Determinants

4) show that $\begin{vmatrix} 1 & -1 & 2 \\ x & y & 1 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} -1 & 1 & y \\ 1 & 3 & x \\ 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} x & 2 & 5 \\ -1 & y & 3 \\ 0 & 0 & 2 \end{vmatrix}$ represent a circle, Find its radius and center.

$$[(y+0+0)-(0-x+0)][(0+0+y)-(0-x+0)] = (2xy+0+0)-(0+0-4)$$

$$\Rightarrow (y+x)(y+x) = 2xy+4 \Rightarrow (y+x)^2 = 2xy+4$$

$$y^2+2xy+x^2 = 2xy+4$$

$$\Rightarrow y^2+x^2 = 4$$

$\therefore$ (radius)$^2 = 4 \Rightarrow$ r=2

$\therefore$ center (0,0)

5) solve using Cramer's rule:-

$$\begin{vmatrix} x & y & 1 \\ 1 & -1 & 1 \\ 13 & 2 & 1 \end{vmatrix} = 0 \quad \& \quad \begin{vmatrix} x & y & 1 \\ 3 & 2 & 1 \\ -6 & -4 & 1 \end{vmatrix} = 0$$

$$(-x+13y+2)-(-13+2x+y)=0$$

$$-x+13y+2+13-2x-y=0$$

$$12y-3x+15=0 \quad \div 3 \Rightarrow 4y-x+5=0 \Rightarrow x=4y+5 \ldots (1)$$

$$(2x-6y-12)-(-12+3y-4x)=0$$

$$2x-6y-12+12-3y+4x=0$$

$$6x-9y=0 \quad \div 3 \Rightarrow 2x-3y=0 \ldots\ldots (2)$$

substituting eq (1) in eq (2)

$$2x-3y=0$$

$$2(4y+5)-3y=0 \Rightarrow 8y+10-3y=0$$

$$5y=-10 \Rightarrow y=\frac{-10}{5} \Rightarrow y=-2$$ substituting in eq (1)

$$x=4y+5$$

$$x=4*-2+5 \Rightarrow x=-8+5 \Rightarrow x=-3$$

Determinants

1) Show that $\begin{vmatrix} x & 1 & 2 \\ y & 0 & -1 \\ 1 & 2 & 0 \end{vmatrix} = 1$ represent a straight line.

Find its slope.

sol:

$$(0+4y-1)-(0+0-2x)=1$$

$$4y-1+2x=1 \Rightarrow 4y+2x=2 \quad \div 2$$

$$2y+x=1$$

$$2\cdot\bar{y}+1=0 \Rightarrow 2\bar{y}=-1 \Rightarrow \bar{y}=\frac{-1}{2}$$

2) show that $\begin{vmatrix} -1 & 1 & y \\ 1 & 3 & x \\ 0 & 2 & 0 \end{vmatrix} = 4$ and $\begin{vmatrix} 1 & 1 & 2 \\ y & x & 1 \\ 0 & 0 & 3 \end{vmatrix} = 6$

represent orthogonal straight lines.

$$(0+0+2y)-(0+0-2x)=4$$

$$2y+2x=4 \quad \div 2 \Rightarrow y+x=2$$

$$\bar{y}+1=0 \Rightarrow \bar{y}=-1$$

$$(3x+0+0)-(0+0+3y)=6$$

$$3x-3y=6 \quad \div 3 \Rightarrow x-y=3$$

$$y=x-3 \Rightarrow \bar{y}=1\cdot 0 \Rightarrow \bar{y}=1$$

$\therefore$ orthogonal straight lines

3) Find the slope of $\begin{vmatrix} x & 2 & 0 \\ 1 & 0 & 1 \\ y & 4 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 1 \\ 0 & -3 & 1 \\ 4 & -2 & 0 \end{vmatrix}$

$$(0+2y+0)-(0-2+4x)=(0+0-8)-(-12+0-2)$$

$$2y+2-4x=-8+14$$

$$2y-4x=6-2 \Rightarrow 2y-4x=4 \quad \div 2$$

$$\Rightarrow y-2x=2 \Rightarrow y=2+2x$$

$$\bar{y}=0+2 \Rightarrow \bar{y}=2$$

١. جمع وطرح المصفوفات

لجمع او طرح مصفوفتين يجب ان تكون المصفوفات متساويتان في عدد الاسطر والاعمدة

Ex. $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 2 & -1 \end{bmatrix}$, $B = \begin{vmatrix} 2 & 5 \\ 1 & 2 \\ 0 & 1 \end{vmatrix}$

find $C = A + B$, $C = A - B$

$$C = A+B = \begin{vmatrix} 3+2 & 1+5 \\ 4+1 & 2+2 \\ 2+0 & -1+1 \end{vmatrix} = \begin{vmatrix} 5 & 6 \\ 5 & 4 \\ 2 & 0 \end{vmatrix}$$

