Chapter 1: The Atomic Models

1-1 Atomic Structure:

In order to develop the band structure of a solid, which will allow us to distinguish between an insulator, a semiconductor, and a metal. We begin with a review of the basic properties of matter leading to discrete electronic energy levels in atoms.

The atom is a basic unit of matter that consists of a dense, central nucleus surrounded by a cloud of negatively charged electrons. The atomic nucleus contains a mix of positively charged protons and electrically neutral neutrons (except in the case of hydrogen-1, which is the only stable nuclide with no neutrons). A group of atoms can remain bound to each other, forming a molecule. An atom containing an equal number of protons and electrons is electrically neutral; otherwise it has a positive or negative charge and is an ion.

1-2 Classical Atomic Models:

A number of atomic models have been proposed to describe the distribution of the positive and the negative charges in the atom, the following are the main models:

1-2-1 Thomson Atomic Model:

The plum pudding model of the atom by **J. J. Thomson**, who discovered the electron in 1897, was proposed in 1904 before the discovery of the atomic nucleus. In this model, the atom is composed of electrons (which Thomson still called "corpuscles", though **G. J. Stoney** had proposed that atoms of electricity be called electrons in 1894) surrounded by a soup of positive charge to balance the electrons' negative charges, like negatively-

charged "plums" surrounded by positively-charged "pudding". The electrons (as we know them today) were thought to be positioned throughout the atom, but with many structures possible for positioning multiple electrons, particularly rotating rings of electrons (Figure 1-1). Instead of a soup, the atom was also sometimes said to have had a "cloud" of positive charge. According to this model, there is nothing to prevent the union of the positive and the negative charges.

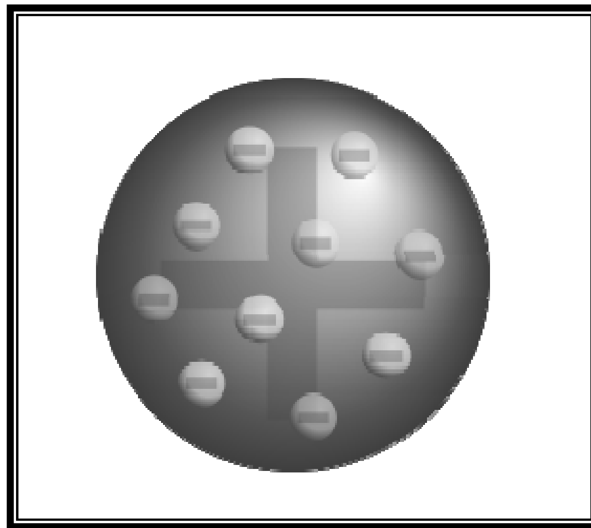


Figure 1-1: Thomson Atomic Model.

1-2-2 Rutherford Atomic Model:

The 1904 Thomson model was disproved by the 1909 gold foil experiment. The **Geiger–Marsden experiment** (also called the **Gold foil experiment** or the **Rutherford experiment**) was an experiment to probe the structure of the atom performed by Hans Geiger and Ernest Marsden in 1909 under the direction of Ernest Rutherford. The unexpected results of the experiment demonstrated for the first time the existence of the atomic nucleus, leading to the downfall of the plum pudding model of the atom, and the development of **the Rutherford (or planetary) model.**

1-2-2-1 The Gold foil experiment (Rutherford experiment):

A beam of accelerated alpha particles (α-particles), generated by the radioactive decay of radioactive elements (α-particles are Helium atoms (He) that lose two electrons and become charged with positive charge (+2e)), was directed normally onto a sheet of very thin gold foil. The gold foil was surrounded by a circular sheet of zinc sulfide (ZnS) which was used as a detector (Figure 1-2): the ZnS sheet would light up when hit with alpha particles.

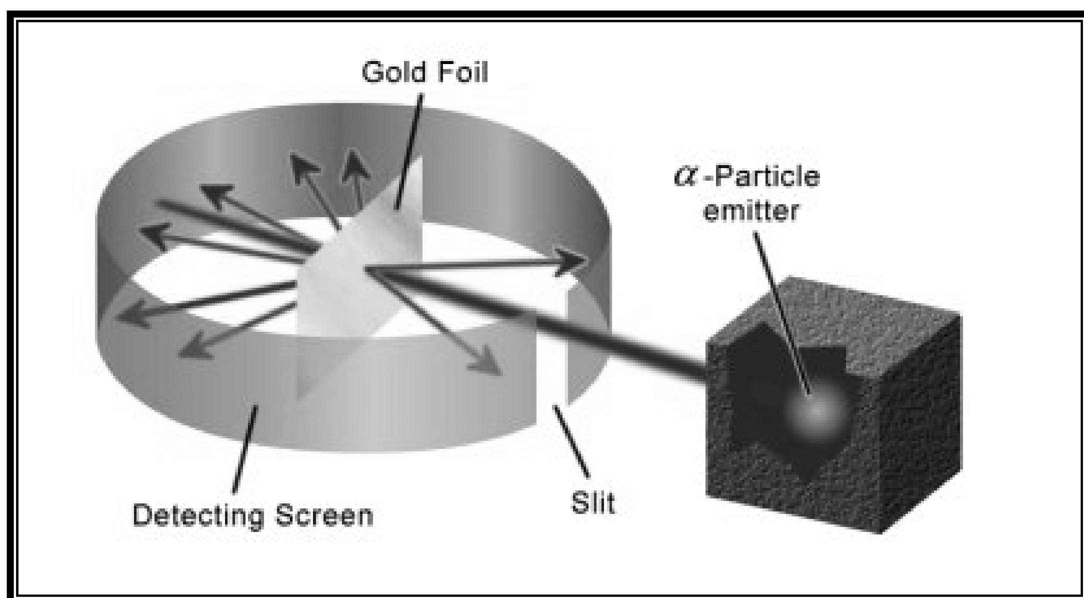


Figure 1-2: Rutherford experiment.

Under the prevailing plum pudding model, the alpha particles should all have been deflected by, at most, a few degrees; measuring the pattern of scattered particles was expected to provide information about the distribution of charge within the atom. However they observed that a very small percentage of particles were deflected through angles much larger than 90 degrees.

Rutherford interpreted the experimental results in a famous 1911 paper. He was able to definitively reject J.J. Thomson's plum pudding model of the atom, since none of Thomson's negative "corpuscles" (i.e. electrons) contained enough charge or mass to deflect alphas strongly, nor did the diffuse positive "pudding" or cloudlike positive charge, in which the electrons were embedded in the plum pudding model.

From the results of this experiment, Rutherford proposed the following model of an atom:

(i) An atom consists of a small and massive central core in which the entire positive charge and almost the whole mass of the atom are concentrated. This core is called the nucleus.

(ii) The nucleus occupies a very small space as compared to the size of the atom and outside of this nucleus, he proposed that the atom was mostly empty space.

(iii) The atom is surrounded by a suitable number of electros so that their total negative charge is equal to the total positive charge on the nucleus and the atom as a whole is electrically neutral.

(iv) The electrons revolve around the nucleus in various orbits just as planets revolve around the sun (Figure 1-3).

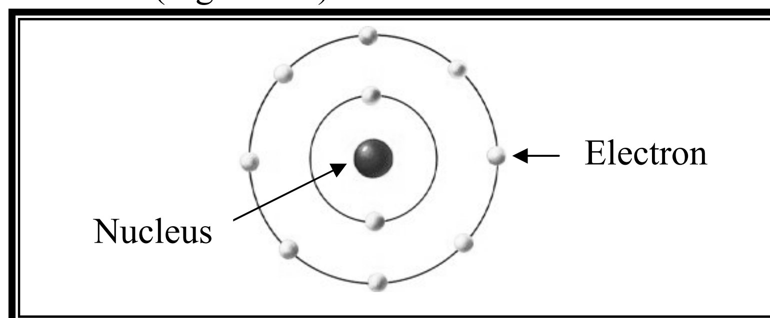


Figure 1-3: Rutherford (planetary) model.

1-2-2-2 Calculating the electron energy depending on Rutherford model:

Because the proton carries practically all the mass of the atom, it will remain substantially immobile, whereas the electron will move about it in a closed orbit. Rutherford assumed that the electron revolves around the nucleus in circular orbits and falls under the influence of two forces (Figure 1-4).

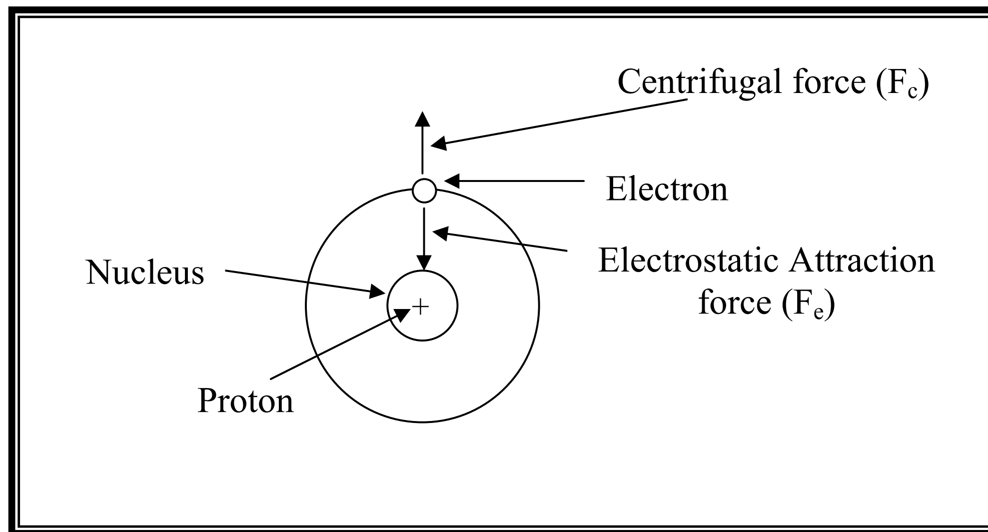


Figure 1-4: Forces on the electron.

1- Electrostatic Attraction force (F_e): This force is resulting from the attraction between the positive charge nucleus and the negative charge electron, this force pulls the electron toward the nucleus. This force can be calculated in the Hydrogen atom, which is the simplest atom consisting of one electron revolving around the nucleus that consists of one proton, using Coulomb's law that leads to the following relation:

$$F_e = \frac{e^2}{4\pi\varepsilon_o r^2} \quad \text{----------- Eq.(1-1)}$$

Where:

e : The electron charge = ($1.602*10^{-19}$ C).

ε_o : space permittivity = (8.85*10^{-12} F/m).

r : radius of the orbit (m).

2- Centrifugal force (F_c): This force is resulting from the circular motion of the electron, this force pushes the electron away from the nucleus, it can be calculated by Newton's second law of motion, as follows:

$$F_c = \frac{mv^2}{r} \quad \text{---------- Eq.(1-2)}$$

Where:

m : The electron mass = (9.109×10^{-31} kg).

v : The electron velocity (m/sec.)

In order to have a stable orbit, the two forces must be equal in magnitude and opposite in direction, hence:

$$F_e = F_c$$

$$\frac{e^2}{4\pi\varepsilon_o r^2} = \frac{mv^2}{r}$$

$$v = \frac{e}{\sqrt{4\pi\varepsilon_o rm}} \quad \text{---------- Eq.(1-3)}$$

The kinetic energy (E_k) of the electron equals:

$$E_k = \frac{mv^2}{2} \quad \text{---------- Eq.(1-4)}$$

Substituting (v) produces:

$$E_k = \frac{e^2}{8\pi\varepsilon_o r} \quad \text{---------- Eq.(1-5)}$$

E_k : Kinetic energy of the electron in joule (J).

In addition, the electron posses a potential energy (E_p) because it lies at a distance (r) from the nucleus:

$$E_p = \frac{-e^2}{4\pi\varepsilon_o r} \quad \text{---------- Eq.(1-6)}$$

E_p : Potential energy of the electron in joule (J).

The total energy equals the kinetic energy plus the potential energy:

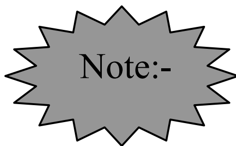
$$E_T = E_k + E_p$$
$$E_T = \frac{-e^2}{8\pi\varepsilon_o r} \quad \text{---------- Eq.(1-7)}$$

E_T : Total energy of the electron in joule (J).

This equation gives the desired relationship between the radius and the energy of the electron. This equation shows that the total energy of the electron is always negative. This result is necessary for the electron in order to remain linked to the atom because, If the total energy is greater than zero then the electron will have enough energy to separate from the nucleus .This expression also shows that the energy of the electron becomes smaller (more negative) as it approaches closer to the nucleus.

This model could not explain the stability of the atom because according to classical electromagnetic theory the electron revolving around the nucleus must continuously radiate energy in the form of electromagnetic radiation and hence it should fall into the nucleus. If the electron is radiating energy, its total energy must decrease by the amount of this emitted energy. As a result the radius r of the orbit must decrease, in accordance with

Eq. (1-7). Consequently, as the atom radiates energy, the electron must move in smaller and smaller orbits, eventually falling into the nucleus. Since the frequency of oscillation depends upon the radius of the circular orbit, the energy radiated would be of a gradually changing frequency. Such a conclusion, however, is incompatible with the sharply defined frequencies of spectral lines.



The energy of the electron is often measured using an energy unit called (**electron volt** (symbol (**eV**); also written **electronvolt**)). This unit is defined as the energy that the electron has when it falls in a (1 V) potential difference, therefore:

$$1eV = 1.602 \times 10^{-19}\ Joule$$

1-2-3 Bohr Atomic Model:

Although the electromagnetic theory agrees with a lot of experimental results, it does not agree with the presence of the atom in the stable state. The reason beyond the failure of the classical physics' laws in explaining the atomic structure is that it deals with things as either waves or particles without any duality. In order to understand the atomic structure, this dual character of wave and particle must be taking into account and this was done by **Bohr** when he proposed his atomic structure that combines between classical and modern physics.

In 1913, **Bohr** postulated two main assumptions:-

The first assumption: The electron revolves around the nucleus continuously and without radiating energy if its orbit contains full number of the electron **De Broglie** wavelengths. This assumption is based on the wave-particle duality of the electron because the electron wavelength is calculated in terms of the classical velocity of the electron as the following equation shows:

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \text{----------- Eq.(1-8)}$$

Where:

λ: De Broglie wavelength (m).

h: Planck's Constant $= 6.625 \times 10^{-34}\ (J.\sec.)$.

p: The electron momentum ($\frac{kg.m}{\sec.}$).

m: The electron mass.

v: The electron velocity.

Substituting v

$$v = \frac{e}{\sqrt{4\pi\varepsilon_o r m}}$$

Gives the following equation:

$$\lambda = \frac{h}{e}\sqrt{\frac{4\pi\varepsilon_o r}{m}} \quad \text{----------- Eq.(1-9)}$$

Since the orbit of the electron is a circular circumference with a radius (r_n) and equals ($2\pi r$), therefore the condition of a stable orbit is:

$$n\lambda = 2\pi r_n \qquad \text{Eq.(1-10)}$$

$n = 1,2,3,\ldots, \infty$

n : **The principal quantum number**.

r_n: The orbit radius that contains (n) wavelengths. Substituting (λ) in the above equation gives:

$$\frac{nh}{e}\sqrt{\frac{4\pi\varepsilon_o r_n}{m}} = 2\pi r_n$$

Hence, the radii of stable orbits of the electron are:

$$r_n = \frac{n^2 h^2 \varepsilon_o}{\pi m e^2} \qquad \text{Eq.(1-11)}$$

Substituting ($n = 1$) gives the smallest orbit (r_1) , in the Hydrogen atom, for example, it equals:

$$r_1 = 0.53 \times 10^{-10}\, m$$

From Eq. (1-11):

$$r_n = \frac{n^2 h^2 \varepsilon_o}{\pi m e^2} = a_o n^2$$
$$a_o = \frac{h^2 \varepsilon_o}{e^2 m \pi} \qquad \text{Eq.(1-12)}$$

a_o (which equals $0.53 \times 10^{-10}\, m$) is called **Bohr radius** or **the atomic radius unit** , it represents the radius of the electron orbit in **the ground state (n =1)** for the Hydrogen atom.

The second allowed orbit for the electron has a radius equals:

$$r_2 = 2.12 \times 10^{-10}\ m$$

All the radii between (r_1) and (r_2) are forbidden and any electron can not stay in an orbit if its radius is ranging between (r_1) and (r_2) because the circumference of the stable orbit equals either the wavelength (λ) of that electron or its multiples ($n\lambda$).

Also, Substituting (r_n) in the equation of the total energy ($E_T = \frac{-e^2}{8\pi\varepsilon_o r}$), gives:

$$E_{Tn} = \frac{-me^4}{8\varepsilon_o^2 n^2 h^2} \quad \text{Eq.(1-13)}$$

This equation shows that there is a definite energy for each orbit and that the energy of the electron is determined by the radius of the orbit (r_n) or by the principal quantum number (n). These energies (E_{Tn}) represent **the energy levels** of the atom.

The lowest energy state ($E_{T1}\ at\ n = 1$) is called **the normal, or ground level**, and the other stationary states of the atom are called **excited, radiating, critical, or resonance levels**.

As (n) increases, the energy gradually approaches to zero and at (n=∞) the energy becomes zero and the electron is detached from the nucleus. The idea of the separate energy levels of the atom interprets the origin of the atomic spectra which will be explained by the second assumption of Bohr.

The electron of the Hydrogen atom occupies the lowest energy level ($n = 1$) which is the closest to the nucleus. From the equation of (E_{Tn}) the energy of this level is (-13.6 eV), this energy is the required energy to

separate the electron from the nucleus of the Hydrogen atom and is called the **ionization potential**. The second energy level ($n = 2$) in the Hydrogen atom is (10.2 eV) from the ground state so in order to raise the electron to this level, the required energy must be supplied by an external source.

The second assumption: It is possible for the electron under the right conditions to transfer from one orbit to another and since these orbits have definite energies; therefore the transition of the electron requires either absorption of energy (if the transition is from a low energy level to a high energy level) or radiation of energy (if the transition is from a high energy level to a low energy level) according to the conservation of energy. As mentioned in the first assumption, the energies of the atomic levels are definite or **quantized**; therefore the absorption or radiation of energy will be quantized too, hence, the change in the energy will equal one quantum (hf) as the following equation shows:

$$E_i - E_f = hf \qquad \text{Eq.(1-14)}$$

Where:

E_i : The energy of the level from which the transition initiates.

E_f : The energy of the level at which the transition terminates.

h : Planck's Constant.

f : The frequency of the radiation emitted from or absorbed by the atom (Hz).

If ($E_i - E_f = +$), then there is a radiation of energy.

If ($E_i - E_f = -$), then there is an absorption of energy.

According to the second assumption of Bohr, the energy is radiated (emitted) from the atom only when the electron moves from a high level to a lower level, this energy equals the difference between the two levels' energies and it will be released once in the form of **photons** (discrete energy packets (i.e. not in a gradual form)). The frequency of the emitted photon equals:

$$f = \frac{E_i - E_f}{h}$$

Chapter 1: The Atomic Models (Continued)

1-2-3-1 Hydrogen spectral series:

A **line spectrum** contains only certain discreet wavelengths of light. Each element gives a characteristic line spectrum, the lines arising from electron transitions within the atom. With one electron and one proton, hydrogen is the simplest element and gives the simplest line spectrum.

The spectral lines of hydrogen correspond to particular jumps of the electron between energy levels. When an electron jumps from a higher energy level to a lower, a photon of a specific wavelength is emitted. The spectral lines are grouped into series according to n_f (the number of the level at which the transition terminates) as shown in figure (1-5).

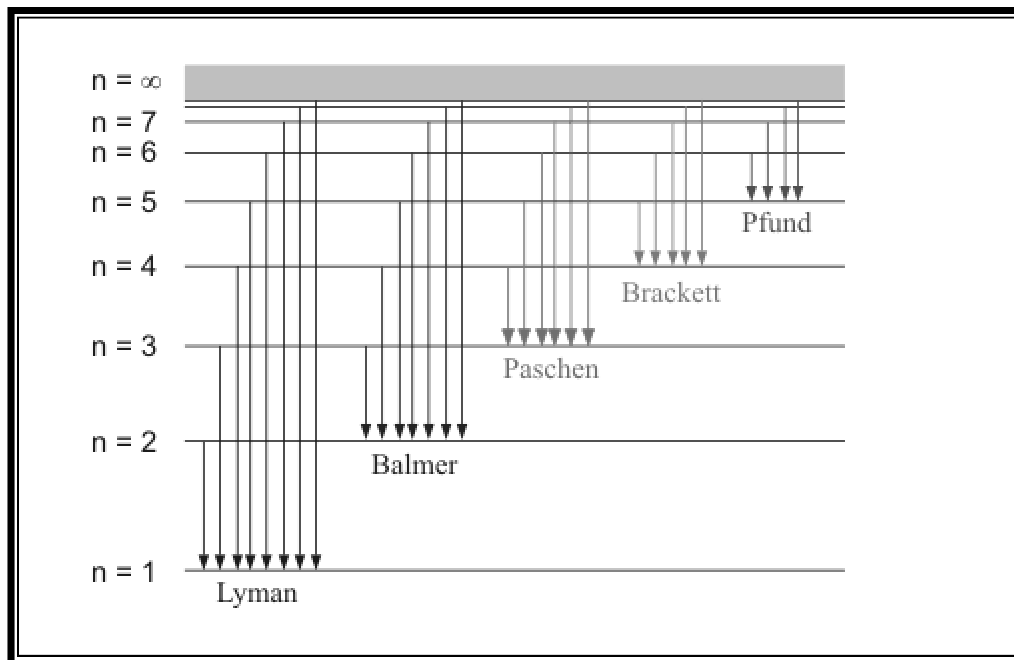


Figure 1-5: Hydrogen spectral series.

The emission spectrum of hydrogen is divided into a number of **spectral series**, with wavelengths given by the **Rydberg formula**.

Having:

$$E_i - E_f = hf$$

Substitute (E_i) and (E_f) by:

$$E_{Tn} = \frac{-me^4}{8\varepsilon_o^{\,2} n^2 h^2}$$

Substitute (f):

$$f = \frac{c}{\lambda} \quad \text{------------ Eq.(1-15)}$$

Hence:

$$\frac{-me^4}{8\varepsilon_o^{\,2} n_i^{\,2} h^2} + \frac{-me^4}{8\varepsilon_o^{\,2} n_f^{\,2} h^2} = h \times \frac{c}{\lambda}$$

$$\frac{1}{\lambda} = \frac{me^4}{8h^3 c \varepsilon_o^{\,2}} \left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}} \right)$$

The result is **Rydberg formula**:

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}} \right) \quad \text{------------ Eq.(1-16)}$$

Where: f : Frequency of emitted photon (Hz).

c : Speed of light. ($c = 2.9979 \times 10^8 \ \frac{m}{\text{sec.}}$).

λ : The wavelength (m).

$\frac{1}{\lambda}$: The wave number (m^{-1}, cm^{-1}).

R : Rydberg constant. ($R = 1.0974 \times 10^7 \ m^{-1}$).

n_f : The number of the level at which the transition terminates.

n_i : The number of the level from which the transition initiates.

The transitions of the electron in the hydrogen atom result in the following spectral series:

1- The **Lyman series:** The wavelengths in the ultra violet (UV) band, resulting from electrons dropping from higher energy levels into the (**n = 1**) orbit. ($n_f = 1 \quad n_i = 2,3,4,....\infty$).

2- The **Balmer series:** The wavelengths in the visible spectrum, resulting from electrons falling from higher energy levels into the (**n = 2**) orbit. ($n_f = 2 \quad n_i = 3,4,5,....\infty$).

3- The **Paschen series:** The wavelengths in the infrared spectrum, resulting from electrons falling from higher energy levels into the (**n = 3**) orbit. ($n_f = 3 \quad n_i = 4,5,6,....\infty$).

4- The **Brackett series:** The wavelengths in the infrared spectrum, resulting from electrons falling from higher energy levels into the (**n = 4**) orbit. ($n_f = 4 \quad n_i = 5,6,7,....\infty$).

5- The **Pfund series:** The wavelengths in the infrared spectrum, resulting from electrons falling from higher energy levels into the (**n = 5**) orbit. ($n_f = 5 \quad n_i = 6,7,8,....\infty$).

1-2-4 The Wave-Mechanics Model:

Although Bohr theory was consistent well with the atoms containing one electron such as hydrogen as well as ions of hydrogen, it was found that this theory can not be applied to the spectrum of complex atoms consisting of two electrons or more, it was also found that some spectral lines decompose into close multiple lines known as (fine structure). This fine structure can not be explained by Bohr theory which assumed the existence of only one orbit for each quantum number (n) while the fine structure refers

to the existence of several levels with slightly different energy for each quantum number (n).

Schrodinger has been able to develop a more comprehensive theory during the period 1925-1926 under the title (**The Wave-Mechanics**). Schrodinger has been able to formulate a differential wave equation to describe the behavior of the electron when it falls under the effect of nucleus attraction. When solving this equation for the electron of the hydrogen atom, **the quantum number (n)** appears automatically as one of the results of solving Schrodinger equation. It has been found that the electrons that have the same principal quantum number (n) clustered around the nucleus in an **atomic shell**.

n	1	2	3	4	5	…
atomic shell	K	L	M	N	O	

The maximum number of electrons that the atomic shell can contain is ($2n^2$).

This atomic shell contains different energy levels, which introduced a new concept that is the presence of the **atomic subshell** because the electron possesses another quantum number which is **the orbital quantum number** (ℓ) that takes the values from zero to (n-1).

ℓ	0	1	2	3	…
atomic subshell	s	p	d	f	

The maximum number of electrons that the atomic subshell can contain is ($2(2\ell+1)$). The orbital quantum number is associated with the angular momentum of the electron.

The third quantum number is called **the magnetic quantum number** (m_ℓ) which is used to describe the orientation in space of a particular orbit. The allowed values of (m_ℓ) are extended between ($+\ell$) and ($-\ell$) passing through zero.

Although the three quantum numbers have appeared spontaneously when solving Schrodinger equation for the hydrogen atom, but the theory remains deficient in giving all the characteristics of the atom without the introduction of a fourth quantum number which is the **Spin quantum number** (m_s) that takes the values ($+\frac{1}{2}$ "spin up") or ($-\frac{1}{2}$ "spin down").

1-2-5 The Exclusion Principle

A law introduced by **Pauli** in 1925. He stated that no two electrons in the atom can have the same set of four quantum numbers (n, ℓ , m_ℓ , and m_s). This statement that no two electrons may occupy the same quantum state is known as **the Pauli exclusion principle**.

Example (1-1): Find the electron velocity in the Hydrogen atom if its total energy equals (-13.6 eV):

Sol:-

Example (1-2): The Hydrogen atom electron moves from the second energy level (n=2) to the first energy level (n=1). Calculate the released energy and the wavelength caused by this movement.

Sol:-

Chapter 2: The Electric field

2-1 The electric field intensity:

The region in which the impact of the electric force on any charged object enters this region is appeared is called **the electric field** and when there are several charged objects in the region there will be several forces and the resultant of the forces is the vector summation of these forces.

All charged objects create an electric field that extends outward into the space that surrounds it. The charge alters that space, causing any other charged object that enters the space to be affected by this field. The strength of the electric field is dependent upon how charged the object creating the field is and upon the distance of separation from the charged object.

