selected.



A heat exchanger can be as simple as two concentric pipes.

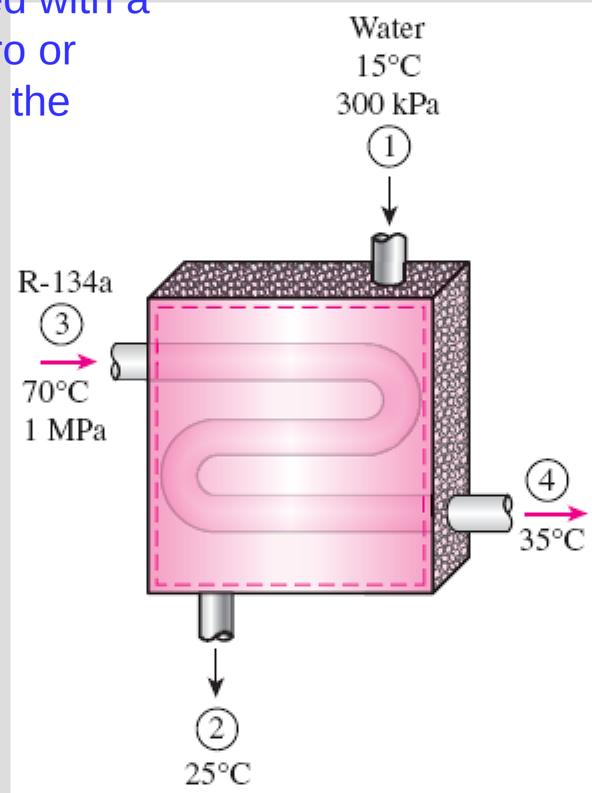
Mass and energy balances for the adiabatic heat exchanger in the figure is:

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_w$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

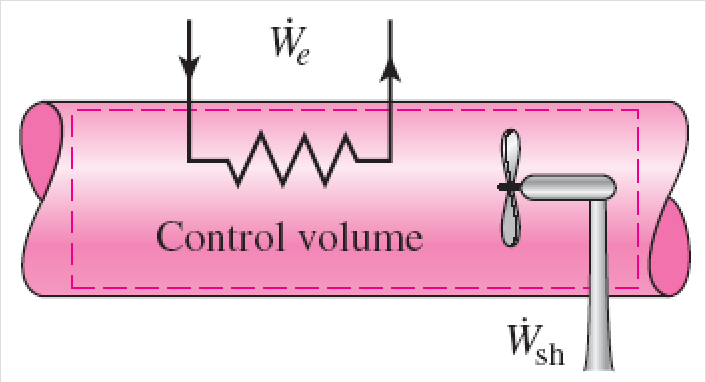
$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4$$

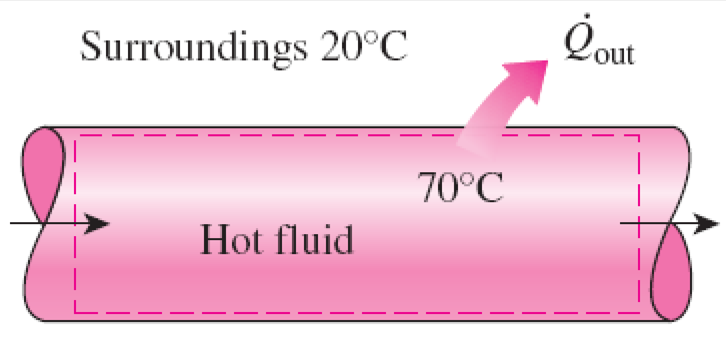


Pipe and duct flow

The transport of liquids or gases in pipes and ducts is of great importance in many engineering applications. Flow through a pipe or a duct usually satisfies the steady-flow conditions.



Pipe or duct flow may involve more than one form of work at the same time.



