1-Introduction

1-1 Introduction to Communication Engineering :

The principle objective of a communication system is to transmit information signals from one point to another .
The information signals may be the result of a voice message , a T.V. picture , a meter reading , or may take on a variety of other formats depending on the specific application .
Communication Engineering involves the analysis , design , and fabrication of an operating system that performs the communication objective .

1-2 Functional Elements of a Communication System :

Fig 1.1 shows a commonly used model for a single- link communication system . Although it suggests a system for communication between two remotely located points , this block diagram is also applicable to remote sensing systems , such as radar or sonar , in which the system input and output may be located at the same site .
Regardless of the particular application and configuration , all information transmission systems involve three major subsystems :
-a transmitter , the channel , and a receiver .
We will now discuss briefly each functional element shown in Fig. 1-1 .

Input Transducer:

The wide variety of possible sources of information results in many different forms for messages . Messages may be analog or digital . The message produced by a source must be converted by a transducer to a form suitable for the particular type of communication system employed .For example, in electrical communications, speech waves are converted by a microphone to voltage variations .Such a converted message is referred to as the message signal .

Transmitter:

The purpose of the transmitter is to couple the message to the channel .
It is often necessary to modulate a carrier wave with the signal from the input transducer .
Modulation is the systematic variation of some attribute of the carrier, such as amplitude , phase , or frequency , in accordance with a function of the message signal .
There are several reasons for using a carrier and modulating it . Important ones are :
(1) for ease of radiation , (2) to reduce noise and interference , (3) for channel assignment , (4) for multiplexing or transmission of several messages over a single channel , and (5) to overcome equipment limitations .

Channel :

The channel can have many different forms ; the most familiar is the channel that exists between the transmitting antenna of a commercial radio station and the receiving antenna of a radio . In this channel , the transmitted signal propagates through the atmosphere , or free space , to the receiving antenna .

Other forms of channels are :

- Transmission lines (such as open two- wire systems and co-axial cables) .
-Optical fiber channels .
- Guided electromagnetic – wave channels .

All channels have one thing in common : the signal undergoes degradation from transmitter to receiver . This degradation results from noise and other undesired signals or interference but also may include other distortion effects as well, such as fading signal levels, multiple transmission paths, and filtering .

Receiver:

The receiver's function is to extract the desired message from the received signal at the channel output and to convert it to a form suitable for the output transducer . Although amplification may be one of the first operations performed by the receiver , where the received signal may be extremely weak , the main function of the receiver is to demodulate the received signal .
Often it is desired that the receiver output be a scaled , possibly delayed , version of the message signal at the modulator input .

Output Transducer:

The output transducer completes the communication system . The device converts the electric signal at its input into the form desired by the system user. The most common output transducer is a loudspeaker . There are many other examples , such as tape recorders, personal computers, meters , and cathode – ray tubes.