The electric field intensity (Electric field strength) (E) is a vector quantity; it has both magnitude and direction. The magnitude of the electric field is simply defined as the force per charge and it is measured in (N/C).

$$E = \frac{F}{q} \qquad \text{------------ Eq.(2-1)}$$

F : Force (N). q : Charge (C).

The electric field is represented by force lines (field lines) directed from the positively charged object to the negatively charged object as shown in figure (2-1). If there is a positive charge in the electric field region, the direction of the force exerted on the charge is the same as the direction of the electric field but if the charge is negative, then the direction of the force is opposite to the direction of the electric field

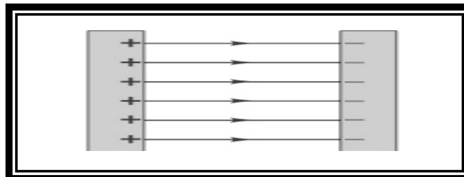


Figure (2-1): The electric field between two plates.

2-2 Coulomb's law:

The attraction or repulsion force between two charges is directly proportional to the product of the two charges values and inversely proportional to the square of distance between them.

$$F = k\frac{q_1 q_2}{r^2} \quad \text{------------ Eq.(2-2)}$$

$q_1,\ q_2$: Charges (C).

r: The distance between the two charges (m).

k: The electric constant.

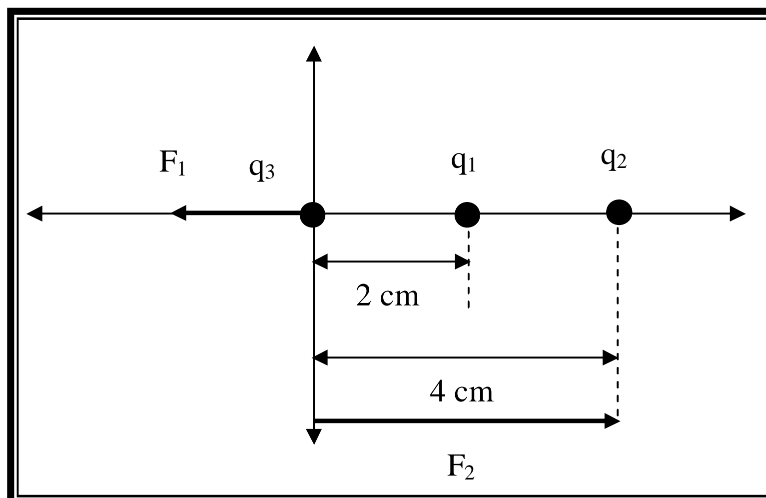
$$k = \frac{1}{4\pi\varepsilon_o} \quad \text{------------ Eq.(2-3)}$$

ε_o: Space permittivity= ($8.85\times10^{-12}\ \frac{C^2}{N.m^2}$).

$$\therefore k = 8.9\times10^{9}\ \frac{N.m^2}{C^2}$$

Example (2-1): Two charges on the positive side of the x-axis, the first charge ($q_1 = 2\times10^{-9} C$) is at (2 cm) from the origin, the second charge ($q_2 = -3\times10^{-9} C$) is at (4 cm) from the origin. Calculate the total force exerted by these two charges on a third one ($q_3 = 5\times10^{-9} C$) at the origin.

Sol:-



2-3 The motion of charged object through the electric field:

The equation of motion for a charged object in an electric field is:

$$F = qE = ma$$
$$\therefore a = \frac{q}{m}E \qquad \text{Eq.(2-4)}$$

m: Mass of the charged object (kg).

a: Acceleration of the charged object (m/s^2).

The Acceleration of any charged object depends on the ratio ($\frac{q}{m}$) which varies according to the charged objects. The Acceleration of a charged object is uniform when the electric field is uniform.

The laws of linear motion are applied to the motion of a charged object in the electric field:

$$v = v_o + at \qquad \text{Eq.(2-5)}$$

$$v^2 = v_o^2 + 2ax \qquad \text{Eq.(2-6)}$$

$$x = v_o t + \frac{1}{2} a t^2$$ ------------ Eq.(2-7)

v_o : initial velocity of the charged object (m/sec.).

v : final velocity of the charged object (m/sec.).

t : time for the charged object's motion (sec.).

x : The displacement that the charged object travels through the electric field (m).

Chapter 2: The Electric field (Continued)

2-4 The Electric Potential:

The potential (V) (volts) of point (B) with respect to point (A) is the work done against the field in taking a unit positive charge from (A) to (B). This definition is valid for a three-dimensional field. For a one-dimensional problem with (A) at (x_o) and (B) at an arbitrary distance (x), it follows that:

$$V = -\int_{x_o}^{x} E\,dx \qquad \text{Eq. (2-8)}$$

Where (E) now represents the X component of the field. Differentiating Eq. (2-8) gives:

$$E = -\frac{dV}{dx} \qquad \text{Eq. (2-9)}$$

The minus sign shows that the electric field is directed from the region of higher potential to the region of lower potential. In three dimensions, the electric field equals the negative gradient of the potential.

The potential energy (E_p) (J) equals the potential (V) multiplied by the charge (q) under consideration, or:

$$E_p = qV \qquad \text{Eq. (2-10)}$$

The total energy (E_T) for a charged object whose mass is (m) and its charge is (q) equals its kinetic energy (E_k) plus its potential energy (E_p):

$$E_T = E_k + E_p$$
$$E_T = \frac{mv^2}{2} + qV \qquad \text{Eq. (2-11)}$$

When the charged object moves from point (P_1) whose potential is (V_1) to point (P_2) whose potential is (V_2), the energy is conserved:

$$\begin{aligned} &Energy\ at\ P_1 = Energy\ at\ P_2 \\ &\frac{1}{2}mv_1^{\ 2} + qV_1 = \frac{1}{2}mv_2^{\ 2} + qV_2 \\ &q(V_1 - V_2) = \frac{1}{2}mv_2^{\ 2} - \frac{1}{2}mv_1^{\ 2} \end{aligned} \quad \text{Eq. (2-12)}$$

If the motion starts from rest from point (P_1), Eq. (2-12) becomes:

$$qV = \frac{1}{2}mv^2 \quad \text{Eq. (2-13)}$$

Example (2-2): Uniform electric field region between two parallel bars (0.3 cm) apart, if a potential difference of (400 v) is applied between them and an electron moves from rest from the negative bar, Calculate:

1) The electron velocity when it reaches the positive bar. 2) The required time. 3) The kinetic energy. 4) The exerted force on the electron.

Sol.:-

2-5 The magnetic force affecting an electric charge moving in a magnetic field:

If a positive electric charge (q) moved with a velocity ($\vec{v}$) in a uniform magnetic field ($\vec{B}$), this charge will be affected by a magnetic force ($\overrightarrow{F_m}$) perpendicular to the plane containing ($\vec{v}$) and ($\vec{B}$).

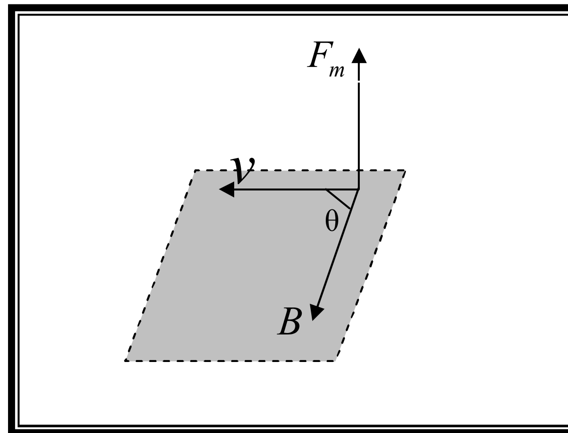


Figure (2-2): Lorentz force.

This force is called **(Lorentz force)** and it is expressed by the following vector relation:

$$\overrightarrow{F_m} = q\,\vec{v} \times \vec{B} \qquad \text{Eq. (2-14)}$$

The magnitude of this force is given by:

$$F_m = qvB\sin\theta \qquad \text{Eq. (2-15)}$$

Where:

F_m: the magnetic force (N), q: charge (C), v: velocity ($\frac{m}{sec.}$), B: magnetic field (in tesla (T), 1 tesla = 10,000 gauss), θ: The angle between the velocity vector ($\vec{v}$) and the magnetic field vector ($\vec{B}$).

The direction of the force is given by the **right hand rule** as shown in figure (2-3) where a positive charge was considered. For a negative charge, the force is in the opposite direction.

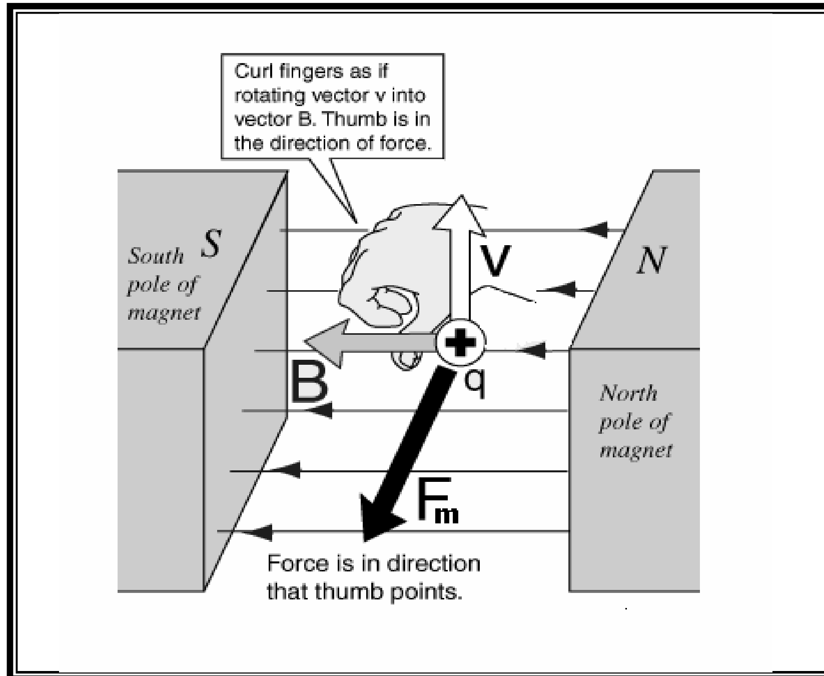


Figure (2-3): Direction of the magnetic force.

If the angle between the velocity vector ($\vec{v}$) and the magnetic field vector ($\vec{B}$) equals (90°), the electric charge will move in a circular path because of the effect of the magnetic force which represents the centripetal force (F_c) required to maintain this circular path.

$$F_m = F_c$$

$$qvB = \frac{mv^2}{r}$$

$$\therefore r = \frac{mv}{qB} \qquad \text{Eq. (2-16)}$$

Where: r: radius of the path (m).

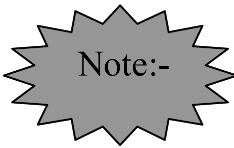
The angular velocity (ω) is given by:

$$v = \omega . r$$

$$\omega = \frac{v}{r} = \frac{v}{\frac{mv}{qB}}$$

$$\therefore \omega = \frac{qB}{m} \quad \text{------------ Eq. (2-17)}$$

ω : The angular velocity ($\frac{rad.}{\sec.}$)



When a charged object moves in a region containing both electric field and magnetic field, it will be affected by both electric force and magnetic force, the total force on the object will be:

$$\vec{F} = \underbrace{q\vec{E}}_{\text{Electric force}} + \underbrace{q\vec{v} \times \vec{B}}_{\text{Magnetic force}} \quad \text{------------ Eq. (2-18)}$$

Notes:-

Example (2-3): An electron enters a magnetic field (0.2 T) with an angle (30°) with respect to the field. If the radius of the motion is (2 m), determine the kinetic energy of the electron in (e.v.).

Sol.:-

Problems

(Atomic models+Electric field)

Q1) For the Hydrogen atom in the ground state, determine:

1) The kinetic energy of the electron.
2) The potential energy of the electron.
3) The total energy of the electron.
4) The required energy to release (remove) the electron from the atom.
5) The required wavelength to release (remove) the electron from the atom.

Q2) Calculate the wavelength of an electron which starts its motion from rest through a potential difference of (500 v), then calculate the wavelength of a proton under the same conditions.

Note: mass of proton= $1.673*10^{-27}$ Kg.

Q3) A photon with a wavelength (600 nm), find:

1) The photon frequency.
2) The photon momentum.
3) The photon energy in (e.v).

Q4) Determine the centripetal force on the Hydrogen atom electron with a wavelength ($3.3* 10^{-10}$ m).

Q5) Calculate the frequency and the wavelength of the electron in Balmer series from the fourth level.

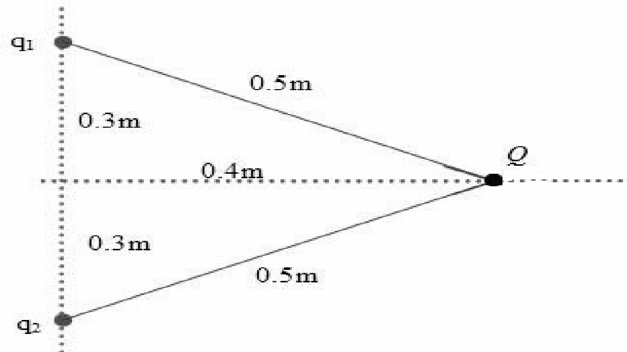
Q6) The electron of the Hydrogen atom moves from a certain level to the first level with a frequency of ($2.93*10^{15}$ Hz). Determine the number of the initial level.

Q7) Determine the maximum frequency and the minimum frequency in:

1) Lyman series.
2) Balmer series.
3) Brackett series.

Q8) Find a relation between the potential difference and the wavelength of an electron which starts its motion from rest.

Q9) In the following figure, two charges: ($q_1 = 2 \times 10^{-6}\ C$) and ($q_2 = 3 \times 10^{-6}\ C$). Find the magnitude and the direction of the resultant force exerted by the two charges on a third one ($Q = 4 \times 10^{-6}\ C$).



Q10) A proton moves in a magnetic field (1.5 T) with an angle (30°) with respect to the field. If the proton velocity equals ($10 \frac{m}{\sec.}$), find: 1) The radius of the path of the proton motion. 2) The angular velocity. 3) The frequency resulting from the proton motion through the magnetic field.

Q11) The wavelength of a photon is twice as the wavelength of another photon; determine the ratio between their energy.

Q12) A uniform electric field between two parallel bars (20 cm apart), an electron moves from rest from the negative bar and reaches the other bar (the positive bar) in ($1.5*10^{-8}$ sec.), determine:

1) The electric field intensity.
2) The electron velocity when it reaches the positive bar.

Chapter 3: Energy Bands in Solids

3-1 Introduction:

The electrons of a single atom revolve around the nucleus in certain orbits, each orbit has a specific constant energy level and each level consists of several secondary levels while in solids, which consist of a large number of close atoms, when these atoms unite to form the crystal any electron does not affected by the charges in its atom only but by the nuclei and the electrons in all other atoms forming the crystal too; therefore, the energy levels of the outer electrons change as a result of the interaction between atoms and instead of the specified energy levels of the single atom, the crystal contains a band of large number of very close energy levels that seems to be continuous, this band is called **the valence band (V.B.)**. The electrons in the valence band are restricted to the atom and do not participate in the electrical conduction. When an electron in the valence band gains enough energy so that it can be separated from the atom, it jumps to the next band which is **the conduction band (C.B.)**. The electrons in the conduction band participate in the electrical conduction. Between the valence band and the conduction band there is a forbidden region that the electron can not exist in, this region is called the **energy gap (E.g.)**. The materials are classified depending on the amount of the energy gap into three types: conductors, insulators and semiconductors.

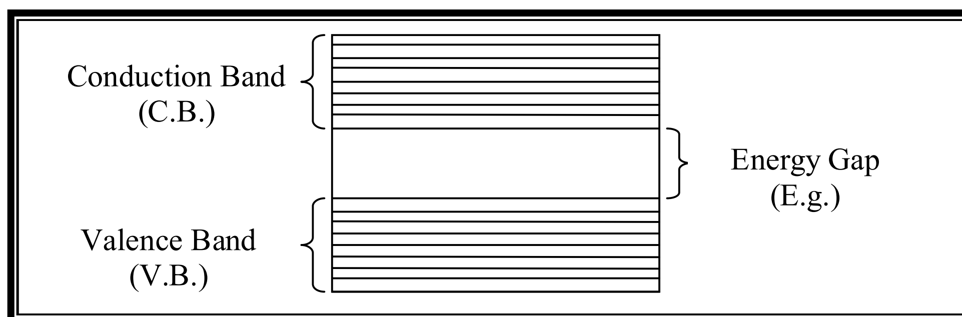


Figure (3-1): Energy bands.

3-2 Conductors:

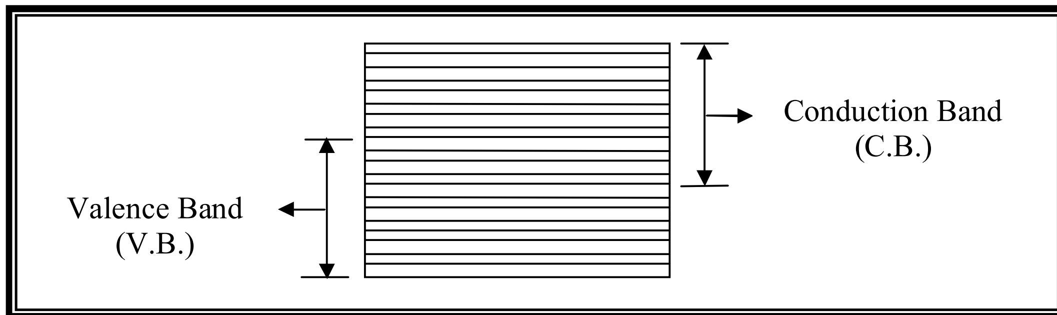


Figure (3-2): Energy-band structure in conductors.

The valence band is overlapped with the conduction band and thus there is no energy gap (Figure (3-2)). The absence of the energy gap in the conductive crystals means that any valence electron will be free to move in the crystal in temperature degrees above the absolute zero as well as to move in response to the electric field if it exists.

At a temperature of absolute zero, the electrons can not move through the crystal because they are all strongly linked to their atoms. Thus, they fill the valence band from the lowest energy level to the highest energy level or in other words, the conduction band at a temperature of absolute zero is empty, this means that there is no enough energy for the electrons to move to the conduction band.

When the temperature increases above the absolute zero, the thermal energy that will be acquired by the electron will enable it to escape from its atom and to transport to the conduction band. The movement of the electron in the conduction band will be random and in all directions with a velocity called **the thermal velocity**. When a potential difference is applied across the conductor, an electric field will be created inside the conductor, this electric field works to accelerate the free electrons in the conduction band due to the force experienced by them and that is equal to:

$$F = qE$$

But the acceleration of electrons in the crystal material will be impeded as a result of the ongoing collision with the atoms vibrating around their positions in the crystal. The electron velocity quickly reaches an average fixed value called **the drift velocity** (v_d) which is related linearly with the intensity of the electric field (E) by **the mobility** (μ) in the given material:

$$v_d = \mu E \quad \text{Eq. (3-1)}$$

v_d: The Drift velocity **($\frac{m}{\sec.}$)**, μ: The mobility ($\frac{m^2}{V.\sec.}$), E: The electric field intensity ($\frac{N}{C}$).

It is worth mentioning that for a conductor at a constant potential (voltage) that raising the temperature of the conductor increases the number of collisions between the electron and the atoms vibrating around their positions in the crystal therefore the drift velocity will decrease and hence the resistance of the conductor will increase.

Conductivity:

If a metal object having a length (L) and a cross section area (A) is connected to a voltage source (V), then a current (I) will pass in the object (Figure (3-3)).

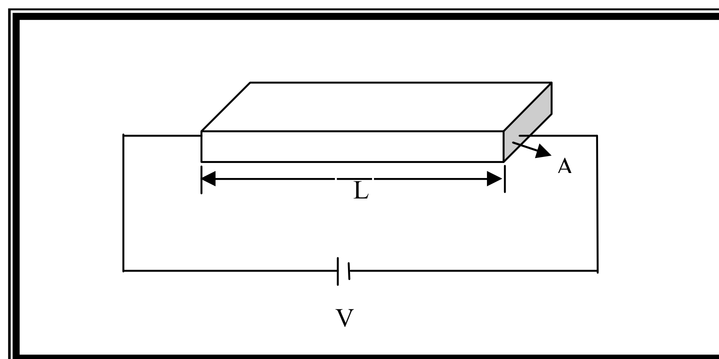


Figure (3-3): Metal object connected to a voltage source.

According to **Ohm's law:**

$$V = IR \quad \text{------------ Eq. (3-2)}$$

R is the resistance of the metal and it is defined as:

$$R = \rho \frac{L}{A} \quad \text{------------ Eq. (3-3)}$$

ρ is resistivity (Ω.m). Eq. (3-2) and Eq. (3-3) leads to:

$$\frac{I}{A} = \frac{1}{\rho}\frac{V}{L}$$

$$\because J = \frac{I}{A} \quad \text{------------ Eq. (3-4)}$$

And

$$\sigma = \frac{1}{\rho} \quad \text{------------ Eq. (3-5)}$$

And

$$E = \frac{V}{L} \quad \text{------------ Eq. (3-6)}$$

$$\therefore J = \sigma E \quad \text{------------ Eq. (3-7)}$$

Where:

J: The **current density** ($\frac{A}{m^2}$) through the cross section of the metal.

E: The electric field intensity ($\frac{V}{m}$).

σ: **Conductivity** ((Ω.m)$^{-1}$).

$$\because \sigma = ne\mu \quad \text{------------ Eq. (3-8)}$$

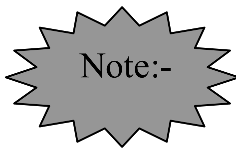
$$\therefore J = ne\mu E \quad \text{------------ Eq. (3-9)}$$

Where:

e: The electron charge (C).

n: The number of free electrons per unit of volume ($\frac{electron}{m^3}$).

From equation (Eq. (3-8)), it is clear that the conductivity (σ) depends on two factors: The density of free electrons in the material (n) and the ability of these electrons to move in the material (μ) under the influence of the electric field.



$$\because\ v_d = \mu E \quad and \quad J = ne\mu E$$

$$\therefore\ \boxed{J = nev_d} \quad \text{------------ Eq. (3-10)}$$

Example (3-1): A metal object having a length (10 m), cross section area (0.5 mm^2), and resistance (0.34 Ω) is connected to a potential difference source (5 V). If the density (concentration) of free electrons equals (8.5*10^{28} electron/m^3), Calculate:

1) The object's conductivity.
2) The electron's mobility.
3) The current density through the object.

Sol.)

3-3 Insulators:

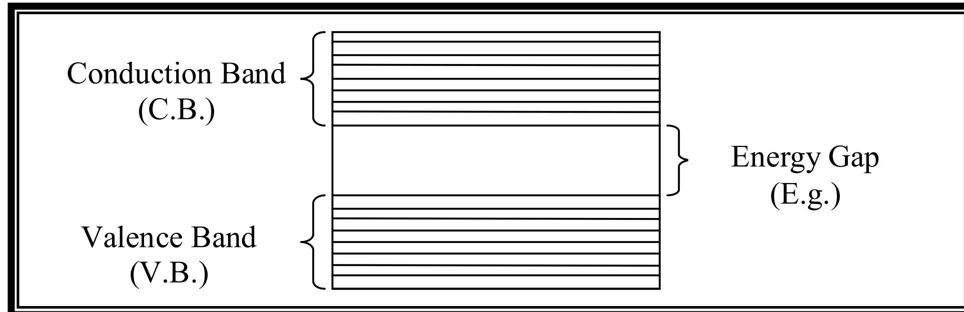


Figure (3-4): Energy-band structure in insulators.

The valence band is separated from the conduction band by a large energy gap amounting to about (5 eV); therefore, the valence electrons can not move to the conduction band unless it receives the enough energy that equals the energy gap.

At normal temperatures, the valence electrons do not have the enough energy to move to the conduction band; therefore, it can be said that the insulator crystal has the following: large energy gap, valence band that is filled with electrons, and an empty conduction band. This explains why there are no free charges in an insulator.

3-4 Semiconductors:

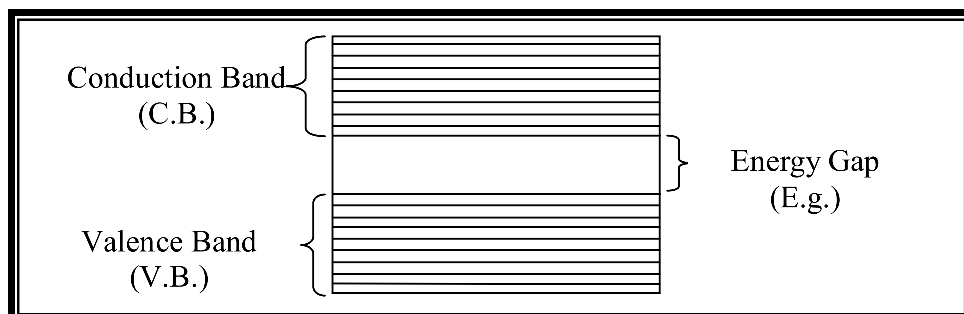


Figure (3-5): Energy-band structure in semiconductors.

The energy-band structure of semiconductors does not differ from that of insulators except in the capacity of the energy gap which in semiconductors equals (1.1 eV) or less. These materials behave like insulators at the absolute zero temperature. At this temperature, the conduction band is empty which means that the electrons have no enough energy to move to the conduction band. At high temperatures, these materials behave as conductors. On the other hand, at room temperature ($27\,^{\circ}C = 300\,K$) some electrons gain the enough energy to move to the conduction band but the resulting current is small and can not be used in most applications; therefore, at room temperature, the semiconductor is not a good insulator nor a good conductor and this is why it is called a semiconductor. Semiconductors can be classified into two types: intrinsic semiconductors and extrinsic semiconductors.

3-4-1 Intrinsic Semiconductors:

Raising the temperature of the semiconductor will increase the energy of the valence electrons, and then the number of the electrons reaching the conduction band will increase. Hence, the conductivity (σ) of these materials will increase with temperature increase.

The elements of the fourth group (group IV) in the periodic table have four valence electrons; the crystals that theses elements form are called **covalent crystals** (Figure (3-6)).

