Ex. Find $\frac{d^2y}{dx^2}$ if $y = t - 1$, $x = 2t^2$

$$\frac{dy}{dt} = 1 \quad , \quad \frac{dx}{dt} = 4t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1}{4t}$$

$$\frac{d^2\left(\frac{dy}{dx}\right)}{dt} = \frac{4t * 0 - 1 * 4}{(4t)^2} = \frac{-4}{16t^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{dt}} = \frac{\frac{-4}{16t^2}}{4t}$$

$$\frac{d^2y}{dx^2} = \frac{-1}{16t^3}$$

when $x = 2t^2 \Rightarrow t = \frac{\sqrt{x}}{\sqrt{2}}$, $t^3 = \frac{\left(x^{\frac{1}{2}}\right)^3}{\left(2^{\frac{1}{2}}\right)^3}$

$$\frac{d^2y}{dx^2} = \frac{-1}{16 * \frac{x^{\frac{3}{2}}}{2^{\frac{3}{2}}}} = \frac{-\sqrt{(2)^3}}{16\sqrt{x^3}}$$

$$= \frac{-2\sqrt{2}}{16\sqrt{x^3}} = \frac{-\sqrt{2}}{8\sqrt{x^3}}$$

Higher derivate المشتقات العليا

Ex. Find $\frac{d^4y}{dx^4}$, if $F(x) = 3x^4 + 2x - 6x^2 - 4$

$$\frac{dy}{dx} = 12x^3 + 2 - 12x$$

$$\frac{d^2y}{dx^2} = 36x^2 - 12$$

$$\frac{d^3y}{dx^3} = 72x$$

$$\frac{d^4y}{dx^4} = 72$$

Ex. Find the third derivative of the function $y = \sqrt{x^3}$

sol.

$$y = x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{3}{2} * \frac{1}{2} x^{-\frac{1}{2}} = \frac{3}{4} x^{-\frac{1}{2}}$$

$$\frac{d^3y}{dx^3} = \frac{3}{4} * \left(-\frac{1}{2}\right) x^{-\frac{3}{2}} = -\frac{3}{8} * x^{-\frac{3}{2}}$$

$$= -\frac{3}{8} * \frac{1}{\sqrt{x^3}}$$

Ex. Find $\frac{d^2y}{dx^2}$ for the function

$$y = x^2 \cdot \sin x$$

$$\frac{dy}{dx} = x^2 \cos x + \sin x * 2x$$

$$\frac{d^2y}{dx^2} = \left[x^2 * (-\sin x) + \cos x * 2x\right] + \left[\sin x * 2 + 2x * \cos x\right]$$

Ex. Fin $\frac{d^3y}{dx}$ For $y = \frac{1}{x} + \sqrt{x^3}$

$$y = \frac{1}{x} + x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = -\frac{1}{x^2} + \frac{3}{2} x^{\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = -\frac{x^2 * 0 - 1 * 2x}{x^4} + \frac{3}{2} * \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{2}{x^3} + \frac{3}{4} x^{-\frac{1}{2}}$$

$$\frac{d^3y}{dx^3} = \frac{x^3 * 0 - 2 * 3x^2}{(x^3)^2} + \frac{3}{4} * \left(-\frac{1}{2}\right) x^{-\frac{3}{2}}$$

$$= \frac{-6}{x^4} - \frac{3}{8\sqrt{x^3}}$$

implicit differentation

الاشتقاق الضمني.

نستخدم هذا النوع من الاشتقاق عند ملاحظة (x,y) في نفس الدالة.

Ex. Find $\frac{dy}{dx}$ For $xy + 2x - 5y = 2$

$$x * \frac{dy}{dx} + y * 1 + 2 - 5\frac{dy}{dx} = 0$$

ننقل الحدود التي تحتوي على $\frac{dy}{dx}$ أي لا يسار، والباقي في الطرف الايمن

$$x\frac{dy}{dx} - 5\frac{dy}{dx} = -y - 2$$

$$\frac{dy}{dx}(x-5) = \frac{-2-y}{}$$

$$\frac{dy}{dx} = \frac{-y-2}{x-5}$$

Ex Find $\frac{dy}{dx}$ For $x^2y^2 = x^2 + y^2$

$$x^2 * 2y\frac{dy}{dx} + y^2 * 2x = 2x + 2y\frac{dy}{dx}$$

$$2yx^2\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - 2xy^2$$

$$\frac{dy}{dx}(2yx^2 - 2y) = 2x - 2xy^2$$

$$\frac{dy}{dx} = \frac{2x - 2xy^2}{2yx^2 - 2y} = \frac{2(x - xy^2}{2(yx^2 - y)}$$

