

$$\text{EX. } \int \frac{\cot^2 \sqrt{x}}{\sqrt{x}} dx.$$

$$\cot^2 \sqrt{x} = \csc^2 \sqrt{x} - 1$$

$$= \int \frac{\csc^2 \sqrt{x} - 1}{\sqrt{x}} dx$$

$$= \int \frac{\csc^2 \sqrt{x}}{\sqrt{x}} dx - \int \frac{1}{\sqrt{x}} dx$$

$$= \int \frac{\csc^2 \sqrt{x}}{\sqrt{x}} dx \times \frac{2}{2} - \int x^{-\frac{1}{2}} dx$$

$$= 2 \times (-\cot \sqrt{x}) - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= -2 \cot \sqrt{x} - 2x^{\frac{1}{2}} + C$$

$$= -2 \cot \sqrt{x} - 2\sqrt{x} + C$$

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$$1) \int \tan x dx = -\ln |\cos x| + c$$

$$2) \int \cot x dx = \ln |\sin x| + c$$

$$3) \int \sec x dx = \ln |\sec x + \tan x| + c$$

$$4) \int \csc x dx = -\ln |\csc x + \cot x| + c$$

$$\text{Ex. } \int \tan(3x+5) dx$$

$$\int \tan(3x+5) dx \neq \frac{3}{3}$$

$$= \frac{1}{3} \int \tan(3x+5) \cdot 3 dx$$

$$= -\frac{1}{3} \ln |\cos(3x+5)| + c$$

$$\text{Ex. } \int \frac{\cos X}{1 - \sin^2 X} dx$$

$$= \int \frac{\cos X}{\cos^2 X} dx = \int \frac{1}{\cos X} dx = \int \sec X dx$$

$$= \ln |\sec X + \tan X| + C$$

$$\text{Ex } \int \frac{\sin X + \cos X}{\cos X} dx$$

$$\int \frac{\sin X}{\cos X} dx + \int \frac{\cos X}{\cos X} dx = \int \tan X dx + \int dx$$

$$= -\ln |\cos X| + X + C$$

* كل قوس او دالة مرفوعة الى اس (-1) فعند
التكامل تُكامل باضافه (ln) شرط توفر مشتقة
داخله لقوس.

$$\text{Ex. } \int \frac{\cot x}{\ln(\sin x)} dx$$

$$= \int [\ln(\sin x)]^{-1} \cdot \frac{\cos x}{\sin x} dx$$

$$\ln(\ln \sin x) + c$$

$$\text{Ex. } \int \frac{1}{(\ln^3 x + 4)} \cdot \frac{dx}{x} \quad \checkmark$$

$$= \int (\ln^3 x + 4)^{-1} \frac{dx}{x} \neq \frac{3}{3}$$

$$= \frac{1}{3} \int (\ln^3 x + 4)^{-1} \frac{3}{x} dx$$

$$= \frac{1}{3} \ln(\ln^3 x + 4) + c$$

$$\text{Ex. } \int \frac{dx}{\sqrt{x^2 - 2x + 1}}$$

$$= \int \frac{dx}{\sqrt{(x-1)(x-1)}} = \int \frac{dx}{\sqrt{(x-1)^2}}$$

$$= \int \frac{dx}{x-1} = \int (x-1)^{-1} dx = \ln(x-1) + c$$

$$\text{Ex. } \int \frac{\cot(\ln x)}{x} dx \quad \checkmark$$

$$\int \cot(\ln x) * \frac{1}{x} dx$$

$$= \ln |\sin(\ln x)| + c$$

$$\text{Ex. } \int (\sin \theta - \cos \theta)^2 d\theta$$

$$= \int (\sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta) d\theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \int (1 - 2 \sin \theta \cos \theta) d\theta$$

$$= \int d\theta - \int 2 \sin \theta \cos \theta d\theta$$

$$= \int d\theta - 2 \int (\sin \theta)' (\cos \theta)' d\theta * \frac{-1}{-1}$$