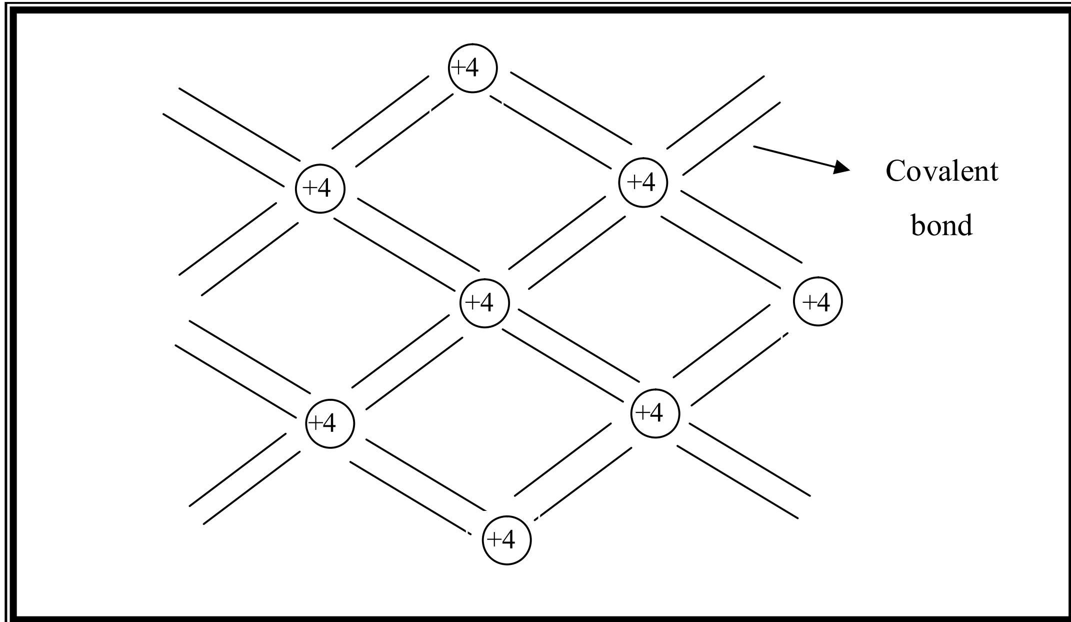


Figure (3-6): Covalent crystal at absolute zero temperature.

If the covalent crystal is exposed to an electric potential or a radiation with sufficient energy or a thermal energy, then this gained energy will break the covalent bonds and transfer the electron to the conduction band to participate in the electrical conduction. The energy required to break the covalent bonds is equal to the value of the energy gap (E_g) or greater. The value of the energy gap equals (0.72 eV) for the germanium crystal (Ge) and (1.1 eV) for the silicon crystal (Si). These two elements are of the most important elements of the fourth group used in the electronics industries. (Si) has (14) electrons while (Ge) has (32) electrons.

The transfer of an electron from the valence band to the conduction band leaves an empty place in the covalent bond that is called the **hole**. The atom now becomes an ion. The hole appears as a positive charge (+e) with an effective mass (m_p) that does not equal the electron mass. The hole is defined as a place ready to receive the electron; therefore, it quickly filled by the adjacent electron. Thus leading to the movement of charges. The motion

of the holes is random and in different directions but when an electric field is applied, the holes will move in the same direction of the field and the opposite direction relative to the electrons' motion. The holes' motion gives a separate current from that resulting from the electrons' motion and the total current equals the holes' current plus the electrons' current. The thermal energy is the main source in generating the **electron-hole pairs**. The conduction process resulting from these charge carriers' motion (the electrons and the holes) is called **intrinsic conduction**.

When an electric field is applied, the carries will move with a velocity that reaches a constant value after a specific time, this constant value of the velocity is called the drift velocity, where:

$$v_h = \mu_h E$$
$$v_e = \mu_e E$$

(h) refers to the holes and (e) to the electrons. For semiconductors:

$$J_e = nev_e$$

and

$$J_h = pev_h$$

(n) represents the density of the free generated electrons and (p) represents the density of the generated holes.

$$J_t = J_e + J_h = nev_e + pev_h \qquad \text{Eq. (3-11)}$$

(J_t) is the total current density. Substituting (v_h) and (v_e) gives:

$$J_t = ne\mu_e E + pe\mu_h E$$

In pure (intrinsic) semiconductors, the density of the electrons (n) in the conduction band equals the density of the holes (p) in the valence band:

$$n = p = n_i \qquad \text{Eq. (3-12)}$$

(n_i): intrinsic carriers concentration:

$$J_t = n_i(\mu_e + \mu_h)eE \qquad \text{Eq. (3-13)}$$

Similarly, the total conductivity (σ_t) in the pure semiconductor is:

$$\sigma_t = \sigma_e + \sigma_h$$
$$\therefore \sigma_t = (\mu_e + \mu_h)n_i e \qquad \text{Eq. (3-14)}$$

Note:-

$$n = p = n_i$$

$$\therefore np = n_i^2 \qquad \text{Eq. (3-15)}$$

This equation is called **the mass action law**.

Note:-

(n_i) can be calculated using the following equation:

$$n_i = (N_c N_v)^{\frac{1}{2}} \exp(-\frac{E_g}{2KT}) \qquad \text{Eq. (3-16)}$$

Where:

T : The absolute temperature (in Kelvin (K)).

K : Boltzman constant:

$$K = 1.381 \times 10^{-23} \left(\frac{J}{K}\right)$$

E_g : Energy gap (J).

N_c : The effective density of the energy levels in the conduction band given by:

$$N_c = 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{\frac{3}{2}} \quad \text{--------- Eq. (3-17)}$$

m_e^* : The effective mass of the electron (kg).

N_v : The effective density of the energy levels in the valence band given by:

$$Nv = 2\left(\frac{2\pi m_p^* KT}{h^2}\right)^{\frac{3}{2}} \quad \text{--------- Eq. (3-18)}$$

m_p^* : The effective mass of the hole (kg).

Example (3-2): Calculate the conductivity of an intrinsic silicon crystal if the number of released electrons at a specific temperature is one electron for each 10^{13} of valence electrons. The silicon density is ($2.33*10^3$ Kg/m^3), the silicon atomic weight is (28.086), the electron mobility is (0.135 m^2/v.sec.) and the hole mobility is (0.048 m^2/v.sec.).

Sol.)

Chapter 3 (Continued)

3-4-2 Extrinsic Semiconductors:

There are two types of extrinsic semiconductors: extrinsic semiconductors with a larger electron concentration than hole concentration (n-type semiconductors) and another having a larger hole concentration than electron concentration (p-type semiconductor). In the two types, the more abundant charge carriers are called **majority carriers** while the less abundant charge carriers are called **minority carriers**. The majority carriers' concentration is controlled by **doping** the semiconductor with **impurity atoms**. The impurity atoms determine the type of the extrinsic semiconductor.

3-4-2-1 n-type semiconductor:

In this semiconductor, the free electrons' concentration is more than the holes' concentration ($n > p$); therefore, the electrons are the majority carriers and the holes are the minority carriers.

The n-type semiconductor is produced by adding atoms of the fifth group's elements of the periodic table (such as **Phosphorus (P)**, **Arsenic (As)**, and **Antimony (Sb)**) to the intrinsic semiconductor. These elements have five electrons in the outer orbital. If one atom of phosphorus, for example, is added to the silicon then this atom replaces the silicon atom, four of the phosphorus electrons will form bonds with the surrounding silicon atoms and the fifth electron remains loose (Figure (3-7)).

The fifth electron of the phosphorus can release from the atom without the need for large energy (about 0.05 eV only). This energy is

twenty times less than the energy required to break the bond (silicon – silicon) (E_g = 1.1 eV).

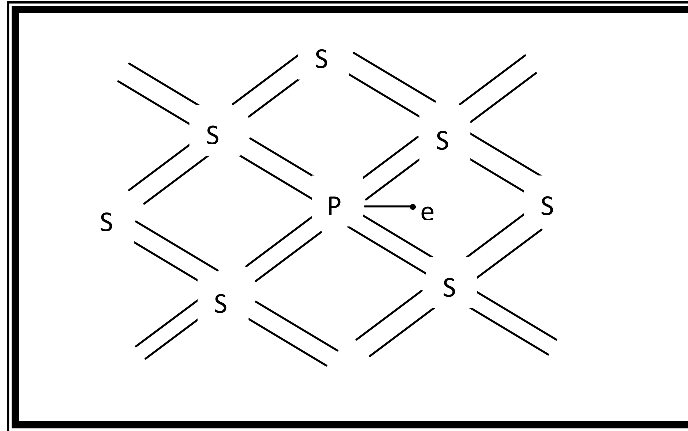


Figure (3-7): Phosphorus atom (P) in a matrix of silicon atoms.

When the fifth electron of the phosphorus atom releases by gaining (0.05 eV), this means that this electron will move to the conduction band and also means that this electron was at a position (0.05 eV) under the edge of the conduction band before it released (figure (3-8)).

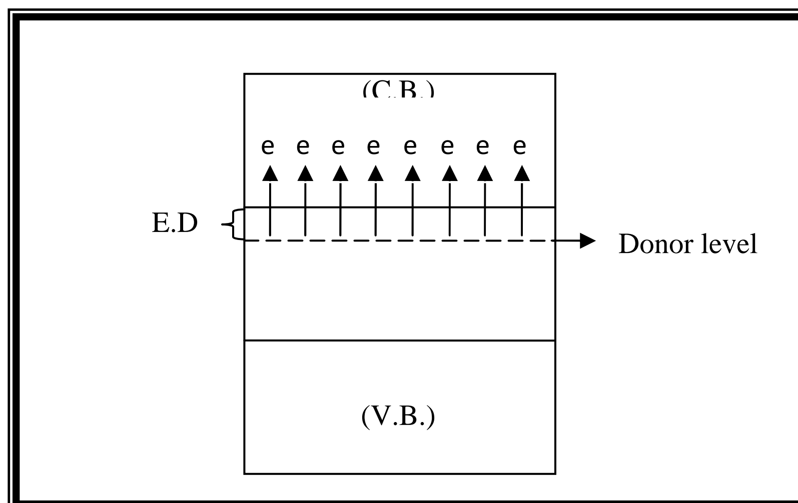


Figure (3-8): Energy-band structure of n-type semiconductor.

If (N_D) of phosphorus atoms give one electron from each atom, this will add ($n = N_D$) of electrons to the conduction band leaving (N_D^+) of ionized phosphorus atoms that are called **donor atoms** because they “donate” an electron to the conduction band. Usually, the concentration of these atoms in the cubic meter of the semiconductor is much higher

than the concentration of the electrons already exist in the conduction band before the doping; therefore, the ionization of these atoms makes the concentration of the electrons in the conduction band equal to the concentration of the donor atoms; therefore, the concentration (density) of the electrons (n) in the semiconductor that is doped with (N_D) of donor atoms is:

$$n = N_D$$

The concentration of the minority carriers (the holes (p)) is calculated as follows:

$$np = n_i^2$$
$$\therefore\ p = \frac{n_i^2}{n} = \frac{n_i^2}{N_D}$$

The total conductivity (σ_t) is calculated as follows:

$$\sigma_t = \sigma_e + \sigma_h$$
$$\sigma_t = ne\mu_e + pe\mu_h$$
$$\because\ n \gg p \quad \therefore\ \sigma_e \gg \sigma_h$$
$$\therefore\ \sigma_t \approx \sigma_e = ne\mu_e$$
$$\therefore\ \sigma_t = N_D e \mu_e$$

3-4-2-2 p-type semiconductor:

If atoms of elements from the third group of the periodic table (such as **Boron (B)**, **Aluminium (Al)**, **Gallium (Ga)**, and **Indium (In)**) are added to the intrinsic semiconductor, the three valence electrons of these atoms will form three bonds with the surrounding semiconductor atoms and the fourth semiconductor atom remains without a bond as in

the following figure (figure (3-9)) in which a boron atom exists between the silicon atoms.

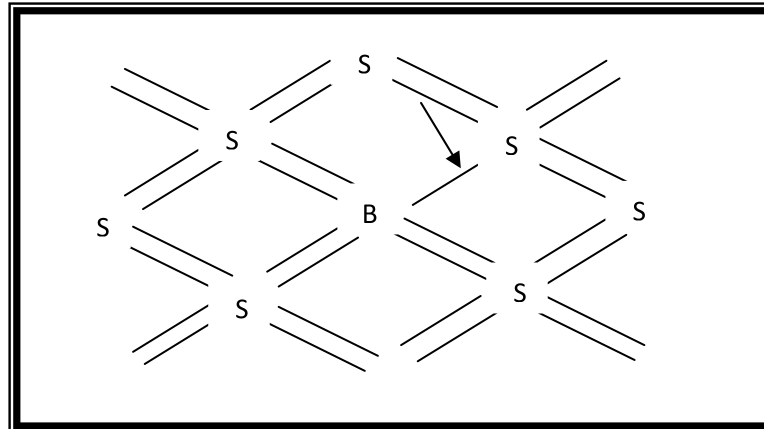


Figure (3-9): Boron atom between the silicon atoms.

An adjacent bond to the boron can break and its electron move to complete the bonds between the boron and the four surrounding silicon atoms; as a result, the boron atom gains a negative charge while a hole will result from the breaking of the adjacent bond. This hole does not result from the movement of the electron from the valence band to the conduction band and no free electron will result from this process because the electron moves to the boron atom. The required energy for this process in silicon is about (0.05 eV) for boron, (0.07 eV) for aluminium, and (0.16 eV) for indium. These atoms are called **acceptor atoms** (N_A) and if all these atoms ionized (N_A^-) then the number of holes will be equal to the acceptor atoms.

$$p = N_A$$

The energy-band structure is shown in the following figure:

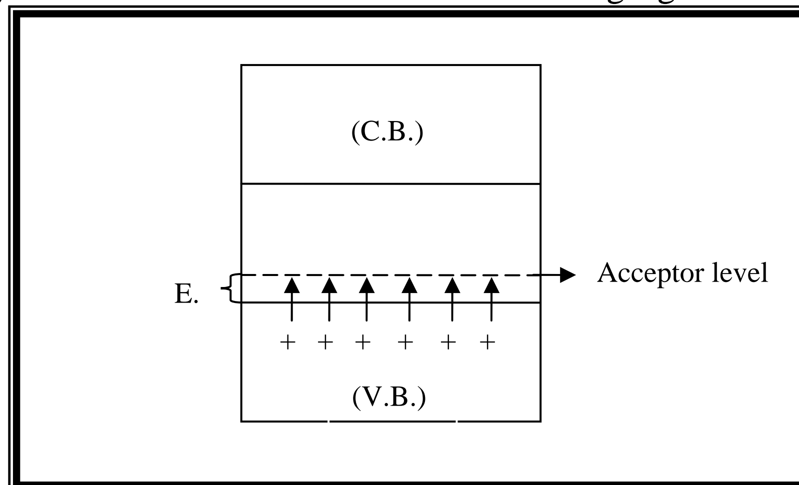


Figure (3-10): Energy-band structure p-type semiconductor.

The concentration of the minority carriers (the electrons (n)) is calculated as follows:

$$np = n_i^2$$
$$\therefore n = \frac{n_i^2}{p} = \frac{n_i^2}{N_A}$$

The total conductivity (σ_t) is calculated as follows:

$$\sigma_t = \sigma_e + \sigma_h$$
$$\sigma_t = ne\mu_e + pe\mu_h$$
$$\because p \gg n \qquad \therefore \sigma_h \gg \sigma_e$$
$$\therefore \sigma_t \approx \sigma_h = pe\mu_h$$
$$\therefore \boxed{\sigma_t = N_A e \mu_h}$$

Example (3-3): A Silicon cube with dimensions (10*10*10) mm contains at a certain temperature (10^{19}) gallium atom in the cubic meter. Calculate the resistance of this cube and compare it to the resistance of an identical cube that contains (10^{19}) arsenic atom in the cubic meter of the Silicon. The hole mobility is (0.05 m^2/V.sec.) and the electron mobility is (0.12 m^2/V.sec.).

Sol.

3-5 Fermi Energy level:

The distribution of electrons in the energy levels that form the energy band depends on a statistics called **Fermi- Diriac distribution function**:

$$f(E) = \frac{1}{1 + \exp .(\frac{E - E_f}{KT})}$$

$f(E)$: **Fermi function** that specifies the probability of the existence of the electron in any energy level having an energy (E). (E_f) is **Fermi energy level** which represents the highest energy level that the electron can occupy at the absolute zero temperature.

At the absolute zero temperature, all the energy levels below (less than) Fermi energy level are filled with electrons while the energy levels above (greater than) Fermi energy level are empty (Figure (3-11)). At the other temperatures, Fermi energy level is defined as the energy level in which the probability of the existence of the electron is (50%).

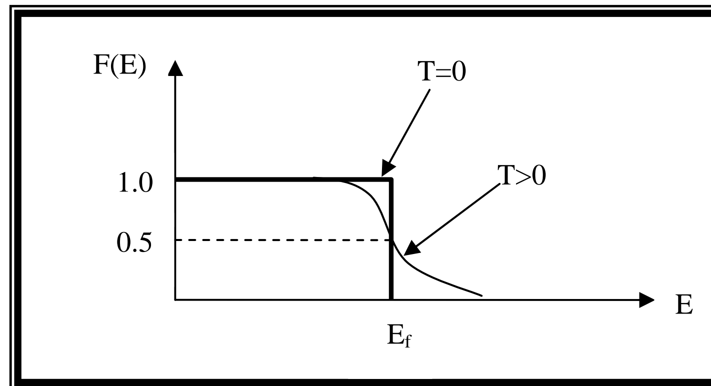


Figure (3-11): Fermi- Diriac statistics.

For the **pure (intrinsic) semiconductor**, Fermi energy level is calculated as follows:

$$E_f = \frac{E_g}{2} + \frac{KT}{2} \ln(\frac{N_v}{N_c})$$

Substituting (N_c) and (N_v):

$$N_c = 2(\frac{2\pi m_e^* KT}{h^2})^{\frac{3}{2}}$$

$$N_v = 2(\frac{2\pi m_p^* KT}{h^2})^{\frac{3}{2}}$$

gives:

$$E_f = \frac{E_g}{2} + \frac{3}{4} KT \ \ln\left(\frac{m_p^*}{m_e^*}\right)$$

The last equation shows that when (m_p^*) and (m_e^*) are equal then Fermi energy level is in the middle of the energy gap ($\frac{E_g}{2}$). Also, if ($T = 0$) then Fermi level is in the middle of the energy gap, in all cases, if the temperature increases or if (m_p^*) differs from (m_e^*) then this will not affect the position of Fermi level very much because of the small logarithmic effect which raises the value of (E_f) with the increase in

temperature because the logarithmic term is usually positive because (m_p^*) is greater than (m_e^*).

For **n-type semiconductor**, Fermi level is calculated by the following equation:

$$E_{fn} = E_c - KT\ln(\frac{N_c}{N_D})$$

E_c is the energy at the edge of the conduction band.

For **p-type semiconductor**, Fermi level is calculated by the following equation:

$$E_{fp} = E_v + KT\ln(\frac{N_v}{N_A})$$

E_v is the energy at the edge of the valence band.

Example (3-4): Calculate the concentration of the electrons and the holes in intrinsic silicon at (300°k). The energy gap equals (1.12 eV) and (N_c=N_v=$2.5 \times 10^{25}\ m^{-3}$). Find the value of Fermi energy level.

Sol.)

3-5 Hall Effect:

If an electric current passed in a semiconductor and a magnetic field is applied perpendicularly to the current direction then an electric field is generated in a perpendicular direction to the current and to the magnetic field, this phenomenon is called **the Hall Effect** (figure (3-12)).

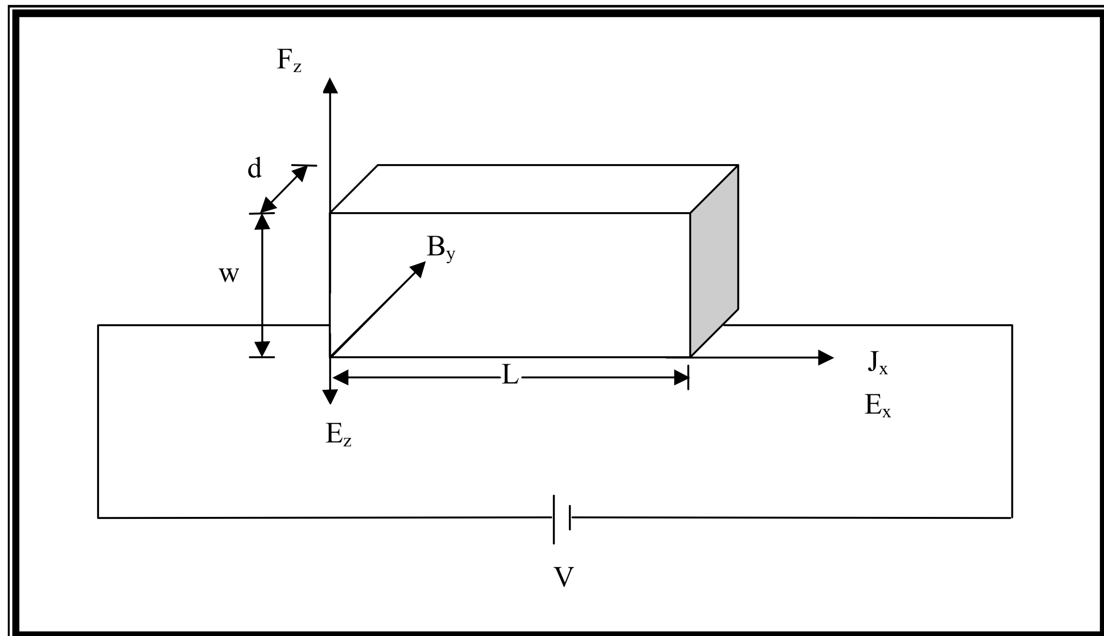


Figure (3-12): The Hall Effect.

If an electric current with a density (J_x) passed in a p-type semiconductor, where the charge carriers are positive (holes) with a density (p), and a magnetic field (B_y) is applied perpendicularly on (J_x). Assume that the velocity of the holes in (x) direction which is caused by the electric field (E_x) is (v_x), then applying a magnetic field (B_y) generates a force that acts on the holes toward the (z) direction:

$$F_z = ev_x B_y$$

This force raises the holes up, which makes the number of holes in the top more than the number at the bottom; this difference generates an

electric field directed downward toward ($-z$) acting on the hole with a force (eE_z), this force equals (F_z) in magnitude:

$$eE_z = ev_x B_y$$

Current density (J_x) resulting from the motion of the holes in (x) direction is:

$$J_x = pev_x$$

$$\therefore E_z = \frac{J_x B_y}{pe}$$

Hall coefficient (R_H**)** is defined is:

$$R_H = \frac{E_z}{J_x B_y} = \frac{1}{pe}$$

Taking the dimensions in consideration:

$$R_H = \frac{\frac{V_H}{w}}{(\frac{I}{wd})B_y}$$

$$\therefore R_H = \frac{V_H d}{IB_y} = \frac{1}{pe}$$

(V_H) represents **Hall voltage** which is the potential difference resulting from the motion of the holes to the top.

For n-type semiconductor, the derivation is in the same way but the direction of hall voltage is in the opposite direction and:

$$R_H = \frac{-1}{ne}$$

The angle (θ) in the following figure is called **Hall angle** where:

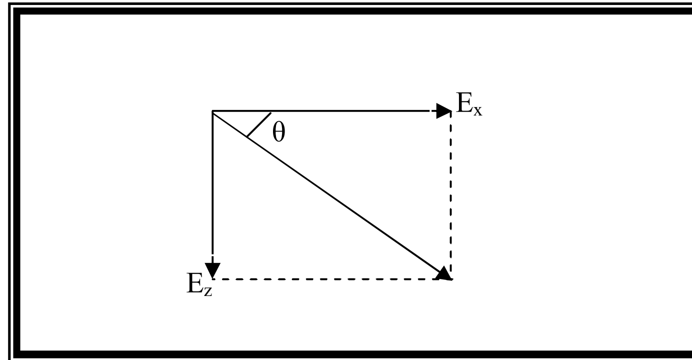


Figure (3-13): Hall angle.

$$\tan\theta = \frac{E_z}{E_x}$$

$$\therefore J_x = \sigma E_x$$

$$\therefore \tan\theta = \frac{J_x B_y R_H}{\frac{J_x}{\sigma}} = \sigma R_H B_y$$

$$\because \mu = \sigma R_H$$

$$\therefore \boxed{\tan\theta = \mu B_y}$$

<u>Example (3-5):</u> The resistivity of extrinsic silicon equals ($9.27\times10^{-3}\,(\Omega m)$) and Hall coefficient equals ($3.84\times10^{-4}\,\frac{m^3}{C}$). Find the concentration and the mobility of the majority carriers.

Sol.)

Problems

Q1) The mobility of the electrons and the holes in an intrinsic germanium at a certain temperature is ($0.36\frac{m^2}{v.\sec.}$) and ($0.17\frac{m^2}{v.\sec.}$) respectively, the concentration of the electrons and the holes equals ($2.5\times10^{19}/m^3$). Calculate the conductivity.

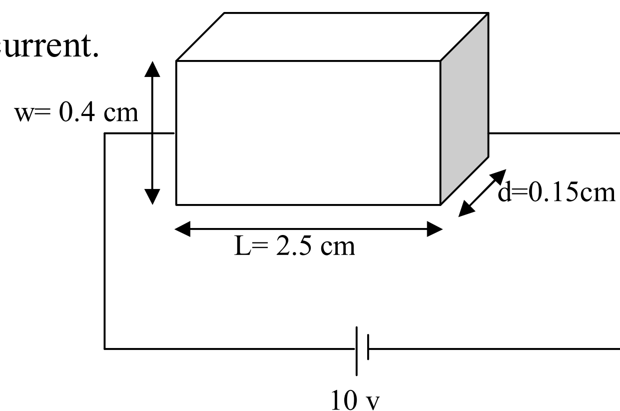
Ans.) 2.12 $(\Omega.m)^{-1}$

Q2) Determine the concentration of the donor impurities in (n-type) germanium whose resistivity is ($100(\Omega.m)$) knowing that the electron mobility equals ($0.36\frac{m^2}{v.\sec.}$).

Ans.) $1.73\times10^{17}/m^3$

Q3) For the pure (intrinsic) germanium in the following figure $\mu_e = 0.36\frac{m^2}{v.\sec.}$, $\mu_h = 0.17\frac{m^2}{v.\sec.}$, calculate:

1) The drift velocity of the electron and the hole. 2) The conductivity of the germanium if the intrinsic carriers concentration equals ($2.5\times10^{19}/m^3$). 3) The total current.



Ans.) $144\frac{m}{\sec.}, 68\frac{m}{\sec.}, 2.12(\Omega m)^{-1}, 0.005A$

Q4) Calculate the resistivity of an intrinsic germanium crystal after adding donor atoms at an average of : 1 donor atom for each 10^6 germanium atoms. Ge density=5323 kg/m^3, Ge atomic weight=72.60, $\mu_e = 0.36 \frac{m^2}{v.\sec}$.

Ans.) $3.93\times10^{-4}\,\Omega m$

Q5) A germanium bar with length (6mm), width (1mm), and depth (0.5mm) has a resistance of (120 Ω). Knowing that the germanium is (p-type), $n_i = 2.5\times10^{19}/m^3$, $\mu_e = 0.36 \frac{m^2}{v.\sec}$, $\mu_h = 0.17 \frac{m^2}{v.\sec}$, calculate:

1) The concentration of the majority and the minority carriers.
2) The conductivity of the minority carriers.

Ans.) $3.6\times10^{21}/m^3, 1.73\times10^{17}/m^3, 9.9\times10^{-3}(\Omega m)^{-1}$

Q6) What is the probability of occupying a place at a level (0.1 ev) above Fermi level at (27°C)?

Ans.) 0.02

Q7) What is the probability of occupying a place at a level (0.2 ev) below Fermi level at (50°C)?

Ans.) 99.92%

Q8) When applying a voltage of (100 mv) across a wire of length (20m), the current density passing through the wire was (312500 A/m^2). Calculate the resistivity of the wire and the electron mobility if the concentration of the free electrons equals ($5.86\times10^{28}/m^3$).

