

The Function (a^x)

Rules of (a^x) For any number of x and y

$$1) a^x \cdot a^y = a^{x+y} \quad \text{if } a > 0$$

$$2) \frac{a^x}{a^y} = a^{x-y}$$

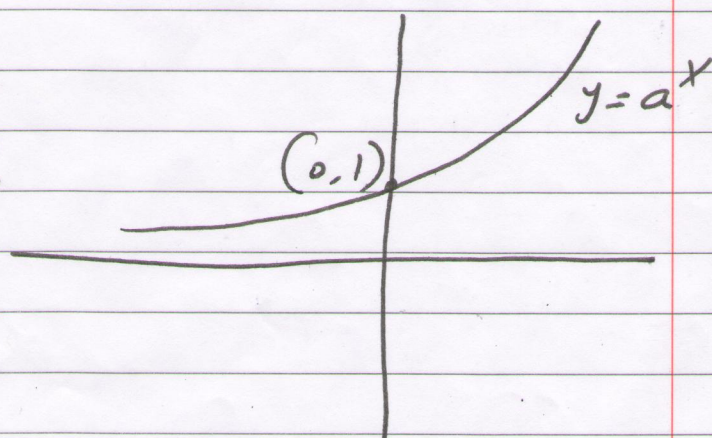
$$3) a^{-x} = \frac{1}{a^x}$$

$$4) (a^x)^y = a^{xy}$$

Graphs of (a^x)

$$D_{a^x} = \mathbb{R}$$

$$R_{a^x} = \{y : y > 0\}$$



قاعدة التفاضل دالة (a^x)

$$\frac{d}{dx} (a^{f(x)}) = a^{f(x)} * f'(x) * \ln a$$

Ex. Find $\frac{dy}{dx}$ for $y = 2^{5x}$

$$\frac{dy}{dx} = 2^{5x} \times 5 \times \ln 2$$

$$= 5 \cdot 2^{5x} \cdot \ln 2$$

Ex. Find $\frac{dy}{dx}$ for $y = 5^{\csc x^3}$

$$\frac{dy}{dx} = 5^{\csc x^3} \times -\csc x^3 \cdot \cot x^3 \cdot 3x^2 \times \ln 5$$

The Logarithmic Functions

الدوال اللوغاريتمية.

$$y = \log_a x \implies x = a^y$$

ex Find y if $y = \log_2 32$

$$\begin{matrix} y \\ 2 \end{matrix} = 32 \implies \begin{matrix} y \\ 2 \end{matrix} = 5$$

$$\therefore y = 5$$

properties of Logarithmic Functions

$$1) \log(x \cdot y) = \log x + \log y$$

$$2) \log\left(\frac{x}{y}\right) = \log x - \log y$$

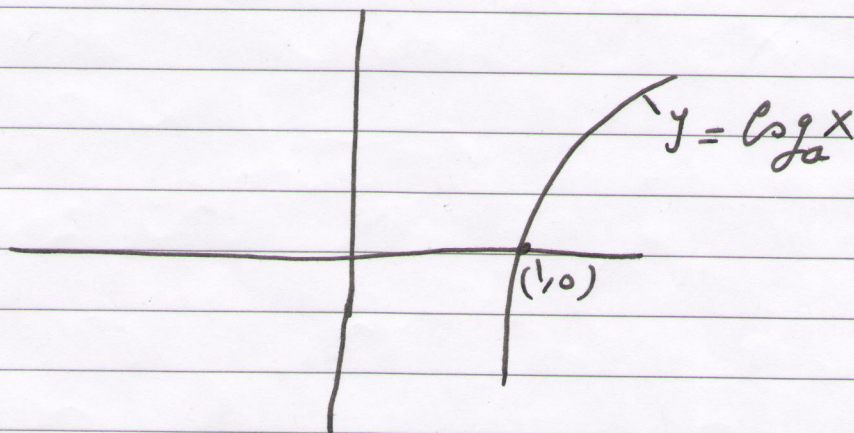
$$3) \log x^a = a \log x$$

$$4) \log_a x = \frac{\ln x}{\ln a}, \quad a \neq 1 \text{ and } x \text{ are positive}$$

