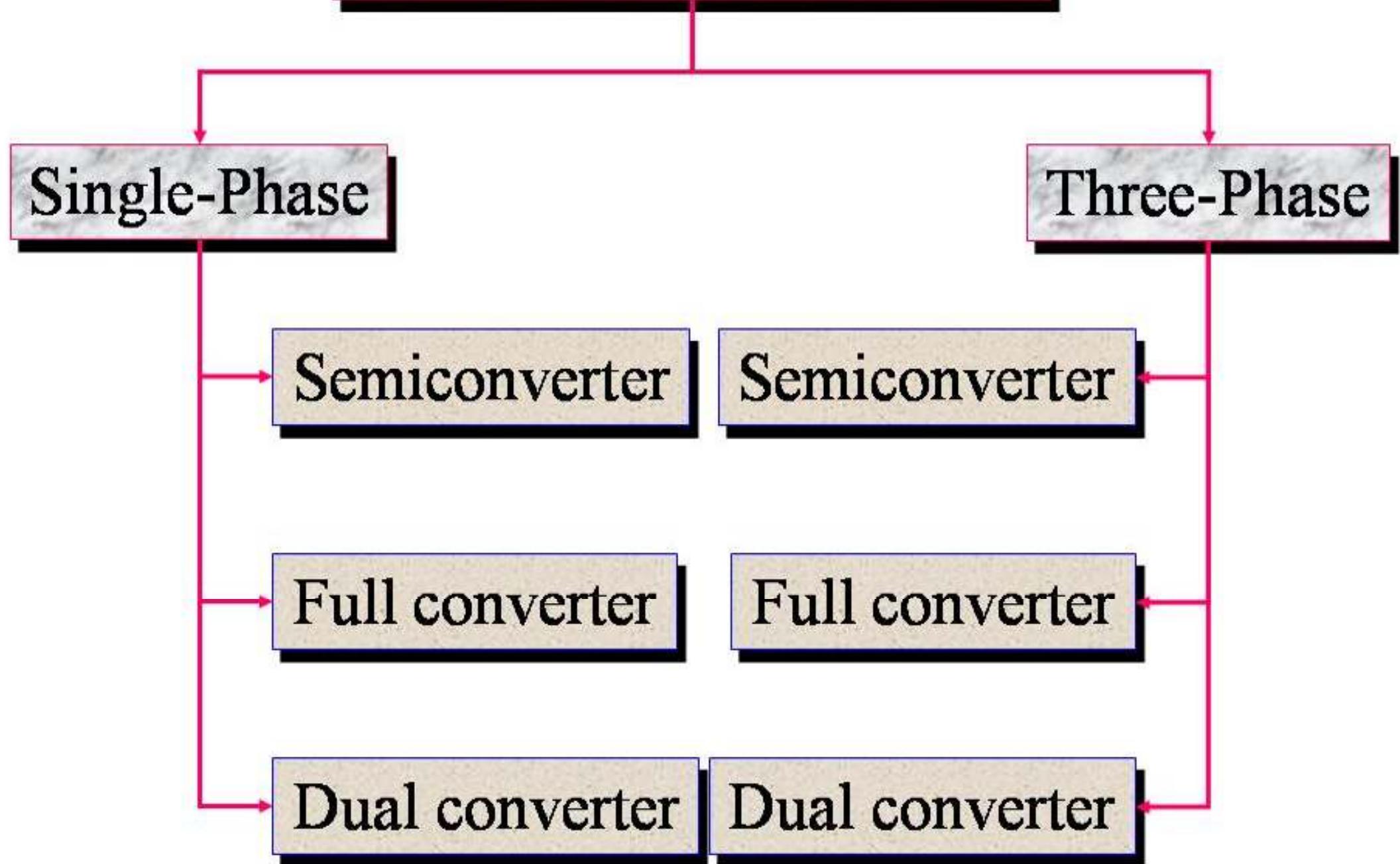


PHASE CONTROLLED CONVERTERS

Phase-Control Converters



Semiconverter

..is a one-quadrant converter and it has one polarity

Full converter

..is a two-quadrant converter and the polarity of its output can be either positive or negative.

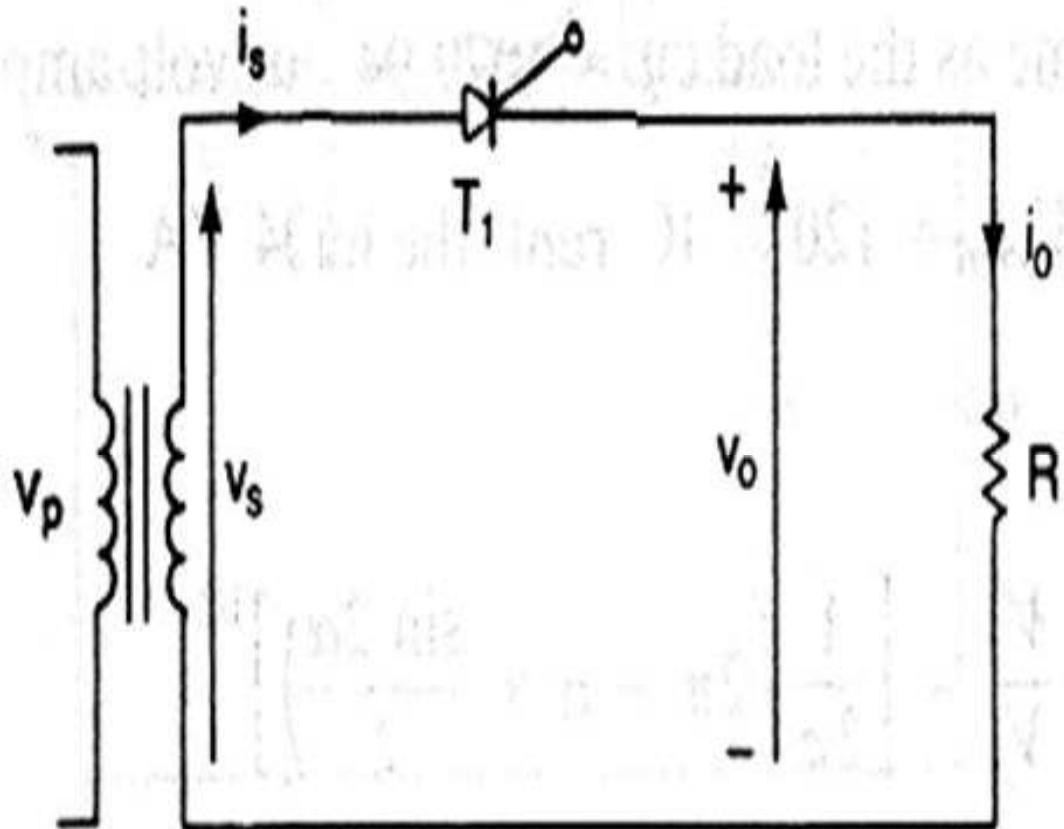
However

the output current of full converter has one polarity only

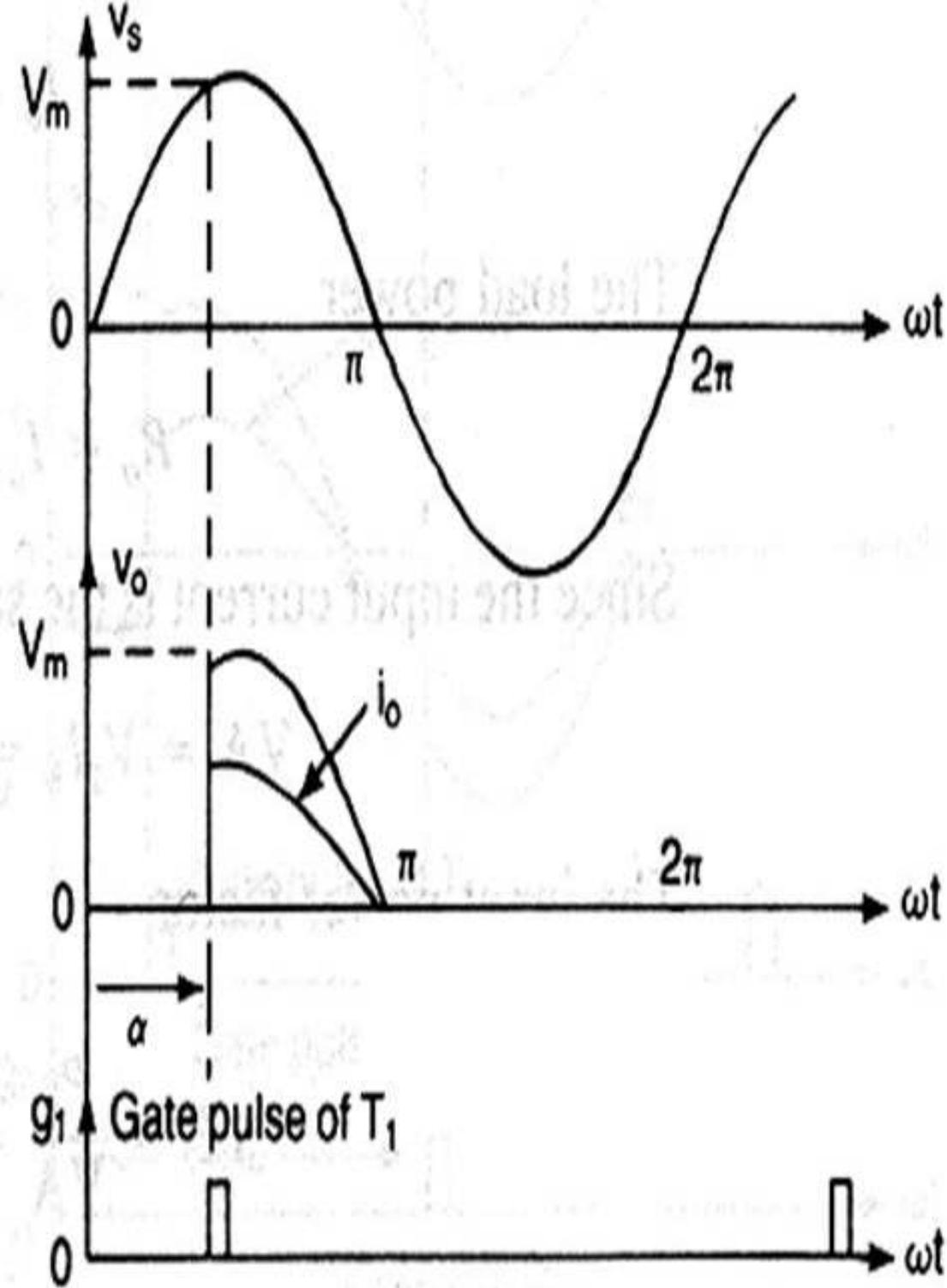
Dual converter

..can operate in four quadrants ; both the output voltage and current can be either positive or negative

Single-Phase thyristor converter with a resistive load



(a) Circuit



(b) Waveforms

Average Output Voltage

$$V_{dc} = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t d(\omega t) = \frac{V_m}{2\pi} (1 + \cos \alpha)$$

Maximum
Output Voltage

$$V_{dm} = \frac{V_m}{\pi}$$

Normalizing
Output Voltage

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5(1 + \cos \alpha)$$

RMS Output Voltage

$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t d(\omega t)} = \frac{V_m}{2} \sqrt{\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right)}$$

If the converter has a purely resistive load of R and the delay angle is , $\alpha = \pi / 2$ determine

(a) the rectification efficiency

(b) the form factor FF

(c) the ripple factor RF

and (d) the peak inverse voltage PIV of thyristor T_1

$$V_{dc} = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\pi} V_m \sin \omega t d(\omega t) = \frac{V_m}{2\pi} \left(1 + \cos \frac{\pi}{2} \right)$$

$$V_{dc} = 0.1592 V_m$$

$$V_{rms} = \frac{V_m}{2} \sqrt{\frac{1}{\pi} \left(\pi - \frac{\pi}{2} + \sin \frac{2 \times \frac{\pi}{2}}{2} \right)} = 0.3536 V_m$$

$$\eta = \frac{V_{dc}^2}{V_{rms}^2} = \frac{(0.1592 V_m)^2}{(0.3536 V_m)^2} = 20.27\%$$

If the converter has a purely resistive load of R and the delay angle is , $\alpha = \pi / 2$ determine

(a) the rectification efficiency

(b) the form factor FF

(c) the ripple factor RF

and (d) the peak inverse voltage PIV of thyristor T₁

$$FF = \frac{V_{rms}}{V_{dc}} = \frac{0.3536V_m}{0.1592V_m} = 2.221$$

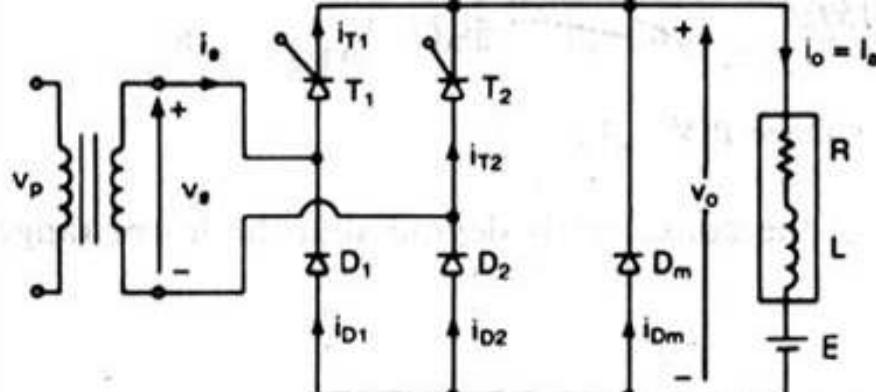
If the converter has a purely resistive load of R and the delay angle is , $\alpha = \pi / 2$ determine

- (a) the rectification efficiency
- (b) the form factor FF
- (c) the ripple factor RF

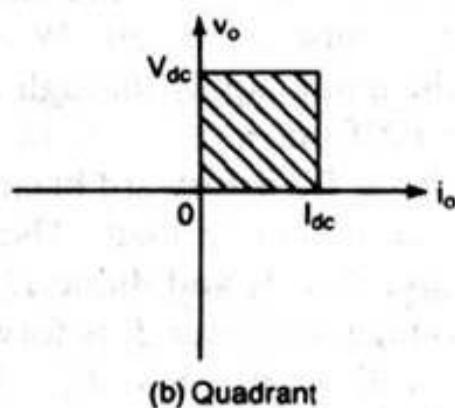
$$RF = \sqrt{FF^2 - 1} = \sqrt{2.221^2 - 1} = 1.983$$

and (d) the peak inverse voltage PIV of thyristor T₁

$$PIV = V_m$$

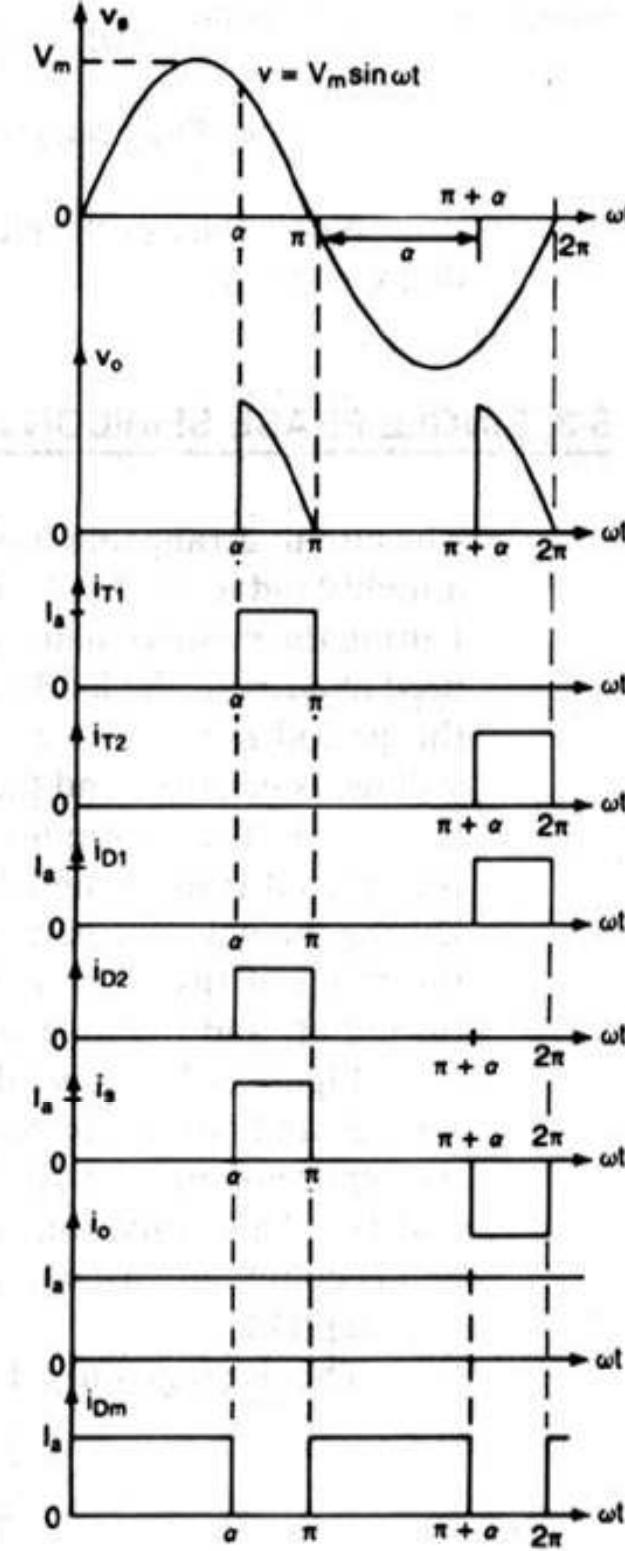


(a) Circuit



(b) Quadrant

Single-Phase Semiconverter



Single-Phase Semiconverter

$$V_{dc} = \frac{2}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t d(\omega t) = \frac{V_m}{\pi} (1 + \cos \alpha)$$

$$V_{rms} = \sqrt{\frac{2}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t d(\omega t)}$$