Ans.) $1.6\times10^{-8}\,\Omega m, 6.67\times10^{-3}\frac{m^2}{v.\sec.}$

Q9) A germanium bar with depth (2mm) carries a current (10mA) across it, a magnetic field (0.1T) is applied perpendicularly to the current, the resulting Hall voltage is (1mv). Find:

1) Hall coefficient. 2) The concentration of the majority carriers. 3) Hall angle knowing that the mobility of the majority carriers is ($0.19 \frac{m^2}{v.\sec.}$).

Ans.) $2\times10^{-3}\frac{m^3}{C}, 3.125\times10^{21}/m^3, 1.088^o$

Q10) A piece of n-type germanium whose dimensions are($10mm\times1mm\times1mm$) is placed in a magnetic field (0.2T).The electrons concentration equals ($7\times10^{21}/m^3$), the potential difference between the two sides of the piece is (1mv), the electrons mobility equals ($0.36\frac{m^2}{v.\sec.}$). Calculate: 1) Hall coefficient. 2) The current passing in the piece. 3) Hall voltage.

Ans.) $-8.9\times10^{-4}\frac{m^3}{C}, 4.03\times10^{-5}A, -7.1\times10^{-6}v$

3-6 The flow of current in semiconductors:

The current resulting from the drift of the charge carrier in **the electric field** is called **the drift current**. There is another type of current that can flow in the semiconductor when there is no electric field, this current flows if the **distribution** of the charges in the material is **irregular**. For example, If the concentration of the electrons in one side of the material is higher than its concentration in the other side then this **concentration gradient** drives the electrons **to diffuse** from the region of

high concentration to the region of low concentration, the resulting current is called the **diffusion current**.

This current depends on the concentration gradient. The motion of the charges, as in the drift case, is affected by the collisions during the motion in the material. The diffusion current resulting from the diffusion of the electrons is directly proportional with the concentration gradient:

$$J_{Dn} = eD_n \frac{dn}{dx}$$

J_{Dn}: Diffusion current density (A/m^2) for the electron.

(D_n) is called **the diffusion constant (diffusion coefficient) for the electron**.

$\frac{dn}{dx}$: Concentration gradient for the electron.

If there is a gradient in the holes' concentration then the diffusion current density equals:

$$J_{Dp} = -eD_p \frac{dp}{dx}$$

J_{Dp}: Diffusion current density (A/m^2) for the hole.

D_p: **The diffusion constant (diffusion coefficient) for the hole**.

$\frac{dp}{dx}$: Concentration gradient for the hole.

The value of (D_n) and (D_p) is a standard for the ability of the electron and the hole to diffuse in the material; therefore, there is a relation between the diffusion constant (D) and the mobility (μ):

$$D = \frac{KT}{e}\mu$$

For the electron:

$$D_n = \frac{KT}{e}\mu_e$$

For the holes:

$$D_p = \frac{KT}{e}\mu_h$$

Applying an electric field (E) on a semiconductor which contains a gradient in the carriers' concentration produces both the drift current and the diffusion current:

$$J_n = \sigma_e E + J_{Dn} = ne\mu_e E + eD_n \frac{dn}{dx}$$

J_n : The density of the total electron current.

$$J_p = \sigma_h E + J_{Dp} = pe\mu_h E - eD_p \frac{dp}{dx}$$

J_p : The density of the total holes current.

3-7 Extra Carriers in Semiconductors:

The semiconductor is often used in the extrinsic case and in a situation of loss of equilibrium in the concentration of the carriers. In equilibrium, the **generation (G)** and the **recombination (R)** of the carriers are equal and the product of the majority carriers and the minority carriers is constant, for example, in a p-type crystal, the number of the holes (p) and the number of the electrons (n) is constant where ($p >> n$) and ($np = n_i^2$) and the generation (the electron-hole pair generation) is equal to the recombination (G = R).

In the loss of equilibrium situation, the concentration of the carriers changes due to an external factor which generates extra carriers above the equilibrium concentration level, for example, if **light** is applied on the semiconductor then the generation of the carriers will increase, for p-type semiconductor, which contains at equilibrium (p) holes and (n) electrons, now assume that this semiconductor from the time instant ($t = 0$) is subjected to an external stimulus (light) which generates extra holes and electrons with a concentration ($\Delta n = \Delta p$) because each generated electron generates a hole, so the number of holes becomes:

$$p_{total} = p + \Delta p$$

And the number of electrons:

$$n_{total} = n + \Delta n$$

In this case, ($n\, p \neq n_i^2$) where ($n\, p > n_i^2$) and the semiconductor is called **unstable semiconductor.**

When the stimulus disappears, the equilibrium level returns to its original level after a short period. Assume that the number of extra electrons

at ($t = 0$) is (Δn_o) and the number of extra holes at this moment is (Δp_o), then the disappearance of the external stimulus (light) at this moment results in the decreasing of the number of the extra carriers with time, the decrease is exponential where the number of the remaining extra electrons is given by the following equation:

$$\Delta n = \Delta n_o \exp \frac{-t}{\tau_e}$$

Where:

Δn: The number of the remaining extra electrons after time (t) from the moment of turning off the light source.

Δn_o: The number of the extra electrons generated at the moment of turning off the light source ($t = 0$).

τ_e: Life time of the electron. (τ_e) represents the time average in which the electron remains free before it recombines.

Similarly, for the holes:

$$\Delta p = \Delta p_o \exp \frac{-t}{\tau_p}$$

τ_p: Life time of the hole.

Example (3-6): The conductivity of a piece of n-type silicon increased from ($5 \times 10^{-4} (\Omega.cm)^{-1}$) to ($(0.3\Omega.cm)^{-1}$) when exposing this piece to a light source. Calculate the number of the excited electrons across the energy gap. $\mu_e = 1500 \frac{cm^2}{V.\sec.}$, $\mu_h = 500 \frac{cm^2}{V.\sec.}$.

Sol.)

Example (3-7): In the previous example, if the electrons life time is ($\tau_e = 10\,\mu\,\text{sec.}$), calculate the concentration of the excited electrons which will remain after ($0.07\,\text{sec.}$) and ($1\,\text{sec.}$) from turning the light source off.

Sol.)

3-8 Diffusion of Extra Carriers:

If a semiconductor that is in equilibrium is injected at its surface ($x = 0$) with extra carriers then these carriers will diffuse inward away from the injected surface as a result of different concentration of carriers between the surface and interior, where the concentration is higher at the surface.

As the injected carriers move into the interior, their number decrease as a result of recombination where the charge carrier has a life time (τ) and during this period, the charge carrier has been penetrated the material to a distance (L). The distance that the charge carrier travels before it combines is called **the diffusion length (L)**.

The concentration is decreased exponentially with the distance according to the following equation:

For the hole:

$$\Delta p = \Delta p_o \exp \frac{-x}{L_p}$$

Where:

Δp: The number of the remaining injected holes after a distance (x) from the injected surface.

Δp_o: The number of the holes injected at the surface ($x = 0$).

L_p: The diffusion length of the hole.

Similarly, for the electrons:

$$\Delta n = \Delta n_o \exp \frac{-x}{L_n}$$

The diffusion length (L) is related to the carriers life time (τ) by the relation:

$$L = \sqrt{D\tau}$$

(D) is the diffusion constant. For the hole:

$$L_p = \sqrt{D_p \tau_p}$$

And for the electron:

$$L_n = \sqrt{D_n \tau_n}$$

Example (3-8): The surface of a semiconductor which contains ($10^{21} \frac{atom}{m^3}$) acceptor atoms is injected with electrons having a concentration of ($10^{15} \frac{electron}{m^3}$), ($n_i$) for this semiconductor equals ($2 \times 10^{16} / m^3$), the electron

diffusion constant equals ($4\times10^{-3}\frac{m^2}{s}$), the electron life time is ($0.1\mu\sec.$).

Find:

1) The electron diffusion length.
2) The depth at which the concentration of the injected electrons becomes ten times the concentration of the electrons in the semiconductor?

Sol.)

3-9 Recombination and Generation of Charge Carriers:

The generation and recombination of electrons and holes in a semiconductor play an important role in their electrical behavior. These processes are defined as:

Recombination: A process whereby electrons and holes are annihilated or destroyed.

Generation: A process whereby electrons and holes are created.

Recombination Processes:

1. **Band-to-Band Recombination**: Also referred to as **direct recombination**, is the direct annihilation of a conduction band electron and a valence band hole. The electron falls from an allowed conduction band state into a vacant valence band state. This process is typically radiative with the excess energy released during the process going into the production of a photon (light).
2. **R-G Center Recombination**: A defect often causes an energy state in the energy gap that can act as traps for carriers. In this process, a conduction band electron gets trapped at this defect (energy state) and then a valence band hole is trapped and recombines with the electron or vice versa. This process is typically nonradiative.
3. **Recombination via Shallow Levels**: Donor and acceptor sites can also act as intermediaries in the recombination process similar to the R-G Center recombination with the exception that the recombination is quite often radiative.
4. **Auger Recombination**: A nonradiative recombination event where a conduction band electron losses energy to another conduction band electron which gains energy to result in one electron recombining with a hole in the valence band and one high energy electron which rapidly losses energy and relaxes back down to the conduction band.

These process are illustrated in the figure below (Figure (3-14)).

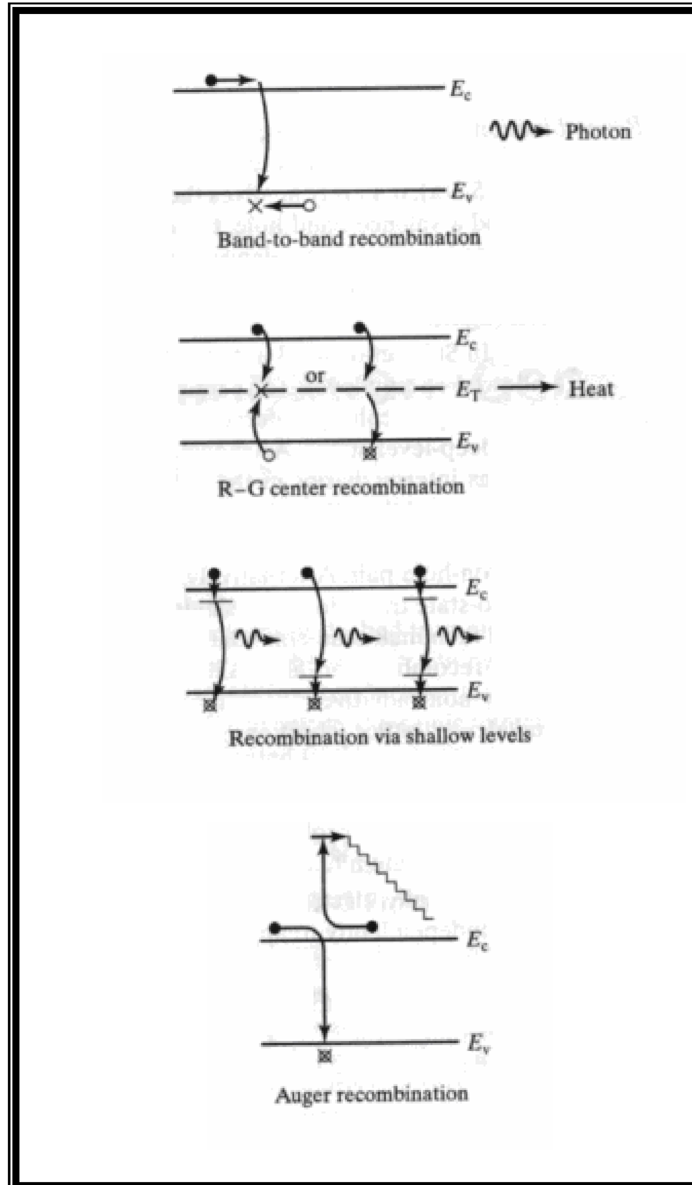


Figure (3-14): Energy-band visualization of recombination processes.

Generation Processes:

The recombination processes can be reversed resulting in generation processes. Band-to-Band generation occurs when an electron from the valence band is excited by light or heat to the conduction band. Generation of an electron and hole by an R-G center intermediary can be done in a couple of ways including and electron from the valence band being excited

to the trap state and then to the conduction band creating a hole in the valence band and electron in the conduction band. Also, an electron in the trap state can be excited to the conduction band while the hole is excited to the valence band. These processes are illustrated in the following figure (Figure (3-15)).

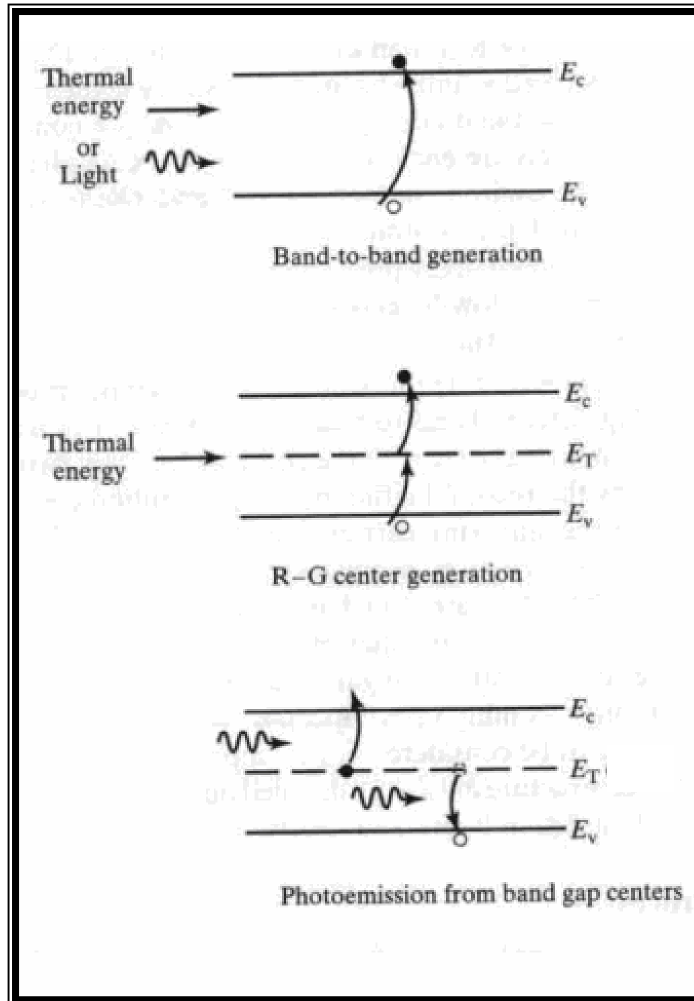


Figure (3-15): Energy-band visualization of generation processes.

Problems

Q1) The square of the intrinsic carriers concentration in a semiconductor material equals ($10^{36}/m^3$), the material is lit by a source which causes an increase in the charge carriers ($\Delta n_o + \Delta p_o$) equals ($3\times10^{18}/m^3$):

A) Assuming that the material is intrinsic and the electrons life time equals (600µsec.). Calculate the total concentration of the electrons in the material after (10^{-5} sec.) from turning the light source off.

B) Assuming that the material is a p-type semiconductor with hole concentration equals ($10^{16}/m^3$), the minority carriers life time equals (200µsec.). Calculate the total concentration of the electrons in the material after (10^{-6} sec.)from turning the light source off.

Ans.) $2.47\times10^{18}/m^3, 1.01\times10^{20}/m^3$

Q2) A sample of negative silicon with doping concentration equals ($10^{16}/cm^3$), when exposing this sample to a light source, the (electron - hole) generation average equals 10^{12} in the second (10^{12} from Δp_o, 10^{12} from Δn_o), if this sample is exposed to the light for (1msec.). Calculate the resistivity after (1msec.) from turning the light source off knowing that the carriers' life time equals (10^{-6}sec.), also calculate the resistivity at the moment of turning the light source off. $\mu_e = 1500\frac{cm^2}{v.\sec.}, \mu_h = 500\frac{cm^2}{v.\sec.}$.

Ans.) 0.416 Ω.cm

Chapter Four: P–N Junction

4-1 Introduction to P-N junction:

P-N junctions are formed by joining n-type and p-type semiconductor materials, as shown below (Figure (4-1)). The p-type region contains majority carriers (holes (p_p)) and minority carriers (electrons (n_p)), the n-type region contains majority carriers (electrons (n_n)) and minority carriers (holes (p_n)). Since the n-type region has a high electron concentration and the p-type has a high hole concentration, electrons diffuse from the n-type side to the p-type side. Similarly, holes flow by diffusion from the p-type side to the n-type side. When the electrons and holes move to the other side of the junction, they leave behind exposed charges on dopant atom sites, which are fixed in the crystal lattice and are unable to move. On the n-type side, positive ion cores are exposed. On the p-type side, negative ion cores are exposed.

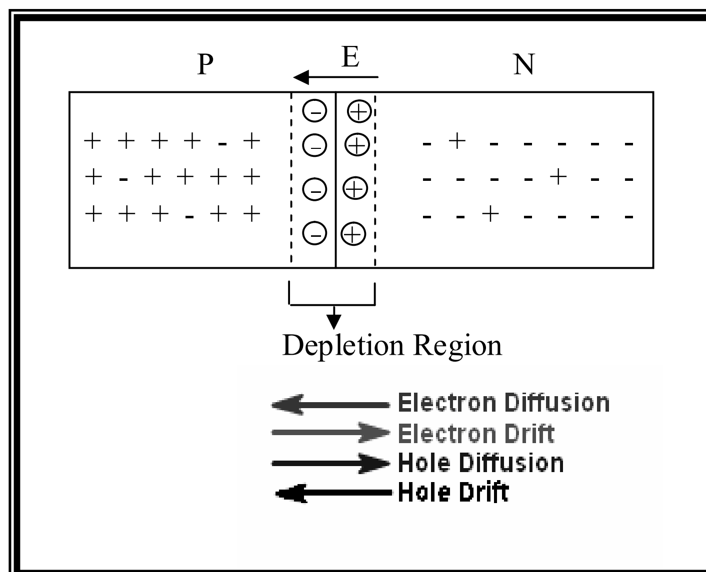


Figure (4-1): P-N junction.

An electric field (E) forms between the positive ion cores in the n-type material and negative ion cores in the p-type material. This region is called

the **"depletion region"** since the electric field quickly sweeps free carriers out, hence the region is depleted of free carriers. A **"built in" or "barrier" potential (V_B)** due to the electric field (E) is formed at the junction. At (300K), the value of (V_B) is (0.7 V) for silicon and (0.3 V) for germanium. The value of (V_B) depends on temperature and the amount of impurities.

The current caused by the diffusion of the majority carriers across the junction is called **the diffusion current**. Minority carriers which reach the edge of the depletion region are swept across it by the electric field in the depletion region. This current is called **the drift current**. **In equilibrium**, the net current from the device is zero. The electron drift current and the electron diffusion current exactly balance out. Similarly, the hole drift current and the hole diffusion current also balance each other out.

4-2 Forward Bias for P-N Junction:

An external voltage ($V_{applied}$) applied to a P-N junction is called **bias**. In forward bias, the positive terminal of the bias battery is connected to the p-type material and the negative terminal of the battery is connected to the n-type material (Figure (4-2)).

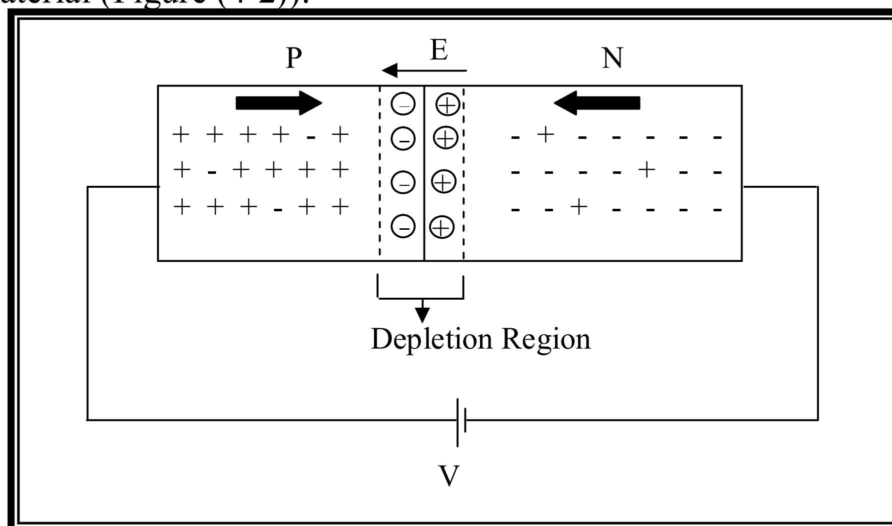


Figure (4-2): Forward Bias for P-N Junction.

The positive potential repels holes toward the junction; at the same time, the negative potential repels electrons toward the junction, therefore, the width of the depletion region decreases. The current resulting from the movement of the holes and the electrons is called **the forward current**. The field of the external voltage opposes the junction field (the field of the barrier potential), therefore, the junction barrier will decrease (the barrier potential decreases from (V_B) to ($V_B - V_{applied}$)) and, therefore, aid high current flow through the junction. Forward bias causes the junction to offer only minimum resistance to the flow of current.

4-3 Reverse Bias for P-N Junction:

The negative battery terminal is connected to the p-type material, and the positive battery terminal to the n-type material (Figure (4-3)).

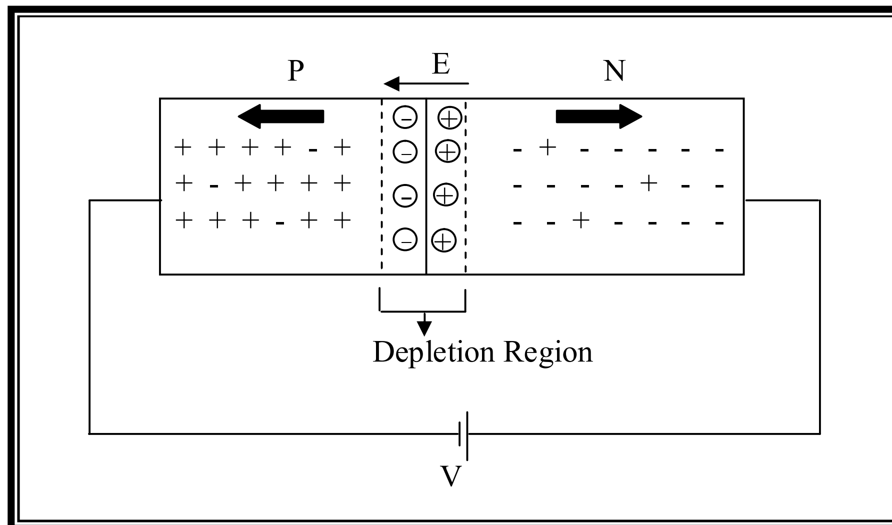


Figure (4-3): Reverse Bias for P-N Junction.

The negative potential attracts the holes away from the edge of the junction barrier on the p side, while the positive potential attracts the electrons away from the edge of the barrier on the n side. This action increases the depletion region width. The electric field of the external

voltage acts in the same direction of the electric field of the barrier potential, therefore, the barrier will increase (the barrier potential increases from (V_B) to ($V_B + V_{applied}$)). Hence, only a very small current will flow through the PN junction in reverse bias, this current is called **the reverse current**.

4-4 Barrier Potential Equation:

$$J_e = \sigma_e E + J_{De}$$

$$J_e = ne\mu_e E + eD_e \frac{dn}{dx}$$

At equilibrium, ($J_e = 0$)

$$eD_e \frac{dn}{dx} = -ne\mu_e E$$

$$\frac{dn}{n} = \frac{-\mu_e}{D_e} Edx$$

$$\because \frac{D_e}{\mu_e} = \frac{KT}{e}$$

$$\therefore \frac{dn}{n} = \frac{-e}{KT} Edx$$

$$\int_{n_p}^{n_n} \frac{dn}{n} = \frac{e}{KT} \int_{x1}^{x2} -Edx$$

$$\because E = -\frac{dV}{dx}$$

$$\therefore dV = -Edx$$

$$\int dV = \int -Edx = V$$

$$\therefore \int_{n_p}^{n_n} \frac{dn}{n} = \frac{e}{KT} V_B$$

$$\ln n]_{n_p}^{n_n} = \frac{e}{KT} V_B$$

$$\ln n_n - \ln n_p = \frac{eV_B}{KT}$$

$$\ln \frac{n_n}{n_p} = \frac{eV_B}{KT}$$

$$\frac{n_n}{n_p} = \exp \frac{eV_B}{KT}$$

$$\therefore n_n = n_p \exp \frac{eV_B}{KT}$$

Similarly, for the holes:

$$p_p = p_n \exp \frac{eV_B}{KT}$$

$$V_B = \frac{KT}{e} \ln(\frac{n_n}{n_p})$$

$$\because n_p \times p_p = ni^2$$

$$n_p \times N_A = ni^2 \Rightarrow n_p = \frac{ni^2}{N_A}$$

$$\therefore V_B = \frac{KT}{e} \ln\left(\frac{N_A N_D}{ni^2}\right)$$

V_B **:** barrier potential (V).

Example (4-1): The conductivity on the two sides of P-N junction is: ($\sigma_n = 1600\,(\Omega.m)^{-1}$) and ($\sigma_p = 0.64\,(\Omega.m)^{-1}$) while the conductivity of pure silicon equals ($\sigma_i = 3.4\times10^{-4}\,(\Omega.m)^{-1}$). Calculate the value of the potential barrier at (300K) assuming that the mobility of the electron in all cases is 3 times the mobility of the hole.

Sol.:

4-5 Depletion Region Width (W):

1) No bias:

$$W = \left[\frac{2\varepsilon V_B (N_D + N_A)}{e N_D N_A}\right]^{\frac{1}{2}}$$

2) Forward bias with voltage ($V_{applied}$):

$$W = \left[\frac{2\varepsilon(V_B - V_{applied})(N_D + N_A)}{eN_D N_A} \right]^{\frac{1}{2}}$$

3) Reverse bias with voltage ($V_{applied}$):

$$W = \left[\frac{2\varepsilon(V_B + V_{applied})(N_D + N_A)}{eN_D N_A} \right]^{\frac{1}{2}}$$

Where:
W : Depletion region width (m).
ε : Electric permittivity for the semiconductor (F/m), where:

$$\varepsilon = \varepsilon_r \varepsilon_o$$

ε_r : The relative permittivity of the material.
ε_o : Vacuum (space) permittivity. (ε_o = $8.85*10^{-12}$ F/m).

The P-N junction acts as a capacitor because the depletion region is free of electric charge like an insulator and on its two sides there are positive and negative charges, the capacitance per unit area of this capacitor is given by:

$$C_j = \frac{\varepsilon}{W}$$

C_j : Capacitance per unit area (F/m^2).

4-6 The Diode:

A **diode** is a two-terminal electronic component that conducts electric current in only one direction. The most common function of a diode is to allow an electric current to pass in one direction (called the diode's forward direction) while blocking current in the opposite direction (the reverse direction).