$$5) \log_e f(x) = \frac{\ln(f(x))}{\ln e} = \ln f(x)$$

لأن e معكوس $\ln$ لذلك احذفها تلغي، لا مضى

$$6) \log_a a = 1, \quad \log_b b = 1$$

Graphs of $\log_a x$ 

لإيجاد مشتقة الدالة اللوغاريتمية نطبق قانون لوبي

$$\frac{d}{dx} (\log_a F(x)) = \frac{1}{\ln a} * \frac{1}{F(x)} * F'(x)$$

Ex. Find $\frac{dy}{dx}$ for $y = \log_4 \sin x$

$$\frac{dy}{dx} = \frac{1}{\ln 4} * \frac{1}{\sin x} * \cos x$$

$$\frac{dy}{dx} = \frac{\cos x}{\sin x} * \frac{1}{\ln 4}$$

$$= \frac{\cot x}{\ln 4}$$

* عند وجود جمع أو طرح في موضوع (log) وطلب حل، المعادلة

فيجب أن نرجمها إلى أصل ونحل، كل بما يقابله

Ex. solve for x ; $\log_{10} x^2 + \log_{10} x = 30$

$$\log_{10} (x^2 \cdot x) = 30$$

$$\log_{10} x^3 = 30$$

$$\log_{10} x^3 = 30 \log_{10} 10$$

$$\log_{10} x^3 = \log_{10} 10^{30}$$

$$\div \log_{10}$$

$$\therefore x^3 = 10^{30}$$

$$x = 10^{10}$$

Ex. solve for x ; $\log_{10} x^{\frac{3}{2}} - \log_{10} \sqrt{x} = 5$

$$\log_{10} \left(\frac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}} \right) = 5 \Rightarrow \log_{10} x = 5$$

$$\log_{10} x = \log_{10} 10^5$$

$$\therefore x = 10^5$$

Ex. solve for x ; $\log_{10}(1+x) = 3$

$$\log_{10}(1+x) = \log_{10} 10^3$$

$$\div \log_{10}$$

$$\therefore (1+x) = 10^3$$

$$1+x = 1000$$

$$x = 1000 - 1 = 999$$

Ex. solve for x ; $\log_5(5^{2x}) = 8$

$$\log_5(5^{2x}) = 8 \log_5 5$$

$$\log_5(5^{2x}) = \log_5(5)^8$$

$$\div \log_5$$

$$\therefore 5^{2x} = 5^8$$

$$2x = 8$$

$$x = \frac{8}{2} = 4$$

Exponential Functions (e^x)

* Rules of (e^x) for any number of (x) and (y) .

$$1) e^x \cdot e^y = e^{x+y}$$

$$2) \frac{e^x}{e^y} = e^{x-y}$$

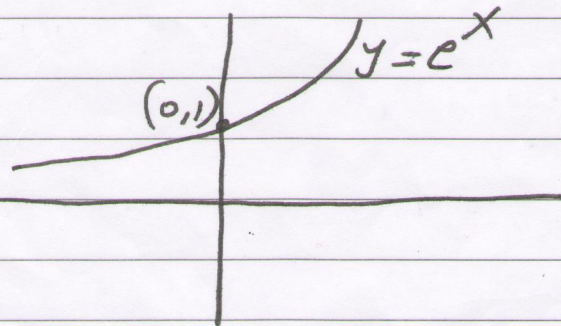
$$3) e^{-x} = \frac{1}{e^x}$$

$$4) \ln e^x = x, \quad e^{\ln x} = x$$

Graphs of (e^x)

$$D_{e^x} = \mathbb{R}$$

$$R_{e^x} = \{y : y > 0\}$$



Exponential Function قاعدة x ، e^x مشتاق

$$\frac{d}{dx} e^{f(x)} = e^{f(x)} \cdot f'(x)$$

Ex. Find $\frac{dy}{dx}$ for $y = e^{\sqrt{1+5x^2}}$

$$\frac{dy}{dx} = e^{\sqrt{1+5x^2}} * \frac{\frac{5}{10} X}{\frac{1}{2\sqrt{1+5x^2}}}$$

$$\frac{dy}{dx} = \frac{5x}{\sqrt{1+5x^2}} \cdot e^{\sqrt{1+5x^2}}$$

Ex. Find $\frac{dy}{dx}$ for $y = e^{\sin x}$

$$\frac{dy}{dx} = e^{\sin x} * \cos x$$

$$= \cos x \cdot e^{\sin x}$$

chain Rules

2-18

قانون السلسلة

١. عندما تكون الدالة y دالة في t ، وحين x دالة في t ،

Ex.
$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

Ex. Find $\frac{dy}{dx}$ if $y = t^5 + 1$ and $t = \sqrt{x}$

$$\frac{dy}{dt} = 5t^4$$

$$\frac{dt}{dx} = \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= 5t^4 \times \frac{1}{2\sqrt{x}}$$

$$\because t = \sqrt{x} \Rightarrow \text{~~2x~~ } x$$

$$\frac{dy}{dx} = 5t^4 \times \frac{1}{2t} = \frac{5}{2} t^3 = \frac{5}{2} (\sqrt{x})^3$$

$$\frac{dy}{dx} = \frac{5}{2} x^{\frac{3}{2}}$$

2. إذا كانت $y = y(t)$ و $x = x(t)$ ، لدينا $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx}$$

Ex. Find $\frac{dy}{dx}$ if $y = 1 - t^2$ and $x = 4t^2$

$$\frac{dy}{dt} = -2t, \quad \frac{dx}{dt} = 8t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2t}{8t} = -\frac{1}{4}$$

3. لإيجاد المشتقة الثانية $\left(\frac{d^2y}{dx^2}\right)$ للقاعدة رقم (2) وهي عندما

يعطى $[y = F(t), x = F(t)]$ فإننا نأخذ عليها من تطبيق

القانون التالي

$$\frac{d^2y}{dx^2} = \frac{\frac{dy}{dx}}{\frac{dx}{dt}}$$