$$= \theta + 2 \frac{\cos^2 \theta}{2} + c$$

$$= \theta + \cos^2 \theta + c$$

$$\text{Ex. } \int \frac{1}{x^2} \csc \frac{1}{x} \cdot \cot \frac{1}{x} dx$$

$$= \int \csc \frac{1}{x} \cdot \cot \frac{1}{x} * \frac{1}{x^2} dx * \frac{-1}{-1}$$

$$= - \int \csc \frac{1}{x} \cdot \cot \frac{1}{x} * \frac{-1}{x^2} dx$$

$$= -(-\csc \frac{1}{x}) + c = \csc \frac{1}{x} + c$$

integration of the e^x and a^x

$$1) \int e^{F(x)} * F'(x) = e^{F(x)} + C$$

$$2) \int a^{F(x)} * F'(x) dx = \frac{a^{F(x)}}{\ln a}$$

$$\text{Ex. } \int e^x dx \\ = e^x + C$$

$$\text{EX } \int e^{3x} dx$$

$$\int e^{3x} dx * \frac{3}{3} = \frac{1}{3} \int e^{3x} * 3 dx$$

$$= \frac{1}{3} e^{3x} + C$$

$$\text{Ex. } \int e^{\sin x} \cos x dx$$

$$= e^{\sin x} + C$$

$$\text{Ex. } \int 2^x dx$$

$$= \frac{2^x}{\ln 2} + C$$

$$\frac{d a^x}{dx} = a^x \cdot \ln a$$

$$\text{Ex. } \int x \cdot 3^{x^2} dx$$

$$= \int x \cdot 3^{x^2} dx \cdot \frac{2}{2}$$

$$= \frac{1}{2} \int 2x \cdot 3^{x^2} dx$$

$$= \frac{1}{2} \cdot \frac{3^{x^2}}{\ln 3} + C$$

$$\text{Ex. } \int e^{\ln 5^x} dx$$

$$= \int e^{\ln 5^x} dx = \int 5^x dx$$

$$= \frac{5^x}{\ln 5} + C$$

$$\text{Ex. } \int \frac{\ln^2 x}{x} dx$$

$$= \int \ln^2 x \times \frac{1}{x} dx$$

$$\frac{\ln^3 x}{3} + C$$

$$\text{Ex. } \int \sec^4 3x dx$$

$$= \int \sec^2 3x \times \sec^2 3x dx$$

$$= \int (\tan^2 3x + 1) \sec^2 3x dx$$

$$= \int \tan^2 3x \times \sec^2 3x dx \times \frac{1}{3} + \int \sec^2 3x dx \times \frac{1}{3}$$

$$= \frac{1}{3} \int \tan^2 3x \times 3 \sec^2 3x dx + \frac{1}{3} \int \sec^2 3x \times 3 dx$$

$$= \frac{1}{3} \times \frac{\tan^3 3x}{3} + \frac{1}{3} \tan 3x + C$$

$$\text{Ex. } \int 3^{\ln^2 x} \frac{dx}{x}$$

$$= \int 3^{2 \ln x} \frac{dx}{x} \quad * \frac{2}{2}$$

$$= \frac{1}{2} \int 3^{2 \ln x} * \frac{2}{x} dx$$

$$= \frac{1}{2} * \frac{3^{2 \ln x}}{\ln 3} + C$$

$$\text{Ex. } \int \frac{dx}{x \ln x}$$

$$= \int \frac{\frac{1}{x} dx}{\ln x} = \int \ln x^{-1} * \frac{1}{x} dx$$

$$= \ln(\ln x) + C$$

$$\text{Ex. } \int \frac{3^{\sin x} + \sec x}{\sec x} dx$$

$$= \int \frac{\sec x}{\sec x} dx + \int \frac{e^{\sin x}}{\sec x} dx$$

$$\sec x = \frac{1}{\cos x}$$

$$= \int \sec^2 x dx + \int \frac{e^{\sin x}}{\frac{1}{\cos x}} dx$$