

Ex. By using vectors show that the segment between $(2, -1, 4)$ and $(5, 2, 1)$ is parallel to that between $(-3, -7, 6)$ and $(3, -1, 0)$

$$\vec{V}_1 = \vec{P_1P_2} = P_2 - P_1 = (3, 3, -3)$$

$$\vec{V}_2 = \vec{P_3P_4} = P_4 - P_3 = (6, 6, -6)$$

$$\theta = \cos^{-1} \frac{\vec{V}_1 \cdot \vec{V}_2}{\|\vec{V}_1\| \cdot \|\vec{V}_2\|}$$

$$\theta = \cos^{-1} \frac{18 + 18 + 18}{\sqrt{9+9+9} * \sqrt{36+36+36}}$$

$$\theta = \cos^{-1} \frac{54}{54} = \cos^{-1} 1$$

$$\theta = 0$$

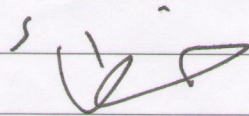
13 up to 10

$$\therefore \vec{V}_1 \parallel \vec{V}_2$$

ex. show that the vector normal to $\vec{A}$ and $\vec{B}$ is normal to $\vec{C}$

$$\vec{A} = 3\vec{i} - \vec{j} + 2\vec{k}, \quad \vec{B} = \vec{i} + 3\vec{j} - 2\vec{k}, \quad \vec{C} = 9\vec{i} + 2\vec{j} + 2\vec{k}$$

$$\vec{A} + \vec{B} = 4\vec{i} + 2\vec{j} + 0\vec{k}$$



$$\theta = \cos^{-1} \frac{\vec{A+B} \cdot \vec{C}}{\|\vec{A+B}\| \cdot \|\vec{C}\|}$$

$$\theta = \cos^{-1} \frac{36 + 4 + 0}{\sqrt{16 + 4} \cdot \sqrt{81 + 4 + 4}}$$

$$\theta = \cos^{-1} \frac{40}{42.17}$$

EX. $\vec{v}_1 (8, 6) \perp \vec{v}_2 (-3, 4)$

$$\vec{v}_1 \cdot \vec{v}_2 = (8 \times -3) + (6 \times 4)$$

$$= -24 + 24$$

$$= 0 \quad \therefore \vec{v}_1 \perp \vec{v}_2$$

EX. Find the angle between $v_1 (8, 6)$,
 $v_2 (-3, 4)$

$$\theta = \cos^{-1} \frac{\vec{v}_1 \cdot \vec{v}_2}{||\vec{v}_1|| ||\vec{v}_2||}$$

$$= \cos^{-1} \frac{(8 \times -3) + (6 \times 4)}{\sqrt{8^2 + 6^2} \times \sqrt{(-3)^2 + (4)^2}}$$

$$\theta = \theta_f = \cos^{-1} \frac{0}{50}$$

$$\theta = 90$$

Ex. By using vectors show that the segment
 $(2, -1, 4)$ and $(5, 2, 1)$ is parallel
 to that between $(-3, -7, 6)$ and $(3, -1, 0)$

$$\vec{P_1P_2} = \vec{V_1} = (5-2)\vec{i} + (2-(-1))\vec{j} + (1-4)\vec{k}$$

$$\vec{V_1} = 3\vec{i} + 3\vec{j} - 3\vec{k}$$

$$\vec{P_3P_4} = \vec{V_2} = (3-(-3))\vec{i} + (-1+7)\vec{j} + (0-6)\vec{k}$$

$$\vec{V_2} = 6\vec{i} + 6\vec{j} - 6\vec{k}$$

$$\vec{V_1} \times \vec{V_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 3 & -3 \\ 6 & 6 & -6 \end{vmatrix}$$

$$= (-18 + 18)\vec{i} - (-18 + 18)\vec{j} + (18 - 18)\vec{k}$$

$$= 0$$

Mean that is parallel

Ex. show that the vector normal to $\vec{A}$ and $\vec{B}$
 is normal to $\vec{C}$

$$\vec{A} = 3\vec{i} - \vec{j} + 2\vec{k}, \quad \vec{B} = \vec{i} + 3\vec{j} - 2\vec{k}$$

$$\vec{C} = 9\vec{i} + 2\vec{j} + 2\vec{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 2 \\ 1 & 3 & -2 \end{vmatrix} = (2-6)\vec{i} - (-6-2)\vec{j} + (9-1)\vec{k}$$

$$\vec{A} \times \vec{B} = -4\vec{i} + 8\vec{j} + 10\vec{k}$$

$$\vec{D} \cdot \vec{C} = 0 = (-36) + (16) + (20) = 0$$

mean that is normal

وهذا يعني أنهما متعامدان.

Ex. if $\vec{A} = 2\vec{i} - 2\vec{k}$ and $\vec{B} = 2\vec{i} - \vec{j} - 3\vec{k}$.