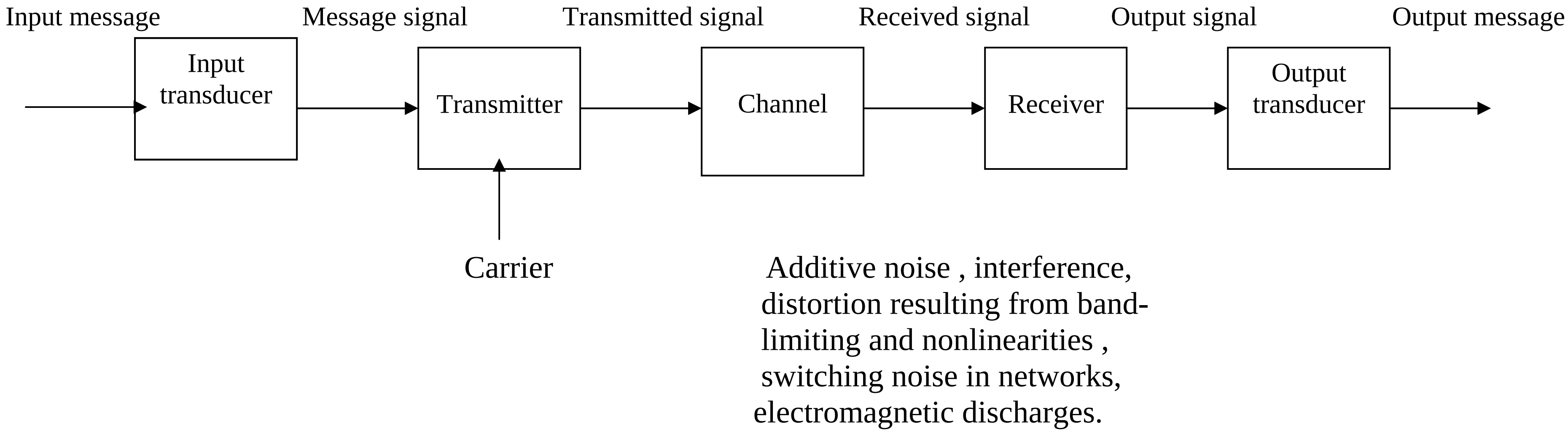


Fig. 1-1The Block Diagram of a Communication System .

2. Signals and Systems

2-1 Introduction :

Signals are time-varying quantities such as voltages or current .
A system is a combination of devices and networks (subsystems) chosen to perform a desired function .Because of the sophistication of modern communication systems , a great deal of analysis and experimentation with trial subsystems occurs before actual building of the desired system . Thus the communications engineer's tools are mathematical models for signals and systems .

2-2Classification of Signals:

2-2-1 Continuous-time and discrete-time signals

By the term continuous-time signal we mean a real or complex function of time s(t), where the independent variable *t* is continuous.

If *t* is a discrete variable, i.e., s(t) is defined at discrete times, then the signal s(t) is a discrete-time signal. A discrete-time signal is often identified as a sequence of numbers ,denoted by {s(n)}, where n is an integer.

2-2-2 Analogue and digital signals :

If a continuous-time signal s(t) can take on any values in a continuous time interval, then s(t) is called an analogue signal.
If a discrete-time signal can take on only a finite number of distinct values, {s(n)}, then the signal is called a digital signal.

2-2-3 Deterministic and random signals :

Deterministic signals are those signals whose values are completely specified for any given time.

Random signals are those signals that take random values at any given times.

2-2-4 Periodic and nonperiodic signals :

A signal s(t) is a periodic signal if $s(t) = s(t + nT_0)$, where T_0 is called the period and the integer $n > 0$.

If $s(t) \neq s(t + T_0)$ for all t and any T_0, then s(t) is a nonperiodic or aperiodic signal.

2-2-5 Power and energy signals :

A complex signal s(t) is a power signal if the average normalized power P is finite, where

$$0 < P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} s(t)s^*(t)dt < \infty$$

and s*(t) is the complex conjugate of s(t).
A complex signal s(t) is an energy signal if the normalized energy E is finite, where

$$0 < E = \int_{-\infty}^{\infty} s(t)s^*(t)\,dt = \int_{-\infty}^{\infty} |s(t)|^2\,dt < \infty$$

In communication systems, the received waveform is usually categorized into the desired part, containing the information signal, and the undesired part, called noise.

2-3 Some Useful Functions :

a: Unit impulse function :

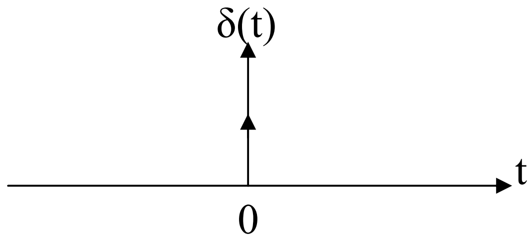
The unit impulse function, also known as the Dirac delta function, δ(t), is defined by :

$$\int_{-\infty}^{\infty} s(t)\delta(t)dt = s(0)$$

An alternative definition is :

$$\int_{-\infty}^{\infty} \delta(t)dt = 1.........and$$

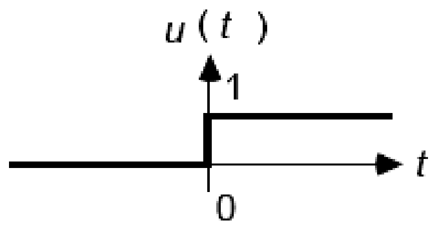
$$\delta(t) = \begin{cases} \infty,......t = 0 \\ 0,........t \neq 0 \end{cases}$$



b: Unit step function :

