Q) Find the equation of all tangents to the circle $x^2+y^2-2x+8y=23$

a) at the point $(3,-10)$ on it

$$x^2+y^2-2x+8y-23=0 \quad \text{---} \quad (1)$$

$$2x+2yy'-2+8y'=0$$

$$(2y+8)y'=2-2x$$

$$y'=\frac{2-2x}{2y+8}=m$$

$$m=\frac{2-2*3}{2*(-10)+8}=\frac{-4}{-12}=\frac{1}{3}$$

$$\frac{y-y_1}{x-x_1}=m$$

$$\frac{1}{3}=\frac{y+10}{x-3}$$

$$x-3=3y+30$$

$$3y-x+33=0$$

b) having slope $=3$

$$y'=\frac{2-2x}{2y+8}$$

$$\frac{2-2x}{2y+8} = 3$$

$$6y + 24 = 2 - 2x$$

$$2x = -22 - 6y$$

$$x = -11 - 3y \quad \text{---} \quad (2)$$

بتعويض (2) في (1)

$$(-11-3y)^2 + y^2 - 2(-11-3y) + 8y - 23 = 0$$

$$121 + 66y + 9y^2 + y^2 + 22 + 6y + 8y - 23 = 0$$

$$10y^2 + 80y + 120 = 0 \qquad \text{بعد التبسيط بـ (10)}$$

$$y^2 + 8y + 12 = 0$$

$$(y+6)(y+2)$$

$$y + 6 = 0 \implies y = -6$$

$$\therefore \quad x = -11 - 3(-6)$$

$$x = -11 + 18 = 7$$

point $(7, -6)$, $m = 3$

$$\frac{y+6}{x-7} = 3 \qquad \text{معادلة المستقيم :}$$

$$3x - y - 27 = 0$$

$y+2=0 \Rightarrow y=-2$

$\therefore x=-11-3(-2)=-5$

$\therefore$ point $(-5,-2)$, $m=3$

$\frac{y+2}{x+5}=3$ معادلة المستقيم

$3x-y+13=0$

Q) Find the equation of the circle passing through the points $(0,0),(4,4),(2,6)$

$(x-h)^2+(y-k)^2=r^2$ معادلة الدائرة

(1) $h^2+k^2=r^2$ For $(0,0)$

(2) $(4-h)^2+(4-k)^2=r^2 \Rightarrow 16-8h+h^2+16-8k+k^2=r^2$ For $(4,4)$

$(2-h)^2+(6-k)^2=r^2 \Rightarrow 4-4h+h^2+36-12k+k^2=r^2$ For $(2,6)$ بالطرح

$16-4-8h+2h+16-36-8k+12k=0$

$4k-6h=8 \Rightarrow 2k-3h=4$

(3) $k=\frac{4+3h}{2}$

بتعويض (3) في (1) , (2)

$$h^2 + \left(\frac{4+3h}{2}\right)^2 = r^2$$

$$4h^2 + 16 + 24h + 9h^2 = 4r^2$$

$$4r^2 = 13h^2 + 24h + 16$$

$$r^2 = \frac{13h^2 + 24h + 16}{4} \quad \text{---} \quad (4)$$

نعوض (4) (3) في (2)

$$32 - 8h + h^2 - 8k + k^2 = r^2$$

$$32 - 8h + h^2 - 8\left(\frac{4+3h}{2}\right) + \left(\frac{4+3h}{2}\right) = \frac{13h^2 + 24h + 16}{4}$$

$$39h^2 + 152h - 16 = 0$$

$$(h+4)(39h-4) = 0$$

if $h+4=0 \Rightarrow h = -4$

$$\therefore k = \frac{4+3*(-4)}{2} = -4$$

نعوض قيم h, k في معادلة (4)

$$r^2 = 128$$

لذا تكون معادلة الدائرة

$$(x+4)^2 + (y+4)^2 = 128$$

or

$$39h - 4 = 0$$

$$h = \frac{4}{39} = 0.01 \approx 0$$

$$\therefore k = \frac{4+3(0)}{2} = \frac{4}{2} = 2$$

$$r^2 = \frac{4 \times 16}{4} = 16$$

وتكون معادلة الدائرة

$$(x-0)^2 + (y-2)^2 = 16$$

Q) Find the line that passes through the point (1, 2) and the point of intersection of the lines $x+2y=3$ and $2x-3y=-1$