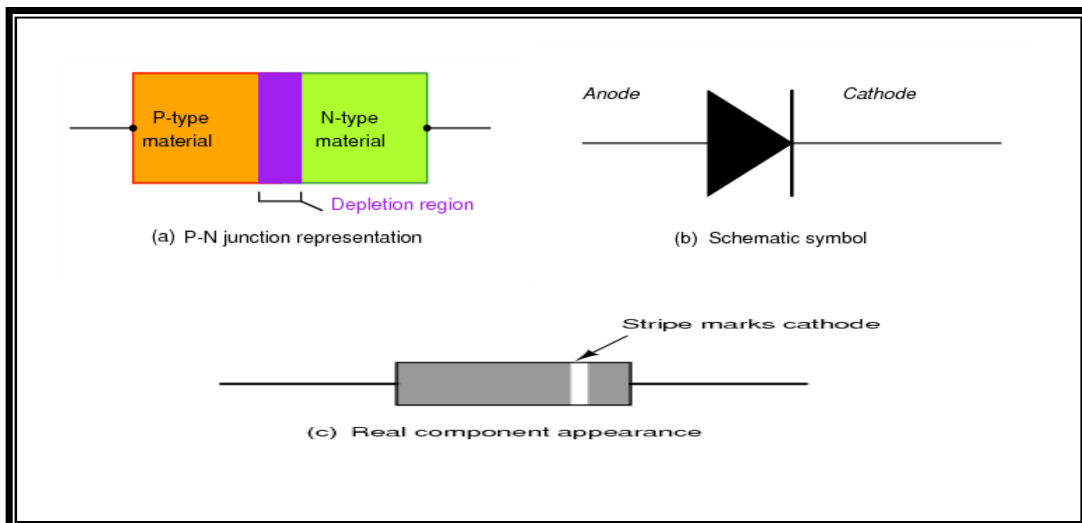


Figure (4-3): Diode representations.

4-7 Current Equation of the diode:

When there is no bias applied on the diode:

$$p_p = p_n \exp\frac{eV_B}{KT} \Rightarrow p_n = p_p \exp\frac{-eV_B}{KT}$$

$$n_n = n_p \exp\frac{eV_B}{KT} \Rightarrow n_p = n_n \exp\frac{-eV_B}{KT}$$

If a forward bias (V) is applied, then the number of holes will increase from (p_n) to (p_e) because the barrier potential is decreased in forward bias case:

$$p_e = p_p \exp \frac{-e(V_B - V)}{KT}$$

Then the increase in the number of holes (Δp_o) is:

$$p_e = p_n + \Delta p_o$$

$$\Delta p_o = p_e - p_n$$

$$\Delta p_o = p_p \exp \frac{-eV_B}{KT}\left[\exp\frac{eV}{KT} - 1\right]$$

$$\therefore \Delta p_o = p_n\left[\exp\frac{eV}{KT} - 1\right]$$

Similarly, the increase in the number of electrons (Δn_o) is

$$\Delta n_o = n_p\left[\exp\frac{eV}{KT} - 1\right]$$

In order to find the holes' current density (J_{po}), we have:

$$J_{po} = -eD_p \frac{d\Delta p}{dx}$$

$$\Delta p = \Delta p_o \exp \frac{-x}{L_p}$$

$$\therefore \frac{d\Delta p}{dx} = -\frac{\Delta p_o}{L_p} \exp \frac{-x}{L_p}$$

$$at\,(x = 0)$$

$$\left(\frac{d\Delta p}{dx}\right)_{x=o} = \frac{-\Delta p_o}{L_p}$$

$$\therefore J_{po} = eD_p \frac{\Delta p_o}{L_p}$$

$$\because \Delta p_o = p_n\left[\exp\frac{eV}{KT} - 1\right]$$

$$\therefore J_{po} = \frac{eD_p p_n}{L_p}\left[\exp\frac{eV}{KT} - 1\right]$$

Similarly, for the electrons, the current density (J_{no}) is:

$$J_{no} = \frac{eD_n n_p}{L_n}\left[\exp\frac{eV}{KT} - 1\right]$$

Then the total current density (J_T) is:

$$J_T = J_{po} + J_{no}$$

$$J_T = e\left[\frac{D_p p_n}{L_p} + \frac{D_n n_p}{L_n}\right]\left[\exp\frac{eV}{KT} - 1\right]$$

The total current (I) is:

$$I = J \times A$$

$$I = \underbrace{Ae\left[\frac{D_p p_n}{L_p} + \frac{D_n n_p}{L_n}\right]}_{\text{Reverse saturation current } (I_s)}\left[\exp\frac{eV}{KT} - 1\right]$$

$$\therefore I = I_s\left[\exp(\frac{eV}{KT}) - 1\right]$$

The last equation can be rewritten in the following form:

$$I = I_s\left[\exp\left(\frac{V}{\eta V_T}\right) - 1\right]$$

Where:

I : The diode current (A).

V : The applied voltage on the diode → (+ V) for forward bias; → (- V) for reverse bias

$$V_T = \frac{KT}{e} = \frac{T}{11600}$$

η : Correction factor → ($\eta = 1$) for germanium; → ($\eta = 2$) for silicon

Example (4-2): P-N junction with the following specifications: The impurities concentration in the P side is ($N_A = 10^{24}/m^3$) and in the N side is ($N_D = 10^{22}/m^3$), the sectional area equals ($10^{-6}\,m^2$), the holes mobility equals ($\mu_p = 0.2\,\frac{m^2}{v.\sec.}$), the electrons mobility equals ($\mu_n = 0.4\,\frac{m^2}{v.\sec.}$), the hole's diffusion length equals ($200\mu m$),the electron's diffusion length equals ($300\mu m$), the relative permittivity equals ($\varepsilon_r = 16$), and the intrinsic carriers concentration equals ($n_i = 10^{19}/m^3$). Calculate the following at 290°K:

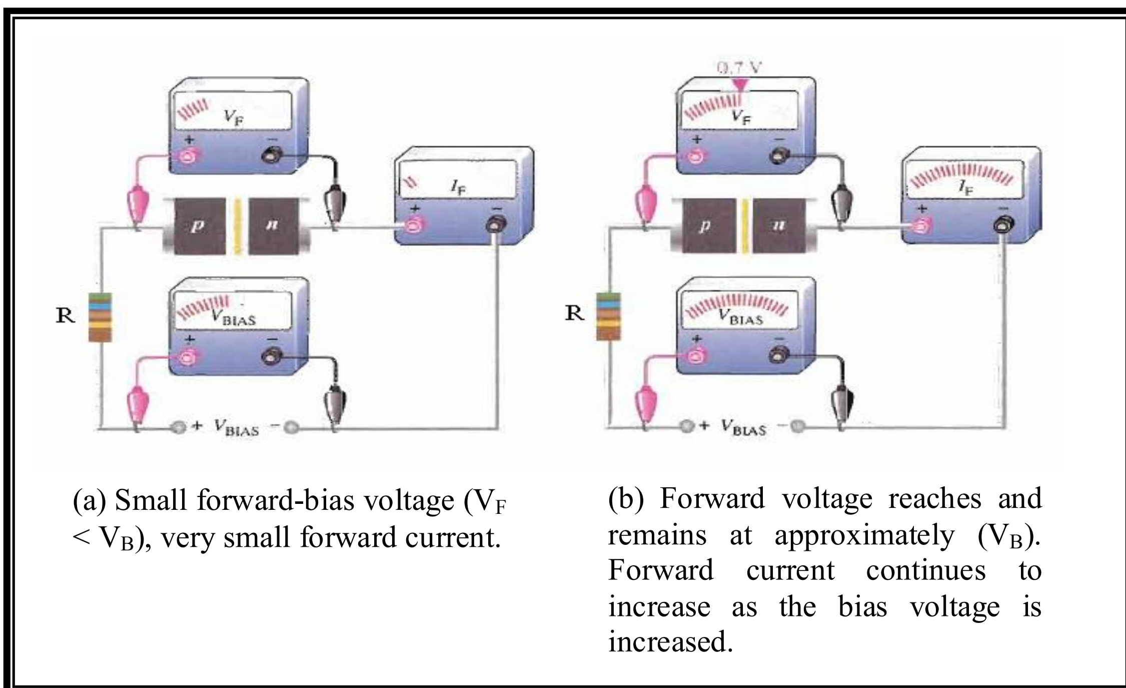
1) The concentration of the majority and the minority carriers in both sides. 2) The conductivity in both sides. 3) The potential barrier. 4) The diffusion constant for the two types of carriers. 5) The reverse saturation current. 6) The current in the junction when applying (0.25 v) in forward bias case. 7) The depletion region width when applying (10 v) in reverse bias case. 8) The depletion region capacitor for (10 v) in reverse bias case.

Sol.

4-7 Voltage-Current Characteristic of a Diode:

V-I Characteristic for Forward Bias:

Figure (4-4) illustrates what happens as the forward-bias voltage is increased positively from 0 V. The resistor is used to limit the forward current to a value that will not overheat the diode and cause damage.



(a) Small forward-bias voltage ($V_F < V_B$), very small forward current.

(b) Forward voltage reaches and remains at approximately (V_B). Forward current continues to increase as the bias voltage is increased.

Figure (4-4) Forward-bias measurements show general changes in V_F and I_F as V_{BIAS} is increased.

With (0 V) across the diode, there is no forward current. As you gradually increase the forward-bias voltage, the forward current and the voltage across the diode gradually increase, as shown in Figure (4-4(a)). A portion of the forward-bias voltage is dropped across the limiting resistor. When the forward-bias voltage is increased to a value where the voltage across the diode reaches approximately the barrier potential, the forward current begins to increase rapidly, as illustrated in Figure (4-4(b)). As you continue to increase the forward-

bias voltage, the current continues to increase very rapidly, but the voltage across the diode increases only gradually above the barrier potential. This small increase in the diode voltage above the barrier potential is due to the voltage drop across the internal dynamic resistance of the semiconductive material.

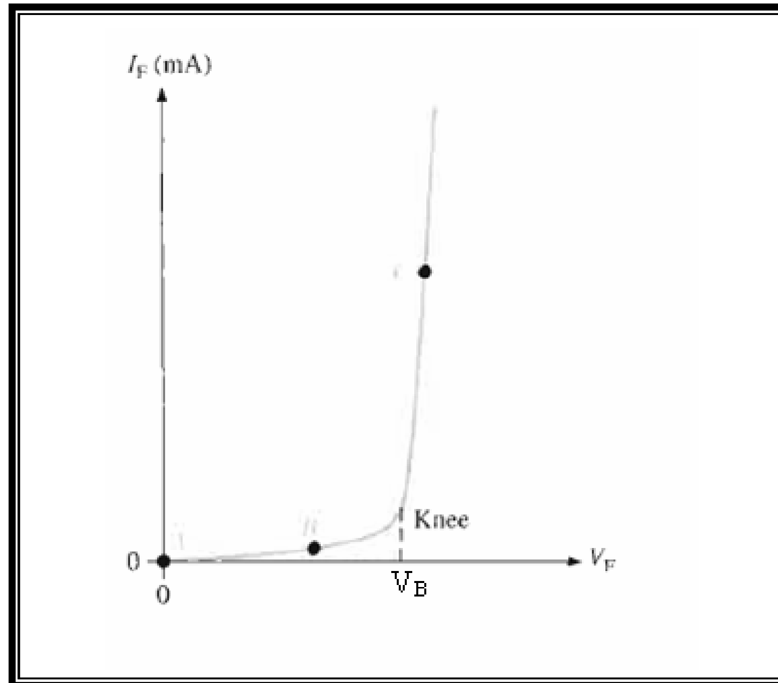


Figure (4-5): V-I characteristic curve for a forward-biased diode.

V-I Characteristic for Reverse Bias:

When a reverse-bias voltage is applied across a diode, there is only an extremely small reverse current (I_R) through the pn junction. With (0 V) across the diode, there is no reverse current. As you gradually increase the reverse-bias voltage, there is a very small reverse current and the voltage across the diode increases. When the applied bias voltage is increased to a value where the reverse voltage across the diode (V_R) reaches **the breakdown value (V_{BR})**, the reverse current begins to increase rapidly. As you continue to increase the bias voltage, the current continues to

increase very rapidly, but the voltage across the diode increases very little above (V_{BR}).

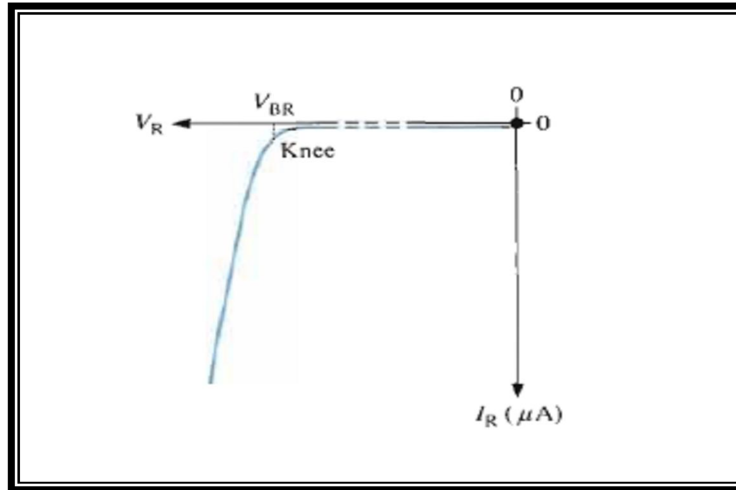


Figure (4-6): V-I characteristic curve for a reverse-biased diode.

The Complete V-I Characteristic Curve:

Combine the curves for both forward and reverse bias to get the complete curve as shown in figure (4-7). Notice that the (I_F) scale is in (mA) compared to the (I_R) scale in (μA).

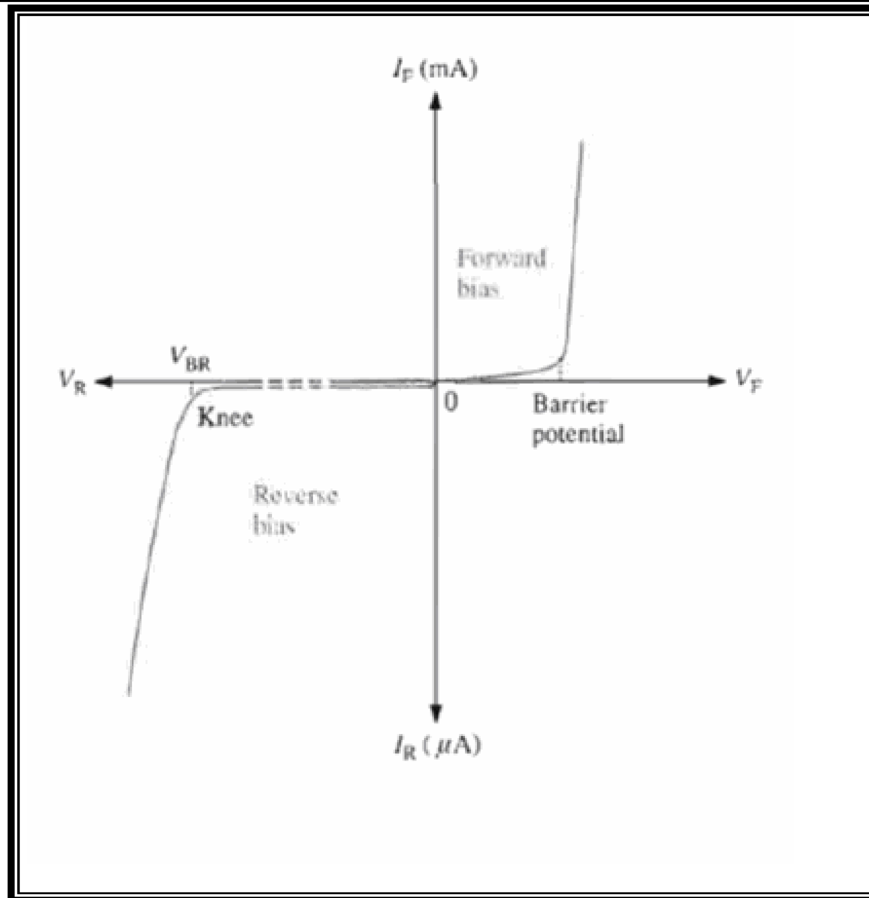


Figure (4-7): The complete V-I characteristic curve for a diode.

Figure (4-8) compares between the V-I characteristic curve for silicon diode and germanium diode.

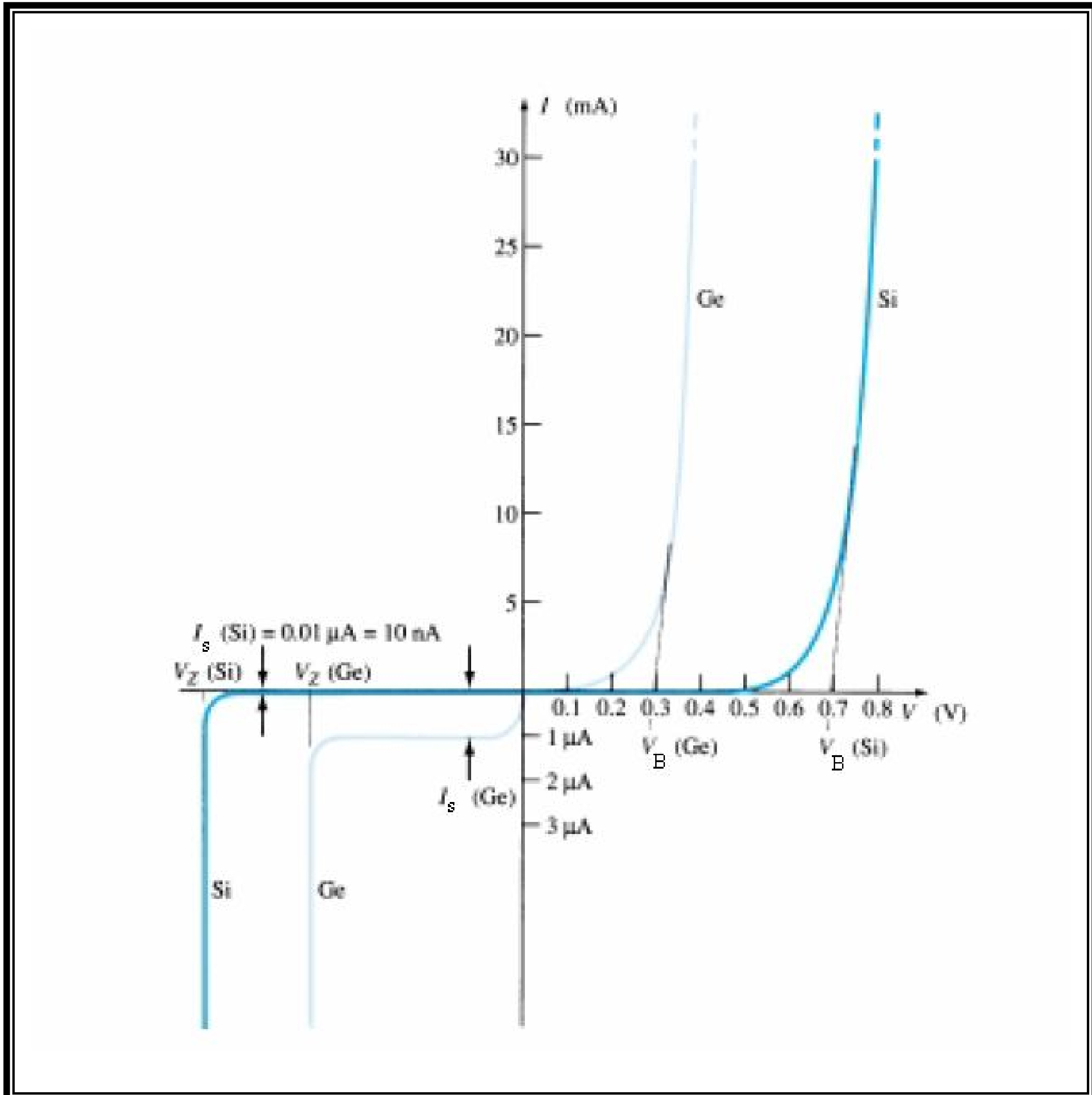


Figure (4-8): Comparison between the V-I characteristic curve for silicon diode and germanium diode.

4-9 Diode Models:

In this section, you will learn how the diode can be modeled for circuit analysis using three levels of complexity.

4-9-1 The Ideal Diode Model:

The ideal model of a diode is a simple switch. When the diode is forward-biased, it acts like a closed (on) switch, as shown in Figure (4-9(a)). When the diode is reverse-biased, it acts like an open (off) switch, as shown in Figure (4-9(b)). The barrier potential, the forward dynamic resistance, and the reverse current are all neglected.

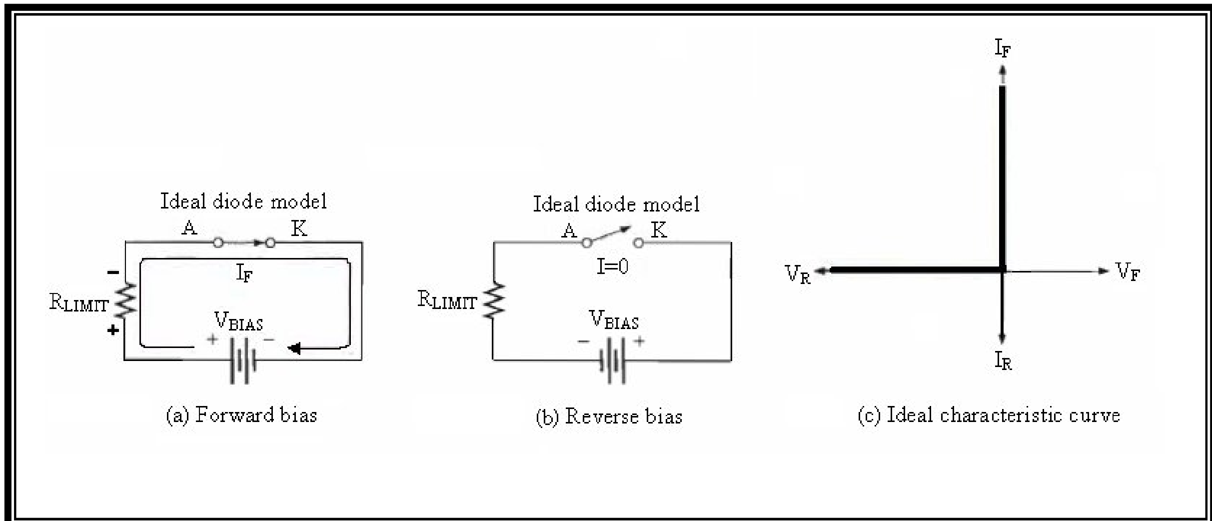


Figure (4-9): The ideal model of a diode.

In Figure (4-9(c)), the ideal V-I characteristic curve graphically depicts the ideal diode operation. Since the barrier potential and the forward dynamic resistance are neglected, the diode is assumed to have a zero voltage across it when forward-biased, as indicated by the portion of the curve on the positive vertical axis.

$$V_F = 0V$$

The forward current is determined by:

$$I_F = \frac{V_{BIAS}}{R_{LIMIT}}$$

Since the reverse current is neglected, its value is assumed to be zero, as indicated in Figure (4-9(c)) by the portion of the curve on the negative horizontal axis.

$$I_R = 0A$$

The reverse voltage equals the bias voltage.

$$V_R = V_{BIAS}$$

4-9-2 The Practical Diode Model:

The practical model adds the barrier potential to the ideal switch mode. When the diode is forward-biased, it is equivalent to a closed switch in series with a small equivalent voltage source equal to the barrier potential with the positive side toward the anode, as indicated in Figure (4-10(a)).

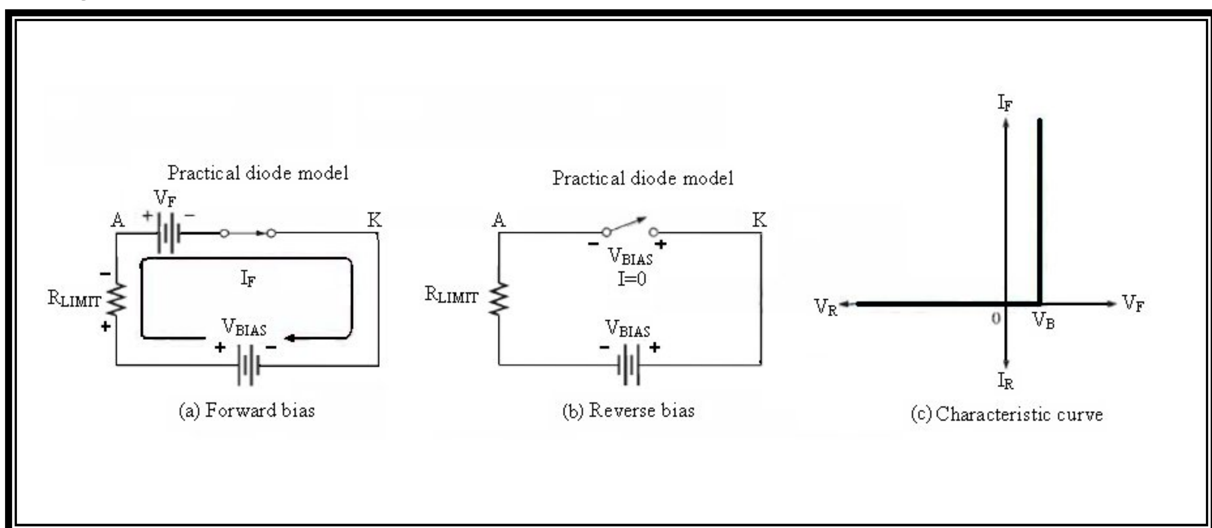


Figure (4-10): The practical model of a diode.

When the diode is reverse-biased, it is equivalent to an open switch just as in the ideal model, as shown in Figure (4-10(b)). The barrier potential does not affect reverse bias, so it is not a factor.

The characteristic curve for the practical diode model is shown in Figure (4-10(c)). Since the barrier potential is included and the dynamic resistance is neglected, the diode is assumed to have a voltage across it when forward-biased, as indicated by the portion of the curve to the right of the origin.

$$V_F = V_B$$

The forward current is determined as follows by first applying Kirchhoff's voltage law to Figure (4-10(a)):

$$V_{BIAS} - V_F - V_{R_{LIMIT}} = 0$$

$$V_{R_{LIMIT}} = I_F R_{LIMIT}$$

Substituting and solving for (I_F),

$$I_F = \frac{V_{BIAS} - V_F}{R_{LIMIT}}$$

The diode is assumed to have zero reverse current, as indicated by the portion of the curve on the negative horizontal axis.

$$I_R = 0A$$

$$V_R = V_{BIAS}$$

4-9-3 The Complete Diode Model:

The complete model of a diode consists of the barrier potential, the small forward dynamic resistance (r_d'), and the large internal reverse resistance (r_R'). The reverse resistance is taken into account

because it provides a path for the reverse current, which is included in this diode model.

When the diode is forward-biased, it acts as a closed switch in series with the barrier potential voltage and the small forward dynamic resistance ($r_d^{'}$), as indicated in Figure (4-11(a)). When the diode is reverse-biased, it acts as an open switch in parallel with the large internal reverse resistance ($r_R^{'}$), as shown in Figure (4-11(b)). The barrier potential does not affect reverse bias, so it is not a factor.