find a vector with length (6) and perpendicular to $\vec{A}$ and $\vec{B}$

$$\vec{C} = a\vec{i} + b\vec{j} + c\vec{k}$$

$$\sqrt{a^2 + b^2 + c^2} = 6$$

$$a^2 + b^2 + c^2 = 36 \quad \text{--- (1)}$$

$$\text{if } \vec{C} \perp \vec{A} \quad \therefore \vec{C} \cdot \vec{A} = 0$$

$$\vec{C} \cdot \vec{A} = a + 0 - 2c = 0$$

$$a = 2c \quad \text{--- (2)}$$

$$\text{if } \vec{C} \perp \vec{B} \quad \therefore \vec{C} \cdot \vec{B} = 0$$

$$\vec{C} \cdot \vec{B} = 2a - b - 3c = 0$$

$$b = 2a - 3c \quad \text{--- (3)}$$

sub (2) in (3)

$$b = c \quad \text{--- (4)}$$

sub (2), (4) in (1)

$$4c^2 + c^2 + c^2 = 36$$

$$6c^2 = 36$$

$$c^2 = \frac{36}{6} = 6$$

$$c = \sqrt{6}$$

$$\therefore b = \sqrt{6}, \quad a = 2\sqrt{6}$$

$$\vec{C} = 2\sqrt{6}i + \sqrt{6}j + \sqrt{6}k$$

Ex. Find unit vector perpendicular
to the plane determined by the points

$$A = (1, -1, 2), B = (2, 0, -1), C = (0, 2, 1)$$

$$\vec{AB} = 1\vec{i} + \vec{j} - 3\vec{k}$$

$$\vec{AC} = -\vec{i} + 3\vec{j} - \vec{k}$$

$$\vec{N} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix} = 8\vec{i} + 4\vec{j} + 4\vec{k}$$

$$u_n = \frac{\vec{N}}{|\vec{N}|} = \frac{8\vec{i} + 4\vec{j} + 4\vec{k}}{\sqrt{64 + 16 + 16}}$$

$$= \frac{8\vec{i} + 4\vec{j} + 4\vec{k}}{\sqrt{96}} = \frac{8\vec{i} + 4\vec{j} + 4\vec{k}}{\sqrt{4^2 \times 6}} = \frac{8\vec{i} + 4\vec{j} + 4\vec{k}}{4\sqrt{6}}$$

$$u_n = \frac{8\vec{i}}{4\sqrt{6}} + \frac{4\vec{j}}{4\sqrt{6}} + \frac{4\vec{k}}{4\sqrt{6}}$$

$$u_n = \frac{2\vec{i}}{\sqrt{6}} + \frac{\vec{j}}{\sqrt{6}} + \frac{\vec{k}}{\sqrt{6}}$$

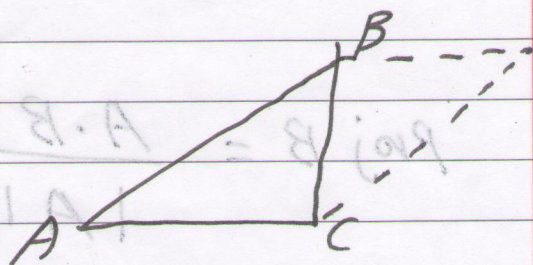
Ex. Find the area of triangle whose

vertices $\vec{A}(1, -1, 0)$, $\vec{B}(2, 1, -1)$ and

$\vec{C}(-1, 1, 2)$

$$\vec{AB} = \vec{i} + 2\vec{j} - \vec{k}$$

$$\vec{AC} = -2\vec{i} + 2\vec{j} + 2\vec{k}$$



$$A = \text{area Triangle} = \frac{1}{2} \square$$

$$= \frac{1}{2} |\vec{AB} \times \vec{AC}|$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ -2 & 2 & 2 \end{vmatrix} = 6\sqrt{2}$$

$$A = \frac{1}{2} \times 6\sqrt{2} = 3\sqrt{2}$$

Ex. Find θ between $\vec{v}_1 = -2\hat{i} + \hat{j} - 6\hat{k}$ 18

$$\vec{v}_2 = -6\hat{i} - \hat{j} - 5\hat{k}$$

$$\vec{v}_1 \cdot \vec{v}_2 = \|\vec{v}_1\| \|\vec{v}_2\| \cos \theta$$

$$\theta = \cos^{-1} \frac{\vec{v}_1 \cdot \vec{v}_2}{\|\vec{v}_1\| \|\vec{v}_2\|}$$

$$\begin{aligned} \vec{v}_1 \cdot \vec{v}_2 &= (-2 \times -6) + (1 \times -1) + (-6 \times -5) \\ &= 12 - 1 + 30 = 41 \end{aligned}$$

$$\|\vec{v}_1\| = \sqrt{4 + 1 + 36} = \sqrt{41}$$

$$\|\vec{v}_2\| = \sqrt{36 + 1 + 25} = \sqrt{62}$$

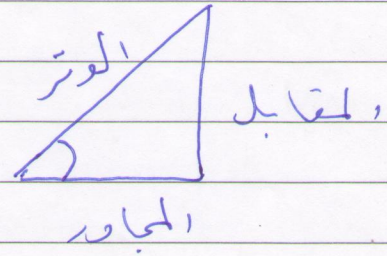
$$\theta = \cos^{-1} \frac{41}{\sqrt{41} \times \sqrt{62}}$$

$$\theta = \cos^{-1} \frac{\sqrt{41}}{\sqrt{62}}$$

$$\theta = (\quad)$$

الدوال، مثلثية

$$1) \sin X = \frac{\text{المقابل}}{\text{الوتر}}$$



$$2) \cos X = \frac{\text{المجاور}}{\text{الوتر}}$$

$$3) \tan X = \frac{\text{المقابل}}{\text{المجاور}} = \frac{\sin X}{\cos X}$$

$$4) \cot X = \frac{\text{المجاور}}{\text{المقابل}} = \frac{1}{\tan X}$$

$$5) \sec X = \frac{\text{الوتر}}{\text{المجاور}} = \frac{1}{\cos X}$$

$$6) \csc X = \frac{\text{الوتر}}{\text{المقابل}} = \frac{1}{\sin X}$$

$$7) \sin^2 X + \cos^2 X = 1$$

$$8) \sec^2 X = 1 + \tan^2 X$$

$$9) \csc^2 X = 1 + \cot^2 X$$