The unit step function u(t) is :

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



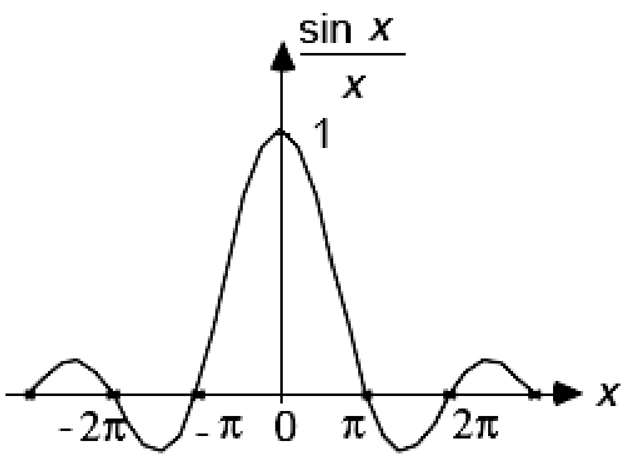
and the unit step function is related to the unit impulse function by **:**

$$u(t) = \int_{-\infty}^{\infty} \delta(t)\, d\, t \qquad \text{and} \qquad \frac{du\ (t\ )}{dt} = \delta(t)$$

c: Sampling function :

A sampling function is denoted by :

$$\mathrm{Sa}(x) = \frac{\sin x}{x}$$



d: Sinc function :

A sinc function is denoted by :

$$\mathrm{sinc}\, x = \frac{\sin \pi\ x}{\pi\, x}$$

Hence,

$$\mathrm{Sa}(x) = \mathrm{sinc}\left(\frac{x}{\pi}\right)$$

e: Rectangular function :

A single rectangular pulse is denoted by :

$$\Pi\left(\frac{t}{T}\right) = \begin{cases} 1, & |t| \le \frac{T}{2} \\ 0, & |t| > \frac{T}{2} \end{cases}$$

f: Triangular function :

$$\Lambda\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \le T \\ 0, & |t| > T \end{cases}$$

2-4 Other Useful Operations :

a; Cross-correlation :

The cross-correlation of two real-valued power waveforms $s_1(t)$ and $s_2(t)$ is defined by :

$$R_{12}(\tau) = \langle s_1(t)\, s_2(t+\tau) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} s_1(t)\, s_2(t+\tau)\, dt$$

If $s_1(t)$ and s_2 (t) are periodic with the same period T_0, then

$$R_{12}(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} s_1(t)\, s_2(t+\tau)\, dt$$

The cross-correlation of two real-valued energy waveforms $s_1(t)$ and $s_2(t)$ is defined by :

$$R_{12}(\tau) = \int_{-\infty}^{\infty} s_1(t)\, s_2(t+\tau)\, dt$$

Correlation is a useful operation to measure the similarity between two waveforms. To compute the correlation between waveforms, it is necessary to specify which waveform is being shifted. In general,
$R_{12}(\tau)$ is not equal to $R_{21}(\tau)$, where

$$R_{21}(\tau) = \langle s_2(t)\, s_1(t+\tau)\rangle.$$

The cross-correlation of two complex waveforms is :

$$R_{12}(\tau) = \langle s_1^*(t)\, s_2(t+\tau)\rangle.$$

b: Auto-correlation :

The auto-correlation of a real-valued power waveform $s_1(t)$ is defined by :

$$R_{11}(\tau) = \langle s_1(t)\, s_1(t+\tau)\rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} s_1(t)\, s_1(t+\tau)\, dt$$

If $s_1(t)$ is periodic with fundamental period T_0, then

$$R_{11}(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} s_1(t)\, s_1(t+\tau)\, dt$$

The auto-correlation of a real-valued energy waveform $s_1(t)$ is defined by :

$$R_{11}(\tau) = \int_{-\infty}^{\infty} s_1(t)\, s_1(t+\tau)\, dt$$

The auto-correlation of a complex power waveform is :

$$R_{11}(\tau) = \langle s_1^*(t) s_1(t+\tau)\rangle.$$

c: <u>Convolution</u> **:**

The convolution of a waveform $s_1(t)$ with a waveform $s_2(t)$ is given by

$$s_3(t) = s_1(t) * s_2(t) = \int_{-\infty}^{\infty} s_1(\lambda)\, s_2(t-\lambda)\, d\lambda$$

$$s_3(t) = s_1(t) * s_2(t) = \int_{-\infty}^{\infty} s_1(\lambda)\, s_2[-(\lambda-t)]\, d\lambda$$

where * denotes the convolution operation. The above equation is obtained by :

