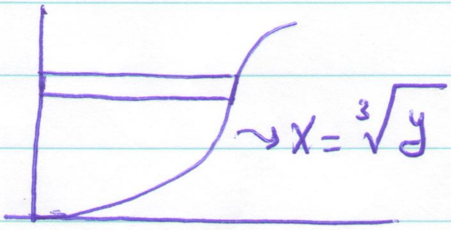
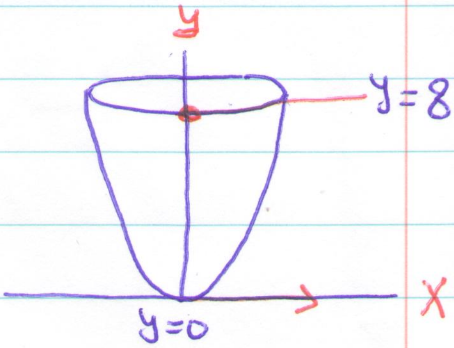
Ex. Find the volume of Soild obtained by the region bounded by $y = x^3$,





$$\text{Volume} = \int_0^8 \pi \left(\sqrt[3]{y}\right)^2 \cdot dy$$

$$= \pi \int_0^8 y^{\frac{3}{2}} \cdot dy$$

$$= \pi \left[\frac{3}{5} y^{\frac{5}{3}} \right]_0^8 = \frac{96\pi}{5}$$

$$\text{EX: } \overbrace{\int_{\frac{\pi}{2}}^{3\pi/2} \frac{d\theta}{\sin\theta} = \int_{\frac{\pi}{2}}^{3\pi/2} \csc\theta \cdot d\theta}$$

$$= -\ln\left|\csc\theta + \cot\theta\right| \Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$-\ln\left|\frac{1}{\sin(\frac{3\pi}{2})} + \frac{\cos(\frac{3\pi}{2})}{\sin(\frac{3\pi}{2})}\right| - (-\ln)\left|\frac{1}{\sin\frac{\pi}{2}} + \frac{\cos(\frac{\pi}{2})}{\sin(\frac{\pi}{2})}\right|$$

$$-\ln\left|\frac{1}{-1} + 0\right| + \ln\left|\frac{1}{1} + 0\right| = 0$$

DERIVATIVES المشتقات

Rules of derivatives قواعد في المشتقة

1) $\frac{d}{dx}(c) = 0$ $c \rightarrow$ constant

2) $\frac{d}{dx}(x)^{n} = n\,x^{n-1}$

3) $\frac{d}{dx}(c * F(x)) = c * \frac{d}{dx}F(x)$

4) $\frac{d}{dx}[F(x) \mp g(x)] = \frac{d}{dx}F(x) \mp \frac{d}{dx}g(x)$

5) $\frac{d}{dx}[F(x) \cdot g(x)] = F(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}F(x)$

6) $= \frac{d}{dx}\left[\frac{F(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}F(x) - F(x) * \frac{d}{dx}g(x)}{[g(x)]^{2}}$

7) $= \frac{d}{dx}[F(x)]^{n} = n * F(x)^{n-1} * \frac{d}{dx}F(x)$

8) $= \frac{d}{dx}(\sin u) = \cos u * \frac{du}{dx}$

9) $= \frac{d}{dx}(\cos u) = -\sin u * \frac{du}{dx}$

) $\frac{d}{dx}(\tan u) = \sec^2 u * \frac{du}{dx}$

11) $\frac{d}{dx}(\cot u) = -\csc^2 u * \frac{du}{dx}$

12) $\frac{d}{dx}(\sec u) = \sec u . \tan u . \frac{du}{dx}$

13) $\frac{d}{dx}(\csc u) = -\csc u * \cot u * \frac{du}{dx}$

14) $\frac{d}{dx}\sqrt{F(x)} = \frac{\frac{dF(x)}{dx}}{2 * \sqrt{F(x)}}$ مشتقة داخل الجذر

EX. Find $\frac{dy}{dx}$ if $y = 2X^4 + 5X^2 + X + 19$

$$\frac{dy}{dx} = 8X^3 + 5X2X + 1$$

$$= 8X^3 + 10X + 1$$

Ex. $y = X \cdot \cos^2(X^3+1)$

$$\frac{dy}{dx} = X.\ 2\cos(X^3+1) * (-\sin(X^3+1) * 3X^2$$
$$+ \cos^2(X^3+1) * 1$$