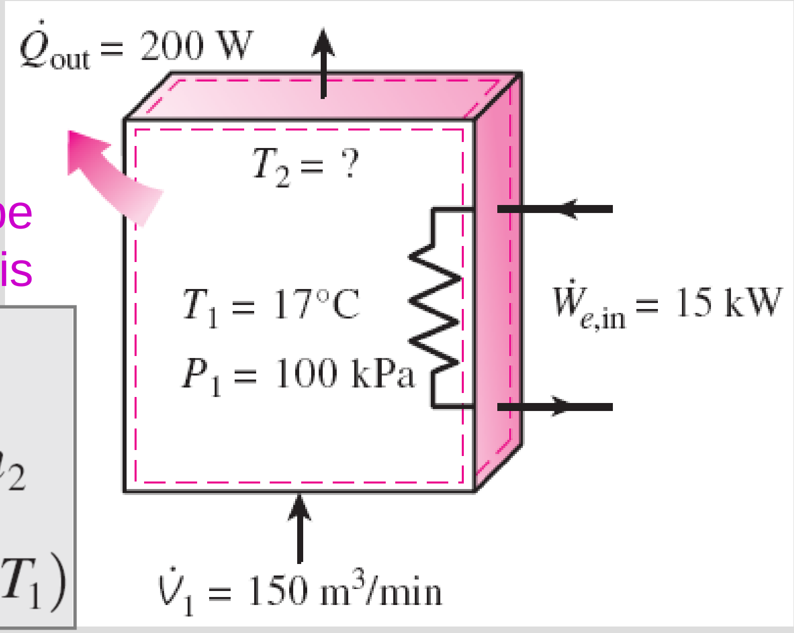
Heat losses from a hot fluid flowing through an uninsulated pipe or duct to the cooler environment may be very significant.

Energy balance for the pipe flow shown in the figure is

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{e,in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2$$

$$\dot{W}_{e,in} - \dot{Q}_{out} = \dot{m}c_p(T_2 - T_1)$$



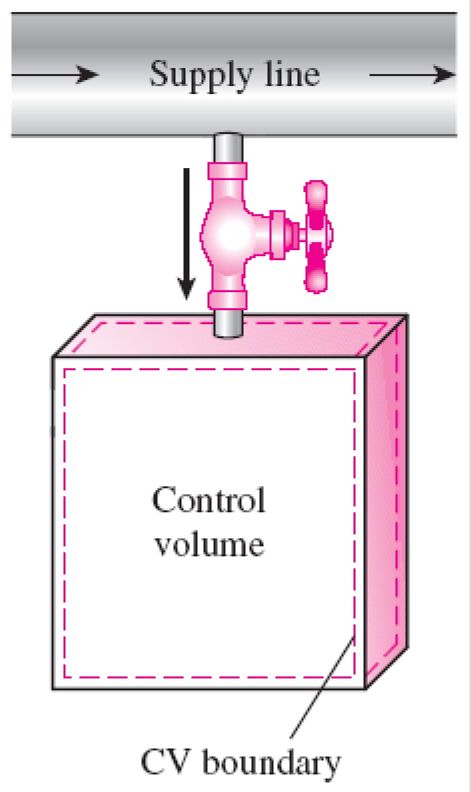
ENERGY ANALYSIS OF UNSTEADY-FLOW PROCESSES

Many processes of interest, involve *changes* within the control volume with time. Such processes are called *unsteady-flow,* or *transient-flow,* processes.

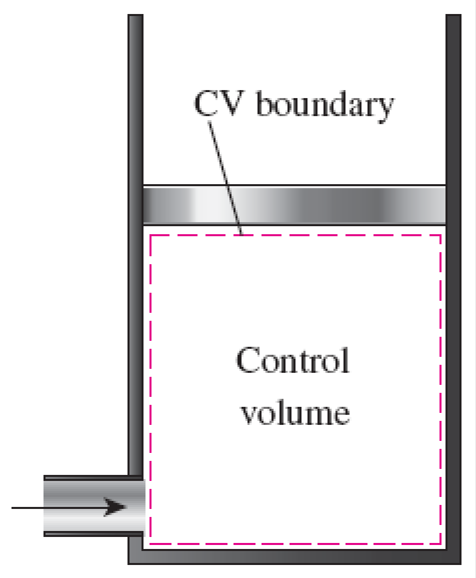
Most unsteady-flow processes can be represented reasonably well by the *uniform-flow process*.

Uniform-flow process: The fluid flow at any inlet or exit is uniform and steady, and thus the fluid properties do not change with time or position over the cross section of an inlet or exit. If they do, they are averaged and treated as constants for the entire process.