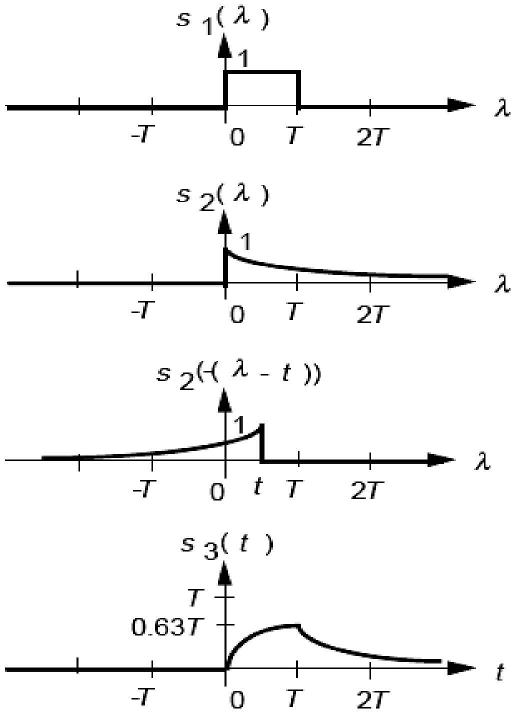
1. Time reversal of $s_2(t)$ to obtain $s_2(-\lambda)$.
2. Time shifting of $s_2(-\lambda)$ to obtain $s_2[-(\lambda-t)]$.
3. Multiplying $s_1(\lambda)$ and $s_2[-(\lambda-t)]$ to form the integrand $s_1(\lambda)\, s_2[-(\lambda-t)]$.

<u>Example 2.1</u> : Convolution of a rectangular waveform

$$s_1(t) = \begin{cases} 1, & 0 < t < T \\ 0, & elsewhere \end{cases}$$

with an exponential waveform

$$s_2(t) = e^{-t/T} u(t).$$



Convolution of a rectangular waveform and an exponential waveform.

2-5 Review of Fourier Series :

2-5-1 The Time-Frequency Concept:

Consider the following set of time functions $s(t) = \{3A \sin \omega_0 t , A \sin 2\omega_0 t\}$. We can represent these functions in different ways by plotting the amplitude versus time t, amplitude versus angular frequency ω, or amplitude versus frequency f.

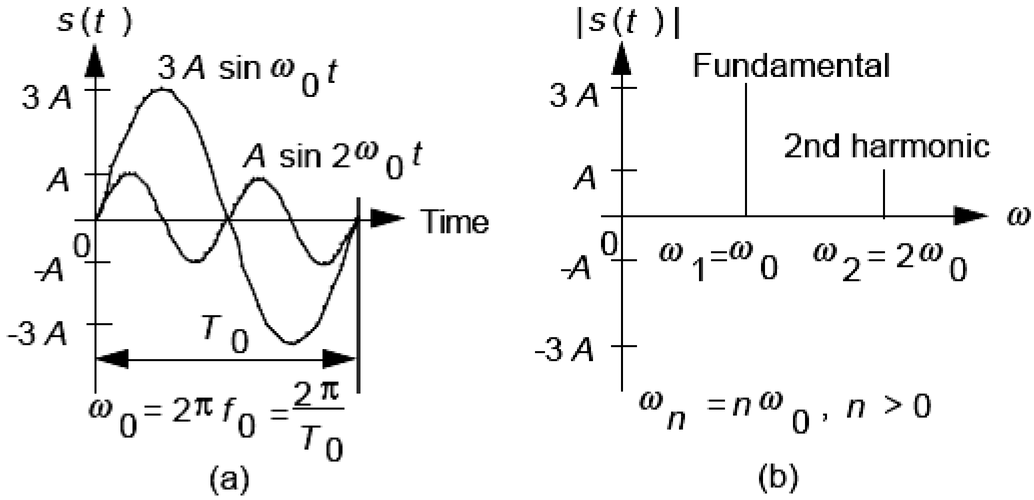


Figure 2.1 (a) Amplitude-time plot, and (b) amplitude-angular frequency plot.

