Ex. $\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\, dx$

$$\int \tanh 2x\, dx = \int \frac{\sinh 2x}{\cosh 2x}\, dx$$

$$= \int \text{———} = \int \cosh^{-1} 2x \sinh 2x\, dx \quad * \frac{2}{2}$$

$$= \frac{1}{2} \int \cosh^{-1} 2x \sinh 2x * 2\, dx$$

$$= \frac{1}{2} \ln|\cosh 2x| + C$$

Ex. $\int \cosh(\ln \cos x)\, dx$

$$= \int \frac{e^{\ln \cos x} + e^{\ln \cos x^{-1}}}{2}\, dx$$

$$= \frac{1}{2} \int (\cos x + \cos^{-1} x)\, dx$$

$$= \frac{1}{2} \int \cos x\, dx + \frac{1}{2} \int \cos^{-1} x\, dx$$

$$= \frac{1}{2} \sin x + \frac{1}{2} \int \frac{1}{\cos x}\, dx = \frac{\sin x}{2} + \frac{1}{2} \int \sec x\, dx$$

$$= \frac{1}{2} \sin x + \frac{1}{2} \ln|\sec x + \tan x| + C$$

Ex. $\int \cosh^2 4x \, dx$

$$= \int \frac{\cosh 8x + 1}{2} \, dx$$

$$= \frac{1}{2} \int (\cosh 8x + 1) \, dx$$

$$= \frac{1}{2} \left[\int \cosh 8x \, dx + \int dx \right.$$

$$= \frac{1}{2} \left[\int \cosh 8x \, dx * \frac{8}{8} + \int dx \right.$$

$$= \frac{1}{2} \left[\frac{1}{8} \sinh 8x + x + c \right]$$

$$= \frac{1}{16} * \sinh 8x + x + c$$

Ex $\int e^{2x} \cosh e^{2x} \, dx$

$$\int \cosh e^{2x} * e^{2x} \, dx * \frac{2}{2}$$

$$= \frac{1}{2} \int \cosh e^{2x} \cdot 2 \cdot e^{2x} \, dx$$

$$= \frac{1}{2} \sinh e^{2x} + c$$

Ex. $\int \cosh^2 x \cdot \sinh^3 x \, dx$

$= \int \cosh^2 x \cdot \sinh^2 x \cdot \sinh x \, dx$

$= \int \cosh^2 x \, (\cosh^2 x - 1) \cdot \sinh x \, dx$

$= \int (\cosh^4 x \cdot \sinh x - \cosh^2 x \cdot \sinh x) \, dx$

$= \int \cosh^4 x \cdot \sinh x \, dx - \int \cosh^2 x \cdot \sinh x \, dx$

$= \frac{\cosh^5 x}{5} - \frac{\cosh^3 x}{3} + C$

Integration التكامل

indefinite integration تكامل غير محدد

definite integration التكامل المحدد

1)) indefinite integration التكامل غير المحدد

Rules of indefinite integration

1) $\int dx = x + c$

2) $\int x^n dx = \frac{x^{n+1}}{n+1}$, $n \neq -1$

3) $\int F(x)^n \cdot F'(x) \cdot dx = \frac{F(x)^{n+1}}{n+1}$, $n \neq -1$

4) $\int F(x) \mp g(x) \cdot dx = \int F(x)\, dx \mp \int g(x)\, dx$

5) $\int k F(x)\, dx = k \int F(x)\, dx$, k is constant

في القاعدة رقم (٢) هي تكامل دالة الدالة والتي تغني
اذا ما عدا ( )
انها كل $[F(x)]$ ففي تكاملها يجب توفر مشتقة
داخل القوس وبعدها تحذف المشتقة وتكامل بإضافة
(١) للاس والقسمة على الاس الجديد

Ex. Find $\int 3x^2\,dx$

$$= 3\int x^2\,dx = 3 * \frac{x^3}{3} + c$$

$$= x^3 + c$$

Ex. find $\int x\sqrt{x^2+1}\,dx$

$$= \int (x^2+1)^{\frac{1}{2}}\, x\,dx$$

$$= \int (x^2+1)^{\frac{1}{2}} \cdot x\,dx * \frac{2}{2}$$

$$= \frac{1}{2}\int (x^2+1)^{\frac{1}{2}}\, 2x\,dx$$

$$= \frac{1}{2} * \frac{(x^2+1)^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{1}{2} * \frac{2}{3} \cdot (x^2+1)^{\frac{3}{2}}$$

$$= \frac{1}{3}(x^2+1)^{\frac{3}{2}} + c$$

Ex. $\int \left(\frac{1}{x^2} + x\right) dx$

$$\int \frac{1}{x^2} dx + \int x \, dx$$

$$= \int x^{-2} dx + \int x \, dx$$

$$= \frac{x^{-1}}{-1} + \frac{x^2}{2} + c$$

$$= \frac{-1}{x} + \frac{1}{2}x^2 + c$$

Ex. $\int (3t + t^{-3})^2 dt$

$$= \int (9t^2 + 6t^{-2} + t^{-6}) dt$$

$$= \int 9t^2 dt + \int 6t^{-2} dt + \int t^{-6} dt$$

$$= 9\frac{t^3}{3} + 6 * \frac{t^{-1}}{-1} + \frac{t^{-5}}{-5} + c$$

$$= 3t^3 - \frac{6}{t} - \frac{1}{5t^5} + c$$

Ex Find $\int \sqrt{x^2 - x^4}\, dx$

$$= \int \sqrt{x^2(1-x^2)}\, dx$$

$$= \int x\sqrt{(1-x^2)}\, dx$$

$$= \int (1-x^2)^{\frac{1}{2}} x\, dx$$

$$= \int (1-x^2)^{\frac{1}{2}} x\, dx * \frac{-2}{-2}$$

$$= \frac{-1}{2} \int (1-x^2)^{\frac{1}{2}} (-2x)\, dx$$

$$= \frac{-1}{2} \frac{(1-x^2)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{-1}{2} * \frac{2}{3} (1-x^2)^{\frac{3}{2}} + C$$

$$= -\frac{1}{3}\sqrt{(1-x^2)^3} + C$$

تكامل الدوال المثلثية

1) $\int \sin x \, dx = -\cos x + c$.

2) $\int \cos x \, dx = \sin x + c$

3) $\int \sec^2 x \, dx = \tan x + c$

4) $\int \csc^2 x \, dx = -\cot x + c$

5) $\int \sec x \cdot \tan x \, dx = \sec x + c$

6) $\int \csc x \cdot \cot x \, dx = -\csc x + c$

في جميع الدوال المثلثية يجب توفر مشتقة الزاوية حتى نكامل

Ex. $\int \cos(7x+1) \, dx$

$$\int \cos(7x+1) \, dx * \frac{7}{7}$$

$$= \frac{1}{7} \int \cos(7x+1) \cdot 7 \, dx = \frac{1}{7} \sin(7x+1) + c$$

Ex. $\int x \cdot \sin(2x^2)\, dx$

$= \int \sin(2x^2) \cdot x * \frac{4}{4}\, dx$

$= \frac{1}{4} \int \sin(2x^2) \cdot 4x\, dx$

$= \frac{-1}{4} \cos(2x^2) + C$

Ex. $\int \cos^2 2x \cdot \sin 2x\, dx$

$= \int \cos^2 2x \cdot \sin 2x\, dx * \frac{-2}{-2}$

$= -\frac{1}{2} \int \cos^2 2x\, (-2 \sin 2x)\, dx$

$= -\frac{1}{2} * \frac{\cos^3 2x}{3} + C$

$= -\frac{1}{6} \cos^3 2x + C$

Ex. $\int \sec^3 x \tan x \, dx$ ✓

$= \int \sec^2 x \underbrace{\sec x \tan x}_{\text{مشتقة المقدار}} \, dx$

$= \frac{\sec^3 x}{3} + c$

Ex. $\int (1 - \sin^2 4x) \cos 4x \, dx$

$= \int (\cos 4x - \sin^2 4x \cos 4x) \, dx$

$= \left[\int \cos 4x \, dx - \int \sin^2 4x \cos 4x \, dx \right] * \frac{4}{4}$

$= \int \cos 4x \, dx * \frac{4}{4} - \int \sin^2 4x \cos 4x \, dx * \frac{4}{4}$

$= \frac{1}{4} \sin 4x - \frac{1}{4} * \frac{\sin^3 4x}{3} + c$

Ex. $\int \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx$ ✓

$\int \sin \sqrt{x} * \frac{1}{\sqrt{x}} \, dx \quad * \frac{2}{2}$

$2 \int \sin \sqrt{x} \frac{1}{2\sqrt{x}} \, dx = -2 \cos \sqrt{x} + c$