Ex. $y = \cot\left(\frac{X}{X+1}\right)$

$$\frac{dy}{dx} = -\csc^2\left(\frac{X}{X+1}\right) * \left[\frac{(X+1)*1 - X*1}{(X+1)^2}\right]$$

$$= -\csc^2\left(\frac{X}{X+1}\right) * \left[\frac{X+1-X}{(X+1)^2}\right]$$

$$= \frac{2}{(X+1)^2}\left[-\csc^2\left(\frac{X}{X+1}\right)\right]$$

EX. $y = \sin^2 x^2 - \cos^3 (3x+1)$

$$\frac{dy}{dx} = 2 \sin x^2 \cdot \cos x^2 * 2x - 3 \cos^2 (3x+1) (-\sin 3x+1) * 3$$

$$= 4x \sin x^2 \cdot \cos x^2 + 9 \cos^2 (3x) \cdot \sin 3x$$

EX $y = [(5-x)(4-2x)]^2$

$$\frac{dy}{dx} = 2[(5-x)(4-2x] * [(5-x) * (-2) + (4-2x) * (-1)]$$

Ex $y = (2x^3 - 3x^2 + 6x)^{-4}$

$$\frac{dy}{dx} = -4(2x^3 - 3x^2 + 6x)^{-5} * (6x^2 - 6x + 6)$$

Ex $y = \frac{12}{x} + \frac{4}{x^3} - \frac{3}{x^4}$

$$\frac{dy}{dx} = \frac{x*0 - 12*1}{x^2} + \frac{x^3*0 - 4*3x^2}{x^6} - \frac{x^4*0 - 3*4x^3}{x^8}$$

$$\frac{dy}{dx} = \frac{-12}{x^2} - \frac{12x^2}{x^6} + \frac{12x^3}{x^8}$$

$$= \frac{12}{x^5} - \frac{12}{x^2} - \frac{12}{x^4}$$

Ex $$y = \frac{(x^2+x)(x^2-x+1)}{x^4}$$

$$y = \frac{\cancel{x}(x+1)(x^2-x+1)}{x^{\cancel{4}3}} = \frac{(x+1)(x^2-x+1)}{x^3}$$

$$\frac{dy}{dx} = \frac{x^3 * [(x+1)(2x-1) + (x^2-x+1) * 1] - \boxed{}}{[x^3]^2}$$

$$(x+1)(x^2-x+1) * 3x^2$$

$$\frac{dy}{dx} = \frac{\overset{x}{\cancel{x^3}}[(x+1)(2x-1)+(x^2-x+1)] - 3\cancel{x^2}(x+1)(x^2-x+1)}{x^{\cancel{6}4}}$$

$$\frac{dy}{dx} = \frac{x[(x+1)(2x-1)+(x^2-x+1)] - 3(x+1)(x^2-x+1)}{x^4}$$

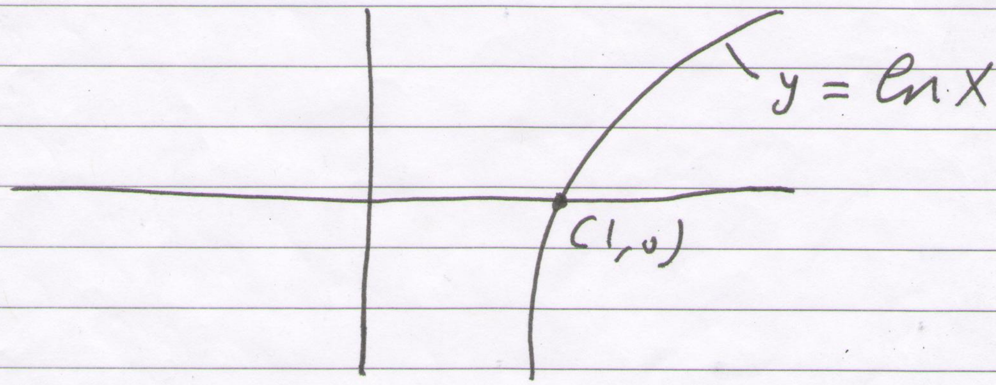
The natural logarithm function

$\ln$ دالة

* Rules of $\ln x$

1) $\ln(x \cdot y) = \ln x + \ln y$
2) $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$
3) $\ln\left(\frac{1}{x}\right) = \ln(1) - \ln(x) = -\ln(x)$
4 $\ln x^n = n \ln x$