Single-Phase Semiconverter (*RL-load*)

Mode 1

$0 \leq \omega t \leq \alpha$

$$L \frac{di_{L1}}{dt} + Ri_{L1} + E = 0$$

$$I_{L1} = i_{L1}(\omega t = \alpha) = I_{L0} e^{-\frac{R\alpha}{L\omega}} - \frac{E}{R} \left[1 - e^{-\frac{R\alpha}{L\omega}} \right]$$

Mode 2

$\alpha \leq \omega t \leq \pi$

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$\theta = \tan^{-1} \frac{\omega L}{R}$$

$$L \frac{di_{L2}}{dt} + Ri_{L2} + E = \sqrt{2} V_s \sin \omega t$$

$$I_{L2} = \frac{\sqrt{2} V_s}{Z} \sin(\omega t - \theta) - \frac{E}{R} \left[I_{L1} + \frac{E}{R} - \frac{\sqrt{2} V_s}{Z} \sin(\alpha - \theta) \right] e^{\frac{R \alpha}{L \omega t}}$$

Single-Phase Semiconverter (*RL-load*)

*RMS Current
for Thyristor*

$$I_R = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} I_{L2}^2 d(\omega t)}$$

*RMS Current
for Thyristor*

$$I_A = \frac{1}{2\pi} \int_{\alpha}^{\pi} i_{L2} d(\omega t)$$

*RMS Output
Current*

$$I_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{\alpha} I_{L1}^2 d(\omega t) + \frac{1}{2\pi} \int_{\alpha}^{\pi} I_{L2}^2 d(\omega t)}$$

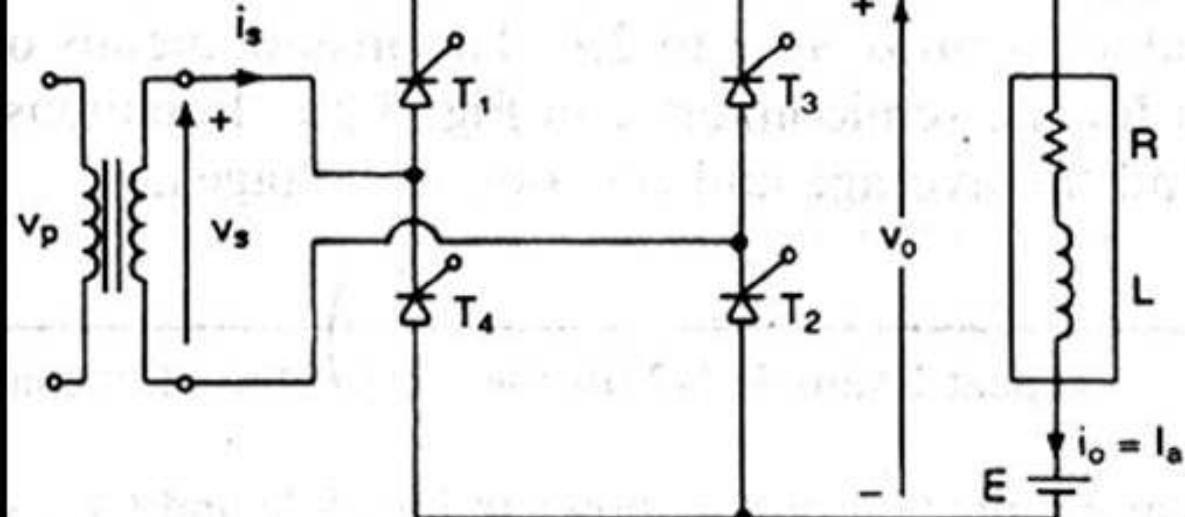
*AVG Output
Current*

$$I_{dc} = \frac{1}{2\pi} \int_0^{\alpha} i_1 d(\omega t) + \frac{1}{2\pi} \int_{\alpha}^{\pi} i_2 d(\omega t)$$

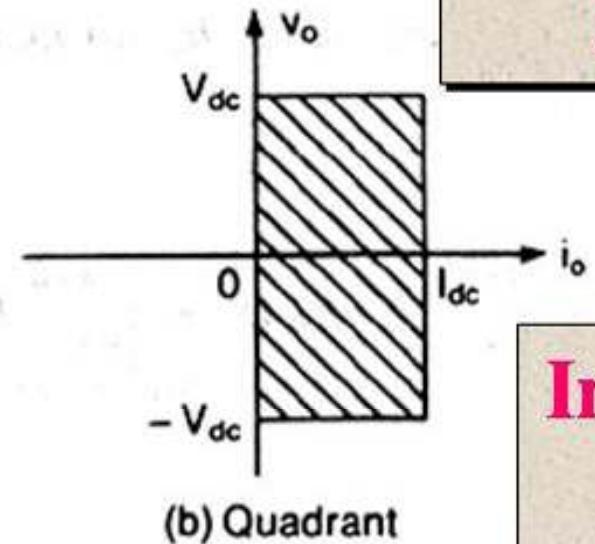
Example : The single-phase semiconverter has an RL load of

$L = 6.5\text{mH}$, $R = 2.5 \text{ Ohm}$, and $E = 10 \text{ V}$. The input voltage is $V_S = 120 \text{ V(rms)}$ at 60 Hz . Determine

- (a) the load current I_{L0} at $\omega t = 0$, and the load current I_{L1} at $\omega t = \alpha = 60^\circ$,
- (b) the average thyristor current I_A
- (c) the rms thyristor current I_R
- (d) the rms output current I_{rms}
- and (e) the average output current I_{dc}



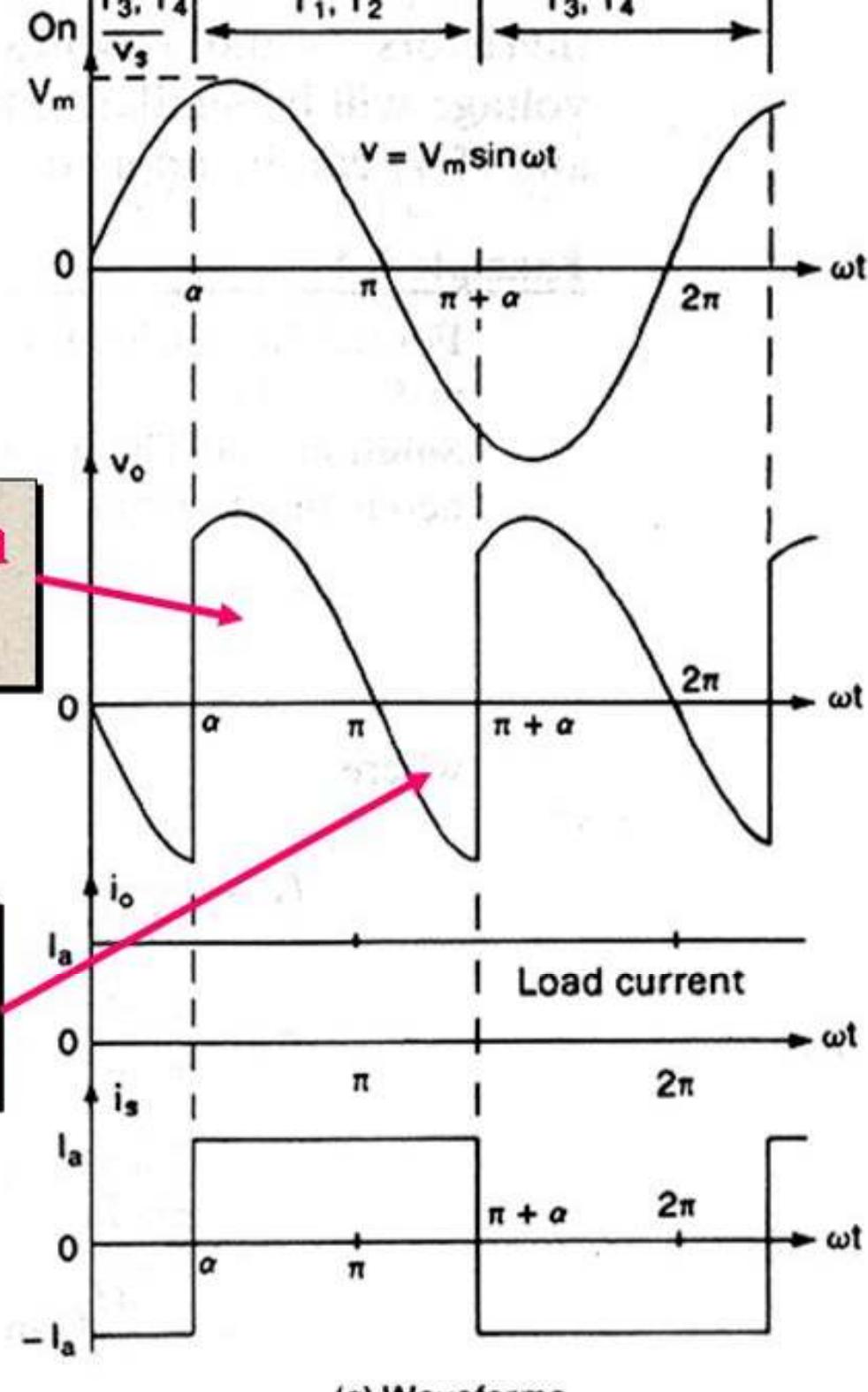
(a) Circuit



(b) Quadrant

**Rectification
Mode**

**Inversion
Mode**



**Single-Phase
Full Converter**

(c) Waveforms

Single-Phase Full Converter

$$V_{dc} = \frac{2}{2\pi} \int_{\alpha}^{\pi+\alpha} V_m \sin \omega t d(\omega t) = \frac{2V_m}{\pi} \cos \alpha$$

$$V_{rms} = \sqrt{\frac{2}{2\pi} \int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2 \omega t d(\omega t)} = \frac{V_m}{\sqrt{2}}$$

Single-Phase Full Converter (*RL-load*)

Mode 1 = Mode 2

$$\theta = \tan^{-1} \frac{\omega L}{R}$$

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$I_L = \frac{\sqrt{2}V_s}{Z} \sin(\omega t - \theta) - \frac{E}{R} + \left[I_{L0} + \frac{E}{R} - \frac{\sqrt{2}V_s}{Z} \sin(\alpha - \theta) \right] e^{\frac{R-\alpha}{L}\omega t}$$

Single-Phase Full Converter (*RL-load*)

*RMS Current
for Thyristor*

$$I_R = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} i_L^2 d(\omega t)}$$

*AVG Current
for Thyristor*

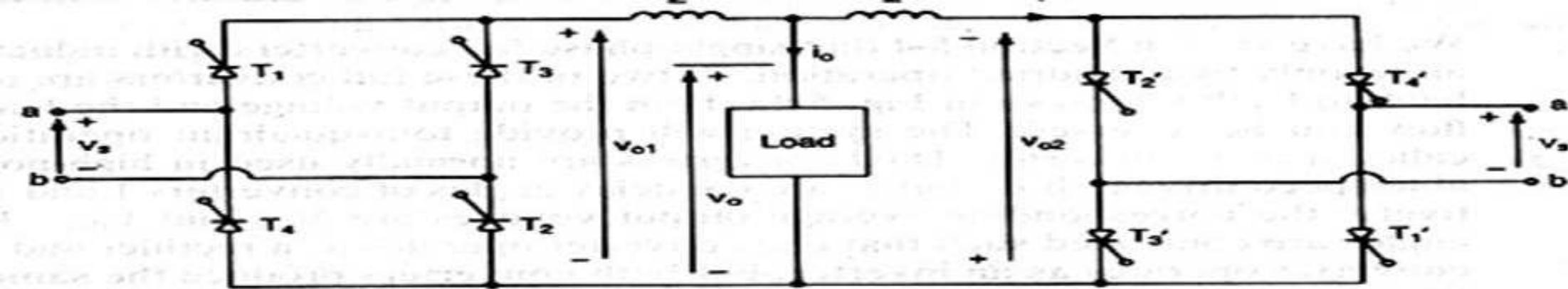
$$I_A = \frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} i_L d(\omega t)$$

*RMS Output
Current*

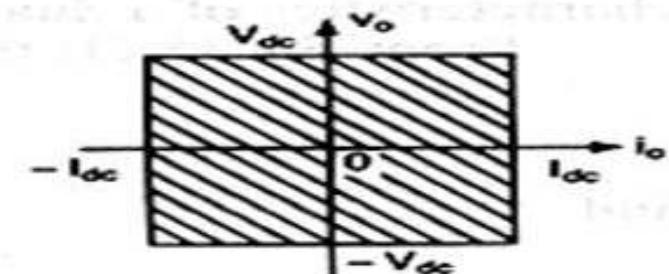
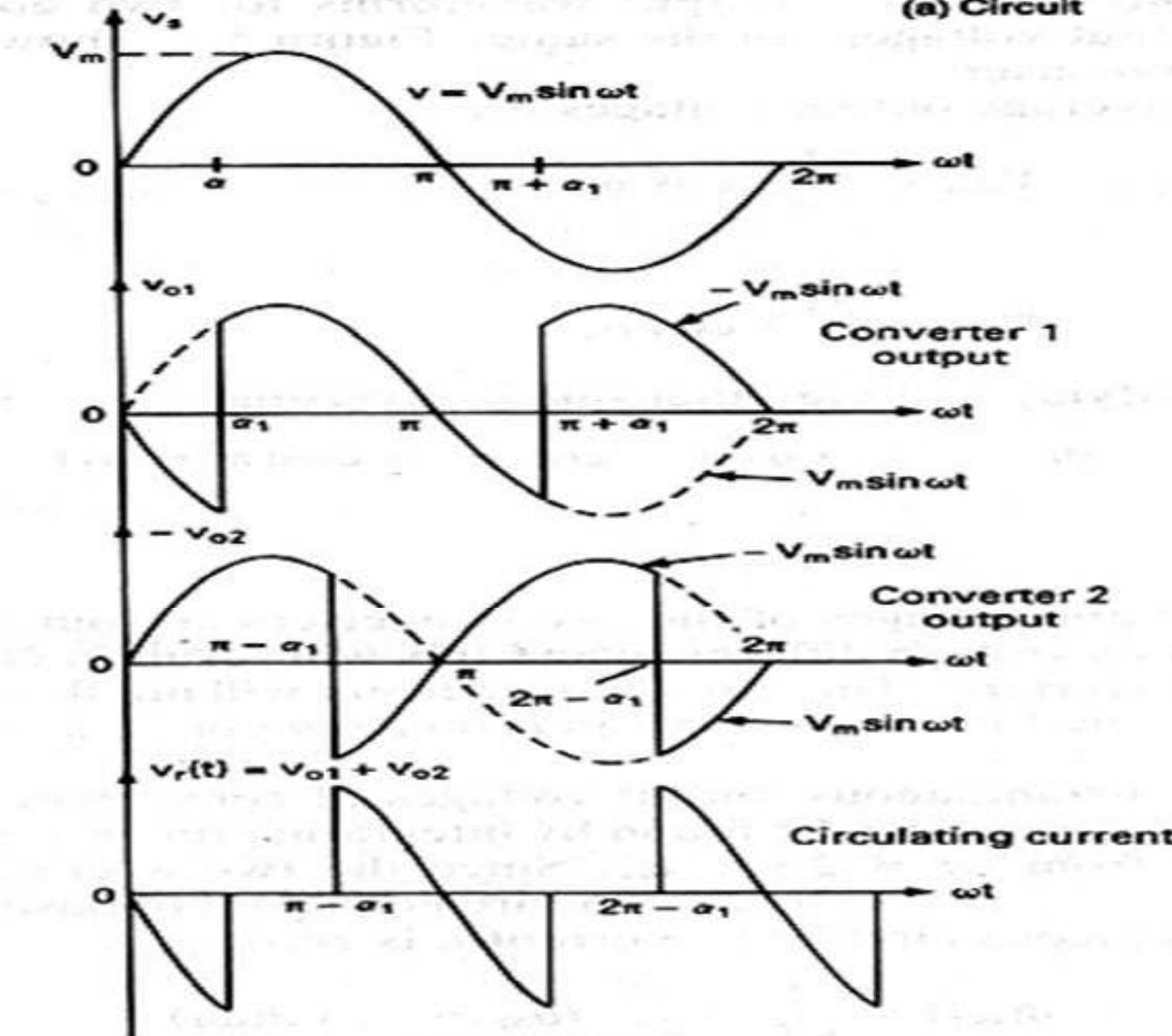
$$I_{rms} = \sqrt{I_R^2 + I_A^2} = \sqrt{2} I_R$$