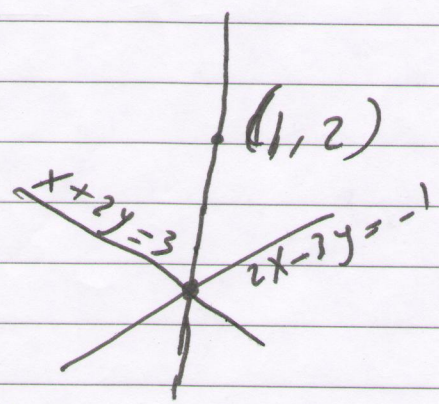
$x+2y=3$

$x = 3-2y$ — (1)

$2x-3y=-1$ — (2)

نعوض (1) في (2)

$2(3-2y)-3y=-1$



$$6 - 4y - 3y = -1$$

$$-7y = -7$$

$$y = 1 \quad \text{---} \quad (3)$$

نعوض (3) في (1)

$$x = 3 - 2 * 1 = 1$$

$$\therefore \text{Point } (1,1)$$

معادلة المستقيم $aX + bY = C$

من النقطة الأولى $(1,2)$

$$a + 2b = C \quad \text{---} \quad (4)$$

من النقطة الثانية $(1,1)$

$$a + b = C \quad \text{---} \quad (5)$$

من تعويض (4) في (5) ينتج

$$a = C$$

$$-b = 0$$

Q) Find the value of (k) such that the line $(k-1)x+(k+1)y-7=0$ is parallel to the line $3x+5y+7=0$

$3x+5y+7=0$

$3+5y'+0=0 \Rightarrow y'=\frac{-3}{5}$

$(k-1)x+(k+1)y=7$

$(k-1)+(k+1)y'=0$

$y'=\frac{-(k-1)}{(k+1)}$

$m_1=m_2=y'$

$\frac{-3}{5}=\frac{-(k-1)}{(k+1)}$

$5k-5=3k+3$

$k=4$

Q) Find the length of the tangent to the circle $x^2+y^2-2x+8y-23=0$ from $(8,-3)$

$$x^2+y^2-2x+8y-23=0$$

$$2x+2yy'-2+8y'=0$$

$$(2y+8)y'=2-2x$$

$$y'=\frac{2-2x}{2y+8}$$

$$m=y=\frac{2-2*8}{2*(-3)+8}$$

$$m=\frac{-14}{2}=-7$$

Q) A Line has the equation $\frac{x}{a}+\frac{y}{b}=1$ write the equation of the Line if it passes through the point $(1,2)$

a) $a=b$

$$\frac{x}{a}+\frac{y}{a}=1$$

$$x+y=a$$

$$1+2=a \Rightarrow a=3$$

$$\frac{x}{3}+\frac{y}{3}=1$$

$$x+y=3$$

a) $ab = 12$

$$a = \frac{12}{b}$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$bx + ay = ab$$

$$bx + \frac{12}{b} y = 12$$

$$b * 1 + \frac{12}{b} * 2 = 12$$

$$b^2 + 24 = 12b \implies b^2 - 12b + 24 = 0$$

~~$(b - 2\sqrt{3})(b - 4\sqrt{3}) = 0$~~

$(b - \quad)(b - \quad) = 0$

Q) write the equation of the circle passing through the points $(2,3), (-1,6)$ with center on the line $2x + 5y + 1 = 0$

$(x-h)^2 + (y-k)^2 = r^2$ معادلة الدائرة

لكون الدائرة تمر بهذه النقاط لذا نعوض النقاط في المعادلة

1) $-(2-h)^2+(3-k)^2=r^2 \Rightarrow 4-4h+h^2+9-6k+k^2=r^2$

2) $-(-1-h)^2+(6-k)^2=r^2 \Rightarrow -1-2h-h^2-36+12k-k^2=-r^2$

بالطرح

$$3-6h-27+6k=0$$

$$6k-6h=24 \Rightarrow k-h=4$$

$$k=4+h \quad \text{---} \quad (3)$$

نعوض (3) في معادلة المستقيم

$$2h+5(4+h)+1=0$$

$$2h+20+5h+1=0$$

$$h=\frac{-21}{7}=-3 \quad \text{---} \quad (4)$$

نعوض (4) في (3)

$$k=4+(-3)=1$$

$\therefore$ center $(-3,1)$

نعوض نقطة الأصل في المعادلة رقم (1)

$$4-4h+h^2+9-6k+k^2=r^2$$
$$4+14+9+9-6+1=r^2$$
$$r^2=29$$

لأنها تحتوي على البعد بين النقطتين

$\therefore$ معادلة الدائرة:

$$(x+3)^2+(y-1)^2=29$$