Graphs of $\ln x$



لايجاد مشتقة دالة ($\ln$) نطبق القاعدة التالية

$$\frac{d}{dx}(\ln u) = \frac{1}{u} * \frac{du}{dx}$$

Ex. find $\frac{dy}{dx}$ for $y = \ln^4 x$

$y = 4 \ln x$

$\frac{dy}{dx} = 4 * \frac{1}{x} * 1 = \frac{4}{x}$

Ex. find $\frac{dy}{dx}$ for $y = \ln(x^2 + \cos x)$

$\frac{dy}{dx} = \frac{1}{x^2 + \cos x} * (2x - \sin x)$

Ex. find $\frac{dy}{dx}$ for $y = \ln(\cos x \cdot \sqrt{\sin x})$.

$y = \ln \cos x + \ln \sqrt{\sin x}$

$y = \ln \cos x + \ln(\sin x)^{\frac{1}{2}}$

$y = \ln \cos x + \frac{1}{2} \ln \sin x$

$\frac{dy}{dx} = \frac{1}{\cos x} * (-\sin x) + \frac{1}{2} * \frac{1}{\sin x} * \cos x$

$= \frac{-\sin x}{\cos x} + \frac{1}{2} \frac{\cos x}{\sin x}$

$= -\tan x + \frac{1}{2} \cot x$

$\frac{1}{\cos x \sqrt{\sin x}}$

$+ \frac{1 \cos^2 x}{2\sqrt{\sin x}}$

$\frac{-\sin x \sqrt{\sin x}}{\cos x \sqrt{\sin x}}$

$+ \frac{1}{2} \frac{\cos x}{\cos x \sin x}$

Ex. find $\frac{dy}{dx}$ For $\ln y^2 = \sin x^2$

$$= \frac{1}{y^2} * 2y \frac{dy}{dx} = \cos x^2 * 2x$$

$$\frac{2\frac{dy}{dx}}{y} = \cos^2 x * 2x$$

$$\cancel{2}\frac{dy}{dx} = \cancel{2}x \cdot y \cos^2 x$$

$$\frac{dy}{dx} = x \cdot y \cos^2 x$$

$$2\ln y = \sin x^2$$

$$\cancel{2}\frac{1}{y}\frac{dy}{dx} = \cos x^2 * \cancel{2}x$$

$$\frac{dy}{dx} = y x \cos x^2$$

* عند وجود متغير أس متغير او دالة بأس دالة وكما يلي

$$y = \begin{cases} x^{x} \\ F(x)^{F(x)} , \; x^{F(x)} \end{cases}$$

ففي هذه الحالة نأخذ ln للطرفين اذا كان الطرف الايمن سيكون من حد واحد.

Ex. find $\frac{dy}{dx}$ for $y = x^{\cos x}$

$$\ln y = \ln x^{\cos x}$$

$$\ln y = \cos x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \left[\cos x * \frac{1}{x} + \ln x * (-\sin x)\right]$$

$$\frac{dy}{dx} = y\left[\frac{\cos x}{x} - \ln x \cdot \sin x\right]$$

$$\frac{dy}{dx} = x^{\cos x}\left[\frac{\cos x}{x} - \ln x \cdot \sin x\right]$$

* اما اذا كان الطرف الايمن يتكون من اكثر من حد واحد

فاننا ندخل $e^{\ln f(x)}$ وبالتالي

Ex. find $\frac{dy}{dx}$ for $y = \sin x^{x} + e^{\ln x}$

$$y = \sin e^{\ln x^{x}} + e^{\ln x} = \sin e^{x \ln x} + x$$

$$\frac{dy}{dx} = \cos e^{x\ln x} * e^{x \ln x} \times \left[x * \frac{1}{x} + \ln x * 2\right] + 1$$

$$\frac{dy}{dx} = \cos e^{x\ln x} \cdot e^{x\ln x} + \cos e^{x\ln x} \cdot e^{x \ln x} \cdot \ln x + 1$$