*AVG Output
Current*

$$I_{dc} = I_A + I_A = 2I_A$$



(a) Circuit



(c) Quadrant

Dual Converter

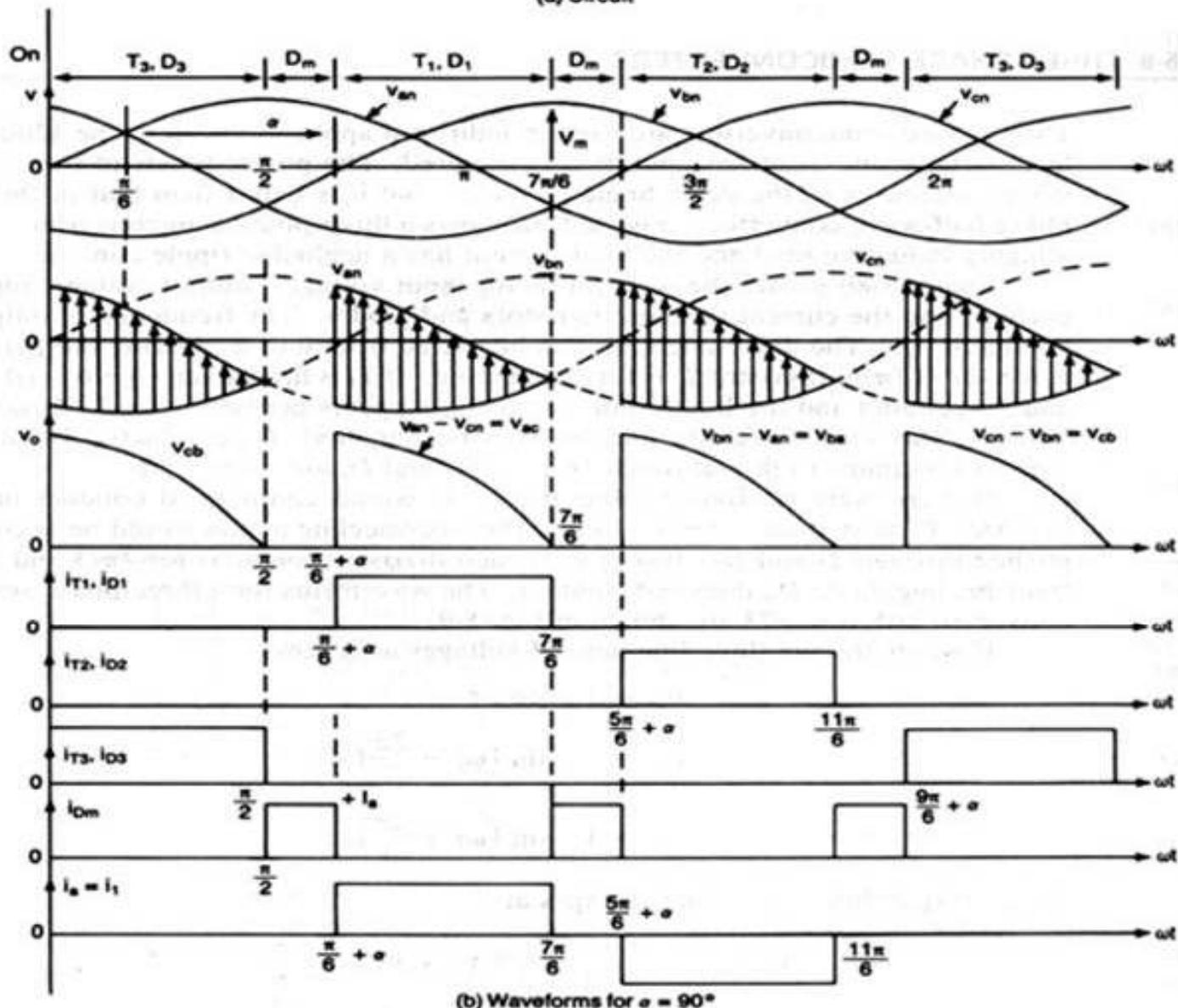
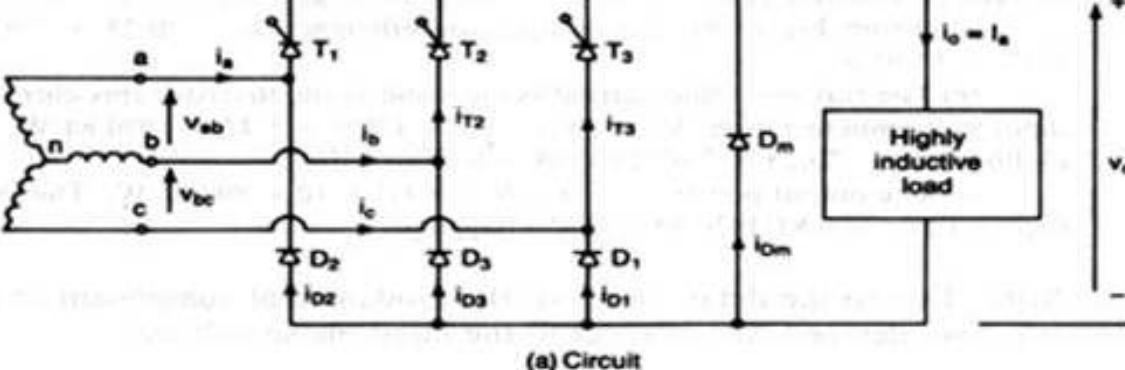
Single-Phase Dual Converter

$$V_{dcl} = \frac{2V_m}{\pi} \cos \alpha_1$$

$$V_{dc2} = \frac{2V_m}{\pi} \cos \alpha_2$$

$$V_{dcl} = -V_{dc2}$$

Three-Phase Semiconverter



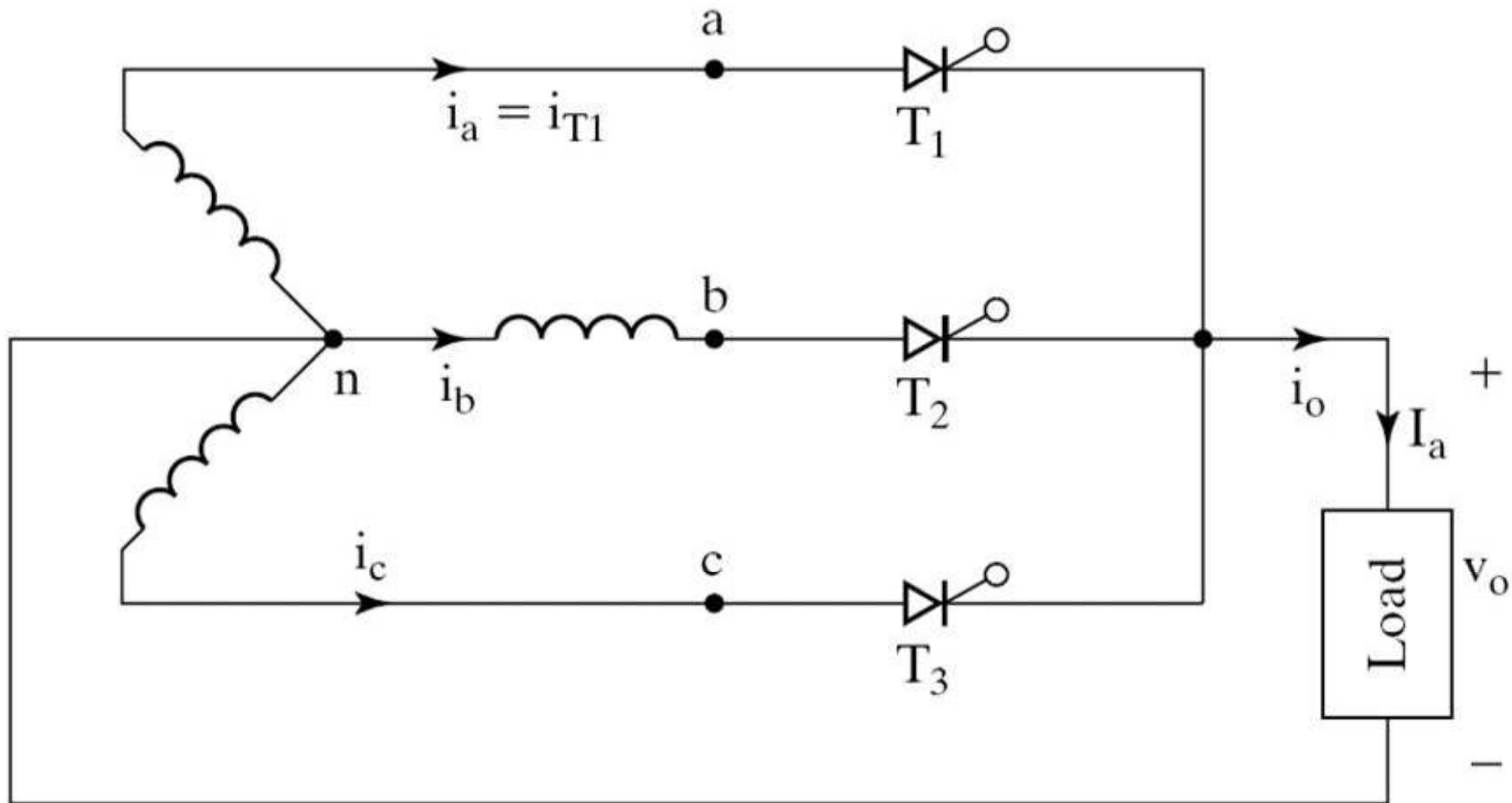
Three Phase Controlled Rectifiers

- Operate from 3 phase ac supply voltage.
- They provide higher dc output voltage.
- Higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing out load voltage and load current.

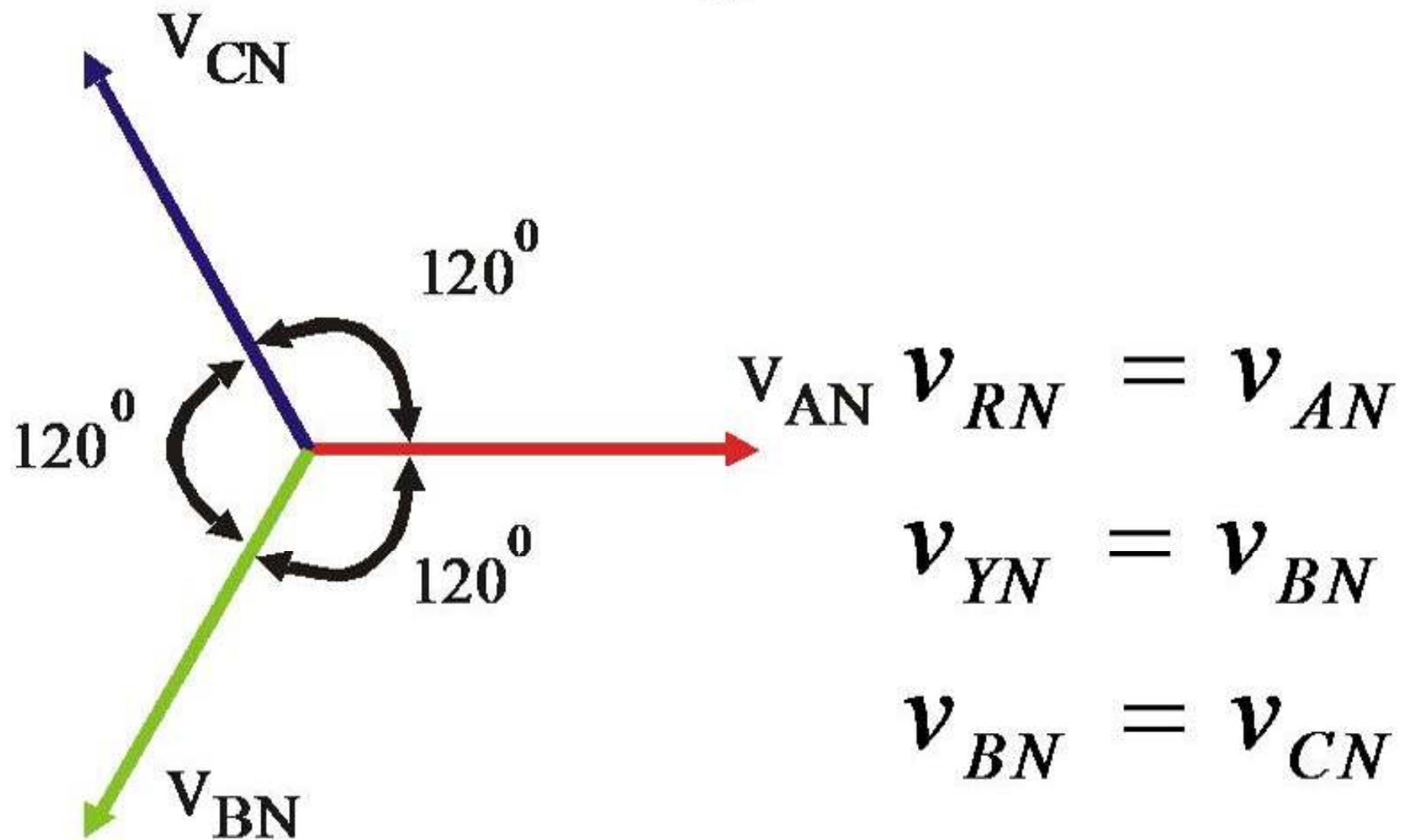
- Extensively used in high power variable speed industrial dc drives.
- Three single phase half-wave converters can be connected together to form a three phase half-wave converter.