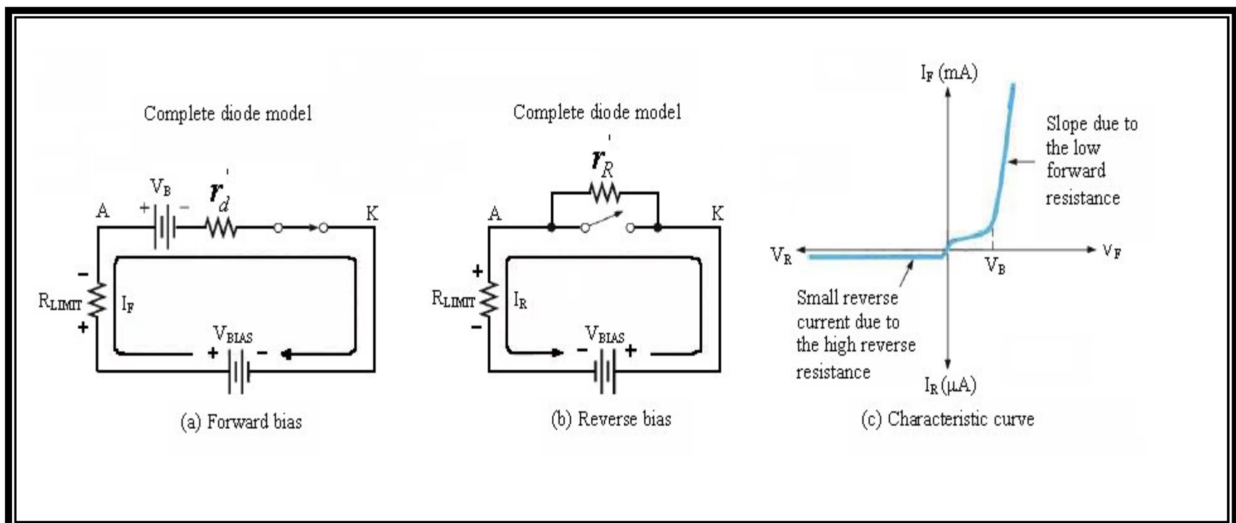


Figure (4-11): The complete model of a diode.

The characteristic curve for the complete diode model is shown in Figure (4-11(c)). Since the barrier potential and the forward dynamic resistance are included, the diode is assumed to have a voltage across it when forward-biased. This voltage (V_F) consists of the barrier potential voltage plus the small voltage drop across the dynamic resistance, as indicated by the portion of the curve to the right of the origin. The curve slopes because the voltage drop due to dynamic

resistance increases as the current increases. For the complete model of a diode, the following formulas apply:

$$V_F = V_B + I_F r_d^{'}$$

$$I_F = \frac{V_{BIAS} - V_B}{R_{LIMIT} + r_d^{'}}$$

The reverse current is taken into account with the parallel resistance and is indicated by the portion of the curve to the left of the origin. The breakdown portion of the curve is not shown because breakdown is not a normal mode of operation for most diodes.

Example (4-3):

(a) Determine the forward voltage and forward current for the Si diode in Figure (4-12(a)) for each of the diode models. Also find the voltage across the limiting resistor in each case. Assume ($r_d^{'} = 10\Omega$) at the determined value of forward current.

(b) Determine the reverse voltage and reverse current for the Si diode in Figure (4-12(b)) for each of the diode models. Also find the voltage across the limiting resistor in each case. Assume $I_R = 1\mu A$.

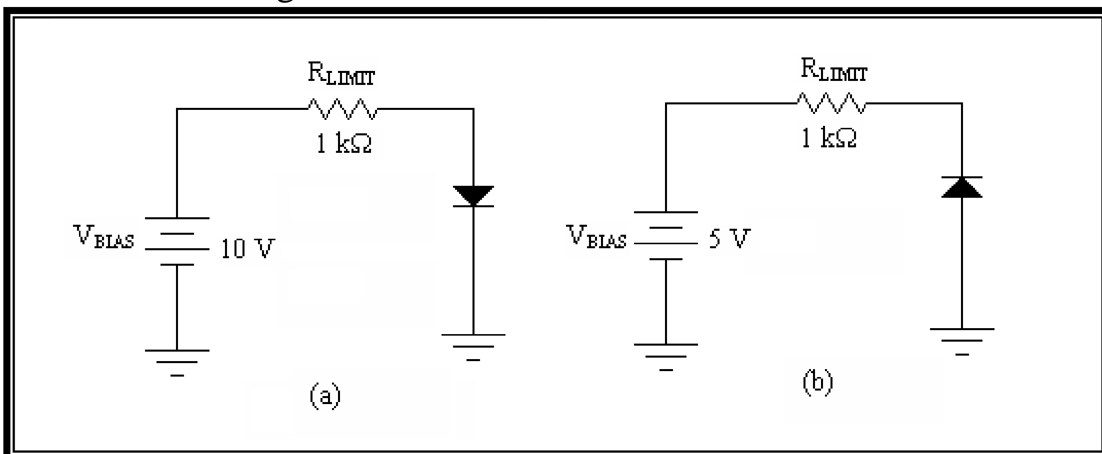


Figure (4-11): Example (4-3).

Sol.

4-10 DC Load Line:

For the following circuit:

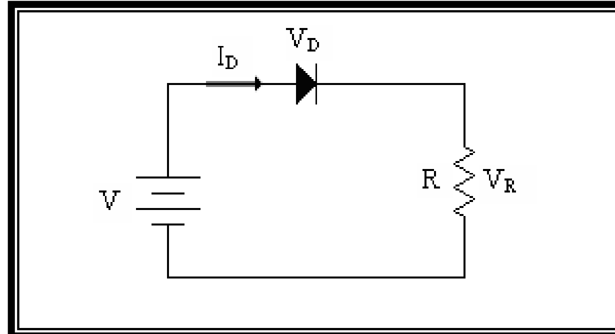


Figure (4-12)

The following equations can be written:

$$V = V_D + V_R$$
$$V = V_D + I_D .R$$

The previous equation represents the equation of a straight line between (V_D) and (I_D). To draw this line:

$$1)\ I_D = 0 \Rightarrow V_D = V$$
$$2)\ V_D = 0 \Rightarrow I_D = \frac{V}{R}$$

If this line is drawn on the V-I characteristic curve, the following figure will be produced:

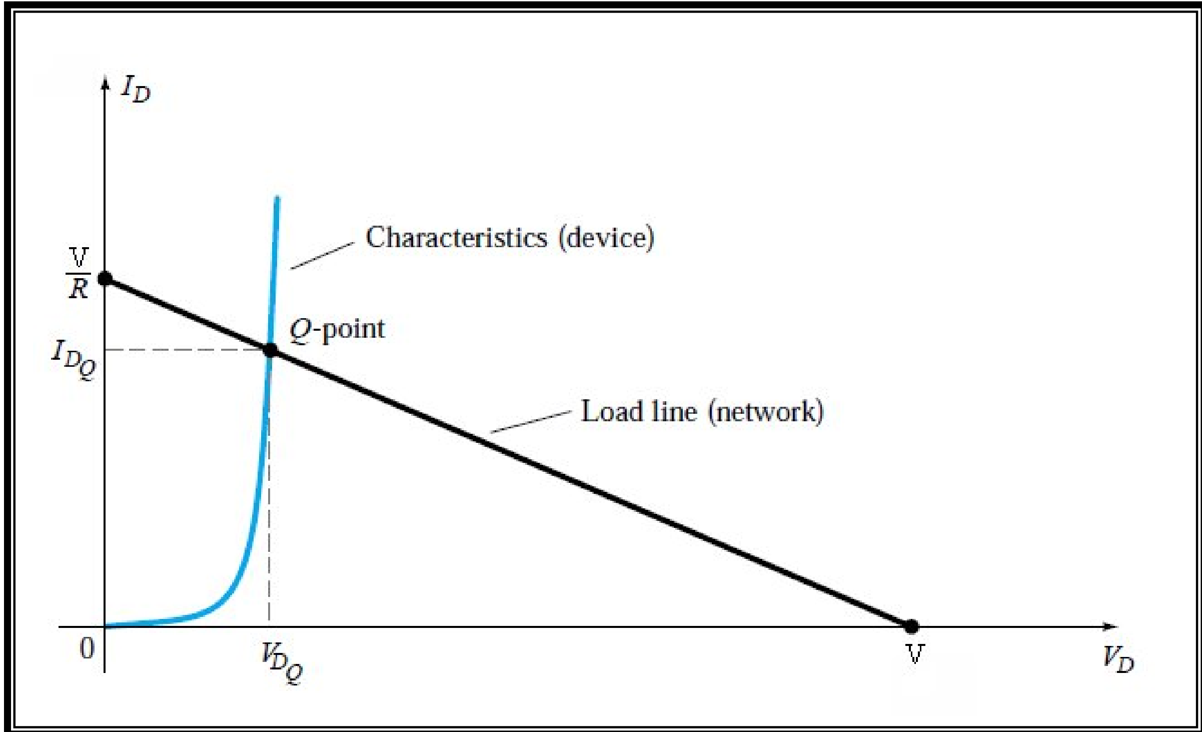


Figure (4-13): Drawing the load line and finding the point of operation.

The intersection of the straight line with the V-I characteristic curve of the diode gives a point that is called **the operating point (the quiescent point (abbreviated "Q-pt."))**. The straight line is called **the load line**. From the operating point we can find the current passing in the circuit and the voltage across the diode by drawing two straight lines, the first line is perpendicular to the (v_D) axis and the second line is parallel to the same axis. For example, for the previous circuit (figure (4-12)) if

$$R = 2\ K\Omega$$
$$V = 20\ V$$

then to draw the load line:

$$I_D = \frac{V}{R} = \frac{20}{2k} = 10 \ mA$$

$$V_D = V = 20 \ V$$

After drawing the V-I characteristic curve and the load line we get the following from the resulting figure:

$$V_Q = 1V \ , I_Q = 9.5 \ mA$$

where (I_Q) is the current passing in the circuit and (V_Q) is the voltage across the diode, (V_R) can be found from:

$$V_R = V - V_D = V - V_Q = 20 - 1 = 19 \ v$$

or from: $$V_R = I_Q . R = 9.5 \times 2 = 19 \ v$$

4-10 RESISTANCE LEVELS:

As the operating point of a diode moves from one region to another the resistance of the diode will also change due to the nonlinear shape of the characteristic curve. It will be demonstrated in the next few paragraphs that the type of applied voltage or signal will define the resistance level of interest. Three different levels will be introduced in this section

4-10-1 DC or Static Resistance:

There are two methods to find the static resistance:

a) Physically:

$$R_{static} = R_{dc} = r_p + r_n$$

$$r_p = \rho_p \frac{L}{A} = \frac{L}{N_A e \mu_h A}$$

$$r_n = \rho_n \frac{L}{A} = \frac{L}{N_D e \mu_e A}$$

b) Graphically:

The application of a dc voltage to a circuit containing a semiconductor diode will result in an operating point on the characteristic curve that will not change with time. The resistance of the diode at the operating point can be found simply by finding the corresponding levels of (V_D) and (I_D) as shown in Figure (4-14) and applying the following equation:

$$R_{dc} = \frac{V_D}{I_D}$$

The dc resistance levels at the knee and below will be greater than the resistance levels obtained for the vertical rise section of the characteristics. The resistance levels in the reverse-bias region will naturally be quite high.

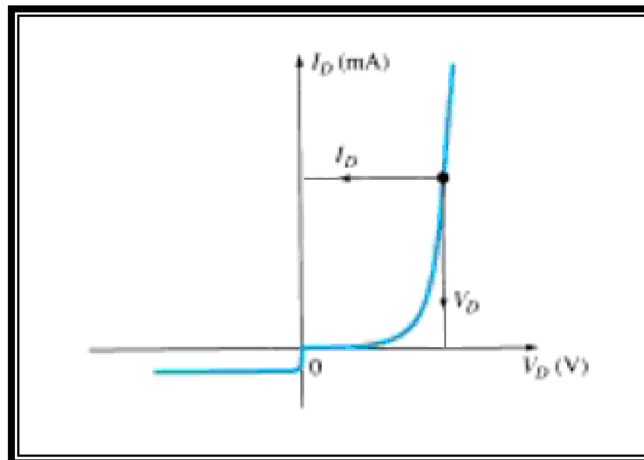


Figure (4-14): Determining the dc resistance of a diode at a particular operating point.

Example (4-4):

Determine the dc resistance levels for the diode of figure (4-15) at

(a) $I_D = 2\,mA$

(b) $I_D = 20\,mA$

(c) $V_D = -10\,V$

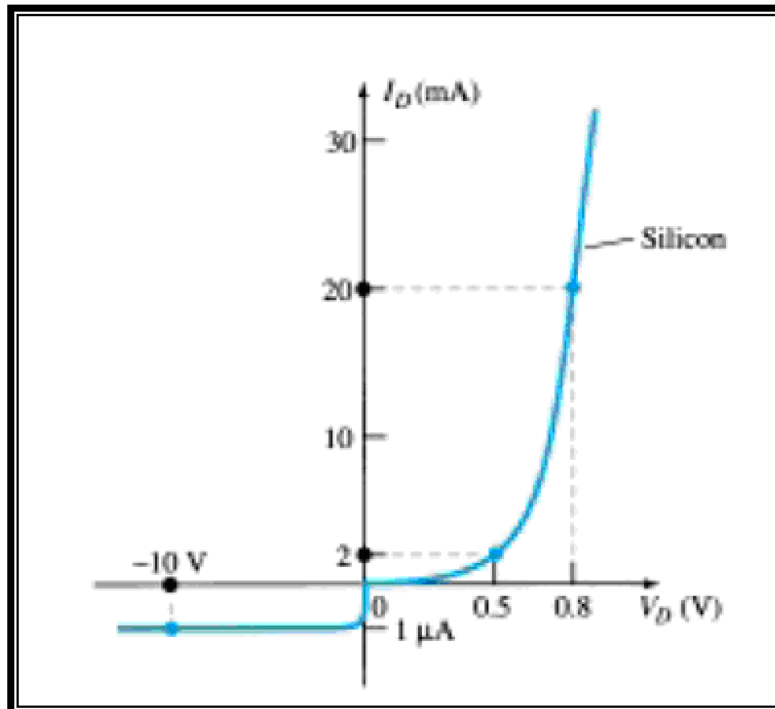


Figure (4-15): Example (4-4).

Sol.

4-10-2 AC or Dynamic Resistance:

There are two methods to find the static resistance:

a) Graphically:

It is obvious from Example (4-4) that the dc resistance of a diode is independent of the shape of the characteristic in the region surrounding the point of interest. If a sinusoidal rather than dc input is applied, the situation will change completely. The varying input will move the instantaneous operating point up and down a region of the characteristics and thus defines a specific change in current and voltage as shown in figure (4-16). With no applied varying signal, the point of operation would be the *Q*-point appearing on figure (4-16) determined by the applied dc levels. The designation *Q-point* is derived from the word *quiescent,* which means "still or unvarying."

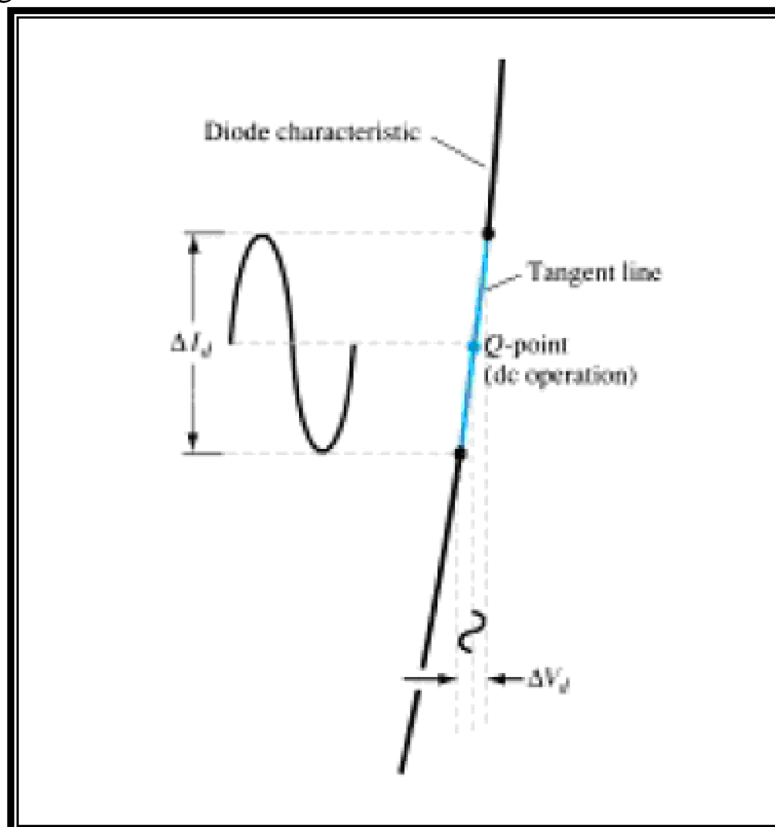


Figure (4-15): Defining the dynamic or ac resistance.

A straight line drawn tangent to the curve through the *Q*-point as shown in Figure (4-16) will define a particular change in voltage and current that can be used to determine the ac or dynamic resistance for this region of the diode characteristics. An effort should be made to keep the change in voltage and current as small as possible and equidistant to either side of the *Q*-point. In equation form,

$$r_{ac} = \frac{\Delta V_d}{\Delta I_d}$$

where Δ signifies a finite change in the quantity.

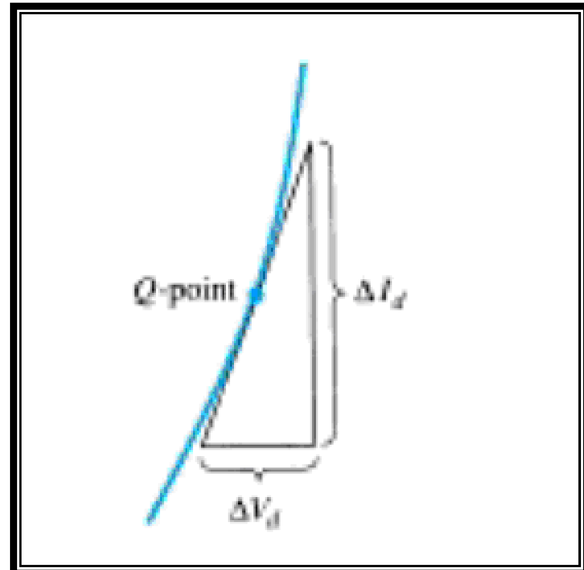


Figure (4-16): Determining the ac resistance at a Q-point.

The steeper the slope, the less the value of ΔV_d for the same change in ΔI_d and the less the resistance. The ac resistance in the vertical-rise region of the characteristic is therefore quite small, while the ac resistance is much higher at low current levels. The discussion above has centered solely on the forward-bias region. In the reverse-bias region we will assume that the change in current along the I_s line is nil from 0 V to the Zener region and the resulting ac resistance is sufficiently high to permit the open-circuit approximation.

Example (4-5):

For the characteristics of Figure (4-17):

(a) Determine the ac resistance at $I_D = 2\,mA$.

(b) Determine the ac resistance at $I_D = 25\,mA$.

(c) Compare the results of parts (a) and (b) to the dc resistances at each current level.

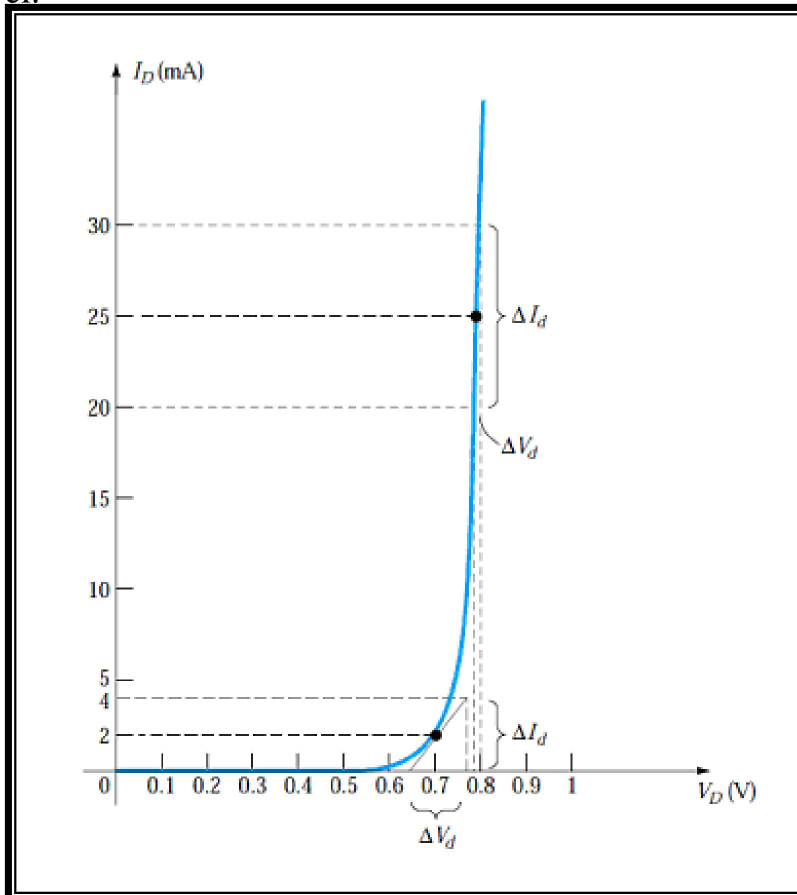


Figure (4-17): Example (4-5).

Sol.

b) Analytically: There is a basic definition in differential calculus which states:

$$r_{dynamic} = r_{a.c} = \frac{1}{\left(\frac{dI}{dV}\right)}$$

$$I = I_s\left[\exp\left(\frac{V}{\eta V_T}\right) - 1\right]$$

$$I = I_s \exp\left(\frac{V}{\eta V_T}\right) - I_s$$

$$I + I_s = I_s \exp\left(\frac{V}{\eta V_T}\right)$$

$$\because\ I >> I_s$$

$$\therefore I = I_s \exp\left(\frac{V}{\eta V_T}\right)$$

$$\frac{dI}{dV} = \frac{I_s \exp\left(\frac{V}{\eta V_T}\right)}{\eta V_T}$$

$$\frac{dI}{dV} = \frac{I}{\eta V_T}$$

$$\therefore r_{a.c} = \frac{dV}{dI} = \frac{\eta V_T}{I}$$

At room temperature (T=300 K)

$$V_T = \frac{T}{11600} = \frac{300}{11600} = 26\,mv$$

$$r_{a.c} = \frac{\eta \times 26mv}{I}$$

For germanium (Ge): $\eta = 1$

$$\therefore r_{a.c} = \frac{26\ mv}{I(mA)}$$

For silicon (Si): $\eta = 2$

$$\therefore r_{a.c} = \frac{2 \times 26\ mv}{I(mA)} = \frac{52\ mv}{I(mA)}$$

4-10-3Average AC Resistance:

If the input signal is sufficiently large to produce a broad swing such as indicated in Figure (4-18), the resistance associated with the device for this region is called the average ac resistance. The average ac resistance is, by definition, the resistance determined by a straight line drawn between the two intersections established by the maximum and minimum values of input voltage. In equation form:

$$r_{av} = \left.\frac{\Delta V_d}{\Delta I_d}\right]_{pt.to\ pt.}$$

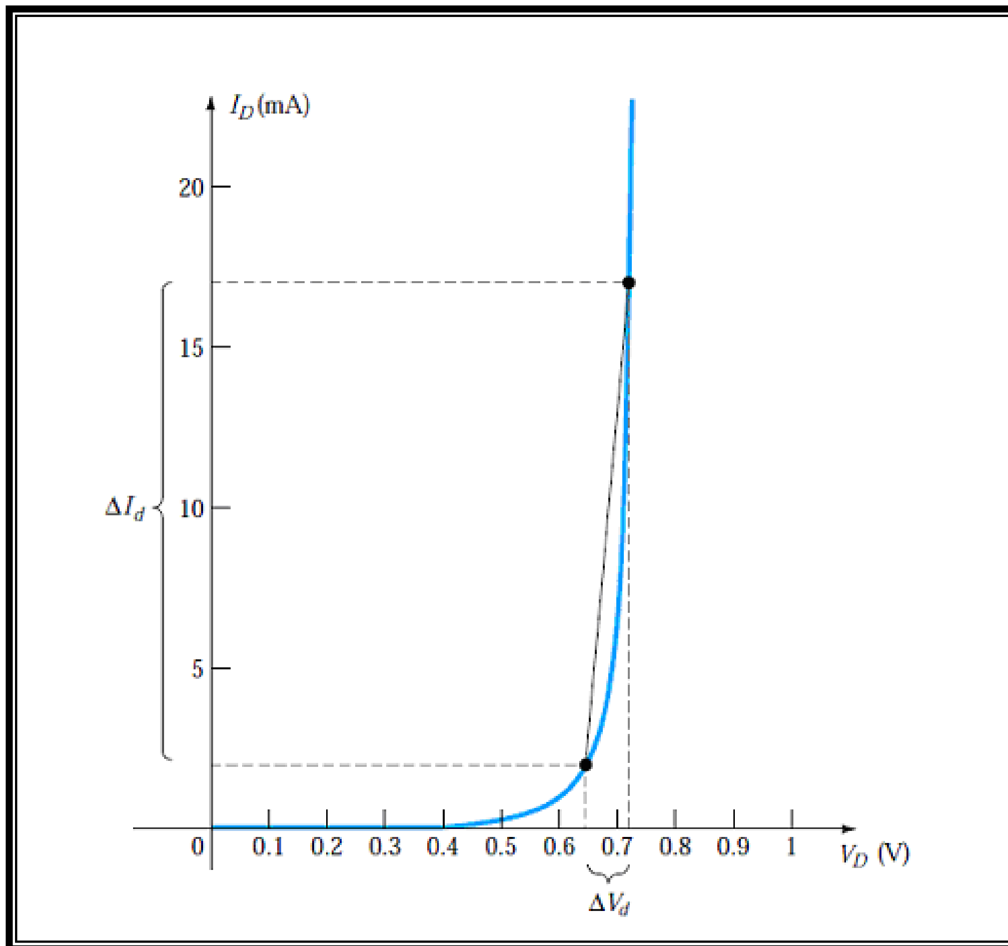


Figure (4-18): Determining the average ac resistance between indicated limits.

For the situation indicated by Figure (4-18),

$$\Delta I_d = 17\,mA - 2\,mA = 15\,mA$$

$$\Delta V_d = 0.725V - 0.65V = 0.075V$$

$$r_{av} = \frac{\Delta V_d}{\Delta I_d} = \frac{0.075V}{15|\,mA} = 5\Omega$$

If the ac resistance (r_{ac}) were determined at $I_d = 2\,mA$ its value would be more than 5Ω , and if determined at 17 mA it would be less. In between the ac resistance would make the transition from the high value at 2 mA to the lower value at 17 mA.

4-11 Temperature Effects on the diode V-I Characteristics:

Diode characteristics are affected by the temperature, this is because temperature affects the barrier voltage (V_B) also, the temperature strongly affects the density of minority carriers (n_p and p_n) which play a key role in determining the reverse saturation current (I_s). For these reasons, we see that the diode current is affected by temperature change in both directions (forward and reverse). In addition, the breakdown voltage (V_{BR}) changes with temperature. The change in V-I characteristics curve of the diode with temperature increase is shown in the following figure.