$$C = A-B = \begin{vmatrix} 3-2 & 1-5 \\ 4-1 & 2-2 \\ 2-0 & -1-1 \end{vmatrix} = \begin{vmatrix} 1 & -4 \\ 3 & 0 \\ 2 & -2 \end{vmatrix}$$

c. ضرب المصفوفات

لضرب اي مصفوفتين يجب ان يتحقق احد الشرطين

أ) يجب ان يكون عدد الاعمدة من المصفوفة الاولى

يساوي عدد الاسطر من المصفوفة الثانية

ب) ان يكون عدد الاسطر من المصفوفة الاولى يساوي

عدد الاعمدة من المصفوفة الثانية

Ex. $A = \begin{vmatrix} 1 & 3 & 2 \end{vmatrix}_{1\times3}$, $B = \begin{vmatrix} 3 & 1 \\ 4 & 2 \\ -2 & 5 \end{vmatrix}_{3\times2}$

$C = A * B$

$A_1\ B_1 \quad A_1\ B_1 \quad A_1\ B_1$

$C_{11} = 1*3 + 3*4 + 2*(-2) = 11$

(C_{11}: السطر الاول من A، العمود الاول من B)

$C_{12} = 1*1 + 3*2 + 2*5 = 17$

$\therefore C = \begin{vmatrix} 11 & 17 \end{vmatrix}$

Ex. $A = \begin{vmatrix} 1 & -1 \\ 2 & 0 \\ -1 & 3 \end{vmatrix}$, $B = \begin{vmatrix} 4 & 0 & -1 & 1 \\ 1 & 1 & 2 & -1 \end{vmatrix}$

3×2 $\qquad$ 2×4

$C = A * B = |AB|_{3 \times 4}$

$C_{11} = 1 * 4 + (-1 * 1) = 3$

$C_{12} = 1 * 0 + (-1 * 1) = -1$

$C_{13} = (1 * -1) + (-1 * 2) = -3$

$C_{14} = 1 * 1 + (-1 * -1) = 2$

$C_{21} = 2 * 4 + 0 * 1 = 8$

$C_{22} = 2 * 0 + 0 * 1 = 0$

$C_{23} = (2 * -1) + 0 * 2 = -2$

$C_{24} = 2 * 1 + (0 * -1) = 2$

$C_{31} = (-1 * 4) + 3 * 1 = -1$

$C_{32} = (-1 * 0) + 3 * 1 = 3$

$C_{33} = (-1 * -1) + 3 * 2 = 7$

$C_{34} = (-1 * 1) + (3 * -1) = -4$

$$C = \begin{vmatrix} 3 & -1 & -3 & 2 \\ 8 & 0 & -2 & 2 \\ -1 & 3 & 7 & -4 \end{vmatrix}$$

EX.

$$A = \begin{vmatrix} 3 & 2 \\ 1 & 1 \\ 0 & 3 \end{vmatrix}_{3\times 2}, \quad B = \begin{vmatrix} 1 & 3 & 1 \\ -1 & 1 & 3 \\ 2 & 4 & 4 \\ 0 & 2 & 5 \end{vmatrix}_{4\times 3}$$

$$C = A \cdot B = \text{[illegible]} = |BA|_{4\times 2}$$

$C_{11} = 1*3 + 3*1 + 1*0 = 6$

$C_{12} = 1*2 + 3*1 + 1*3 = 8$

$C_{21} = (-1*3) + 1*1 + 3*0 = -2$

$C_{22} = (-1*2) + 1*1 + 3*3 = 8$

$C_{31} = 2*3 + 4*1 + 4*0 = 10$

$C_{32} = 2*2 + 4*1 + 4*3 = 20$

$C_{41} = 0*3 + 2*1 + 5*0 = 2$

$C_{42} = 0*2 + 2*1 + 5*3 = 17$

$$C = \begin{vmatrix} 6 & 8 \\ -2 & 8 \\ 10 & 20 \\ 2 & 17 \end{vmatrix}$$

Even and odd Functions

الدوال الزوجية والفردية

* A Functions $y = F(x)$ is an_even if

$F(-x) = F(x)$ for every x.

