$$= \int \sec^2 x \, dx + \int e^{\sin x} \cdot \cos x \, dx$$

$$= \tan x + e^{\sin x} + C$$

Ex. $\int \frac{e^{\sqrt{x}} - 1}{\sqrt{x}} \, dx$

$$= \int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx - \int \frac{1}{\sqrt{x}} \, dx$$

$$= \int e^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}} \, dx * \frac{2}{2} - \int x^{-\frac{1}{2}} \, dx$$

$$= 2 \int e^{\sqrt{x}} * \frac{1}{2\sqrt{x}} \, dx - \frac{x^{\frac{1}{2}}}{\frac{1}{2}}$$

$$= 2e^{\sqrt{x}} - 2x^{\frac{1}{2}} + C$$

* عند وجود التكامل بالصيغة التالية

$$\sqrt{1 \mp \sin x} \quad , \quad \sqrt{1 \mp \cos x}$$

فاننا نضرب بالعامل المرافق

Ex. $\int \sqrt{1+\sin y}\, . \, dy$

$$= \int \sqrt{1+\sin y} * \frac{\sqrt{1-\sin y}}{\sqrt{1-\sin y}}\, dy$$

$$= \int \frac{\sqrt{1-\sin^2 y}}{\sqrt{1-\sin y}}\, dy$$

$$= \int \frac{\sqrt{\cos^2 y}}{\sqrt{1-\sin y}}\, dy$$

$$= \int \cos y \, (1-\sin y)^{-\frac{1}{2}}\, dy * \frac{-1}{-1}$$

$$= -\int (1-\sin y)^{-\frac{1}{2}} (-\cos y)\, dy$$

$$= - * \frac{(1-\sin y)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= -2\,(1-\sin y)^{\frac{1}{2}} + c = -2\sqrt{1-\sin y} + c$$

Ex. $\int \sqrt{1-\cos 3x}\, dx$

$$\int \sqrt{1-\cos 3x} * \frac{\sqrt{1+\cos 3x}}{\sqrt{1+\cos 3x}}\, dx$$

$$= \int \frac{\sqrt{1-\cos^2 3x}}{\sqrt{1+\cos 3x}}\, dx = \int \frac{\sqrt{\sin^2 3x}}{\sqrt{1+\cos 3x}}\, dx$$

$$= \int (1+\cos 3x)^{-\frac{1}{2}} \cdot \sin 3x\, dx \quad \frac{-3}{-3}$$

$$= -\frac{1}{3} \int (1+\cos 3x)^{-\frac{1}{2}} (-3\sin 3x)\, dx$$

$$= -\frac{1}{3} * \frac{(1+\cos 3x)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= -\frac{2}{3}(1+\cos 3x)^{\frac{1}{2}} + c = -\frac{2}{3}\sqrt{1+\cos 3x} + c$$

* عند وجود التكامل بالصيغ التالية

$$\frac{x}{x \pm 1} \quad , \quad \frac{x}{\sqrt{x} \pm 1} \quad , \quad \frac{x}{1 \pm \sqrt{x}}$$

نقوم باضافة (1) وطرح واحد من البسط.

Ex. $\int \frac{x}{x-1} \, dx$

$$\int \frac{x+1-1}{x-1} \, dx = \int \frac{(x-1)+1}{x-1} \, dx$$

$$= \int \frac{x-1}{x-1} \, dx + \int \frac{1}{x-1} \, dx$$

$$= \int dx + \int (x-1)^{-1} \, dx$$

$$= x + \ln(x-1) + c$$

Ex. $\int \frac{dx}{\sin x \cos x}$

$$\int \frac{dx}{\sin x \cos x} * \frac{2}{2} = 2 \int \frac{dx}{2 \sin x \cos x} \, dx$$

$$= 2 \int \frac{dx}{\sin 2x} = 2 \int \csc 2x \, dx$$

$$= -2 \ln |\csc 2x + \cot 2x| + c$$

# Limits النهايات

## تعريفات (قواعد) حول النهايات

1) $\lim_{x \to a} c = c$ نهاية العدد الثابت = العدد الثابت نفسه

2) $\lim_{x \to c} [F(x) \pm g(x)] = \lim_{x \to c} F(x) \pm \lim_{x \to c} g(x)$

3) $\lim_{x \to c} AF(x) = A \lim_{x \to c} F(x)$ $A \to$ constant

4) $\lim_{x \to c} [F(x) \cdot g(x)] = \lim_{x \to c} F(x) \cdot \lim_{x \to c} g(x)$

5) $\lim_{x \to c} \frac{F(x)}{g(x)} = \frac{\lim_{x \to c} F(x)}{\lim_{x \to c} g(x)}$

6) $= \lim_{x \to \infty} \frac{1}{x} = \frac{1}{\infty} = 0$ , $\lim_{x \to 0} \frac{x}{\text{عدد}} = 0$

7) $\lim_{x \to 0} \frac{1}{x} = \frac{1}{0} = \infty$

8) $\lim_{x \to 0} \sin x = 0$ , $\lim_{x \to 0} \cos x = 1$

9) $\lim_{x \to 0} \frac{\sin x}{x} = 1$ , $\lim_{x \to 0} \frac{x}{\sin x} = 1$

10) $\lim_{x \to \infty} \frac{\sin x}{x} = 0$ , $\lim_{x \to \infty} \frac{\cos x}{x} = 0$