$\omega_0 = 2\pi/T_0$ is called the fundamental angular frequency and $\omega_2 = 2\omega_0$ is called the second harmonic of the fundamental. In general, $\omega_n = n\omega_0$ is said to be the nth harmonic of the fundamental, where $n > 1$.

In communication engineering we are interested in steady-state analysis much of the time.

The Fourier series provides a useful model for analysing the frequency content and the steady-state network response for periodic input signals.

2-5-2 Trigonometric (Quadrature) Fourier Series :

A periodic time function s(t) over the interval

$$a - \frac{T_0}{2} < t < a + \frac{T_0}{2}$$

may be represented by an infinite sum of sinusoidal waveforms

$$s(t) = \frac{a_0}{T_0} + \frac{2}{T_0} \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \qquad (2.1)$$

where T_0 is the period of the fundamental frequency f_0 and $f = 1/T_0$. This is called the trigonometric (quadrature) Fourier series representation of the time function s(t). The coefficients a_n and b_n are given by :

$$a_n = \int_{a-T_0/2}^{a+T_0/2} s(t) \cos n\omega_0 t \, dt, \; n \geq 0 \qquad (2.2)$$

and

$$b_n = \int_{a-T_0/2}^{a+T_0/2} s(t) \sin n\omega_0 t \, dt, \; n > 0 \qquad (2.3)$$

The choice of a is arbitrary, and it is usually set to 0.

Many forms of the trigonometric Fourier series may be written. For example,

$$s(t) = a'_0 + \sum_{n=1}^{\infty} (a'_n \cos n\omega_0 t + b'_n \sin n\omega_0 t) \qquad (2.4)$$

is commonly used. The coefficients a'_n and b'_n are given by :

$$a'_n = \frac{2}{T_0} \int_{a-T_0/2}^{a+T_0/2} s(t) \cos n\omega_0 t \, dt, \; n \geq 0 \qquad (2.5)$$

and

$$b'_n = \frac{2}{T_0} \int_{a-T_0/2}^{a+T_0/2} s(t) \sin n\omega_0 t \, dt, \; n > 0 \qquad (2.6)$$

Example 2.2 **:** Find the trigonometric Fourier series for the periodic time function $s(t)$ shown in Figure 2.2.

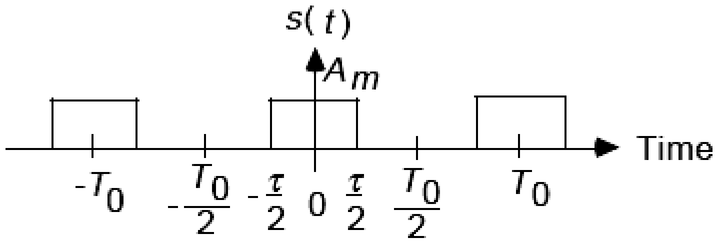


Figure 2.2 A periodic rectangular waveform.

$$a_n = \int_{-T_0/2}^{T_0/2} s(t)\cos n\omega_0 t\, dt$$

$$a_n = \int_{-\tau/2}^{\tau/2} A_m \cos n\omega_0 t\, dt$$

$$a_n = 2A_m \frac{\sin n\omega_0 \tau/2}{n\omega_0} = A_m \tau \frac{\sin n\omega_0 \tau/2}{n\omega_0 \tau/2}$$

$$b_n = \int_{-T_0/2}^{T_0/2} s(t)\sin n\omega_0 t\, dt = 0$$

Therefore ,

$$s(t) = \frac{A_m \tau}{T_0} + \frac{2}{T_0}\sum_{n=1}^{\infty} \left(A_m \tau \frac{\sin n\omega_0 \tau/2}{n\omega_0 \tau/2}\right)\cos n\omega_0 t.$$

2-5-3 Exponential (Complex or Phasor) Fourier Series :