**Three Phase
Half Wave Converter
(3-Pulse Converter)
with
RL Load
Continuous & Constant
Load Current Operation**

Three Phase Half Wave Converter



Vector Diagram of Three Phase Supply Voltages



Three Phase Supply Voltage Equations

We define three line to neutral voltage (3 phase voltage) as follow

$$v_{RN} = v_{an} = V_m \sin \omega t;$$

V_m = Max. Phase Voltage

$$v_{YN} = v_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3} \right)$$

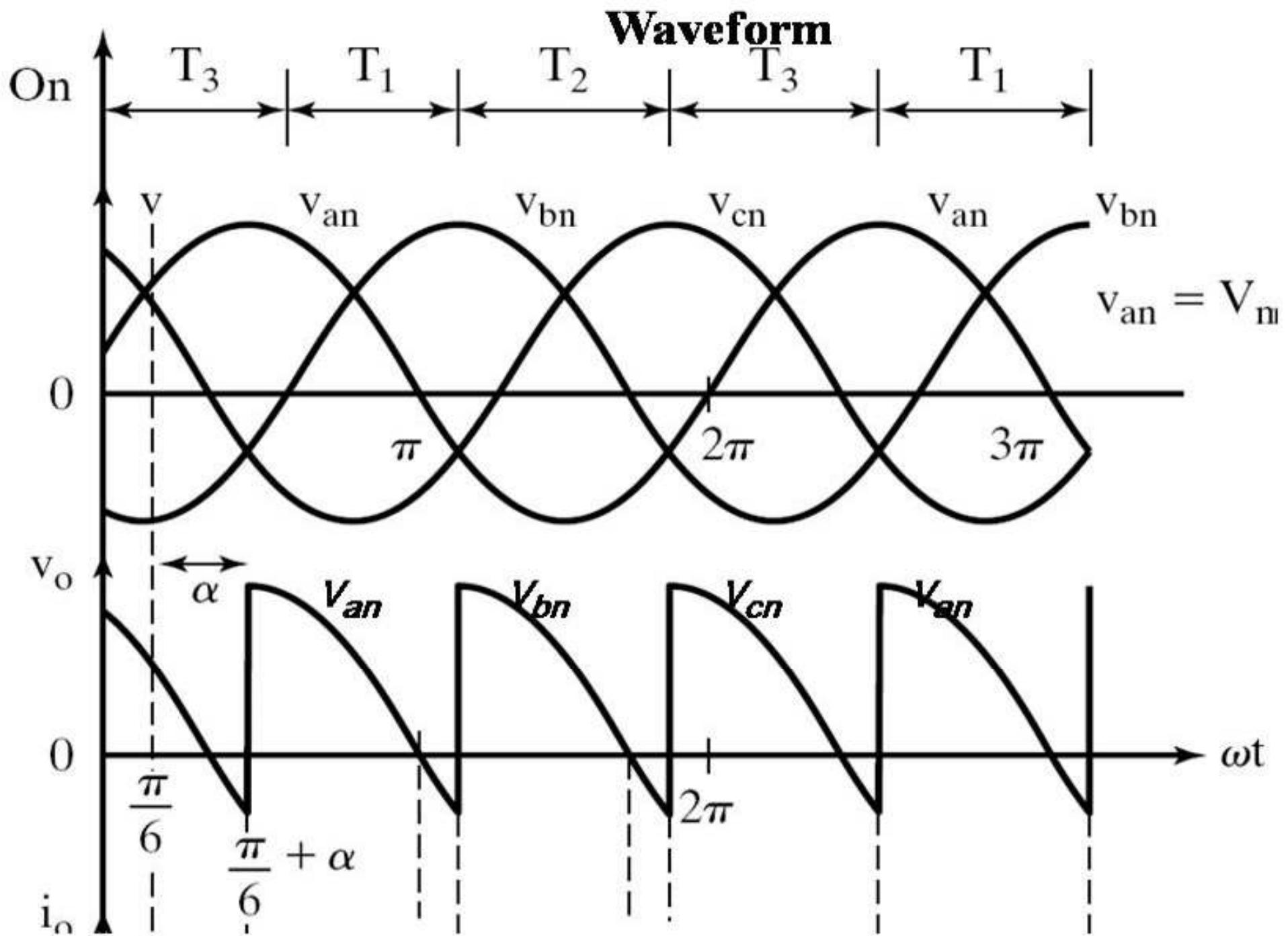
$$= V_m \sin (\omega t - 120^\circ)$$

$$v_{BN} = v_{cn} = V_m \sin \left(\omega t + \frac{2\pi}{3} \right)$$

$$= V_m \sin (\omega t + 120^\circ)$$

$$= V_m \sin (\omega t - 240^\circ)$$

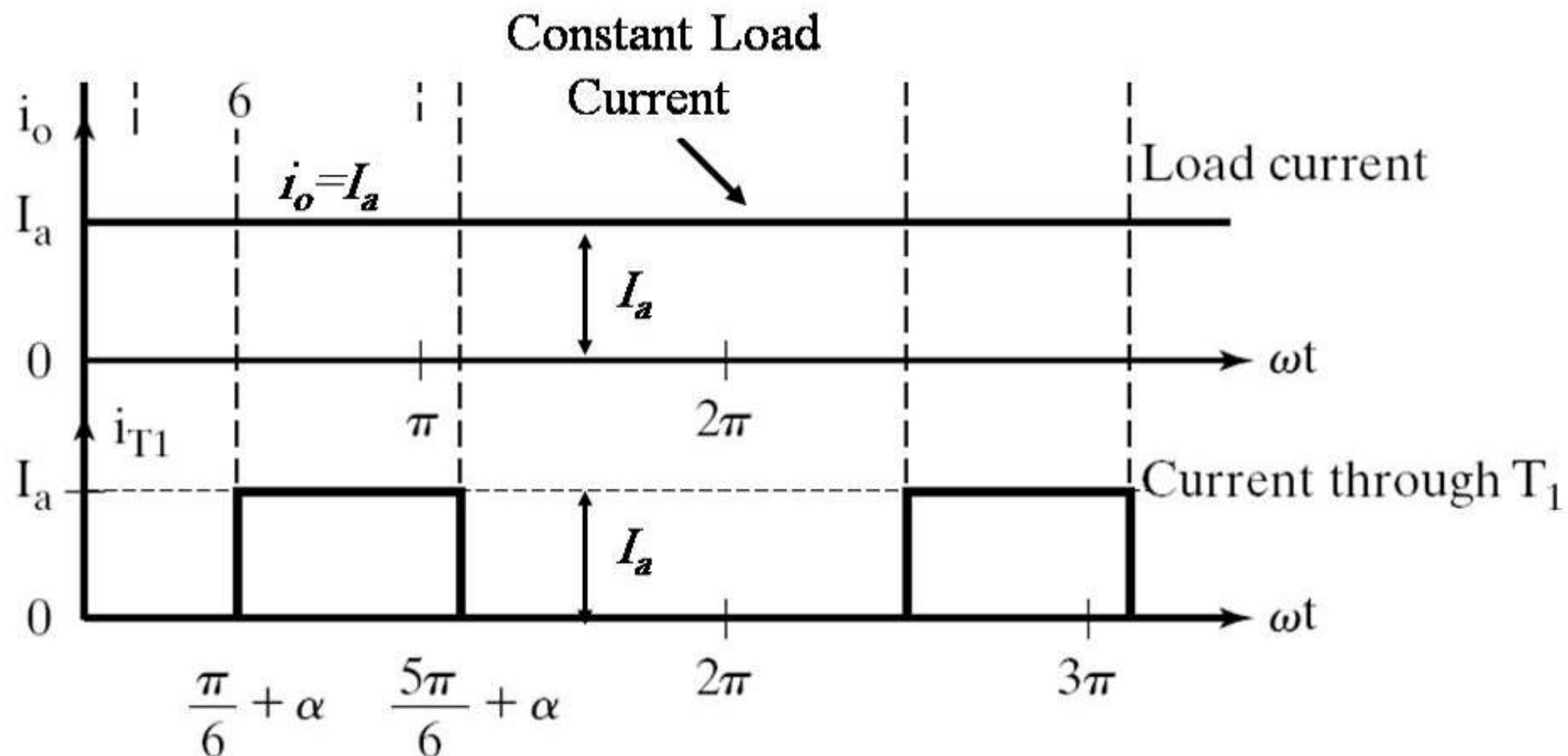
Three Phase Half Wave Converter



Three Phase Half Wave Converter

Waveform

Each thyristor conducts for $2\pi/3$ (120°)



**To Derive an
Expression for the
Average Output Voltage of a
3-Phase Half Wave Converter with
RL Load for Continuous Load
Current**

T₁ is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha\right) = (30^\circ + \alpha)$

T₂ is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha\right) = (150^\circ + \alpha)$

T₃ is triggered at $\omega t = \left(\frac{7\pi}{6} + \alpha\right) = (270^\circ + \alpha)$

Each thyristor conducts for 120° or $\frac{2\pi}{3}$ radians

Three Phase Half Wave Converter

$$V_{dc} = \frac{3V_m}{2\pi} \left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \sin \omega t \cdot d(\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[(-\cos \omega t) \Big|_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos\left(\frac{5\pi}{6} + \alpha\right) + \cos\left(\frac{\pi}{6} + \alpha\right) \right]$$

Three Phase Half Wave Converter

Note from the trigonometric relationship

$$\cos(A+B) = (\cos A \cdot \cos B - \sin A \cdot \sin B)$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos\left(\frac{5\pi}{6}\right)\cos(\alpha) + \sin\left(\frac{5\pi}{6}\right)\sin(\alpha) \right. \\ \left. + \cos\left(\frac{\pi}{6}\right)\cos(\alpha) - \sin\left(\frac{\pi}{6}\right)\sin(\alpha) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos(150^\circ)\cos(\alpha) + \sin(150^\circ)\sin(\alpha) \right. \\ \left. + \cos(30^\circ)\cos(\alpha) - \sin(30^\circ)\sin(\alpha) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos(180^\circ - 30^\circ) \cos(\alpha) + \sin(180^\circ - 30^\circ) \sin(\alpha) \right. \\ \left. + \cos(30^\circ) \cdot \cos(\alpha) - \sin(30^\circ) \sin(\alpha) \right]$$

Note: $\cos(180^\circ - 30^\circ) = -\cos(30^\circ)$

$$\sin(180^\circ - 30^\circ) = \sin(30^\circ)$$

$$\therefore V_{dc} = \frac{3V_m}{2\pi} \left[+\cos(30^\circ) \cos(\alpha) + \cancel{\sin(30^\circ) \sin(\alpha)} \right. \\ \left. + \cos(30^\circ) \cdot \cos(\alpha) - \cancel{\sin(30^\circ) \sin(\alpha)} \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} [2\cos(30^\circ)\cos(\alpha)]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[2 \times \frac{\sqrt{3}}{2} \cos(\alpha) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} [\sqrt{3} \cos(\alpha)] = \frac{3\sqrt{3}V_m}{2\pi} \cos(\alpha)$$

$$V_{dc} = \frac{3V_{Lm}}{2\pi} \cos(\alpha)$$

Where $V_{Lm} = \sqrt{3}V_m$ = Max. line to line supply voltage

Three Phase Half Wave Converter

The maximum average or dc output voltage is obtained at a delay angle $\alpha = 0$ and is given by

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3} V_m}{2\pi}$$

Where V_m is the peak phase voltage.

And the normalized average output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos \alpha$$

Three Phase Half Wave Converter

The rms value of output voltage is found by using the equation

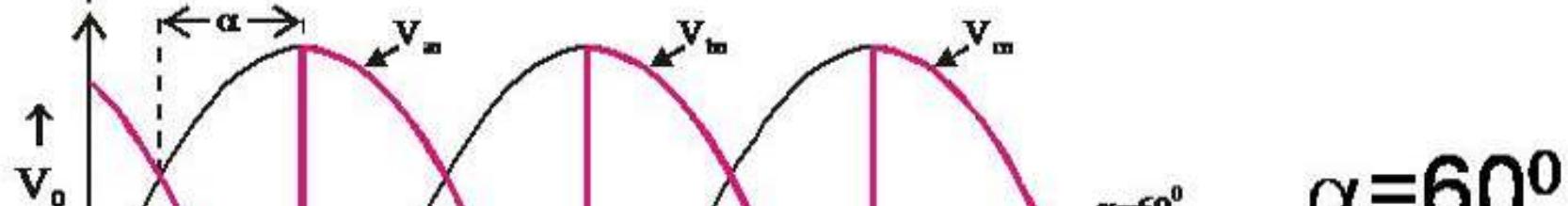
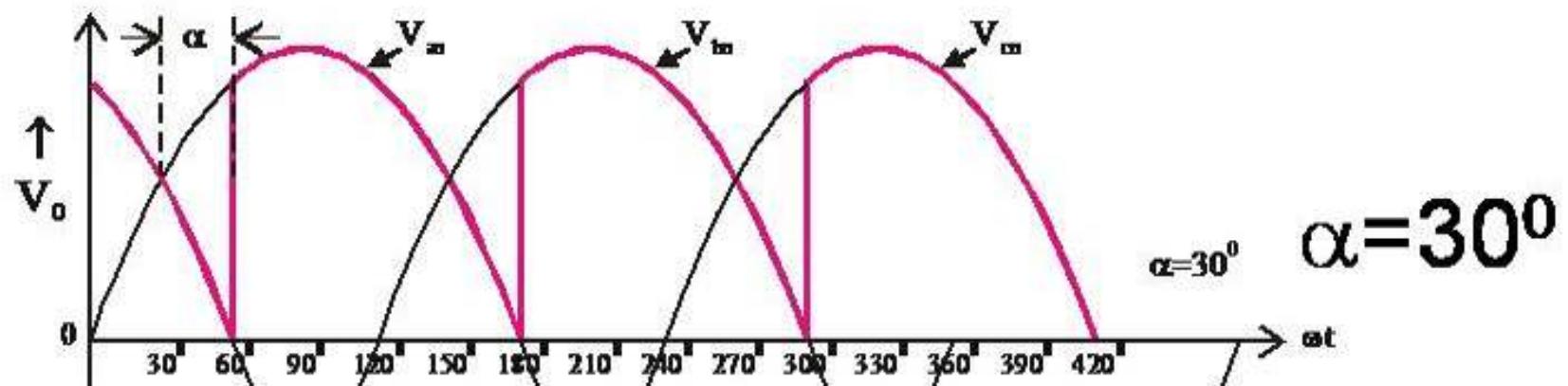
$$V_{O(RMS)} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2 \omega t \cdot d(\omega t) \right]^{\frac{1}{2}}$$

and we obtain

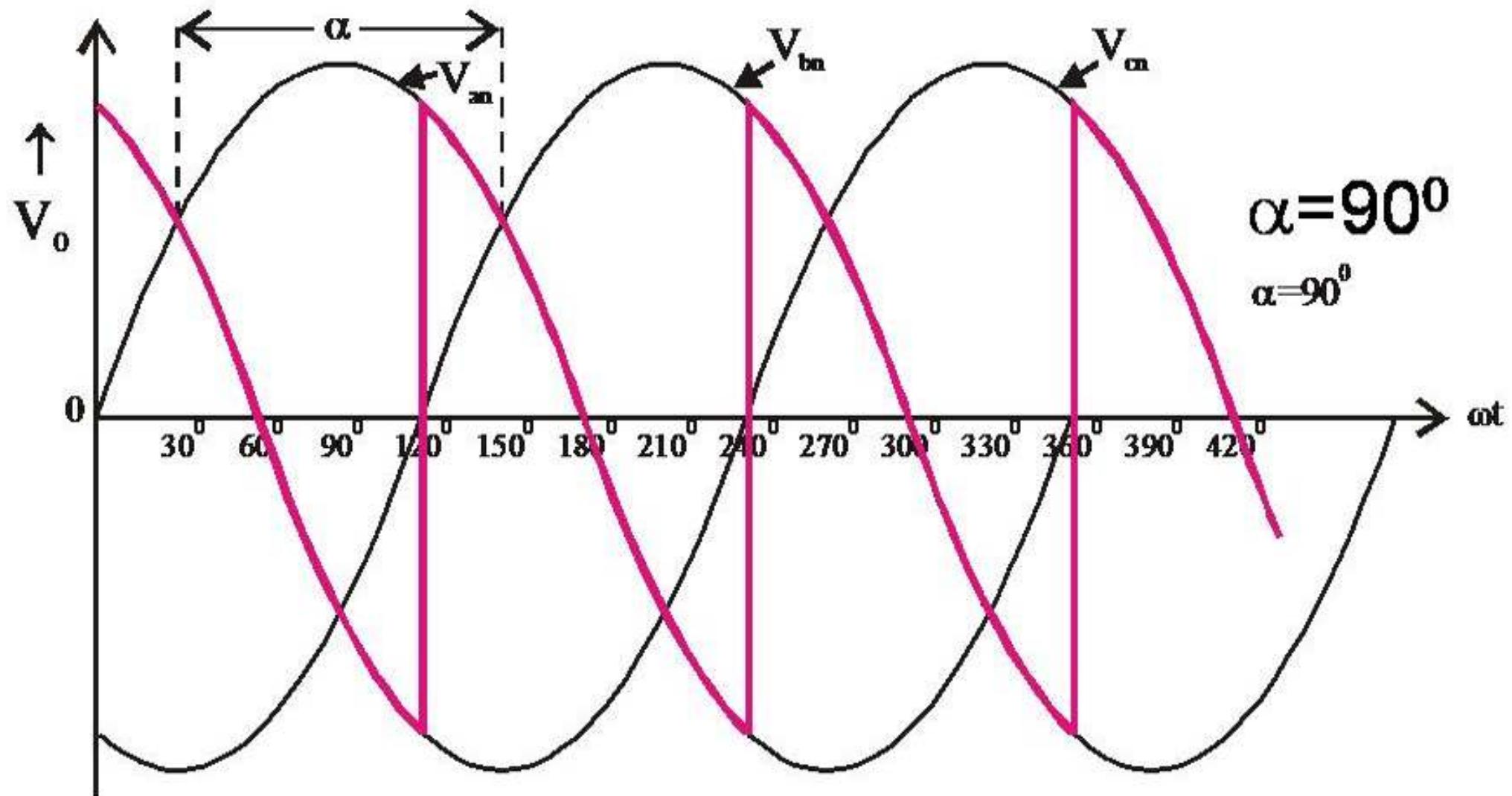
$$V_{O(RMS)} = \sqrt{3}V_m \left[\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha \right]^{\frac{1}{2}}$$

