الدوال الزائدية تتكون

1) $\sinh x = \dfrac{e^{x} - e^{-x}}{2}$

2) $\cosh x = \dfrac{e^{x} + e^{-x}}{2}$

3) $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{\frac{e^{x} - e^{-x}}{2}}{\frac{e^{x} + e^{-x}}{2}} = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$

4) $\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$

5) $\operatorname{sech} x = \dfrac{2}{e^{x} + e^{-x}}$

6) $\operatorname{csch} x = \dfrac{2}{e^{x} - e^{-x}}$

مشتقات الدوال الزائدية

1) $\frac{d}{dx}(\sinh x) = \cosh x$

2) $\frac{d}{dx}(\cosh x) = \sinh x$

3) $\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$

4) $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

5) $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \cdot \tanh x$

6) $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \cdot \coth x$

Ex. Find $\frac{dy}{dx}$ For $y = 2^{\sinh(\ln \cosh x)}$

$$\frac{dy}{dx} = 2^{\sinh(\ln \cosh x)} * \cosh(\ln \cosh x)$$

$$* \frac{1}{\cosh x} * \sinh x * \ln 2$$

$$= 2^{\sinh(\ln \cosh x)} \cdot \cosh(\ln \cosh x) * \frac{\sinh x}{\cosh x} \cdot \ln 2$$

$$= \quad '' \qquad '' \qquad \tanh x \cdot \ln 2$$

Ex. $y = \ln \left| \tanh \frac{x}{2} \right|$

$$\frac{dy}{dx} = \frac{1}{\tanh \frac{x}{2}} * \operatorname{sech}^2 \frac{x}{2} * \frac{1}{2}$$

$$= \frac{\operatorname{sech}^2 \frac{x}{2}}{2 \tanh \frac{x}{2}} = \frac{\frac{1}{\cosh^2 \frac{x}{2}}}{2 \cdot \frac{\sinh \frac{x}{2}}{\cosh \frac{x}{2}}}$$

$$= \frac{1}{2 \sinh \frac{x}{2} \cosh \frac{x}{2}} = \frac{1}{\sinh x} = \operatorname{csch} x$$

Ex. $y = \operatorname{sech} x^{x} + \pi^{\sinh x}$

$y = \operatorname{sech} e^{\ln x^{x}} + \pi^{\sinh x}$

$= \operatorname{sech} e^{x\ln x} + \pi^{\sinh x}$

$$\frac{dy}{dx} = -\operatorname{sech} e^{x\ln x} \cdot \tanh e^{x\ln x} * e^{x\ln x} * \left(x * \frac{1}{x} + \ln x * 1\right) + \pi^{\sinh x} * \cosh x * \ln \pi$$

$$= -\operatorname{sech} e^{x\ln x} \cdot \tanh e^{x\ln x} - \operatorname{sech} e^{x\ln x} \cdot \tanh e^{x\ln x} * e^{x\ln x} \cdot \ln x + \pi^{\sinh x} \cdot \cosh x \cdot \ln \pi$$

Ex. Find $\frac{dy}{dx}$ For $y = x^{\tanh 3x}$

$\ln y = \ln x^{\tanh 3x}$

$\ln y = \tanh 3x \ln x$

$$\frac{1}{y}\frac{dy}{dx} = \tanh 3x * \frac{1}{x} + \ln x * \operatorname{sech}^2 3x * 3$$

$$\frac{dy}{dx} = \frac{\frac{\tanh 3x}{x} + 3\ln x \cdot \operatorname{sech}^2 3x}{\frac{1}{y}}$$

$$= y\left(\frac{\tanh 3x}{x} + 3\ln x \cdot \operatorname{sech}^2 3x\right)$$

$$\frac{dy}{dx} = x^{\tanh 3x} \left(\frac{\tanh 3x}{x} + 3\ln x \cdot \operatorname{sech}^2 3x \right)$$