The time function s(t) may be represented over the interval

$$a - \frac{T_0}{2} < t < a + \frac{T_0}{2}$$

by the equivalent exponential (complex or phasor) Fourier series

$$s(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \qquad (2.7)$$

where the coefficients c_n are given by :

$$c_n = \int_{a-T_0/2}^{a+T_0/2} s(t)\, e^{-jn\omega_0 t}\, dt \qquad (2.8)$$

c_0 is equivalent to the dc value of the waveform s(t).
c_n is, in general, a complex number. Furthermore, it is a phasor since it is the coefficient of $e^{jn\omega_0 t}$.
The complex Fourier series is easier to use for analytical problems.
Many forms of the complex Fourier series may be written. For example,

$$s(t) = \sum_{n=-\infty}^{\infty} c'_n e^{jn\omega_0 t} \qquad (2.9)$$

is commonly used. The coefficients c'_n are given by :

$$c'_n = \frac{1}{T_0} \int_{a-T_0/2}^{a+T_0/2} s(t)\, e^{-jn\omega_0 t}\, dt \qquad (2.10)$$

<u>Example 2.3</u>**:** Find the complex Fourier series for the periodic time function s(t) shown in Figure 2.2.

$$c_n = \int_{-T_0/2}^{T_0/2} s(t)\, e^{-jn\omega_0 t}\, dt$$

$$c_n = \int_{-\tau/2}^{\tau/2} A_m\, e^{-jn\omega_0 t}\, dt$$

$$c_n = A_m \frac{e^{jn\omega_0 \tau/2} - e^{-jn\omega_0 \tau/2}}{jn\omega_0}$$

$$c_n = 2A_m \frac{\sin n\omega_0 \tau/2}{n\omega_0} = A_m \tau \frac{\sin n\omega_0 \tau/2}{n\omega_0 \tau/2}$$

Therefore,

$$s(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \left(A_m \tau \frac{\sin n\omega_0 \tau/2}{n\omega_0 \tau/2}\right) e^{jn\omega_0 t}.$$

The frequency spectrum is shown in Figure 2.3.

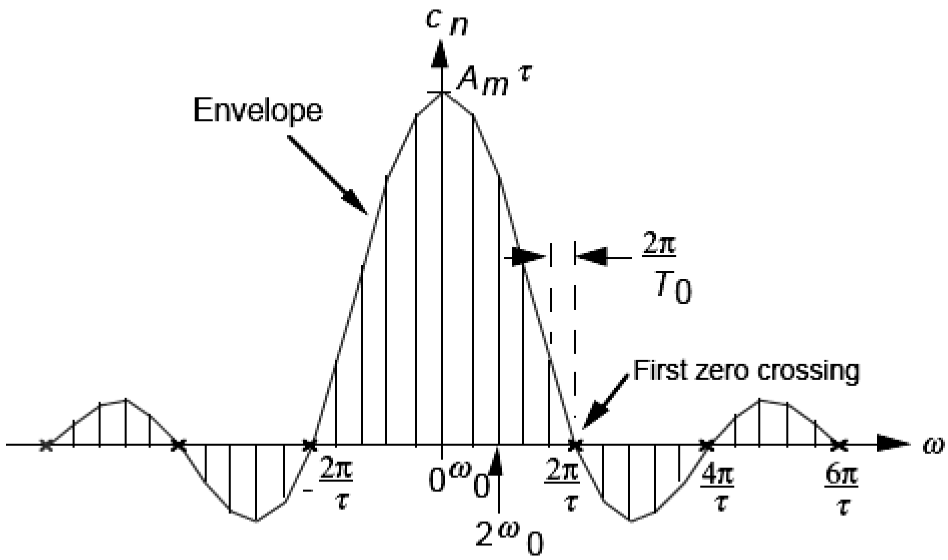


Figure 2.3 Frequency spectrum of a periodic rectangular waveform.

Figure 2.4 shows the effect on the frequency spectrum of smaller τ

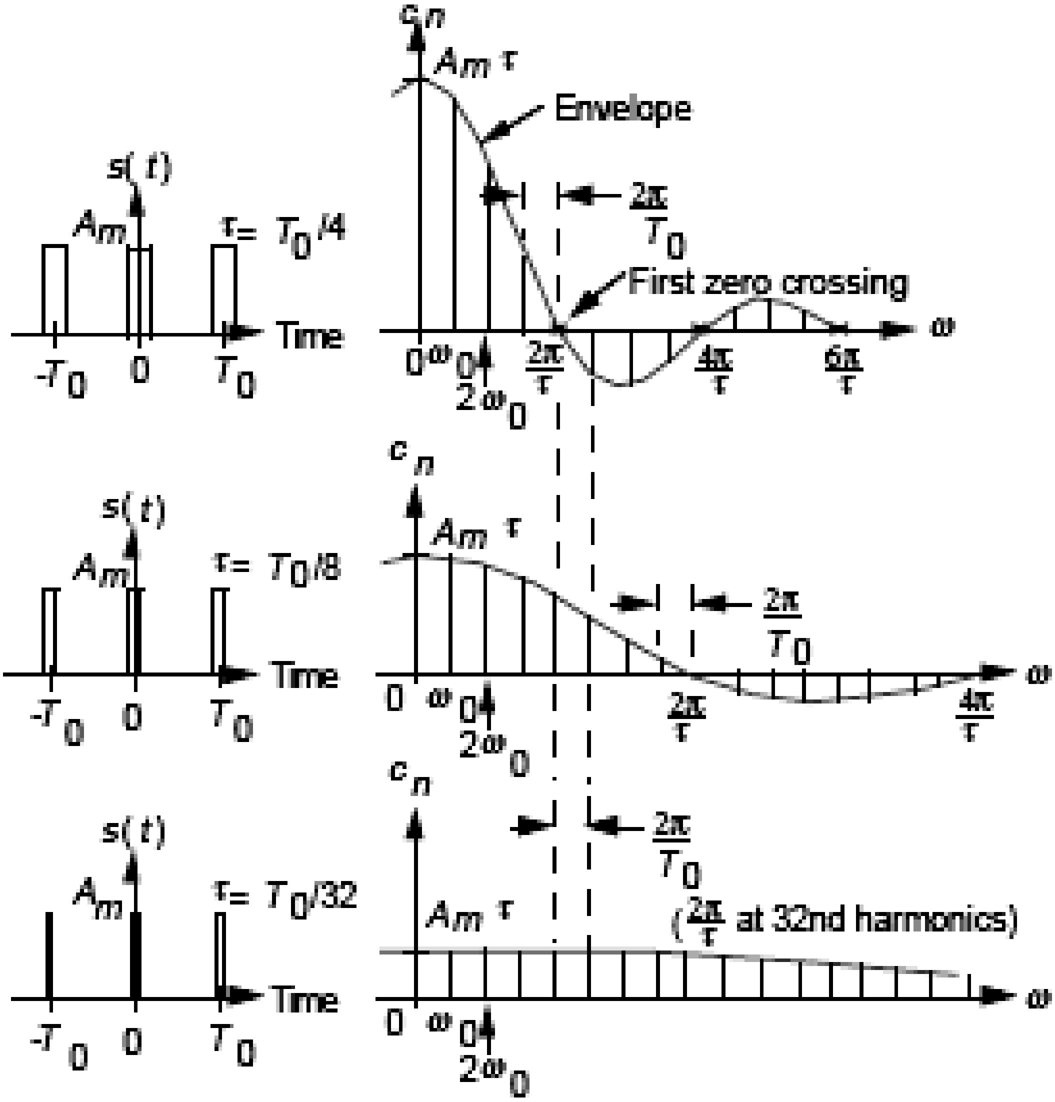


Figure 2.4 Effect on frequency spectrum of smaller τ.

If the bandwidth B is specified as the width of the frequency band of a waveform from zero frequency to the first zero crossing, then B = 1/τ Hz.
If we let the pulse width τ in Figure 2.4 go to zero and the amplitude Am go to infinity with Amτ = 1, all spectral lines in the frequency domain have unity length. Figure 2.5 shows the periodic unit impulses and the frequency spectrum of the periodic unit impulses . The bandwidth becomes infinite.