**3 Phase Half Wave
Controlled Rectifier Output
Voltage Waveforms For RL Load
at
Different Trigger Angles**

Different Trigger Angles

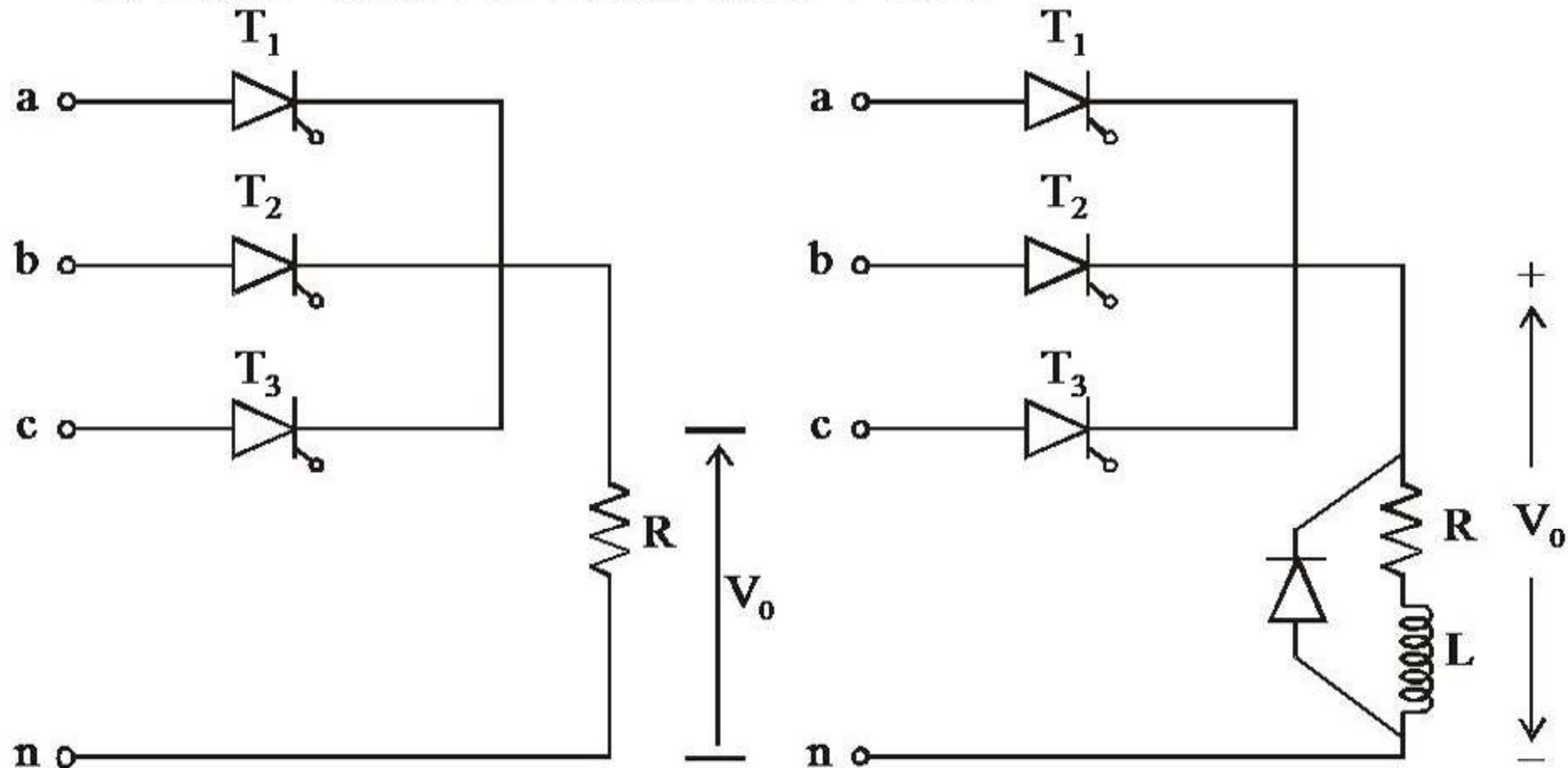


Different Trigger Angles



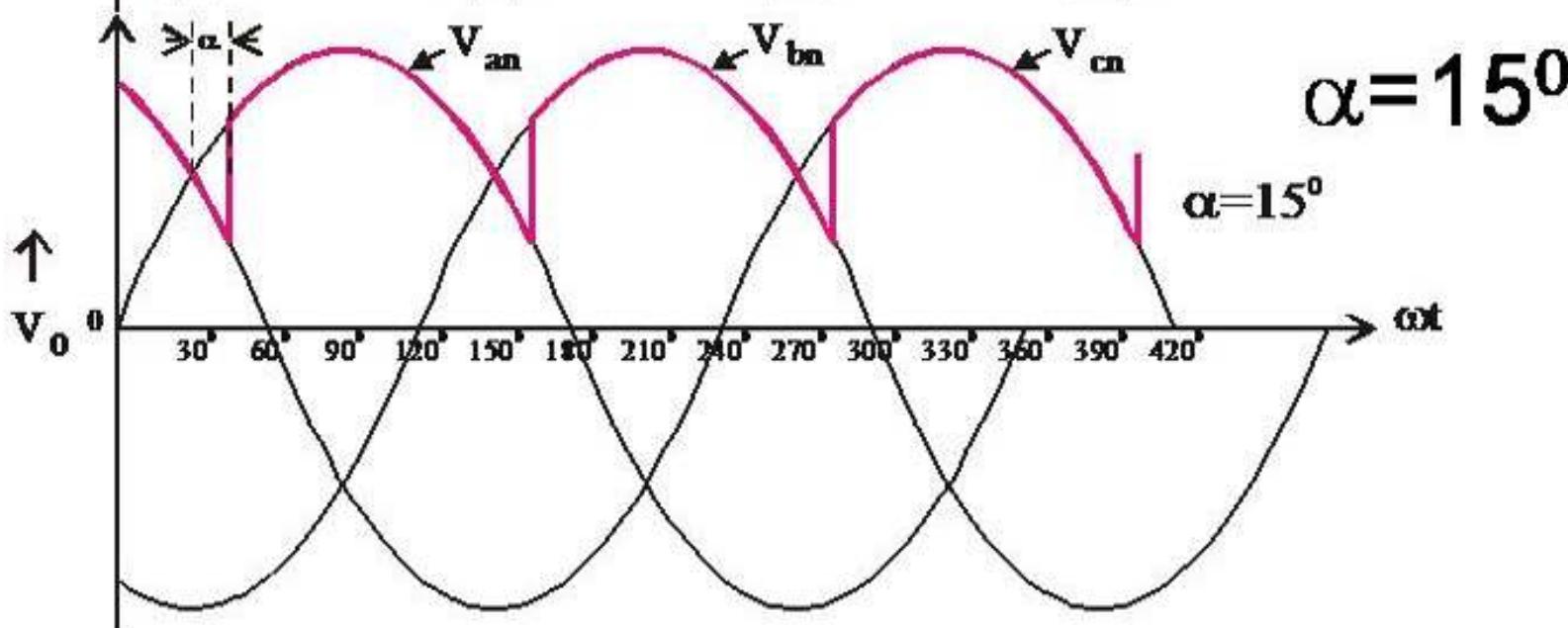
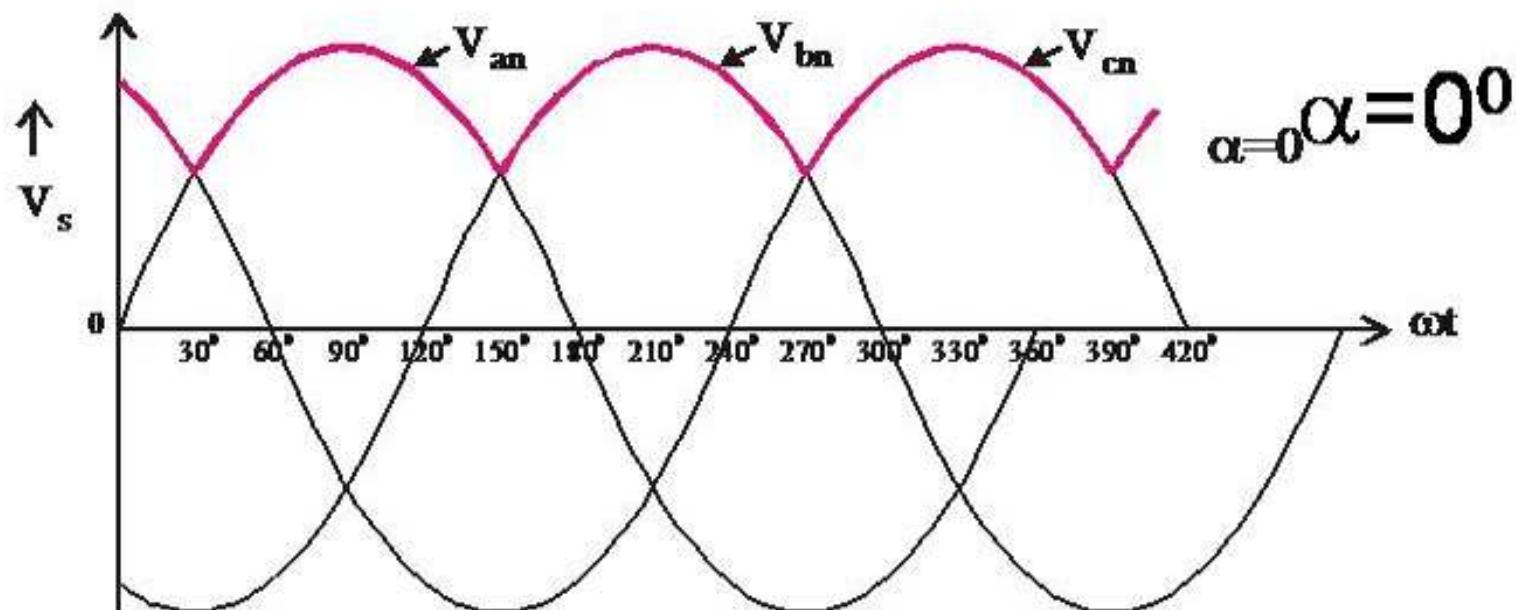
Three Phase Half Wave Converter

3 Phase Half Wave Controlled Rectifier With R Load and RL Load with FWD

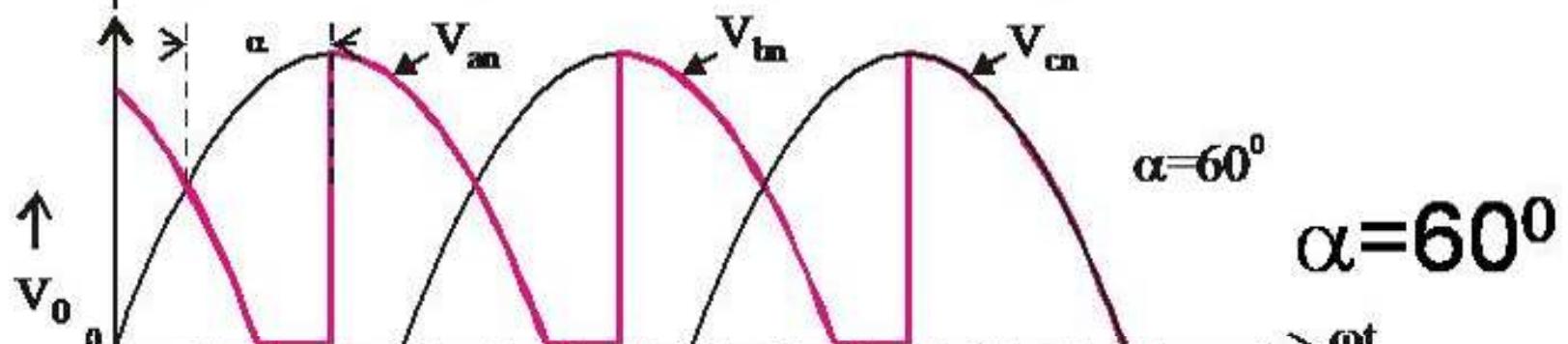
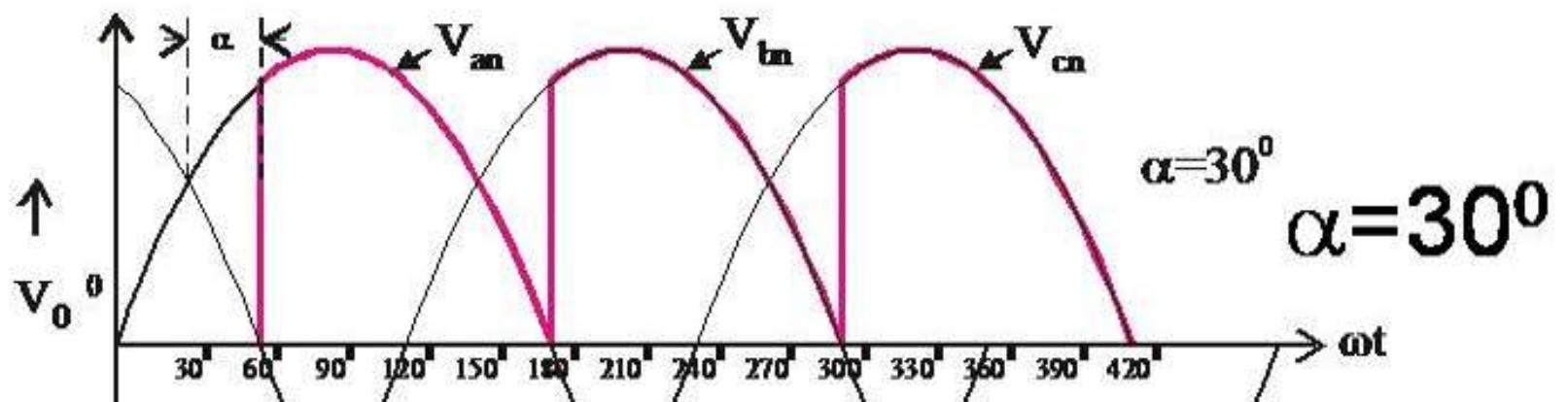


3 Phase Half Wave
Controlled Rectifier Output Voltage
Waveforms For R Load
or RL Load with FWD
at
Different Trigger Angles

Different Trigger Angles



Different Trigger Angles



To Derive An
Expression For The Average Or
Dc Output Voltage Of A
3 Phase Half Wave Converter With
Resistive Load
Or
RL Load With FWD

T_1 is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha\right) = (30^\circ + \alpha)$

T_1 conducts from $(30^\circ + \alpha)$ to 180° ;

$$v_o = v_{an} = V_m \sin \omega t$$

T_2 is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha\right) = (150^\circ + \alpha)$

T_2 conducts from $(150^\circ + \alpha)$ to 300° ;

$$v_o = v_{bn} = V_m \sin(\omega t - 120^\circ)$$

T_3 is triggered at $\omega t = \left(\frac{7\pi}{6} + \alpha \right) = (270^\circ + \alpha)$