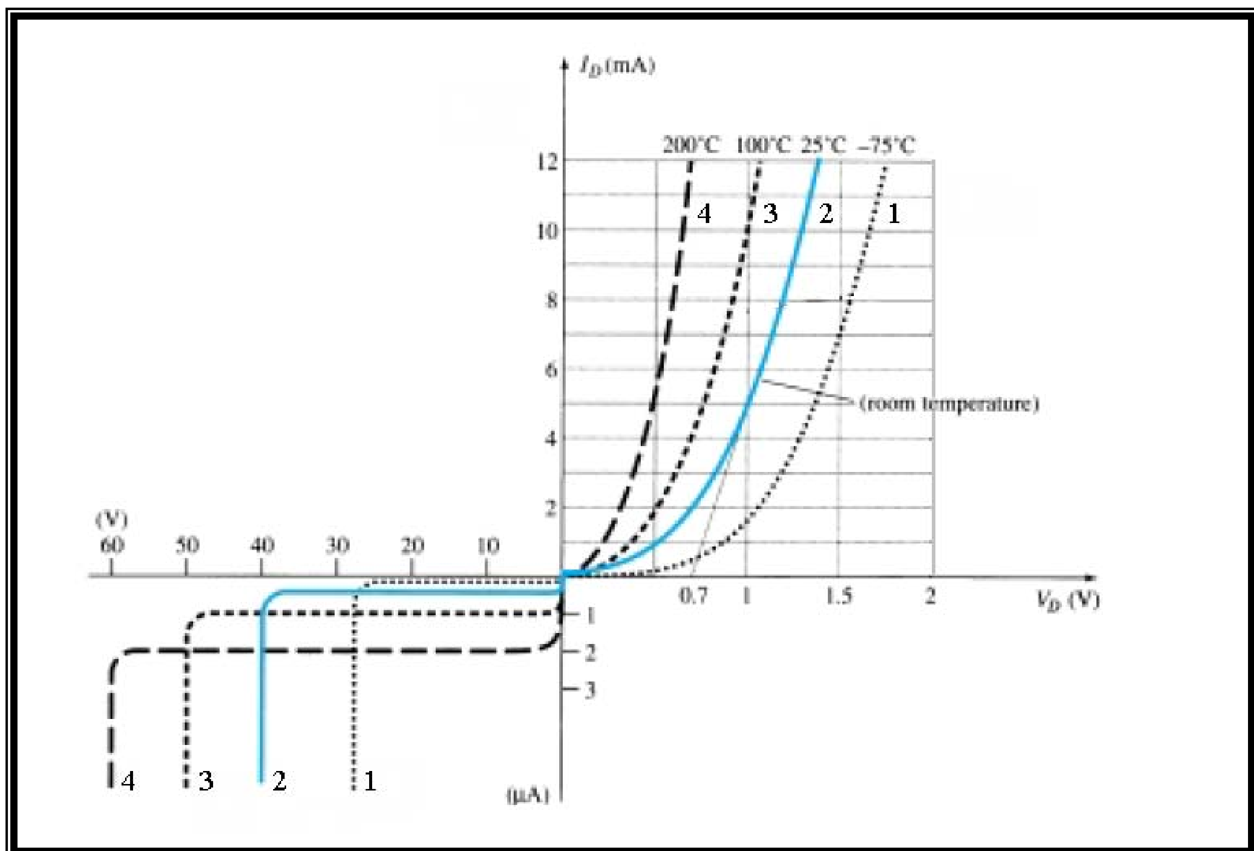


Figure (4-21): Variation in diode characteristics with temperature change.

4-12 Breakdown in Diode:

Normally, the reverse current is so small that it can be neglected. However, if the external reverse-bias voltage is increased to a value called the breakdown voltage, the reverse current will drastically increase. The high reverse-bias voltage imparts energy to the free minority electrons so that as they speed through the p region, they collide with atoms with enough energy to knock valence electrons out of orbit and into the conduction band. The newly created conduction electrons are also high in energy and repeat the process. If one electron knocks only two others out of their valence orbit during its travel through the p region, the numbers quickly multiply. As these high-energy electrons go through the depletion region, they have enough energy to go through the n region as conduction electrons, rather than combining with holes. The multiplication of conduction electrons just discussed results in a very high reverse current that can damage the diode because of excessive heat dissipation.

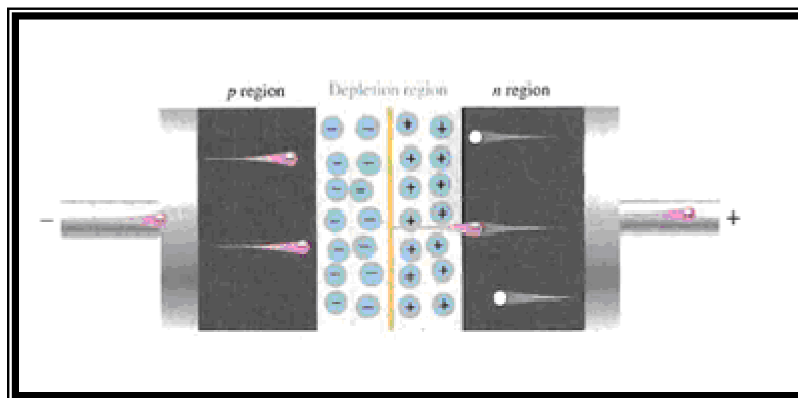


Figure (4-22): Breakdown in Diode.

4-13 Diode Applications

4-13-1 Clipping:

There are a variety of diode networks called **clippers** that have the ability to “clip” off a portion of the input signal without distorting the remaining part of the alternating waveform. Depending on the orientation of the diode, the positive or negative region of the input signal is “clipped” off. There are two general categories of clippers: **series** and **parallel**. The series configuration is defined as one where the diode is in series with the load, while the parallel variety has the diode in a branch parallel to the load.

4-13-1-a: Series Clipping:

Example (4-6): Sketch the output waveform for the following circuits:

1)

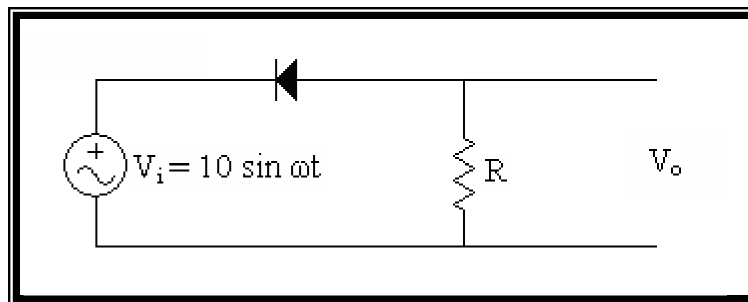


Figure (4-23): Example (4-6-1).

Sol.:

2)

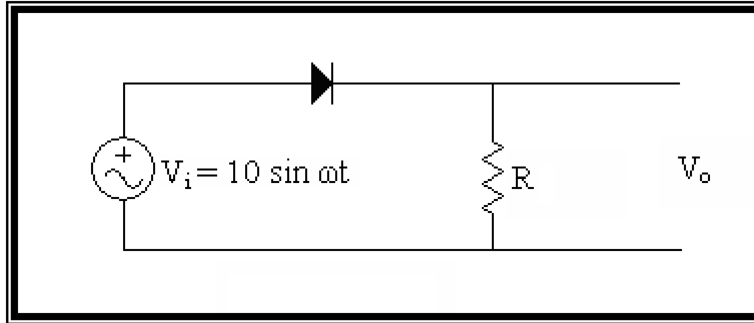


Figure (4-24): Example (4-6-2).

Sol.:

3)

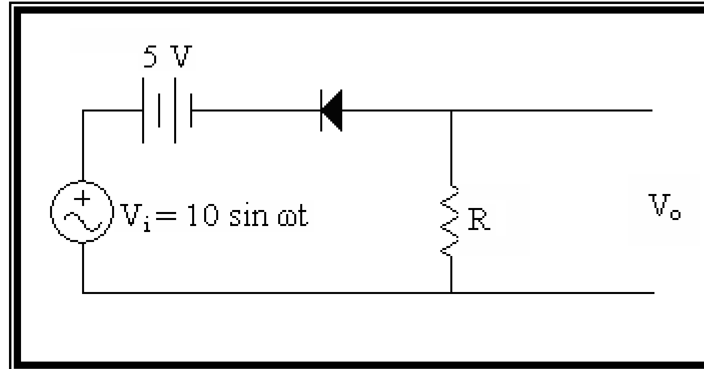


Figure (4-25): Example (4-6-3).

Sol.:

4)

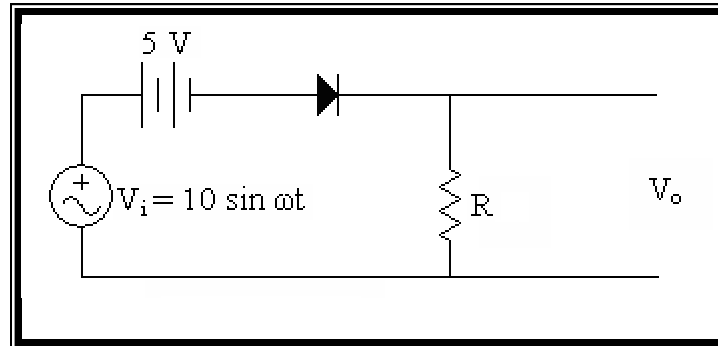


Figure (4-26): Example (4-6-4).

Sol.:

4-13-1-b: Parallel Clipping:

Example (4-7): Sketch the output waveform for the following circuits:

1)

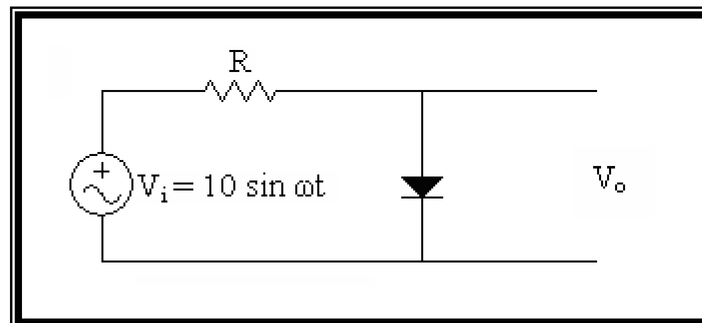


Figure (4-27): Example (4-7-1).

Sol.:

2)

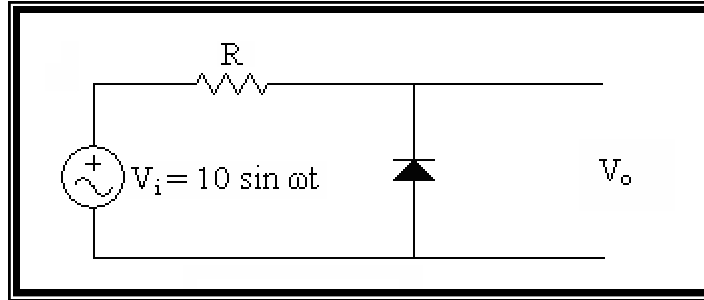


Figure (4-28): Example (4-7-2).

Sol.:

3)

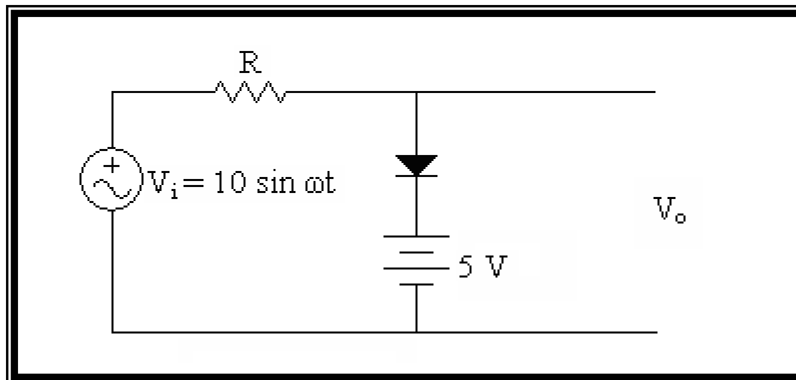


Figure (4-29): Example (4-7-3).

Sol.:

4)

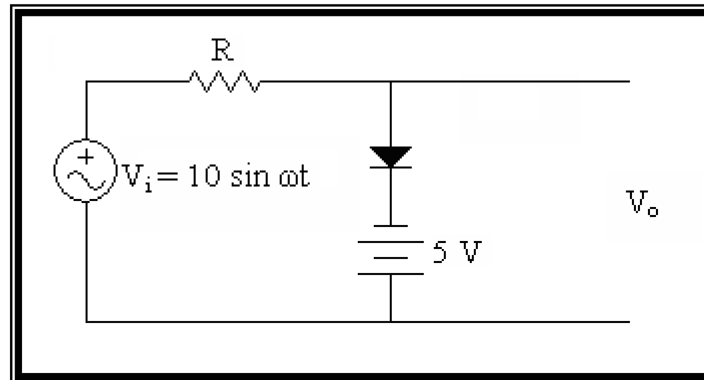


Figure (4-30): Example (4-7-4).

Sol.:

Example (4-8): Sketch the output waveform for the following circuits:

1)

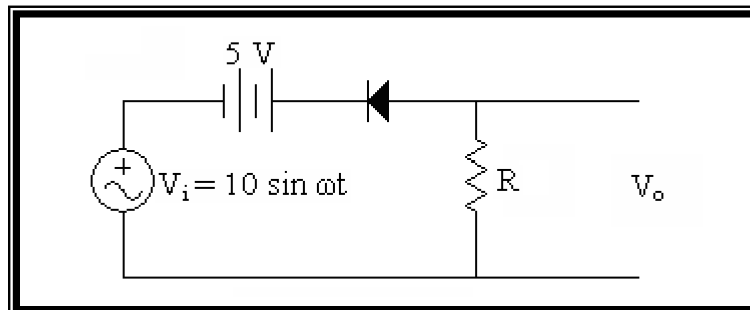


Figure (4-31): Example (4-8-1).

Sol.:

2)

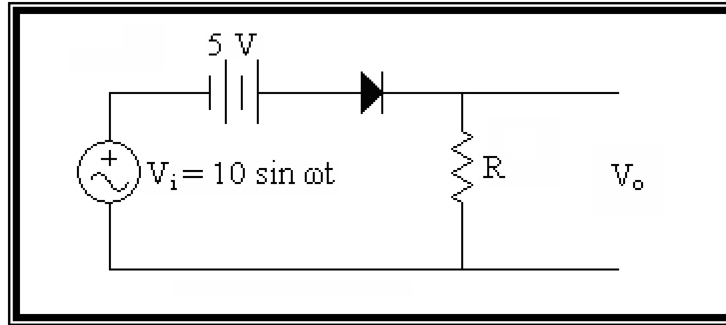


Figure (4-32): Example (4-8-2).

Sol.:

3)

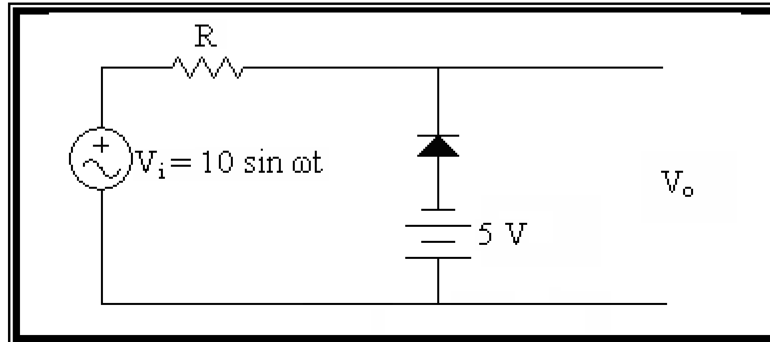


Figure (4-33): Example (4-8-3).

Sol.:

4)

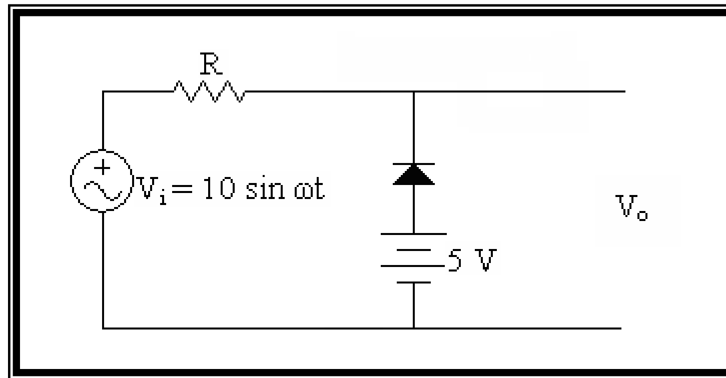


Figure (4-34): Example (4-8-4).

Sol.:

5)

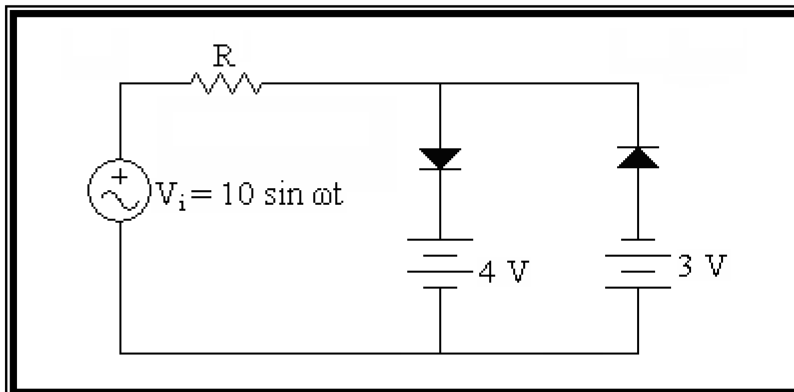


Figure (4-35): Example (4-8-5).

Sol.:

4-13-2 Clamping:

Clamping is a process of giving a D.C. level to the A.C. input signal, this level can be positive or negative or zero depending on the value and polarity of the D.C. source. The simple clamper contains a diode, capacitor, resistor, A.C. source and D.C. source.

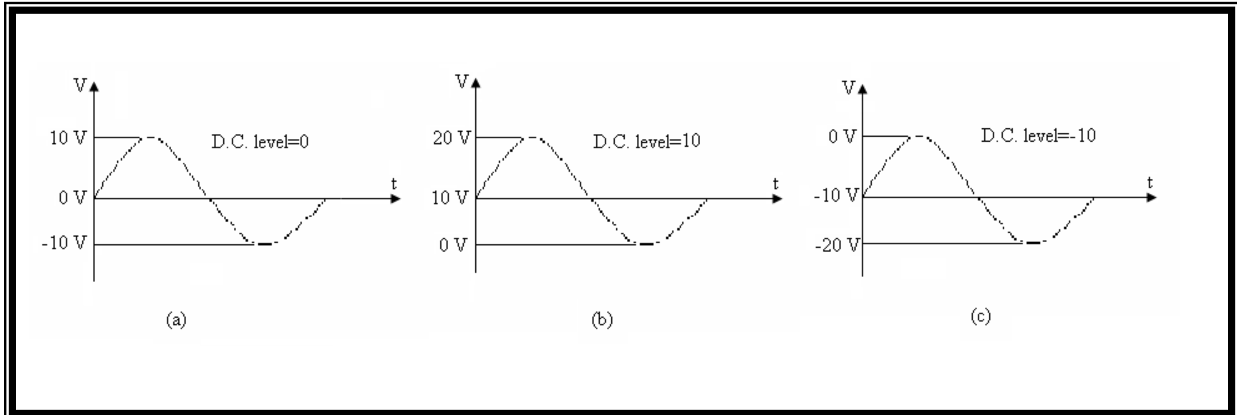


Figure (4-36): DC levels: (a) Zero D.C. level. (b) Positive D.C. level. (c) Negative D.C. level.

Analysis of Clamping circuits:

Example (4-9): Sketch the output waveform for the following circuit:

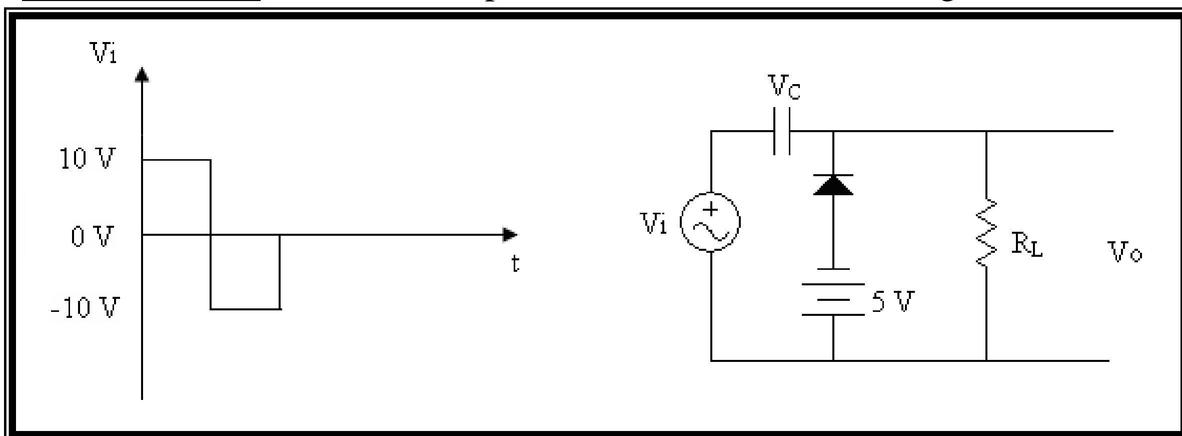


Figure (4-37): Example (4-9).

Sol.:

1) Start with the part of the input signal which makes the diode ON (forward biased), draw the equivalent circuit, calculate Vc and Vo:

2)

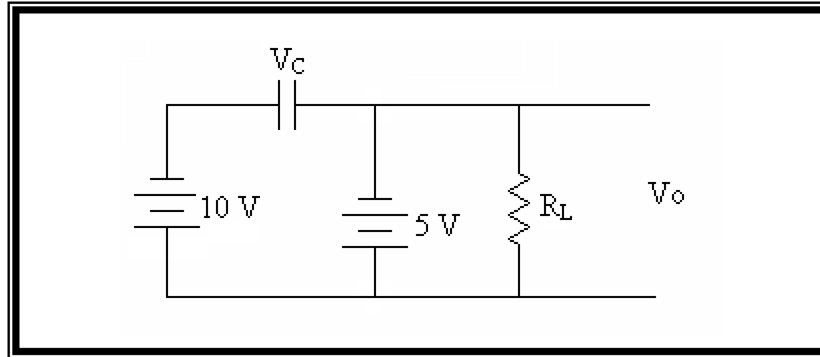


Figure (4-38): Example (4-9-1).

-10+Vc+5= 0

Vc= 5 V

Vo= -5 V

2) Consider the other part of the input signal (which makes the diode off (reverse biased)), draw the equivalent circuit and calculate Vo using Vc from step 1.

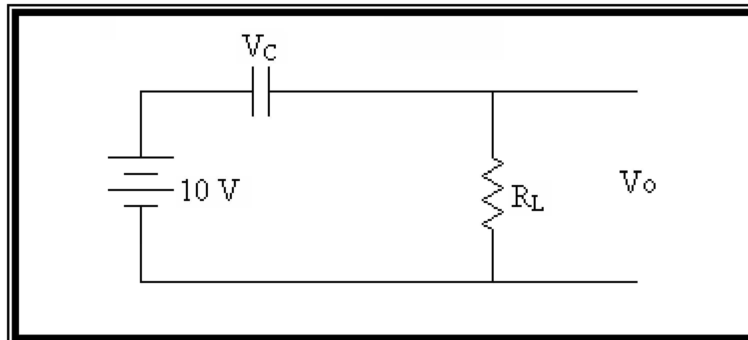


Figure (4-39): Example (4-9-2).

10+5-Vo= 0

Vo= 15 V

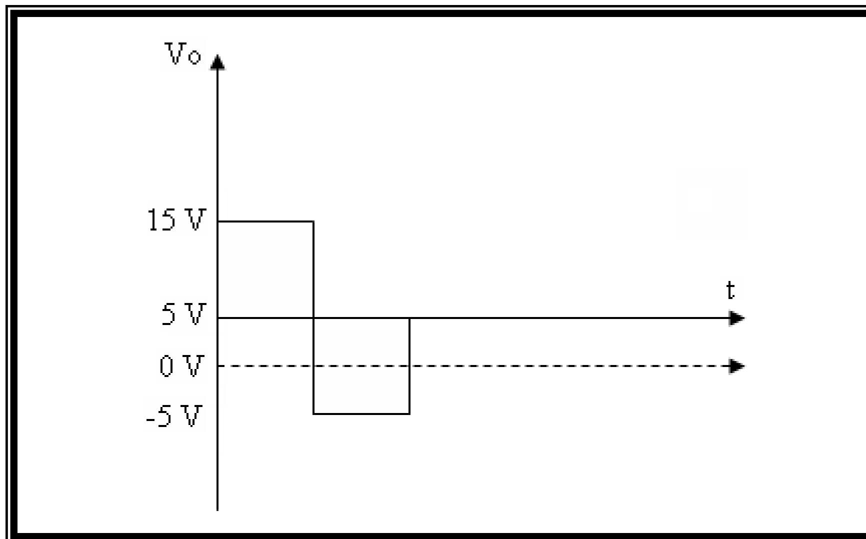


Figure (4-40): Output voltage waveform of the circuit in example (4-9).

3) Check that

Vo p-p= Vi p-p

4-13-3 Rectification:

A.C. to D.C. conversion.

4-13-3-a Half –Wave Rectifier (H.W.R.):

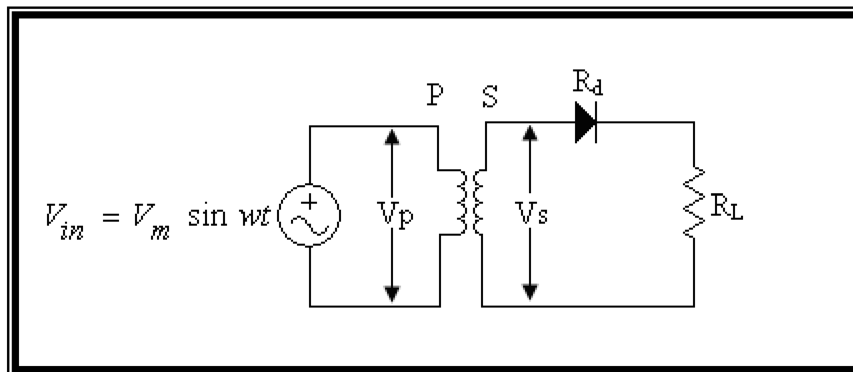


Figure (4-41): Half –Wave Rectifier circuit.

