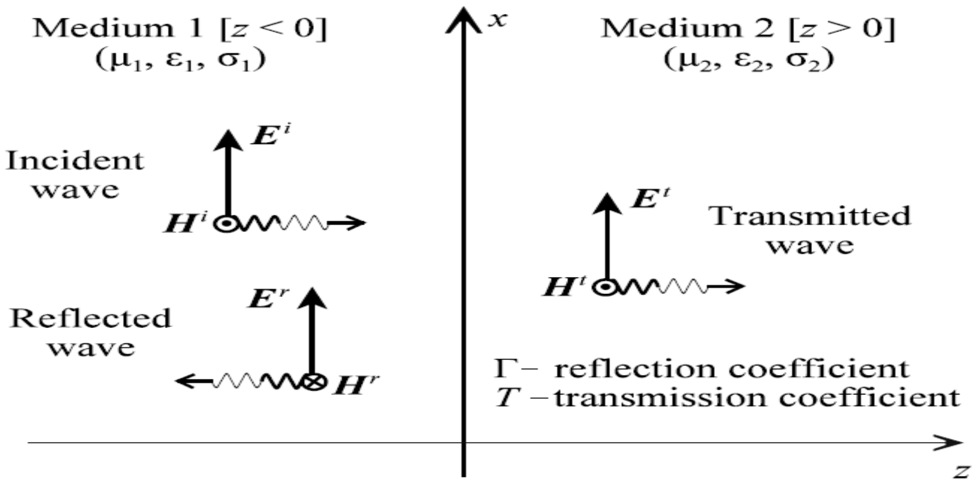
Plane Wave Reflection, Transmission at a Planar Interface

We have considered plane waves in an infinite homogeneous medium. A natural question would arise: what happens if a plane wave hits some object? Such object can be either dielectric or conductor.

To answer this question, we need to use boundary conditions. We study first normal incidence on the boundary.

Normal incidence – propagating waves



Incident wave fields

$$\boldsymbol{E^i} = E_o e^{-\gamma_1 z} \boldsymbol{a_x}$$

$$\boldsymbol{H^i} = \frac{E_o}{\eta_1} e^{-\gamma_1 z} \boldsymbol{a_y}$$

Reflected wave fields

$$\boldsymbol{E^r} = \Gamma E_o e^{\gamma_1 z} \boldsymbol{a_x}$$

$$\boldsymbol{H^r} = -\Gamma \frac{E_o}{\eta_1} e^{\gamma_1 z} \boldsymbol{a_y}$$

Transmitted wave fields

$$\boldsymbol{E^t} = T E_o e^{-\gamma_2 z} \boldsymbol{a_x}$$

$$\boldsymbol{H^t} = T \frac{E_o}{\eta_2} e^{-\gamma_2 z} \boldsymbol{a_y}$$

Boundary Conditions

$$E_x^i + E_x^r = E_x^t \qquad at \quad z = 0 \qquad \Rightarrow \qquad 1 + \Gamma = T$$

$$H_x^i + H_x^r = H_x^t \qquad at \quad z = 0 \qquad \Rightarrow \qquad \frac{1 - \Gamma}{\eta_1} = \frac{T}{\eta_2}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \text{(Reflection coefficient)}$$

$$T = \frac{2\eta_2}{\eta_2 + \eta_1} \qquad \text{(Transmission coefficient)}$$

$$\vec{\Gamma} = \frac{E_r}{E_i} = \Gamma e^{j\theta_\Gamma}$$

$$\vec{\mathrm{T}} = \frac{E_t}{E_i} = \mathrm{T} e^{j\theta_\mathrm{T}}$$

This is a general solution for reflection and transmission of a normally incident wave at the interface of an arbitrary material, where $\boldsymbol{\eta}$ is the intrinsic impedance of the material. We now consider two special cases of this result.

Case 1

If the region 1 is a lossless dielectric [$\boldsymbol{\sigma_1 = 0}$, $\boldsymbol{\eta_1} = \sqrt{\frac{\mu_1}{\varepsilon_1}}$, $\boldsymbol{\alpha_1 = 0}$, $\boldsymbol{\gamma_1 = j\beta_1}$]

the region 2 is Perfect Conductor[$\boldsymbol{\sigma_2} = \infty$, $\boldsymbol{\eta_2 = 0}$, $\boldsymbol{\alpha_2 = \beta_2} \rightarrow \infty$]

Case 2

If the region 1 is a lossless dielectric $[\boldsymbol{\sigma_1} = \mathbf{0},\ \boldsymbol{\eta_1} = \sqrt{\frac{\mu_1}{\varepsilon_1}}\ ,\boldsymbol{\alpha_1} = \mathbf{0}\ ,\boldsymbol{\gamma_1} = \boldsymbol{j\beta_1}]$

the region 2 is a lossless dielectric$[\boldsymbol{\sigma_2} = \mathbf{0},\ \boldsymbol{\eta_2} = \sqrt{\frac{\mu_2}{\varepsilon_2}}\ ,\boldsymbol{\alpha_2} = \mathbf{0}\ ,\boldsymbol{\gamma_2} = \boldsymbol{j\beta_1}]$

if $\boldsymbol{\eta_2} > \boldsymbol{\eta_1}$ then

$1<T\ 2<$and $0 < \Gamma < 1$

If medium 1 is lossless dielectric (that is, $\sigma = 0$), the standing-wave ratio is defined as

$$\mathbf{SWR} = \rho = \frac{|E_{\max}^{(1)}|}{|E_{\min}^{(1)}|} = \frac{1 + \Gamma}{1 - \Gamma}$$

Boundary conditions

There are four basic rules for boundary conditions at the surface between two different materials:

1. The tangential components of electric field intensity are continuous across the boundary.

$$E_{t1} = E_{t2}$$

2. The normal components of electric flux density are discontinuous at the boundary by an amount equal to the surface-charge density on the boundary.

$$D_{n1} = D_{n2} + p_s$$

3. The tangential components of magnetic field intensity are discontinuous at the boundary by an amount equal to the surface-current density on the boundary.

$$H_{t1} = H_{t2} + J_s$$

4. The normal components of magnetic flux density are continuous across the boundary.

$$B_{n1} = B_{n2}$$