EX. prove that $\sinh 2x = 2 \sinh x \cdot \cosh x$

الطرف الايمن $= 2 \cdot \sinh x \cdot \cosh x$

$$= \cancel{2} \cdot \frac{e^x - e^{-x}}{\cancel{2}} * \frac{e^x + e^{-x}}{2}$$

$$= \frac{1}{2} (e^x - e^{-x}) \cdot (e^x + e^{-x})$$

$$= \frac{1}{2} \left[e^{2x} + e^0 - e^0 - e^{-2x} \right]$$

$$= \frac{1}{2} (e^{2x} - e^{-2x})$$

$= \sinh 2x$ الطرف الايسر

EX. solve for x ; $\cosh 2x = 3$

$$\frac{e^{2x} + e^{-2x}}{2} = 3 \implies e^{2x} + \frac{1}{e^{2x}} = 6$$

$$\frac{(e^x)^4 + 1}{(e^x)^2} = 6$$

$$(e^x)^4 + 1 = 6(e^x)^2$$

$$e^{x^4} + 1 = 6 \cdot e^{x^2}$$

$$\ln e^{x^4} + \ln 1 = 6 \ln e^{x^2}$$

$$x^4 \ln e + \ln 1 = 6x^2 \ln e$$

$$\ln e = 1, \quad \ln 1 = 0$$

$$x^4 + 0 = 6x^2$$

$$x^4 = 6x^2$$

$$x^2 = 6$$

$$x = \mp\sqrt{6}$$

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تكامل الدوال الزائدية

1) $\int \sinh x \, dx = \cosh x + c$

2) $\int \cosh x \, dx = \sinh x + c$

3) $\int \operatorname{sech}^2 x \, dx = \tanh x + c$

4) $\int \operatorname{csch}^2 x \, dx = -\coth x + c$

5) $\int \operatorname{sech} x \cdot \tanh x \, dx = -\operatorname{sech} x + c$

6) $\int \operatorname{csch} x \cdot \coth x \, dx = -\operatorname{csch} x + c$

في تكامل الدوال الزائدية يجب توفر مشتقة الزاوية.

EX. Find $\int \sinh(2x+1)\,dx$

$$\int \sinh(2x+1)\,dx * \frac{2}{2}$$

$$= \frac{1}{2}\int \sinh(2x+1) \cdot 2\,dx$$

$$= \frac{1}{2}\cosh(2x+1) + c$$

EX. $\displaystyle\int \frac{\sinh x}{\cosh^4 x}\,dx$

$$\int \cosh^{-4} x \, \sinh x \, dx =$$

$$= \frac{\cosh^{-3} x}{-3} + c$$

$$= \frac{-1}{3} * \frac{1}{\cosh^3 x} = -\frac{1}{3}\operatorname{sech}^3 x + c$$

EX. $\displaystyle\int \frac{\ln(\sinh x)}{\tanh x}\,dx$

$$= \int \ln(\sinh x) * \frac{1}{\frac{\sinh x}{\cosh x}} \cdot dx$$

$$= \int \ln(\sinh x) * \frac{\cosh x}{\sinh x}\, dx$$

$$= \frac{\ln^2(\sinh x)}{2} + c \quad = \frac{1}{2} \ln^2(\sinh x) + c$$

Ex. $\int \operatorname{sech}(\ln x)\, dx$

$$\int \frac{1}{\cosh(\ln x)} = \int \frac{2}{e^{\ln x} + e^{-\ln x}}\, dx$$

$$= 2 \int \frac{1}{e^{\ln x} + e^{-\ln x}} = 2 \int \frac{1}{x + x^{-1}}\, dx$$

$$\boxed{= \int \frac{2}{x}\, dx + \int \frac{2}{x^{-1}}\, dx}$$

$$= \int 2x^{-1}\, dx + \int 2x\, dx$$

$$= 2 \ln x + 2 * \frac{x^2}{2} + c$$

$$= 2 \ln x + x^2 + c$$

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EX. $\int \frac{dx}{\cosh 4x + \sinh 4x}$

$$\int \frac{dx}{e^{4x}} = \int e^{-4x}\,dx * \frac{-4}{-4}$$

$$= -\frac{1}{4}\int e^{-4x}(-4)\,dx = \frac{-1}{4}e^{-4x} + c$$

EX $\int e^{x} \sinh 2x\,dx$

$$\int e^{x} * \frac{e^{2x} - e^{-2x}}{2}\,dx$$

$$= \frac{1}{2}\int (e^{3x} - e^{-x})\,dx$$

$$= \frac{1}{2}\left[\int e^{3x}\,dx - \int e^{-x}\,dx\right]$$

$$= \frac{1}{2}\left[\int e^{3x}\,dx * \frac{3}{3} - \int e^{-x}\,dx * \frac{-1}{-1}\right.$$

$$= \frac{1}{2}\left[\frac{1}{3}\int e^{3x} * 3\,dx + \int e^{-x}(-1)\,dx\right]$$

$$= \frac{1}{2}\left[\frac{1}{3}e^{3x} + e^{-x}\right] + c = \frac{1}{6}e^{3x} + \frac{1}{2}e^{-x} + c$$