Ex. Find $\frac{dx}{dy}$ For $\sqrt{x \cdot y} + 1 = y$

$$\frac{x * \frac{dy}{dx} + y * 1}{2\sqrt{x \cdot y}} + 0 = \frac{dy}{dx}$$

$$\frac{x\frac{dy}{dx} + y}{2\sqrt{x \cdot y}} = \frac{dy}{dx}$$

$$x\frac{dy}{dx} + y = \frac{dy}{dx} * 2\sqrt{x \cdot y}$$

$$\frac{dy}{dx}2\sqrt{x \cdot y} - x\frac{dy}{dx} = y$$

$$\frac{dy}{dx} = \frac{y}{2\sqrt{x \cdot y} - x}$$

* لو يزاد مشتقة ثلاث حدود مضروبة مع بعضها نفرض ان حدين منها حد واحد ونحله بقاعدة ضرب دالتين

Ex. find $\frac{dy}{dx}$ for $y = x^3 \cdot e^{-2x} \cdot \cos 5x$

$y = \underbrace{x^3 \cdot e^{-2x}}_{\text{حد واحد}} \cdot \cos 5x$

$$\frac{dy}{dx} = x^3 \cdot e^{-2x} * -5\sin 5x + \cos 5x\left[x^3 * e^{-2x} * -2 + e^{-2x} * 3x^2\right]$$

$$\frac{dy}{dx} = -5x^3 \cdot e^{-2x} \sin 5x - 2\cos 5x \cdot x^3 \cdot e^{-2x} + 3\cos 5x \cdot x^2 \cdot e^{-2x}$$

حل المعادلات بدلالة ln

Ex. solve for x ; $\ln x^2 = 4$ ①

لكون $\ln e = 1$ بضرب طرفي المعادلة الذي ليس به ln

نضرب به بـ $\ln e$

$$\ln x^2 = 4 \ln e$$

$$\ln x^2 = \ln e^4$$

$$\therefore x^2 = e^4$$

$$x = e^2$$

② عند وجود جمع او طرح في موضوع (ln) وطلب حل المعادلة

(solve for x) فاننا نرجعه الى اصله

Ex. solve for x ; $\ln 4x - 3\ln x^2 = \ln 2$

$$\ln\left(\frac{4x}{(x^2)^3}\right) = \ln 2$$

$$\ln \frac{4x}{x^6} = \ln 2$$

$$\ln\left(\frac{4}{x^5}\right) = \ln 2 \Rightarrow \frac{4}{x^5} = 2$$

$$x^5 = 2 \Rightarrow x = \sqrt[5]{2}$$

3) اذا كان طرفي المعادلة رقم ذات اس x نأخذ ln على الطرفين

Ex. solve for x ; $4^{-x} = 3^{x+2}$

$$\ln 4^{-x} = \ln 3^{x+2}$$

$$-x \ln 4 = (x+2) \ln 3$$

$$-x \ln 4 = x \ln 3 + 2 \ln 3$$

$$-x \ln 4 - x \ln 3 = 2 \ln 3$$

$$-x (\ln 4 + \ln 3) = 2 \ln 3$$

$$-x \ln 4 * 3 = 2 \ln 3$$

$$-x \ln 12 = 2 \ln 3$$

$$x = \frac{-2 \ln 3}{\ln 12}$$

Ex. Solve for x; $e^{2x} + 2e^{2x} + 1 = 9$

$$e^{2x} + e^{2x} = 9 - 1$$

$$2e^{2x} = 8$$

$$e^{2x} = 4 \quad \Rightarrow \quad 2e^{x} = 4$$

$$e^{x} = 2$$

$$\ln e^{x} = \ln 2$$

$$x \ln e = \ln 2$$

$$x = \ln 2$$

Ex. $y = \tan^3\left(\frac{x+1}{x-1}\right)$

$$\frac{dy}{dx} = 3\tan^2\left(\frac{x+1}{x-1}\right) * \sec^2\left(\frac{x+1}{x-1}\right) * \left[\frac{(x-1)*1 - (x+1)*1}{(x-1)^2}\right]$$

$$= 3\tan^2\left(\frac{x+1}{x-1}\right) * \sec^2\left(\frac{x+1}{x-1}\right) * \left[\frac{-2}{(x-1)^2}\right]$$

$$= \frac{-6}{(x-1)^2}\tan^2\left(\frac{x+1}{x-1}\right) * \sec^2\left(\frac{x+1}{x-1}\right)$$

Ex. $y = \tan^{\frac{1}{2}}\sqrt{2x+7}$

$$\frac{dy}{dx} = \frac{1}{2}\tan^{-\frac{1}{2}}\sqrt{2x+7} \cdot \sec^2\sqrt{2x+7} * \frac{2}{2\sqrt{2x+7}}$$

$$\frac{dy}{dx} = \frac{\sec^2\sqrt{2x+7}}{2\tan^{\frac{1}{2}}\sqrt{2x+7} * \sqrt{2x+7}}$$