Charging of a rigid tank from a supply line is an unsteady-flow process since it involves changes within the control volume.



The shape and size of a control volume may change during an unsteady-flow process.



Mass balance

$$m_{in} - m_{out} = \Delta m_{system} \qquad \Delta m_{system} = m_{final} - m_{initial}$$

$$m_i - m_e = (m_2 - m_1)_{CV} \qquad i = \text{inlet},\ e = \text{exit},\ 1 = \text{initial state, and } 2 = \text{final state}$$

Energy balance

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

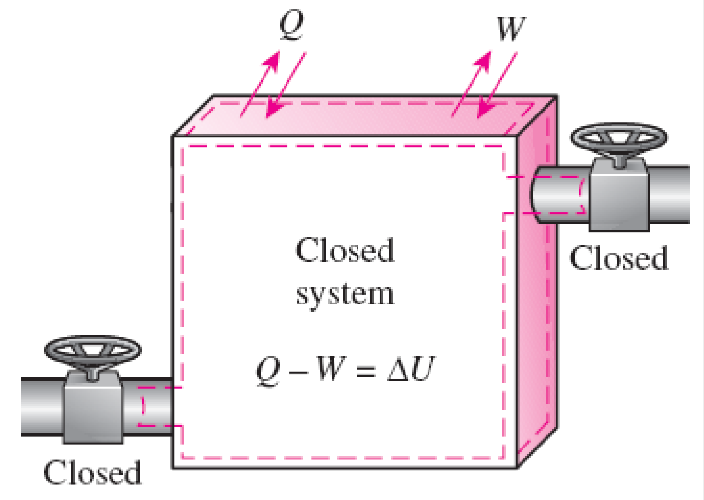
$$\left(Q_{in} + W_{in} + \sum_{in} m\theta\right) - \left(Q_{out} + W_{out} + \sum_{out} m\theta\right) = (m_2 e_2 - m_1 e_1)_{system}$$

$$\theta = h + \text{ke} + \text{pe}$$

$$e = u + \text{ke} + \text{pe}$$

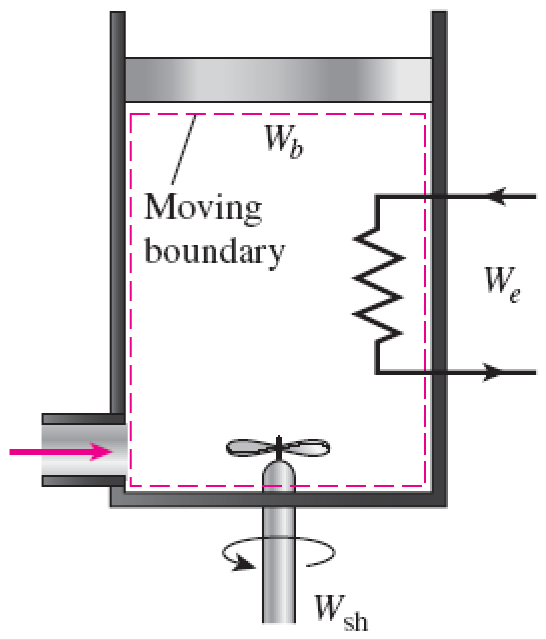
$$Q - W = \sum_{out} mh - \sum_{in} mh + (m_2 u_2 - m_1 u_1)_{system}$$

$$Q = Q_{net,in} = Q_{in} - Q_{out} \qquad W = W_{net,out} = W_{out} - W_{in}$$



The energy equation of a uniform-flow system reduces to that of a closed system when all the inlets and exits are closed.

A uniform-flow system may involve electrical, shaft, and boundary work all at once.



Summary

- Conservation of mass
  - ✓ Mass and volume flow rates
  - ✓ Mass balance for a steady-flow process
  - ✓ Mass balance for incompressible flow
- Flow work and the energy of a flowing fluid
  - ✓ Energy transport by mass
- Energy analysis of steady-flow systems
- Some steady-flow engineering devices
  - ✓ Nozzles and Diffusers
  - ✓ Turbines and Compressors
  - ✓ Throttling valves
  - ✓ Mixing chambers and Heat exchangers
  - ✓ Pipe and Duct flow
- Energy analysis of unsteady-flow processes