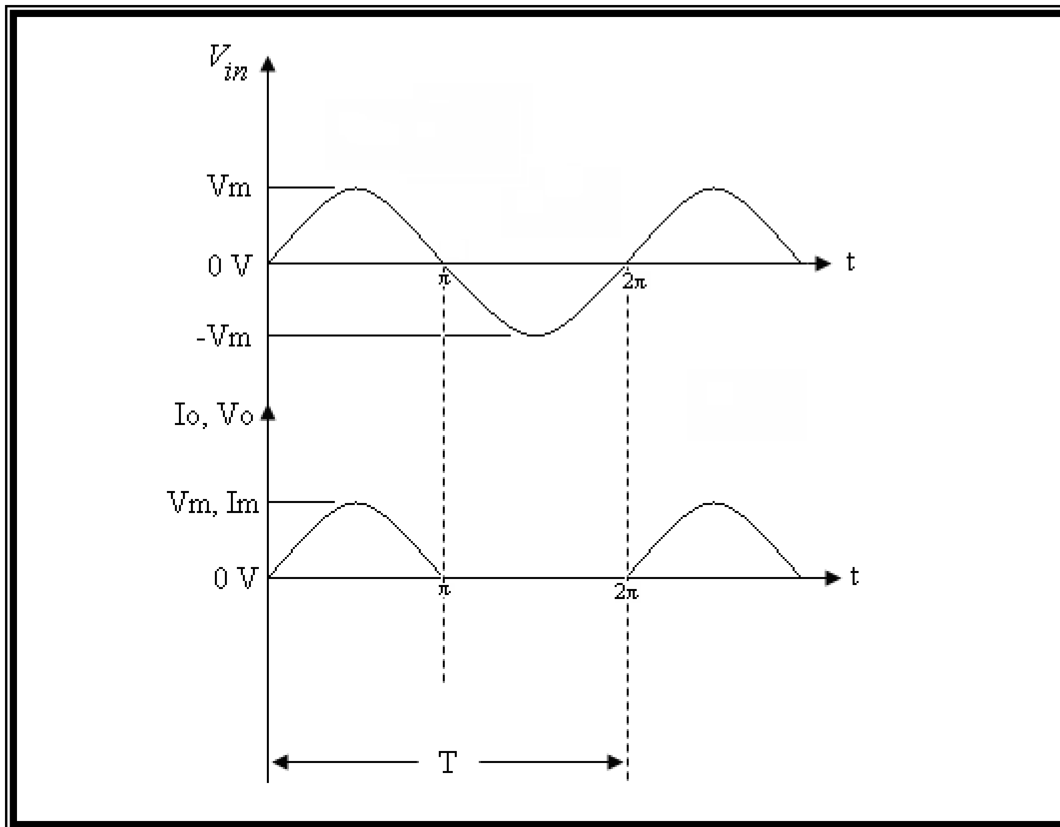


Figure (4-42): output of Half –Wave Rectifier.

Transformation ratio of the transformer:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

V_p : Voltage across the primary winding.

V_s : Voltage across the secondary winding.

N_p : Number of turns of the primary winding.

N_s : Number of turns of the secondary winding.

☆ **The average value of the current (I_{av} or I_{dc}):**

$$I_{av} = I_{dc} = \frac{1}{2\pi}\int_{0}^{2\pi} I_m \sin\alpha \, d\alpha$$

$$= \frac{I_m}{2\pi}\left[\int_{0}^{\pi} \sin\alpha \, d\alpha + \int_{\pi}^{2\pi} 0 \, d\alpha\right]$$

$$= \frac{I_m}{2\pi}\left[-\cos\alpha\right]_0^{\pi}$$

$$= \frac{I_m}{2\pi}\left[-\cos\pi + \cos 0\right]$$

$$= \frac{I_m}{2\pi}\left[1+1\right] = \frac{I_m}{2\pi}\times 2$$

$$\boxed{I_{av} = I_{dc} = \frac{I_m}{\pi}}$$

Where:

$$\boxed{I_m = \frac{V_m}{R_L + R_d}}$$

$$For\ ideal\ diode \Rightarrow R_d = 0 \ \therefore I_m = \frac{V_m}{R_L}$$

$$For\ Si\ diode \Rightarrow I_m = \frac{V_m - 0.7}{R_L + R_d}$$

☆ **The output power (the D.C. power delivered to the load):**

$$\boxed{P_{dc} = I_{dc}^2 \times R_L}$$

☆ **The input power (The A.C. power input to the rectifier):**

$$P_{a.c} = I^2_{r.m.s.} \times (R_d + R_L)$$

☆ **The root mean square value of the current ($I_{r.m.s.}$):**

$$I_{r.m.s.} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d\alpha} \qquad i = I_m \sin\alpha$$

$$I_{r.m.s.} = \sqrt{\frac{1}{2\pi}[\int_0^{\pi} I_m^2 \sin^2\alpha \, d\alpha + \int_{\pi}^{2\pi} 0 \, d\alpha]}$$

$$I_{r.m.s.} = \frac{I_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\int_0^{\pi}\sin^2\alpha \, d\alpha}$$

$$I_{r.m.s.} = \frac{I_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\int_0^{\pi}\frac{(1-\cos 2\alpha)}{2} d\alpha}$$

$$I_{r.m.s.} = \frac{I_m}{\sqrt{2}\times\sqrt{2}}\sqrt{\frac{1}{\pi}\int_0^{\pi}(1-\cos 2\alpha) d\alpha}$$

$$I_{r.m.s.} = \frac{I_m}{2}\sqrt{\frac{1}{\pi}[\alpha - \frac{\sin 2\alpha}{2}]_0^{\pi}}$$

$$I_{r.m.s.} = \frac{I_m}{2}\sqrt{\frac{1}{\pi}[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}]}$$

$$I_{r.m.s.} = \frac{I_m}{2}\sqrt{\frac{1}{\pi}[\pi - 0]}$$

$$\therefore I_{r.m.s.} = \frac{I_m}{2}$$

☆ The Rectification Efficiency (η):

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{d.c.}}{P_{a.c.}}$$

$$\eta = \frac{I_{d.c.}^2 \times R_L}{I_{r.m.s.}^2 \times (R_d + R_L)}$$

$$\eta = \frac{\frac{I_m^2}{\pi^2} \times R_L}{\frac{I_m^2}{4} \times (R_d + R_L)}$$

$$\boxed{\eta = \frac{40.6}{1 + \frac{R_d}{R_L}} \%}$$

$$For\ ideal\ diode \Rightarrow R_d = 0 \ \therefore \eta = 40.6\%$$

Which is the highest efficiency that can be obtained from the half-wave rectifier.

4-13-3-b Full-Wave Rectifier(F.W.R):

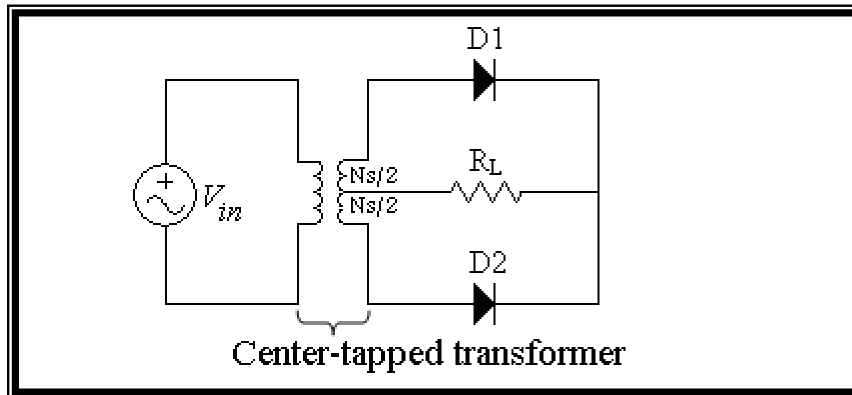


Figure (4-43): Center-tapped transformer full-wave rectifier..

During the positive half cycle of the input signal, D1 will be forward biased while D2 will be reverse biased, therefore the current (i1) will flow only through D1. During the negative half cycle of the input signal, D1 will be reverse biased while D2 will be forward biased and passing a current (i2). Notice that (i1) and (i2) are both in the same direction.

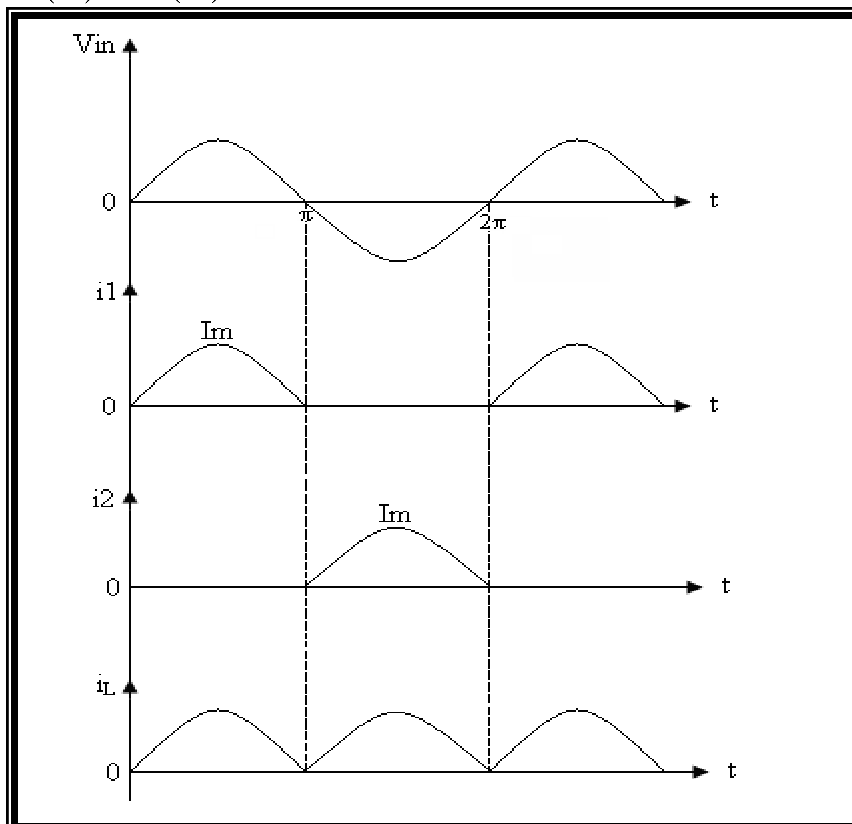


Figure (4-44): output of Half –Wave Rectifier.

☆ **The average value of the current (I_{av} or I_{dc}):**

$$I_{av} = I_{dc} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\alpha \, d\alpha$$

$$= \frac{I_m}{\pi}\left[-\cos\alpha\right]_0^{\pi}$$

$$= \frac{I_m}{\pi}\left[1+1\right]$$

$$\therefore I_{av} = I_{dc} = \frac{2I_m}{\pi}$$

Where:

$$I_m = \frac{V_m}{R_L + R_d}$$

$$For\ ideal\ diode \Rightarrow R_d = 0 \ \therefore I_m = \frac{V_m}{R_L}$$

$$For\ Si\ diode \Rightarrow I_m = \frac{V_m - 0.7}{R_L + R_d}$$

☆ **The output power (the D.C. power delivered to the load):**

$$P_{dc} = I_{dc}^2 \times R_L$$

☆ **The input power (The A.C. power input to the rectifier):**

$$P_{ac} = I_{rms}^2 \times (R_d + R_L)$$

☆ **The root mean square value of the current ($I_{r.m.s.}$):**

$$I_{r.m.s.} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2 \alpha \, d\alpha}$$

⋮

⋮

⋮

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

☆ **The Rectification Efficiency (η):**

$$\eta = \frac{P_{out}(dc)}{P_{in}(ac)}$$

$$= \frac{I_{dc}^2 \times R_L}{I_{rms}^2 (R_d + R_L)}$$

$$= \frac{(\frac{2I_m}{\pi})^2 \times R_L}{(\frac{I_m}{\sqrt{2}})^2 \times (R_d + R_L)}$$

$$\boxed{\eta = \frac{81.2}{1 + \frac{R_d}{R_L}} \%}$$

$$For\,ideal\,diode \Rightarrow R_d = 0 \therefore \eta = 81.2\ \%$$

Which is the highest efficiency that can be obtained from the full-wave rectifier.

Example (4-10): A half wave rectifier circuit contains a diode with a resistance (R_d=1000 Ω) and a load resistance ($R_L=10^4$ Ω). The applied voltage is (V_{in}= 200 sin 100πt). Calculate:

1) The maximum current in the circuit. 2) The average current value. 3) The root mean square value of the current. 4) The input power. 5) The output power. 6) The rectification efficiency.

Sol.:

Example (4-11): for the following full wave rectifier circuit, calculate:

1) The maximum current passing in the load. 2) The average current passing in the load. 3) The average voltage across the load. 4) The root mean square value of the current. 5) The rectification efficiency.

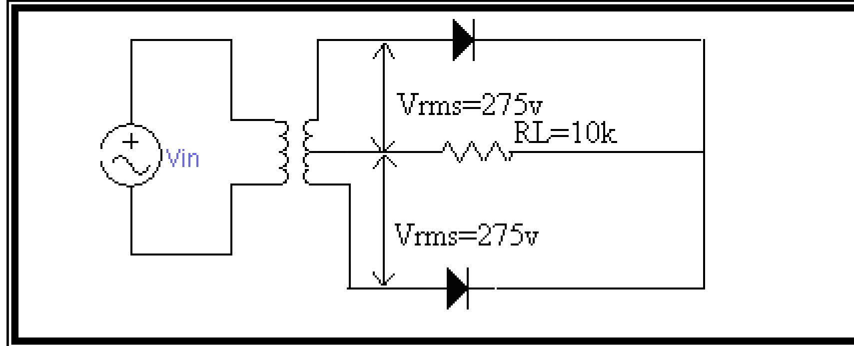


Figure (4-45): Example (4-11).

Sol.:

4-13-4 Zener Diode:

The symbol for a zener diode is shown in figure (4-46). A zener diode is a P-N junction device that is designed for operation in the reverse-breakdown region. The breakdown voltage of a zener diode is set by carefully controlling the doping level during manufacture. Recall, from the discussion of the diode characteristic curve, that when a diode reaches reverse breakdown, its voltage remains almost constant even though the current changes drastically. Zener diode has two modes of operation: A.C. clipping and D.C. voltage regulation.

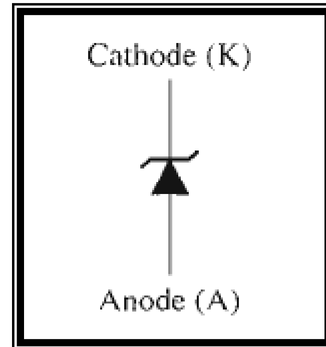


Figure (4-46): Zener diode symbol.

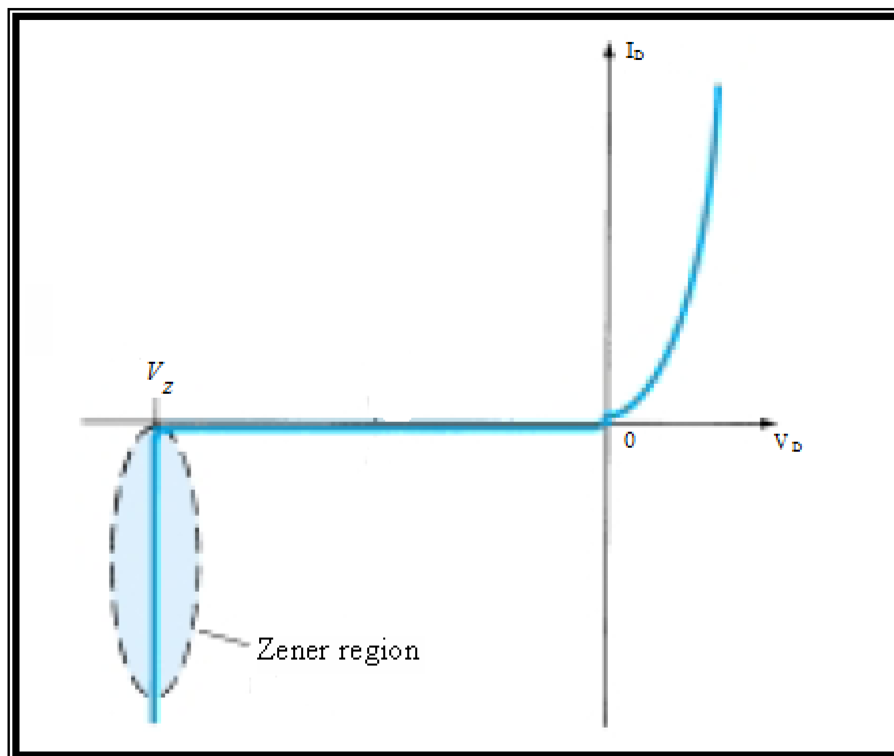


Figure (4-47): Zener region.

4-13-4-a AC Clipping:

1) In forward bias case, it is similar to normal diode.

2) In reverse bias case, it allows the passage of input voltages less than (V_z) and prevents the greater voltages, therefore it works as a constant voltage source of value (V_z).

Example (4-12): Sketch the output waveform for the following circuits:

1)

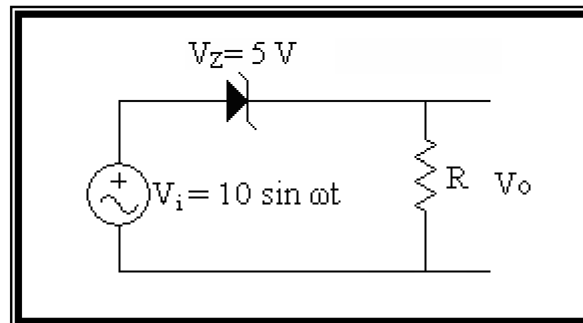


Figure (4-48): Example (4-12-1).

2)

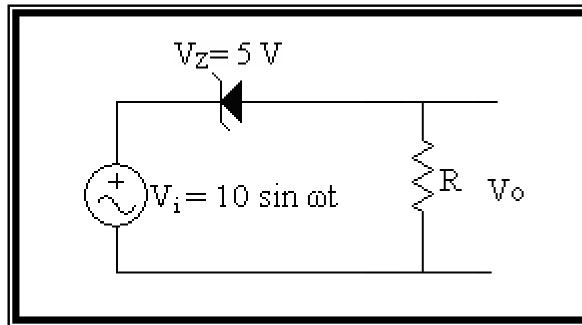


Figure (4-49): Example (4-12-2).

3)

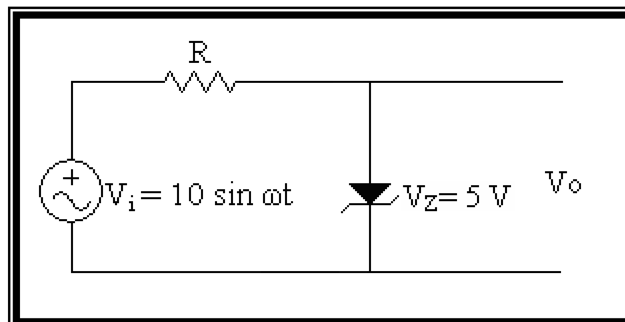


Figure (4-50): Example (4-12-3).

4)

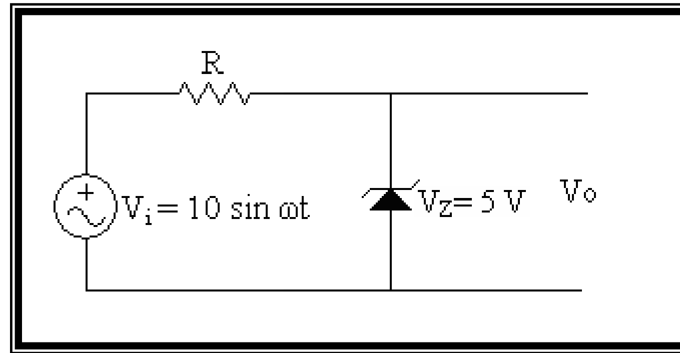


Figure (4-51): Example (4-12-4).

5)

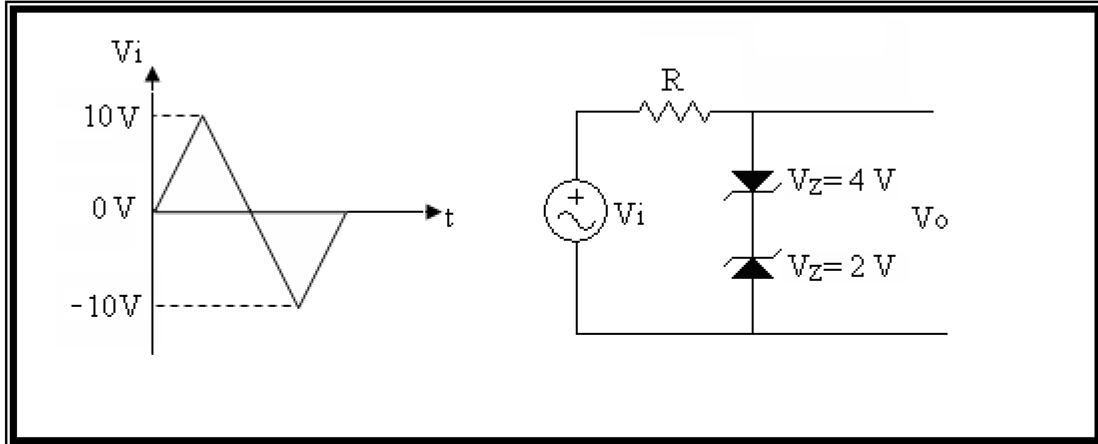


Figure (4-52): Example (4-12-5).

4-13-4-b DC Voltage Regulation:

Zener diode is used to keep the voltage across the load constant however the load is changed or the input voltage is changed. The following circuit is used.

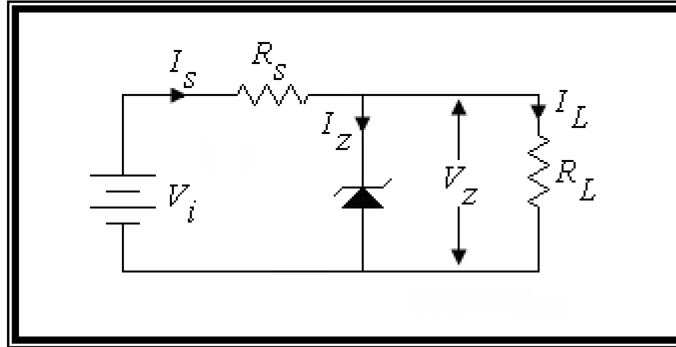


Figure (4-53): Voltage Regulation using zener diode.

$$V_i = I_s R_s + V_z \ldots\ldots\ldots\ldots(1)$$
$$I_s = I_z + I_L \ldots\ldots\ldots\ldots(2)$$
$$I_L = \frac{V_z}{R_L} \ldots\ldots\ldots\ldots(3)$$
$$P_z = V_z I_z \ldots\ldots\ldots\ldots(4)$$
$$P_{z_{max.}} = V_z I_{z_{max.}}$$

In this circuit, zener diode is used to protect a certain load in the following cases:-

1- Variable Vi & Fixed R_L:

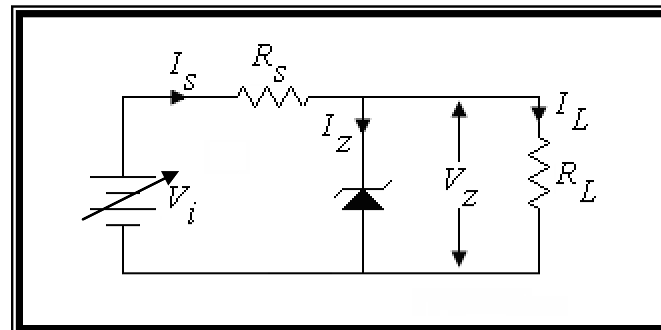


Figure (4-54): Voltage Regulation using zener diode (case 1).

$$I_L = \frac{V_z}{R_L} \Rightarrow Const.$$

$$I_{s_{max.}} = I_{z_{max.}} + I_L$$

$$I_{s_{min.}} = I_{z_{min.}} + I_L$$

$$V_{i_{max.}} = I_{s_{max.}} R_s + V_z$$

$$V_{i_{min.}} = I_{s_{min.}} R_s + V_z$$

2- Variable R_L & Fixed V_i:

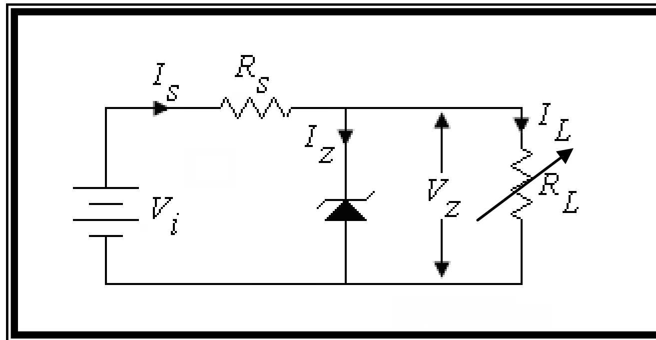


Figure (4-55): Voltage Regulation using zener diode (case 2).

$$I_s = \frac{V_i - V_z}{R_s} \Rightarrow const.$$

$$I_s = I_{z_{max.}} + I_{L_{min.}}$$

$$I_s = I_{z_{min.}} + I_{L_{max.}}$$

$$I_{L_{max.}} = \frac{V_z}{R_{L_{min.}}}$$

$$I_{L_{min.}} = \frac{V_z}{R_{L_{max.}}}$$

3- Variable V_i & Variable R_L

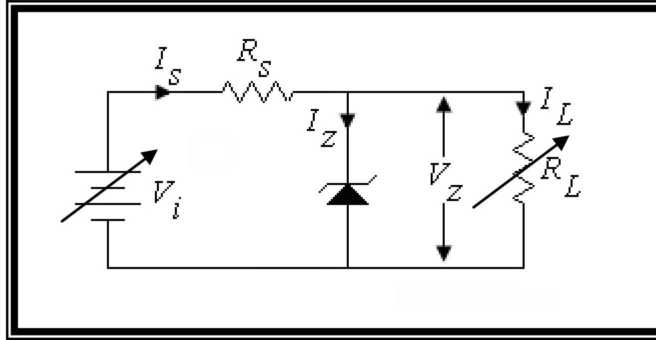


Figure (4-56): Voltage Regulation using zener diode (case 3).

$$I_{s_{max.}} = I_{z_{max.}} + I_{L_{min.}}$$

$$I_{s_{max.}} = \frac{V_{i_{max.}} - V_z}{R_s}$$

$$I_{L_{min.}} = \frac{V_z}{R_{L_{max.}}}$$

$$I_{s_{min.}} = I_{z_{min.}} + I_{L_{max.}}$$

$$I_{s_{min.}} = \frac{V_{i_{min.}} - V_z}{R_s}$$

$$I_{L_{max.}} = \frac{V_z}{R_{L_{min.}}}$$

Example (4-13): it is required to regulate the load voltage to be (10 v) using zener diode that works with maximum power (20 W). If (R_s=3Ω), (R_L=10 Ω), ($I_{z_{min.}}$ =50 mA), Determine the possible input voltages range.

Sol.:

Example (4-14): It is required to maintain the output voltage constant at (20 v) using zener diode whose current is varying between ($5mA \rightarrow 150mA$). If (R_s=50Ω), the applied voltage (V_i=40 v), determine the range of the load resistance (R_L).

Sol.:

Example (4-15): A 12 v zener diode is used to regulate the voltage across the load resistance. The applied voltage (V_i) varies between (19v – 30 v), the load resistance (R_L) varies between (100Ω-200Ω), R_s=50Ω. 1) Calculate the maximum and the minimum current passing in zener diode. 2) The maximum dissipated power in the resistance (R_s) and in zener diode.

Sol.: