7) اذا كانت جميع العناصر الواقعة فوق او تحت القطر الرئيسي كلها اصفار فان قيمة ذلك المحدد تساوي حاصل ضرب عناصر القطر الرئيسي فقط

$$|B| = \begin{vmatrix} 1 & 1 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{vmatrix} = 1*2*3 = 6$$

or

$$\begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 5 & 3 \end{vmatrix} = 1*2*3 = 6$$

8) عند ضرب عناصر اي صف او اي عمود في مقدار ثابت وجمع الناتج مع عناصر اي صف او عمود اخر فان قيمة المحدد لا تتغير

نضرب ب $2X$ ← ونجمع الناتج ←

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 0 \\ 1 & -1 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 7 & 6 \\ 1 & -1 & 4 \end{vmatrix}$$

EX. Find by propertiy (8) $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ -3 & 2 & 1 \end{vmatrix}$

$(-2)\,X \rightarrow$, $+ \rightarrow$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ -3 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ -3 & 2 & 1 \end{vmatrix}$$

$$= -1 \begin{vmatrix} 1 & 2 \\ -3 & 2 \end{vmatrix} = -1\,(2+6) = -8$$

EX.

$$\begin{vmatrix} 1 & 1 & x & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & x & 1 \\ 1 & x & -2 & 2 \end{vmatrix} = \int_0^{\pi} \cos^3\theta \, d\theta$$

$$= +1 \begin{vmatrix} 1 & 2 & 3 \\ 0 & x & 1 \\ x & -2 & 2 \end{vmatrix} + (-1) \begin{vmatrix} 1 & x & 4 \\ 1 & 2 & 3 \\ 0 & x & 1 \end{vmatrix} = \int_0^{\pi} \cos\theta \cos^2\theta \, d\theta$$

$$= \left[(2x + 2x) - (3x^2 - 2)\right] - \left[(2 + 4x) - (x + 3x)\right]$$

$$4x - 3x^2 + 2 - 2$$

$$4x - 3x^2 = \int_0^{\pi} \cos\theta \, (1 - \sin^2\theta) \, d\theta$$

$$= \int_0^{\pi} \cos\theta - \int_0^{\pi} \cos\theta \sin^2\theta \, d\theta$$

$$= \sin\theta \Big|_0^{\pi} - \frac{\sin^3\theta}{3} \Big|_0^{\pi}$$

$$= 0$$

$$4x - 3x^2 = 0$$

$$\therefore \quad x = 0$$

EX.

$$\begin{vmatrix} \cos X & \frac{\tan X}{5} & \sec X \\ 5 & 1 & 7 \\ 5 & 0 & 2 \end{vmatrix} = 0$$

$$-\frac{\tan X}{5}\begin{vmatrix} 5 & 7 \\ 5 & 2 \end{vmatrix} + 1\begin{vmatrix} \cos X & \sec X \\ 5 & 2 \end{vmatrix}$$

$$-\frac{\tan X}{5}(10-35) + (2\cos X - 5\sec X) = 0$$

$$5\tan X + 2\cos X - 5\sec X = 0$$

$$5\frac{\sin X}{\cos X} + 2\cos X - 5\frac{1}{\cos X} = 0 \qquad * \cos X$$

$$5\sin X + 2\cos^2 X - 5 = 0$$

$$5\sin X + 2(1-\sin^2 X) - 5 = 0$$

$$5\sin X + 2 - 2\sin^2 X - 5 = 0$$

$$-2\sin^2 X + 5\sin X - 3 = 0 \qquad * -1$$

$$+2\sin^2 X - 5\sin X + 3 = 0$$

$$(2\sin X - 3)(\sin X - 1) = 0$$

$2\sin X - 3 = 0$ or $\sin X - 1 = 0$

$\sin X = \frac{3}{2}$

$\sin X = 1$

$X = 90 = \frac{\pi}{2}$

Ex. solve the following system of equations by Gramer's rule.

$$2\ln x + a^{y} - e^{z} = 2$$

$$\ln x - a^{y} + e^{z} = 1$$

$$2\ln x + 2a^{y} + e^{z} = 5$$

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} \ln x \\ a^{y} \\ e^{z} \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$

$$D = 2(-3) - 1(3) + 2(0)$$

$$-6 - 3 = -9$$

$$\ln x = \frac{D_{\ln x}}{D} = \frac{\begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 5 & 2 & 1 \end{vmatrix}}{-9} = \frac{-9}{-9} = 1$$

$$\ln x = 1$$

$$e^{\ln x} = e^{1}$$

$$x = e^{1}$$

$$a^{y} = \frac{D_{a^y}}{D} = \frac{\begin{vmatrix} 2 & 2 & -1 \\ 1 & 1 & 1 \\ 2 & 5 & 1 \end{vmatrix}}{-9} = \frac{-9}{-9} = 1$$

$$\ln a^{y} = \ln 1$$

$$y \ln a = 0 \Rightarrow y = \frac{0}{\ln a} \Rightarrow y = 0$$

12) prove that $\begin{vmatrix} {}^{+}a & b & c & d \\ {}^{-}b & a & c & d \\ {}^{+}a & b & d & c \\ {}^{-}b & a & d & c \end{vmatrix} = 0$

Left side

$$= {}^{+}(a)\begin{vmatrix} a & c & d \\ b & d & c \\ a & d & c \end{vmatrix} - b\begin{vmatrix} b & c & d \\ b & d & c \\ a & d & c \end{vmatrix} + a\begin{vmatrix} b & c & d \\ a & c & d \\ a & d & c \end{vmatrix} - b\begin{vmatrix} b & c & d \\ a & c & d \\ b & d & c \end{vmatrix}$$

$$= a[(\cancel{acd} + ac^2 + bd^2) - (ad^2 + bc^2 + \cancel{acd})] - b[(\cancel{bcd} + bd^2 + ac^2) - (ad^2 + bc^2 + \cancel{bcd})] + a[(bc^2 + \cancel{acd} + ad^2) - (\cancel{acd} + ac^2 + bd^2)] - b[(bc^2 + \cancel{bcd} + ad^2) - (\cancel{cbd} + ac^2 + bd^2)]$$

$$= \cancel{a^2c^2} + \cancel{abd^2} - \cancel{a^2d^2} - \cancel{abc^2} - \cancel{b^2d^2} + \cancel{abc^2} + \cancel{abd^2} + \cancel{b^2c^2} + \cancel{abc^2} + \cancel{a^2d^2} - \cancel{a^2c^2} - \cancel{abd^2} - \cancel{b^2c^2} - \cancel{abd^2} + \cancel{abc^2} + \cancel{b^2d^2} = 0$$

$\therefore$ left side = right side

11) Evaluate the following determinants :-

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$$\begin{vmatrix}2&-1&3&1\\1&3&1&2\\3&-1&2&-1\\-2&2&1&-2\end{vmatrix}$$

$$= -(1)\begin{vmatrix}1&3&1\\3&-1&2\\-2&2&1\end{vmatrix} + (2)\begin{vmatrix}2&-1&3\\3&-1&2\\-2&2&1\end{vmatrix} - (-1)\begin{vmatrix}2&-1&3\\1&3&1\\-2&2&1\end{vmatrix} + (-2)\begin{vmatrix}2&-1&3\\1&3&1\\3&-1&2\end{vmatrix}$$

$$= -[(-1-12-6)-(2+4+9)] + 2[(-2+18+4)-(6+8-3)] + 1[(6+2+6)-(-18+4-1)]$$
$$-2[(12-3-3)-(27-2-2)]$$

$$= -(-19-15) + 2(20-11) + 1(14+15) - 2(6-23)$$

$$= \cancel{34} + 18 + 29 - \cancel{34} \quad = 47$$

$$\begin{vmatrix}2&3&4&5\\3&1&3&1\\2&1&4&3\\1&2&3&4\end{vmatrix}$$

$$= -(3)\begin{vmatrix}3&3&1\\2&4&3\\1&3&4\end{vmatrix} + 1\begin{vmatrix}2&4&5\\2&4&3\\1&3&4\end{vmatrix} - 1\begin{vmatrix}2&4&5\\3&3&1\\1&3&4\end{vmatrix} + 2\begin{vmatrix}2&4&5\\3&3&1\\2&4&3\end{vmatrix}$$

$$= -3(63-55) + 1(74-70) - 1(73-69) + 2(86-74)$$

$$= \cancel{-24} + \cancel{4} - \cancel{4} + \cancel{24} \quad = 0$$

$$\begin{vmatrix}1&2&3&4\\0&1&2&3\\0&0&2&1\\0&0&3&2\end{vmatrix} = +1\begin{vmatrix}1&2&3\\0&2&1\\0&3&2\end{vmatrix}$$

$$= (4+0+0) - (0+0+3)$$

$$= 4-3 \Rightarrow = 1$$

$$\therefore \quad 10^3 * 0 * 1 = 0$$

$$\Delta x = \begin{vmatrix} 3 & -1 & 1 \\ 1 & -2 & 0 \\ 2 & 1 & 0 \end{vmatrix} = (0+0+1)-(-4+0+0) = 1+4 \Rightarrow \Delta x = 5$$

$$\Delta y = \begin{vmatrix} 3 & 3 & 1 \\ -1 & 1 & 0 \\ 3 & 2 & 0 \end{vmatrix} = (0+0-2)-(3+0+0) = -2-3 \Rightarrow \Delta y = -5$$

$$\Delta z = \begin{vmatrix} 3 & -1 & 3 \\ -1 & -2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = (-12-3-3)-(-18+2+3) = -18+12 \Rightarrow \Delta z = -5$$

$$x = \frac{\Delta x}{\Delta} \Rightarrow x = \frac{5}{5} \Rightarrow x = 1$$

$$y = \frac{\Delta y}{\Delta} \Rightarrow y = \frac{-5}{5} \Rightarrow y = -1$$

$$z = \frac{\Delta z}{\Delta} \Rightarrow z = \frac{-5}{5} \Rightarrow z = -1$$

$$kx + ky = y + 1 \Rightarrow kx + (k-1)y = 1$$

$$(k-1)x + ky + 1 = 0 \Rightarrow (k-1)x + ky = -1$$

$$\Delta = \begin{vmatrix} k & k-1 \\ k-1 & k \end{vmatrix} = k^2 - (k-1)^2 \Rightarrow k^2 - (k^2 - 2k + 1)$$

$$\cancel{k^2} - \cancel{k^2} + 2k - 1 \Rightarrow \Delta = 2k - 1$$

$$\Delta x = \begin{vmatrix} 1 & k-1 \\ -1 & k \end{vmatrix} = k - (-k+1) \Rightarrow k + k - 1 \Rightarrow \Delta x = 2k - 1$$

$$\Delta y = \begin{vmatrix} k & 1 \\ k-1 & -1 \end{vmatrix} = -k - (k-1) \Rightarrow -k - k + 1 \Rightarrow \Delta y = -(2k-1)$$

$$x = \frac{\Delta x}{\Delta} \Rightarrow x = \frac{\cancel{2k-1}}{\cancel{2k-1}} \Rightarrow x = 1$$

$$y = \frac{\Delta y}{\Delta} \Rightarrow y = \frac{-\cancel{(2k-1)}}{\cancel{2k-1}} \Rightarrow y = -1$$

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10) solve the following system of equations by cramer's rule:-

$2x+y=z+2 \Rightarrow 2x+y-z=2$

$x+z=y+7 \Rightarrow x-y+z=7$

$2x+2y+z=4 \Rightarrow 2x+2y+z=4$

$$\Delta=\begin{vmatrix}2&1&-1\\1&-1&1\\2&2&1\end{vmatrix}=(-2+2-2)-(2+1+4)=-2-7 \Rightarrow \Delta=-9$$

$$\Delta x=\begin{vmatrix}2&1&-1\\7&-1&1\\4&2&1\end{vmatrix}=(-2+4-14)-(4+4+7)=-12-15 \Rightarrow \Delta x=-27$$

$$\Delta y=\begin{vmatrix}2&2&-1\\1&7&1\\2&4&1\end{vmatrix}=(14+4-4)-(-14+2+8)=14+4 \Rightarrow \Delta y=18$$

$$\Delta z=\begin{vmatrix}2&1&2\\1&-1&7\\2&2&4\end{vmatrix}=(-8+14+4)-(-4+4+28)=10-28 \Rightarrow \Delta z=-18$$

$$x=\frac{\Delta x}{\Delta} \Rightarrow x=\frac{\cancel{-27}^{\,3}}{\cancel{-9}} \Rightarrow x=3$$

$$y=\frac{\Delta y}{\Delta} \Rightarrow y=\frac{\cancel{18}^{\,2}}{\cancel{-9}} \Rightarrow y=-2$$

$$z=\frac{\Delta z}{\Delta} \Rightarrow z=\frac{\cancel{-18}^{\,2}}{\cancel{-9}} \Rightarrow z=2$$

$3x+z=y+3 \Rightarrow 3x-y+z=3$

$x-2(y+x)=1 \Rightarrow -x-2y+0z=1$

$x+y=2-2x \Rightarrow 3x+y+0z=2$

$$\Delta=\begin{vmatrix}3&-1&1\\-1&-2&0\\3&1&0\end{vmatrix}=(0+0-1)-(-6+0+0)=-1+6 \Rightarrow \Delta=5$$

8) prove that $\begin{vmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{vmatrix} = (x_2-x_1)(x_3-x_1)(x_3-x_2)$

left side:-

$R_2 - R_1$ & $R_3 - R_1$

$$\begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & x_2-x_1 & x_2^2-x_1^2 \\ 0 & x_3-x_1 & x_3^2-x_1^2 \end{vmatrix} = (x_2-x_1)(x_3-x_1)\begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & 1 & x_2-x_1 \\ 0 & 1 & x_3-x_1 \end{vmatrix}$$

$$= (x_2-x_1)(x_3-x_1)\left[(x_3-x_1)-(x_2-x_1)\right]$$

$$= (x_2-x_1)(x_3-x_1)(x_3-\cancel{x_1}-x_2+\cancel{x_1})$$

$$= (x_2-x_1)(x_3-x_1)(x_3-x_2) = \text{right side.}$$

9) find (K) such that the following system fail to have asolution.

$x+y=z+3$, $kx+2z=y+3$, $x+2y=z+4$

$x+y-z=3$

$kx-y+2z=3$

$x+2y-z=4$

∴ fial to have asolution ∴ $\Delta=0$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ k & -1 & 2 \\ 1 & 2 & -1 \end{vmatrix}$$

$0=(+1-2k+2)-(+1-k+4)$

$0=3-2k-5+k$

$0=-k-2$

$\Longrightarrow \boxed{k=-2}$

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6) Find the value of (K) & (h) for which the system :-

$2x + hy = 8$ & $x + 3y = K$

ⓐ has infinitely many solutions.

ⓑ has no solution.

$\Delta = \begin{vmatrix} 2 & h \\ 1 & 3 \end{vmatrix} \Rightarrow \Delta = 6 - h$

$\Delta x = \begin{vmatrix} 8 & h \\ K & 3 \end{vmatrix} \Rightarrow 24 - hK \quad \therefore x = \frac{\Delta x}{\Delta} \Rightarrow x = \frac{24 - hK}{6 - h}$

$\Delta y = \begin{vmatrix} 2 & 8 \\ 1 & K \end{vmatrix} \Rightarrow \Delta y = 2K - 8 \quad \therefore y = \frac{\Delta y}{\Delta} \Rightarrow y = \frac{2K - 8}{6 - h}$

ⓐ many solutions $\Rightarrow 0 = 6 - h \Rightarrow h = 6$

AND $\frac{2K-8}{6-h} = 0 \Rightarrow 2K - 8 = 0 \Rightarrow 2K = 8 \Rightarrow K = 4$

$\therefore K \neq 4$

ⓑ no solution $\Rightarrow \therefore \Delta = 0$

$0 = 6 - h \Rightarrow h = 6$

$\frac{24 - hK}{6 - h} = 0 \Rightarrow 24 - 6K = 0 \Rightarrow 6K = 24$

$\Rightarrow K = \frac{24}{6} \Rightarrow K = 4$

7) Prove that $\begin{vmatrix} b+c & a & 1 \\ c+a & b & 1 \\ b+a & c & 1 \end{vmatrix} = 0$

left side :-

$C_1 + C_2$ $\begin{vmatrix} a+b+c & a & 1 \\ a+b+c & b & 1 \\ a+b+c & c & 1 \end{vmatrix} = (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$

$= (a+b+c) * 0$

$= 0$ = right side.