$\therefore F(-x) = F(x)$ even

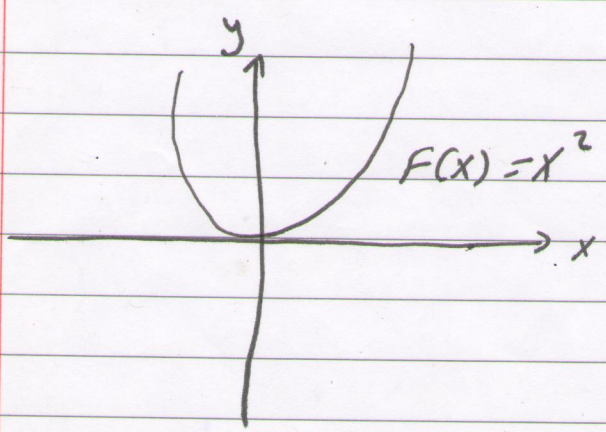
* A Functions $y = F(x)$ is an_odd if

$F(-x) = -F(x)$ for every x

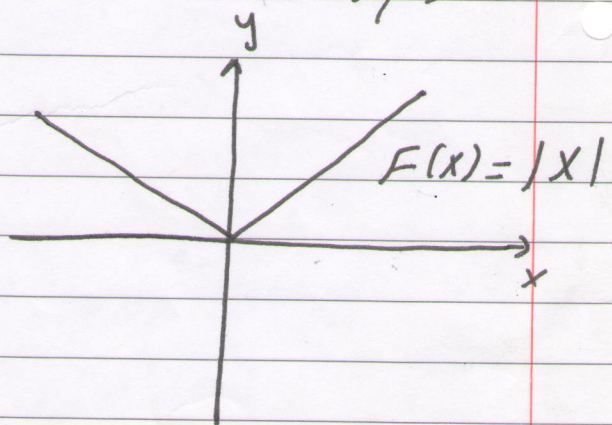
$\therefore F(-x) = -F(x)$ odd

الدوال الزوجية (even) هي الدوال المتناظرة حول محور (y)

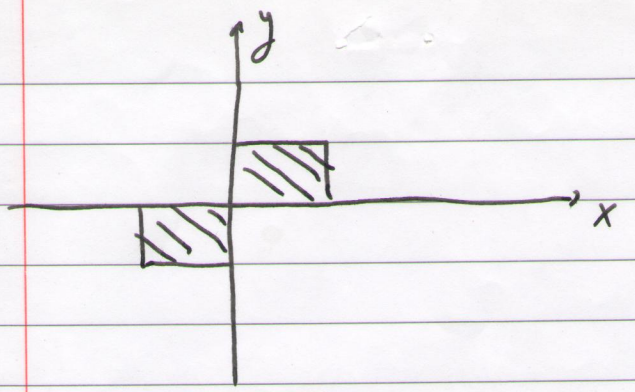
بينما الدوال الفردية (odd) فتكون متناظرة حول نقطة الأصل



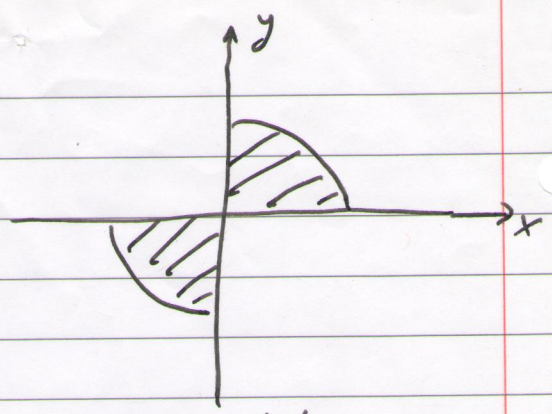
even



even



odd



odd

Ex. show that the functions even or odd ① $y = x^2$, ② $y = \frac{2}{x} + x$

① $y = x^2$

$F(-x) = (-x)^2$

$= x^2$

$\therefore F(x) = x^2 = F(-x)$ is even

2) $F(x) = y = \frac{2}{x} + x$

$F(-x) = \frac{2}{-x} - x$

$F(-x) = -\left(\frac{2}{x} + x\right)$

$F(-x) = -F(x)$ is odd

Ex. show that the functions even or odd $F(x) = \tan x$

$$F(x) = \frac{\sin(x)}{\cos x}$$

$$F(-x) = \frac{\sin(-x)}{\cos(-x)}$$

$$F(-x) = \frac{-\sin x}{\cos x} = -\tan x$$

$F(-x) = -F(x)$ is odd

Hyperbolic Function الدوال الزائدية

الدوال الزائدية، خاصة بالدوال الزائدية، المتطابقات

1) $\cosh^2 x + \sinh^2 x = 1$

2) $\sinh 2x = 2 \sinh x \cdot \cosh x$

3) $\tanh^2 x = 1 - \operatorname{sech}^2 x$

4) $\operatorname{csch}^2 x = \coth^2 x - 1$

5) $\sinh^2 x = \dfrac{\cosh 2x - 1}{2}$

6) $\cosh^2 x = \dfrac{\cosh 2x + 1}{2}$

7) $\cosh(x) + \sinh(x) = e^x$

8) $\cosh(x) - \sinh(x) = e^{-x}$