11) $\lim_{x\to 0} \frac{\tan ax}{ax} = 1$ , $\lim_{x\to 0} \tan x = 0$

12) $\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$ , $\lim_{x\to 0} \frac{x}{\tan x} = 1$

هناك حالات غير معروفة منها

$$\left[ \frac{0}{0}, 0^2, \infty \div \infty, \infty, \frac{\infty}{\infty} \right]$$

Ex. $\lim_{x\to 1} \frac{x^2+1}{2x-3} = \frac{1+1}{2-3} = \frac{2}{-1} = -2$

Ex. $\lim_{x\to -1} \frac{x^2-1}{x+1} = \frac{0}{0}$ ×

$$\lim_{x\to -1} \frac{(x-1)(x+1)}{(x+1)} = -2$$

Ex. $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

$$\lim_{x\to 4} \frac{\sqrt{x}-2}{(\sqrt{x}-2)(\sqrt{x}+2)} =$$

$$\lim_{x\to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$$

Ex. $\lim_{x \to 2} \frac{x^2 - 4}{x^3 - 8}$

$$\lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)}$$

$$\lim_{x \to 2} \frac{x+2}{x^2+2x+4} = \frac{2+2}{4+4+4} = \frac{4}{12} = \frac{1}{3}$$

Ex. $\lim_{x \to 2} \left( \frac{1}{x-2} - \frac{4}{x^2-4} \right)$

$$\lim_{x \to 2} \left( \frac{1}{x-2} - \frac{4}{(x-2)(x+2)} \right)$$

$$\lim_{x \to 2} \frac{1}{x-2} \left( 1 - \frac{4}{x+2} \right)$$

$$\lim_{x \to 2} \frac{1}{x-2} \left( \frac{x+2-4}{x+2} \right)$$

$$\lim_{x \to 2} \frac{1}{x-2} \left( \frac{x-2}{x+2} \right)$$

$$\lim_{x \to 2} \frac{1}{x+2} = \frac{1}{2+2} = \frac{1}{4}$$

Ex. $\lim_{y\to 0} 2y\cot 3y$

$$2\lim_{y\to 0} y\cot 3y = 2\lim_{y\to 0} y\frac{\cos 3y}{\sin 3y} * \frac{3}{3}$$

$$\frac{2}{3}\lim_{y\to 0}\frac{3y}{\sin 3y} \overset{=1}{\cdot} \lim_{y\to 0}\cos 3y$$

$$\frac{2}{3}\lim_{y\to 0}\cos 3y = \frac{2}{3} * 1 = \frac{2}{3}$$

Ex. $\lim_{x\to 0}\frac{x}{\tan 3x}$

$$\lim_{x\to 0}\frac{x}{\tan 3x} * \frac{3}{3}$$

$$\frac{1}{3}\lim_{x\to 0}\frac{3x}{\tan 3x} \overset{=1}{=} \frac{1}{3} * 1 = \frac{1}{3}$$

ملاحظة حتى وجود دالة كسرية في موضوع النهايات نعمل لإغلب نضرب في المرافق

Ex. $\lim_{x\to 2}\frac{4-x^2}{3-\sqrt{x^2+5}}$

$$\lim_{x\to 2}\frac{4-x^2}{3-\sqrt{x^2+5}} * \frac{3+\sqrt{x^2+5}}{3+\sqrt{x^2+5}}$$

$$\lim_{x \to 2} \frac{(4 - x^2)(3 + \sqrt{x^2 + 5})}{9 - (x^2 + 5)}$$

$$\lim_{x \to 2} \frac{(4 - x^2)(3 + \sqrt{x^2 + 5})}{(4 - x^2)}$$

$$\lim_{x \to 2} (3 + \sqrt{x^2 + 5}$$

$$= 3 + \sqrt{4 + 5} = 3 + \sqrt{9} = 3 + 3 = 6$$

Ex. $\lim_{x \to \frac{\pi}{2}} \sec x - \tan x$

$$\lim_{x \to \frac{\pi}{2}} \frac{1}{\cos x} - \frac{\sin x}{\cos x}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} * \frac{1 + \sin x}{1 + \sin x}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{1 - \sin^2 x}{\cos x (1 + \sin x)}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x}{\cos x (1 + \sin x)} = \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{1 + \sin x}$$

$$= \frac{0}{1 + 1} = \frac{0}{2} = 0$$

عندما تكون الدالة مقتربة من ∞ فاننا ننظر الى اعلى

اس لـ (x) ونقسم عليه جميع حدود الدالة ، لانتباه

للكميات غير المعروفة

Ex. $\lim_{x\to\infty} \dfrac{5x^3+2x^2+1}{x^3+3x}$

$$\lim_{x\to\infty} \frac{\frac{5x^3}{x^3}+\frac{2x^2}{x^3}+\frac{1}{x^3}}{\frac{x^3}{x^3}+\frac{3x}{x^3}}$$

$$= \frac{5+\frac{2}{\infty}+\frac{1}{\infty}}{1+\frac{3}{\infty}} = \frac{5}{1} = 5$$

Ex. $\lim_{x\to\infty} \dfrac{\sin x + x}{\cos x + x}$

$$\lim_{x\to\infty} \frac{\frac{\sin x}{x}+\frac{x}{x}}{\frac{\cos x}{x}+\frac{x}{x}}$$

$$= \frac{\lim_{x\to\infty} \frac{\sin x}{x}+\frac{x}{x}}{\lim_{x\to\infty} \frac{\cos x}{x}+\frac{x}{x}} = \frac{0+1}{0+1} = \frac{1}{1} = 1$$