T_3 conducts from $(270^\circ + \alpha)$ to 420° ;

$$v_o = v_{cn} = V_m \sin(\omega t - 240^\circ)$$

$$= V_m \sin(\omega t + 120^\circ)$$

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\alpha+30^0}^{180^0} v_o \cdot d(\omega t) \right]$$

$v_o = v_{an} = V_m \sin \omega t$; for $\omega t = (\alpha + 30^\circ)$ to (180°)

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\alpha+30^0}^{180^0} V_m \sin \omega t \cdot d(\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[\int_{\alpha+30^0}^{180^0} \sin \omega t \cdot d(\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos \omega t \right] \begin{matrix} 180^\circ \\ \alpha + 30^\circ \end{matrix}$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos 180^\circ + \cos(\alpha + 30^\circ) \right]$$

$\because \cos 180^\circ = -1$, we get

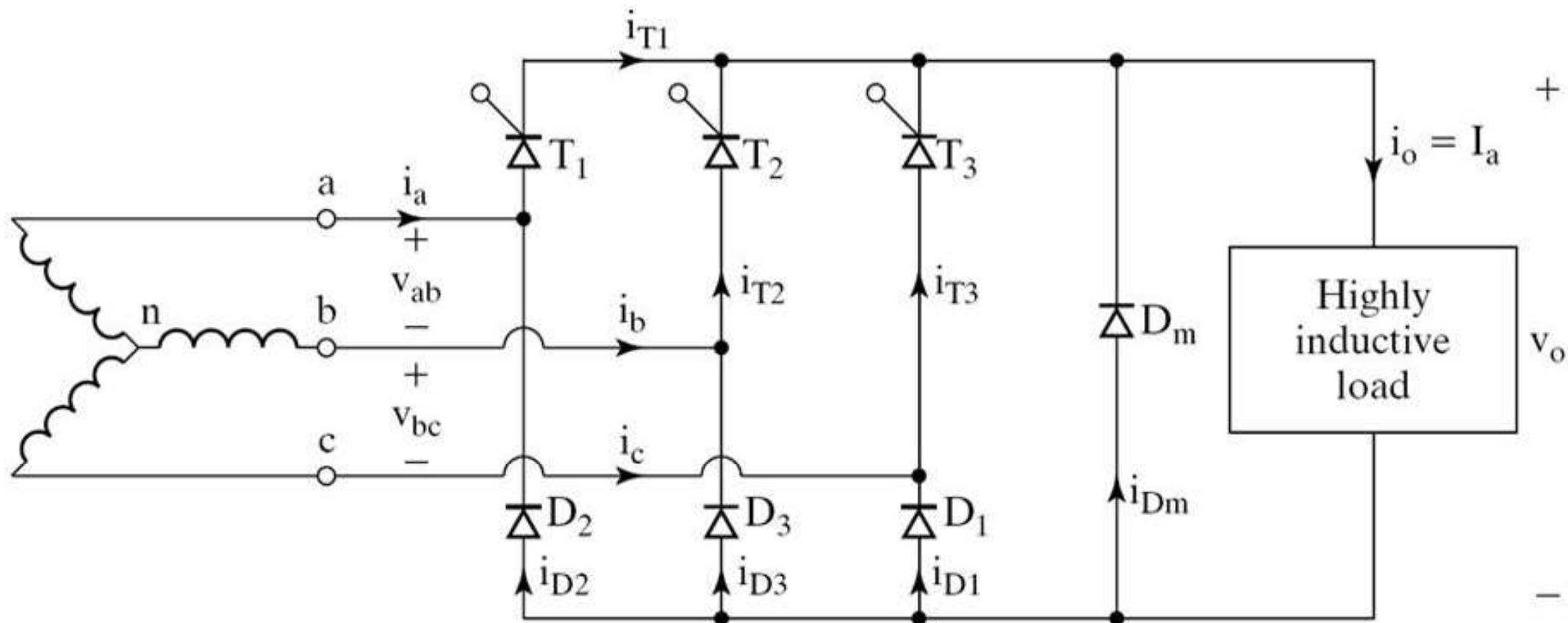
$$V_{dc} = \frac{3V_m}{2\pi} \left[1 + \cos(\alpha + 30^\circ) \right]$$

Three Phase Semiconverters

- 3 Phase semiconverters are used in Industrial dc drive applications upto 120kW power output.
- Single quadrant operation is possible.
- Power factor decreases as the delay angle increases.
- Power factor is better than that of 3 phase half wave converter.

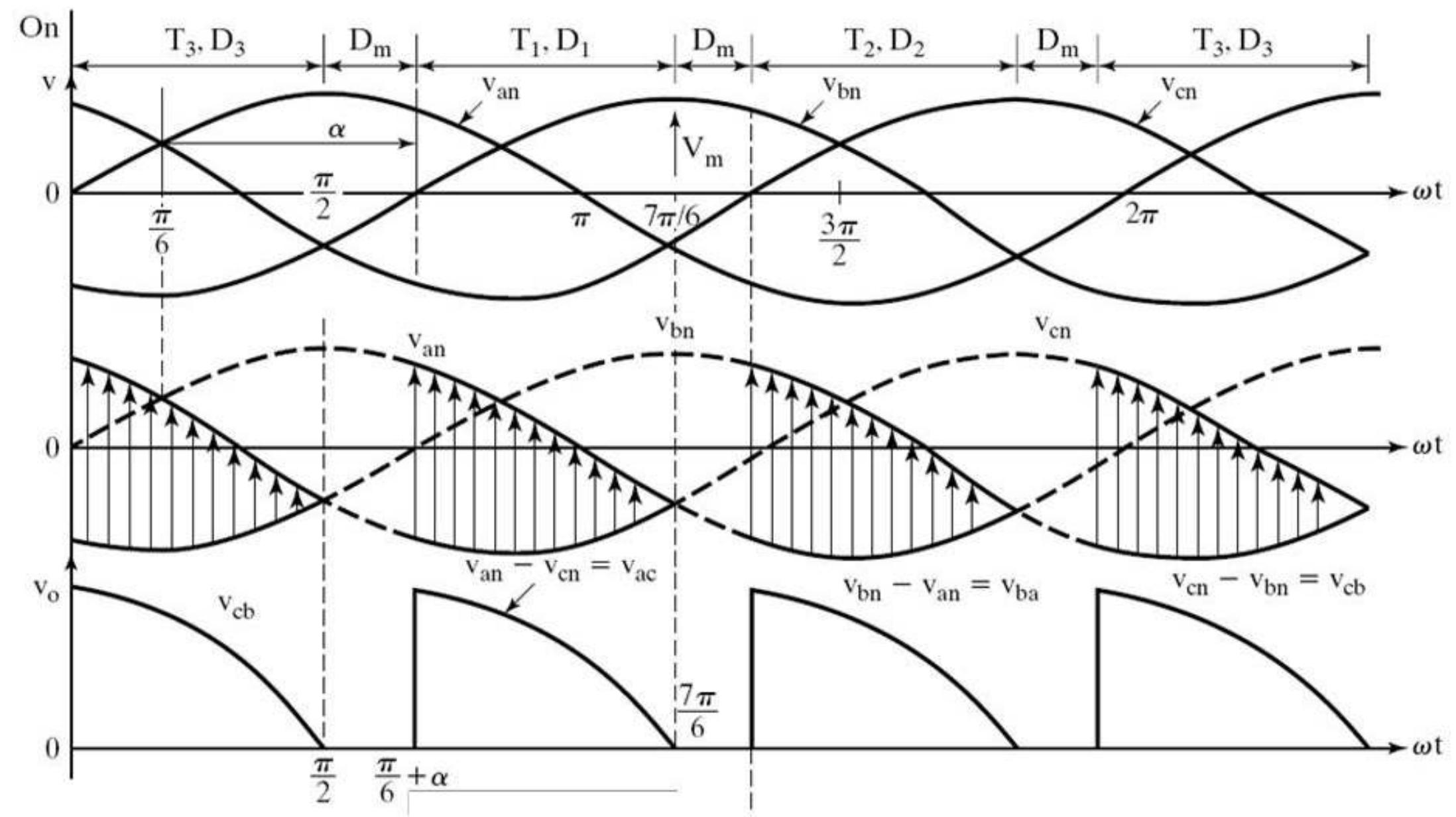
**3 Phase
Half Controlled Bridge Converter
(Semi Converter)
with Highly Inductive Load &
Continuous Ripple free Load
Current**

Three Phase Half Controlled Bridge Converter (Semi Converter)

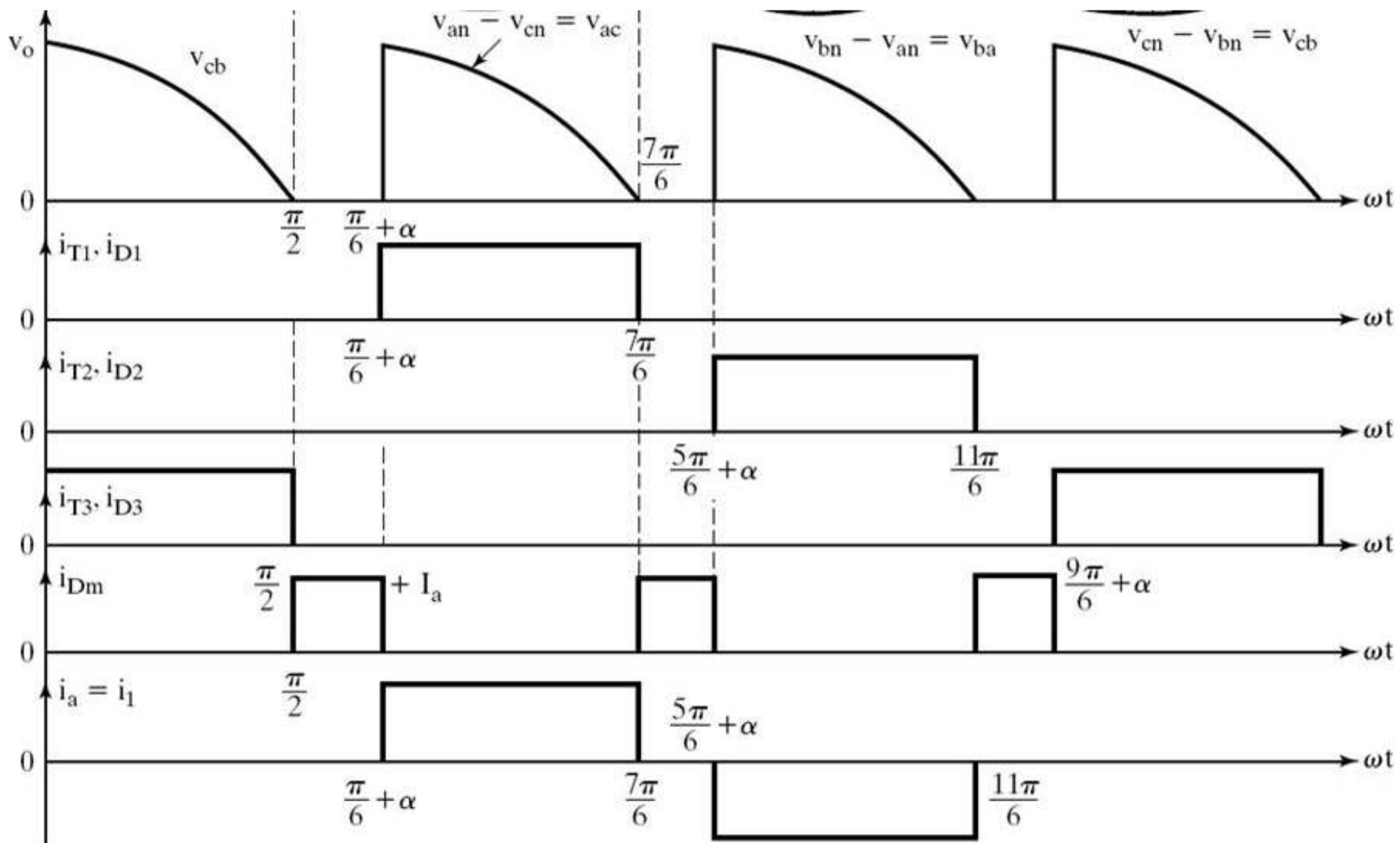


Waveforms of 3 Phase Semiconverter for $\alpha > 60^\circ$

Waveforms of 3 Phase Semiconverter for $\alpha > 60^\circ$



Waveforms of 3 Phase Semiconverter for $\alpha > 60^\circ$



Three Phase Semi Converter

3 phase semiconverter output ripple frequency of output voltage is $3f_s$

The delay angle α can be varied from 0 to π

During the period

$$30^\circ \leq \omega t < 210^\circ$$

$$\frac{\pi}{6} \leq \omega t < \frac{7\pi}{6}, \text{ thyristor } T_1 \text{ is forward biased}$$

Three Phase Semi Converter

If thyristor T_1 is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha\right)$,
 T_1 & D_1 conduct together and the line to line voltage
 v_{ac} appears across the load.

At $\omega t = \frac{7\pi}{6}$, v_{ac} becomes negative & FWD D_m conducts.

The load current continues to flow through FWD D_m ;
 T_1 and D_1 are turned off.

Three Phase Semi Converter

If FWD D_m is not used the T_1 would continue to conduct until the thyristor T_2 is triggered at

$\omega t = \left(\frac{5\pi}{6} + \alpha \right)$, and Free wheeling action would be accomplished through T_1 & D_2 .

If the delay angle $\alpha \leq \frac{\pi}{3}$, each thyristor conducts for $\frac{2\pi}{3}$ and the FWD D_m does not conduct.

Three Phase Semi Converter

We define three line neutral voltages

(3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t ; \quad V_m = \text{Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin(\omega t - 120^\circ)$$

$$\begin{aligned} v_{BN} = v_{cn} &= V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin(\omega t + 120^\circ) \\ &= V_m \sin(\omega t - 240^\circ) \end{aligned}$$

V_m is the peak phase voltage of a wye-connected source.

Three Phase Semi Converter

$$v_{RB} = v_{ac} = (v_{an} - v_{cn}) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{6}\right)$$

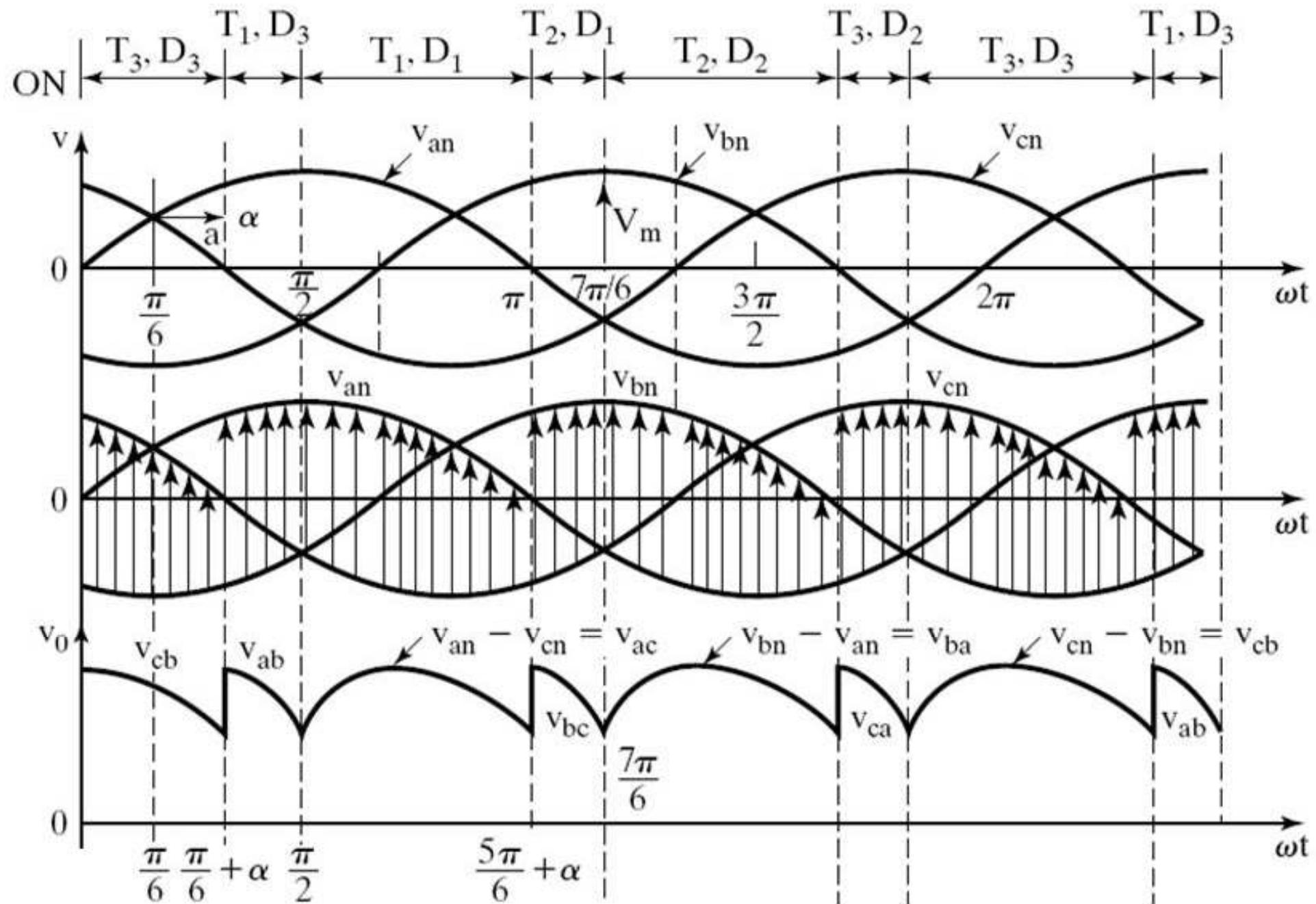
$$v_{YR} = v_{ba} = (v_{bn} - v_{an}) = \sqrt{3}V_m \sin\left(\omega t - \frac{5\pi}{6}\right)$$

$$v_{BY} = v_{cb} = (v_{cn} - v_{bn}) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

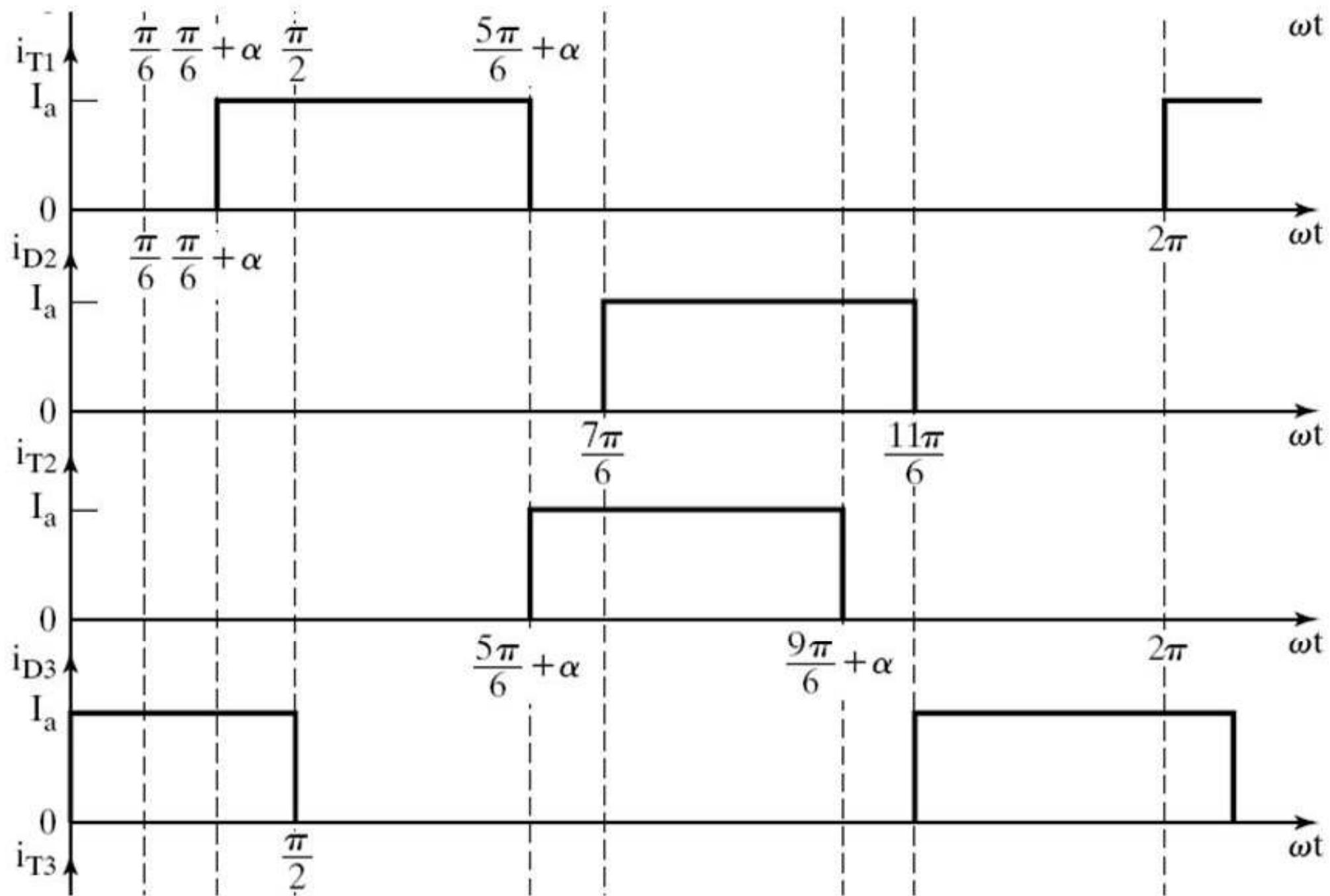
$$v_{RY} = v_{ab} = (v_{an} - v_{bn}) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

Wave forms of 3 Phase Semiconverter for $\alpha \leq 60^0$

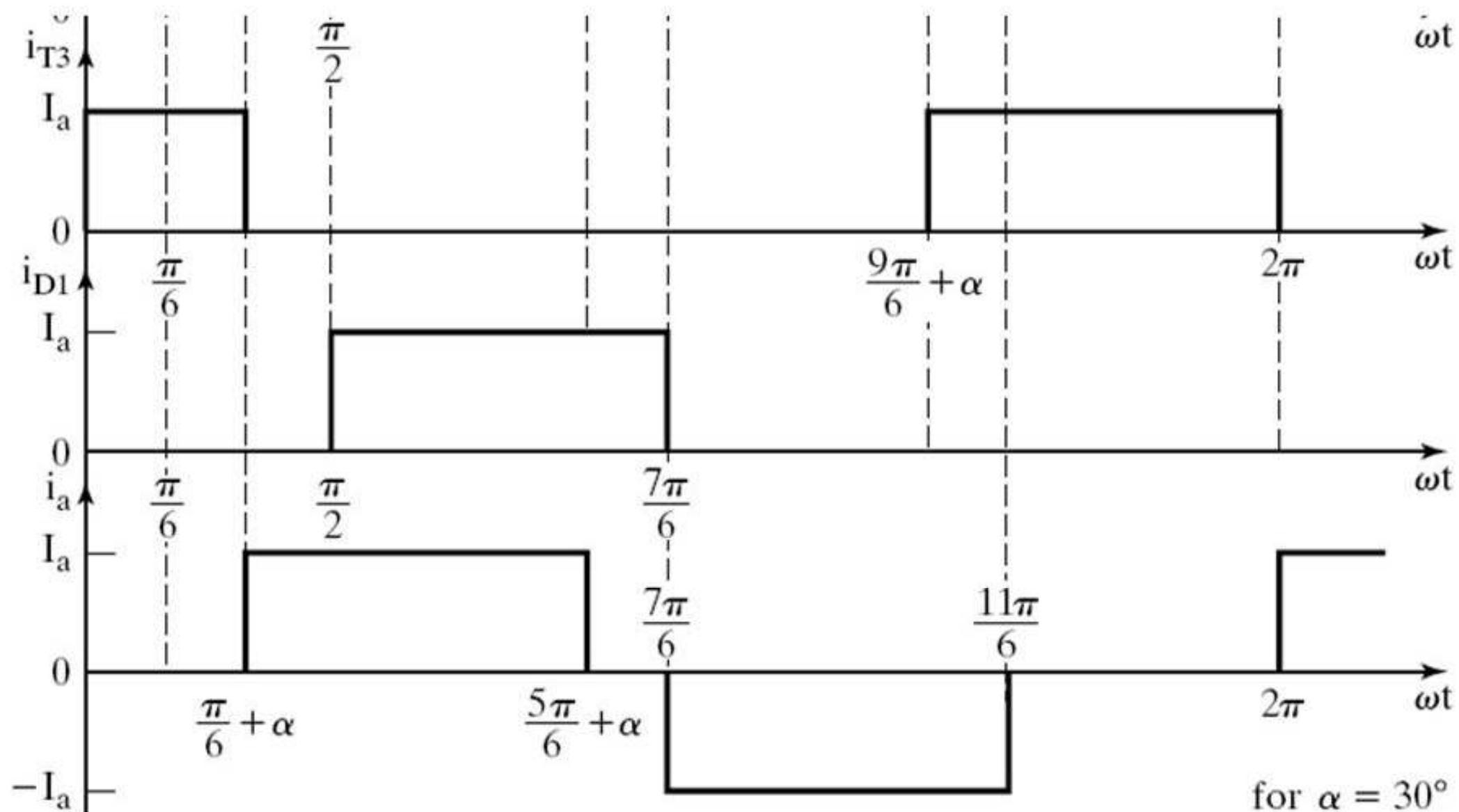
Waveforms of 3 Phase Semiconverter for $\alpha < 60^\circ$



Waveforms of 3 Phase Semiconverter for $\alpha < 60^\circ$



Waveforms of 3 Phase Semiconverter for $\alpha < 60^\circ$



To derive an
Expression for the
Average Output Voltage
of 3 Phase Semiconverter
for $\alpha > \pi / 3$
and Discontinuous Output Voltage

For $\alpha \geq \frac{\pi}{3}$ and discontinuous output voltage:

the Average output voltage is found from

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\pi/6+\alpha}^{7\pi/6} v_{ac} \cdot d(\omega t) \right]$$

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\pi/6+\alpha}^{7\pi/6} \sqrt{3} V_m \sin\left(\omega t - \frac{\pi}{6}\right) d(\omega t) \right]$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}(1 + \cos\alpha)$$

$$V_{dc} = \frac{3V_{mL}}{2\pi}(1 + \cos\alpha)$$

$V_{mL} = \sqrt{3}V_m$ = Max. value of line-to-line supply voltage

The maximum average output voltage that occurs at a delay angle of $\alpha = 0$ is

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}V_m}{\pi}$$

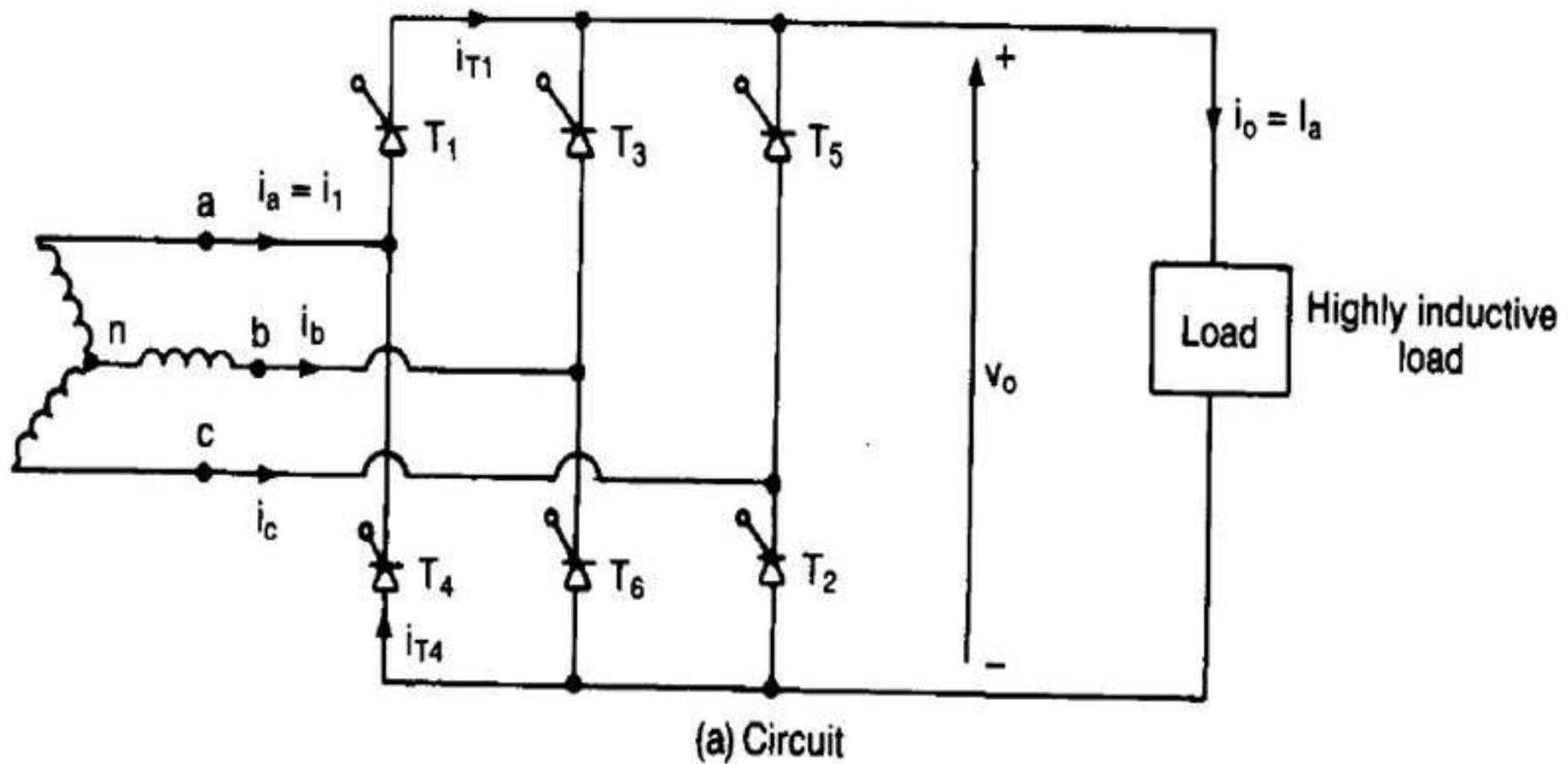
The normalized average output voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5(1 + \cos\alpha)$$

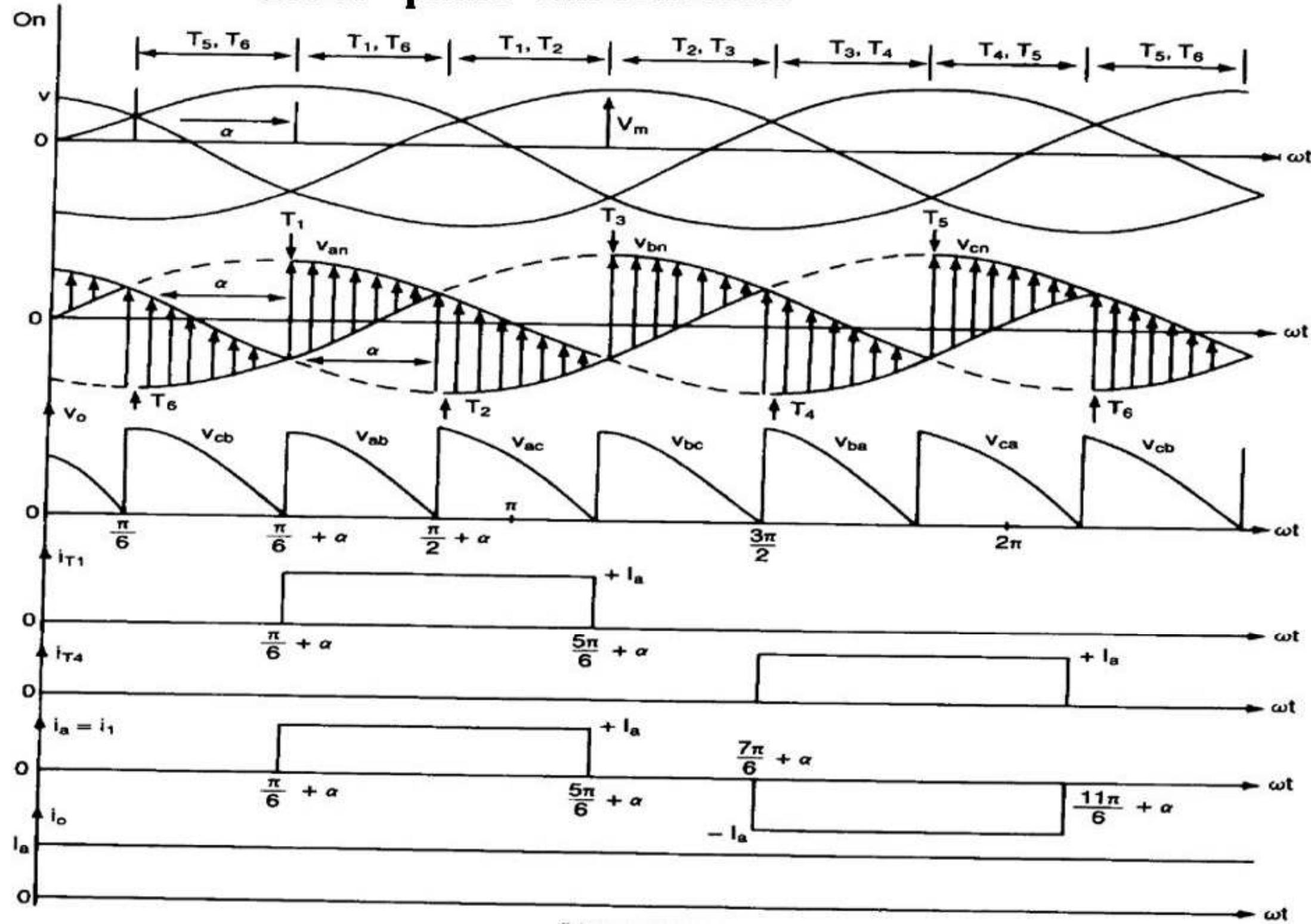
The rms output voltage is found from

$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} v_{ac}^2 d(\omega t) \right]^{1/2}$$

Three-phase full converter



Three-phase full converter



Three -phase full converter

The average output voltage is found from

$$\begin{aligned}V_{dc} &= \frac{3}{\pi} \int_{\pi/6+\alpha}^{\pi/2+\alpha} v_{ab} d(\omega t) = \frac{3}{\pi} \int_{\pi/6+\alpha}^{\pi/2+\alpha} \sqrt{3} V_m \sin \left(\omega t + \frac{\pi}{6} \right) d(\omega t) \\&= \frac{3 \sqrt{3} V_m}{\pi} \cos \alpha\end{aligned}$$

The maximum average output voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = \cos \alpha$$

The normalized average output voltage is

$$V_{dm} = 3 \sqrt{3} V_m / \pi.$$

Three -phase full converter

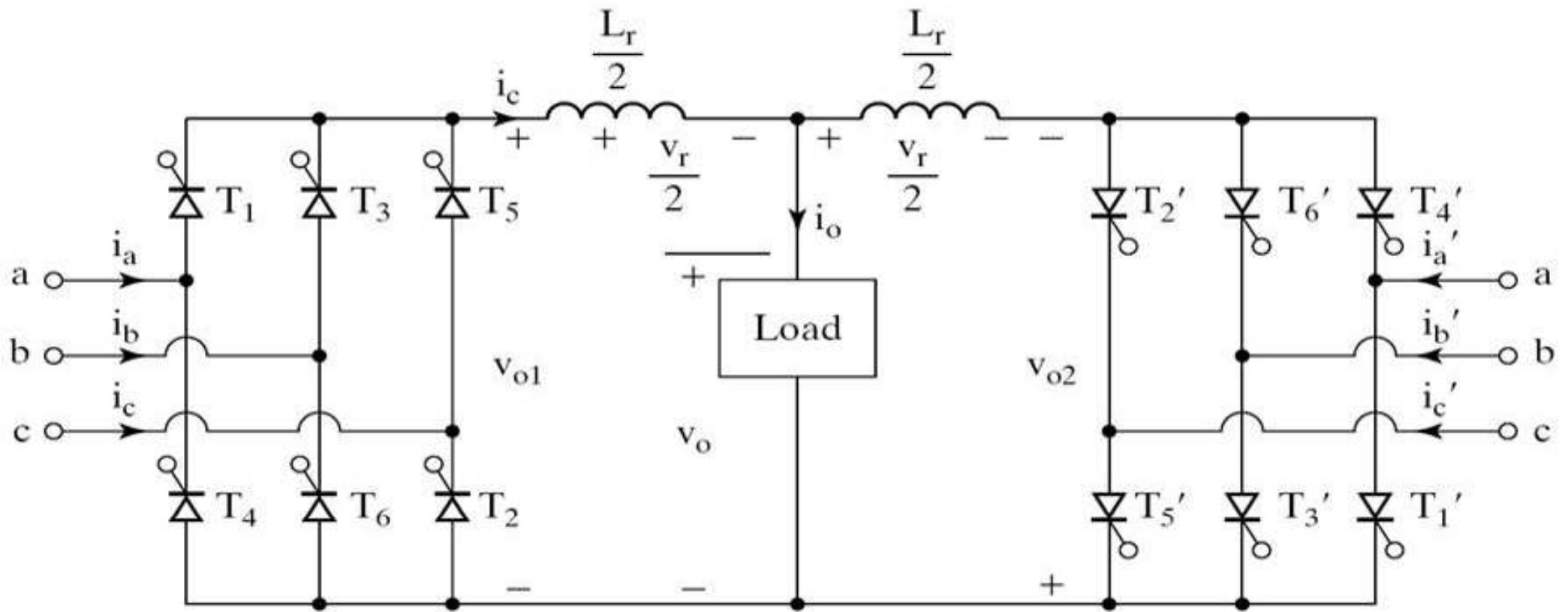
The rms value of output voltage is found from

$$\begin{aligned}V_{\text{rms}} &= \left[\frac{3}{\pi} \int_{\pi/6 + \alpha}^{\pi/2 + \alpha} 3V_m^2 \sin^2 \left(\omega t + \frac{\pi}{6} \right) d(\omega t) \right]^{1/2} \\&= \sqrt{6} V_m \left(\frac{1}{4} + \frac{3\sqrt{3}}{8\pi} \cos 2\alpha \right)^{1/2}\end{aligned}$$

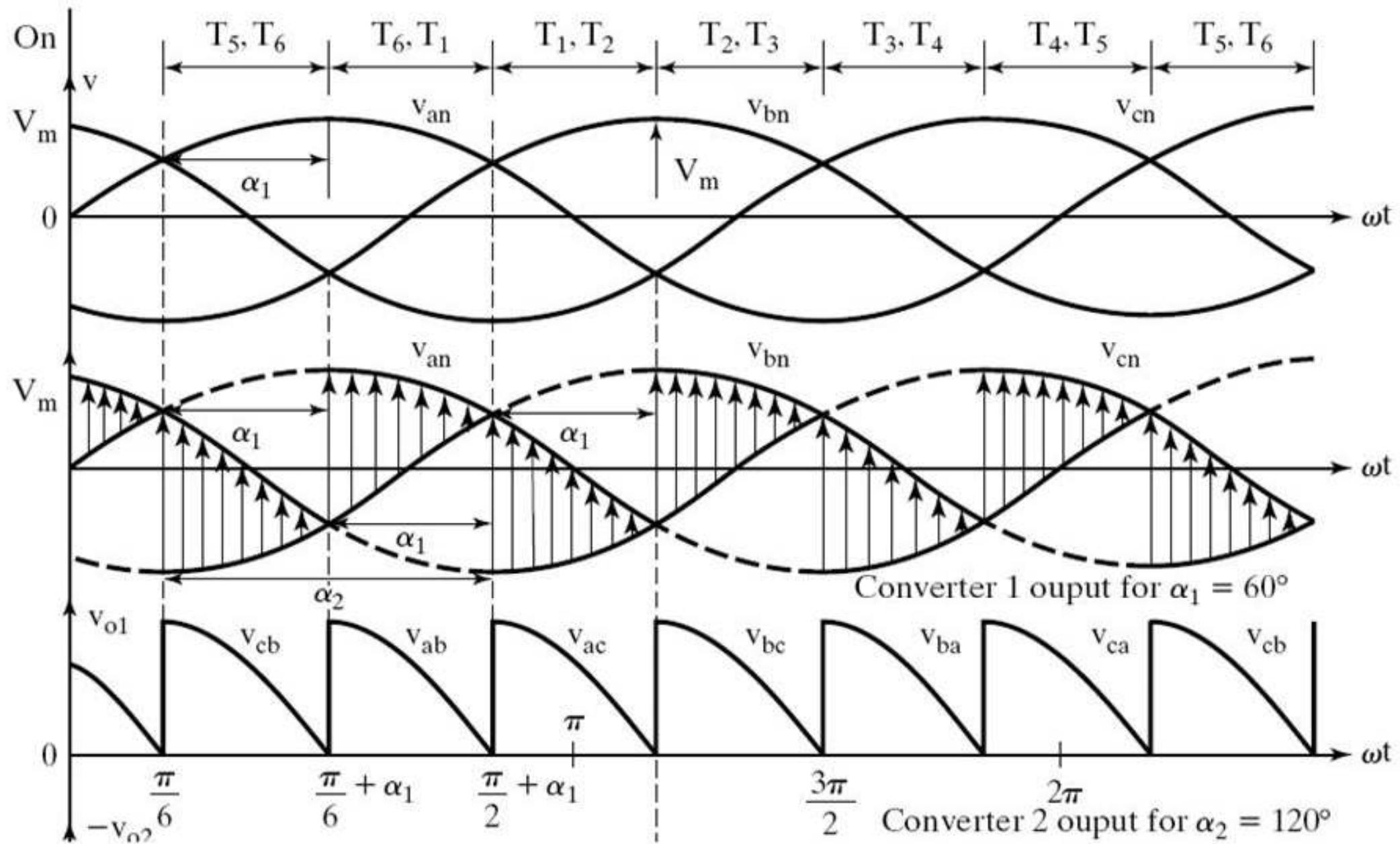
Three Phase Dual Converters

- For four quadrant operation in many industrial variable speed dc drives , 3 phase dual converters are used.
- Used for applications up to 2 mega watt output power level.
- Dual converter consists of two 3 phase full converters which are connected in parallel & in opposite directions across a common load.

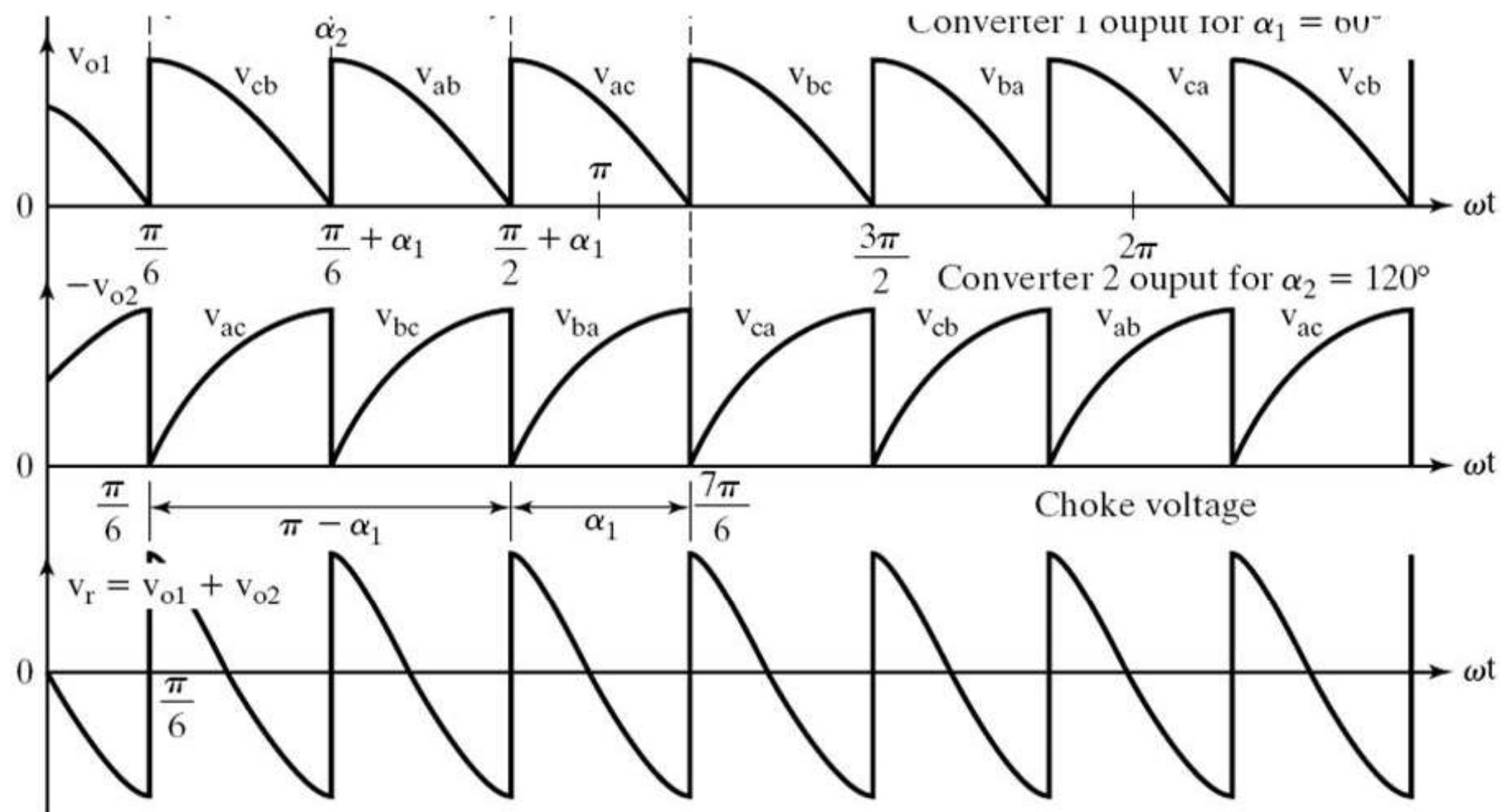
Three Phase Dual Converters



Three Phase Dual Converters



Three Phase Dual Converters



Three Phase Dual Converters

We define three line neutral voltages
(3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t ;$$

V_m = Max. Phase Voltage

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin(\omega t - 120^\circ)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin(\omega t + 120^\circ)$$

$$= V_m \sin(\omega t - 240^\circ)$$

Three Phase Dual Converters

We define three line neutral voltages
(3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t ;$$

V_m = Max. Phase Voltage

$$v_{YN} = v_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3} \right) = V_m \sin (\omega t - 120^\circ)$$

$$v_{BN} = v_{cn} = V_m \sin \left(\omega t + \frac{2\pi}{3} \right) = V_m \sin (\omega t + 120^\circ)$$

$$= V_m \sin (\omega t - 240^\circ)$$

Three Phase Dual Converters

To obtain an Expression for the Circulating Current

If v_{O1} and v_{O2} are the output voltages of converters 1 and 2 respectively, the instantaneous voltage across the current limiting inductor during the interval $(\pi/6 + \alpha_1) \leq \omega t \leq (\pi/2 + \alpha_1)$ is given by

Three Phase Dual Converters

$$v_r = v_{o1} + v_{o2} = v_{ab} - v_{bc}$$

$$v_r = \sqrt{3}V_m \left[\sin\left(\omega t + \frac{\pi}{6}\right) - \sin\left(\omega t - \frac{\pi}{2}\right) \right]$$

$$v_r = 3V_m \cos\left(\omega t - \frac{\pi}{6}\right)$$

The circulating current can be calculated by using the equation

Three Phase Dual Converters

$$i_r(t) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6} + \alpha_1}^{\omega t} v_r \cdot d(\omega t)$$

$$i_r(t) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6} + \alpha_1}^{\omega t} 3V_m \cos\left(\omega t - \frac{\pi}{6}\right) \cdot d(\omega t)$$

$$i_r(t) = \frac{3V_m}{\omega L_r} \left[\sin\left(\omega t - \frac{\pi}{6}\right) - \sin \alpha_1 \right]$$

$$i_{r(\max)} = \frac{3V_m}{\omega L_r}$$

Three Phase Dual Converters

Four Quadrant Operation