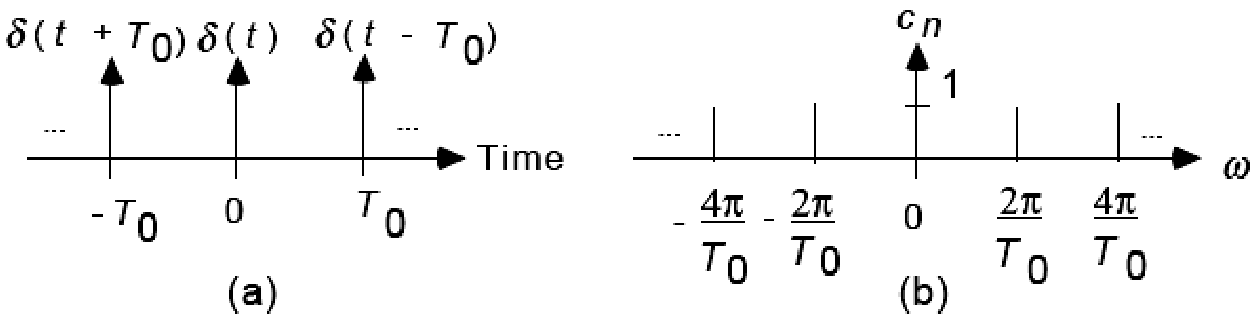


Figure 2.5 (a) Periodic unit impulses, and (b) frequency spectrum.

<u>Properties of the Complex Fourier Series</u> :

1. If s(t) is real, then $c_n = c^*_{-n}$

2. If s(t) is real and even, s(t) = s(-t) , then Img[c_n] = 0

3. If s(t) is real and odd, s(t) = - s(-t) , then Re[c_n] = 0

4. The complex Fourier-series coefficients of a real waveform are related to the quadrature Fourier-series coefficients by :

$$c_n = \begin{cases} a_n - jb_n, & n > 0 \\ a_0, & n = 0 \\ a_{-n} + jb_{-n}, & n < 0 \end{cases}$$

$$|c_n| = \sqrt{a_n^2 + b_n^2}$$

represents the amplitude spectrum and

$$\angle c_n = \theta_n = \tan^{-1} \frac{-b_n}{a_n}$$

represents the phase spectrum of the real waveform.

The equivalence between the Fourier series coefficients is demonstrated in Figure 2.6.

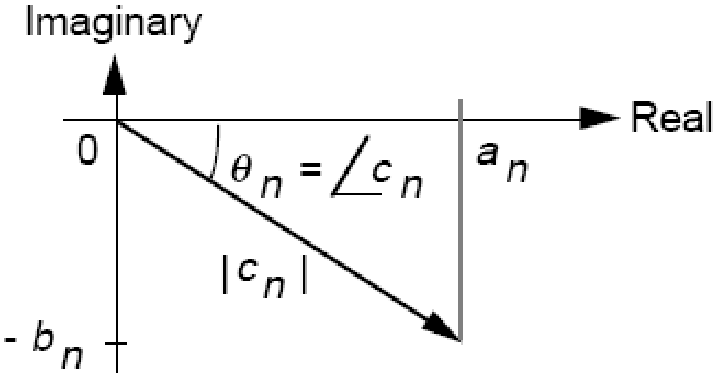


Figure 2.6 Fourier series coefficients, *n* > 1.

<u>Parseval's Theorem for the Fourier Series :</u>

Parseval's Theorem for the Fourier series states that, if s(t) is a periodic signal with period T_0, then the average normalized power (across a 1Ω resistor) of s(t) is :

$$P = \frac{1}{T_0} \int_{a-T_0/2}^{a+T_0/2} |s(t)|^2 \, dt = \sum_{n=-\infty}^{\infty} \left| \frac{c_n}{T_0} \right|^2$$

If s(t) is real, |s(t)| is simply replaced by s(t).

<u>References</u>

[1] M. Schwartz, Information Transmission, Modulation, and Noise, 4/e, McGraw- Hill, 1990.
[2] P. H. Young, Electronic Communication Techniques, 4/e, Prentice-Hall, 1998.
[3] L. W. Couch II, Digital and Analog Communication Systems, 5/e, Prentice Hall,1997.
[4] H. P. Hsu, Analog and Digital Communications, McGraw-Hill, 1993.

Problems :

1- Find the F.S. expansion of the signal defined by :

$$f(t)=\begin{cases}V\text{............}0\le \omega t\le \pi \\ -V\text{..........}\pi \le \omega t\le 2\pi\end{cases}$$

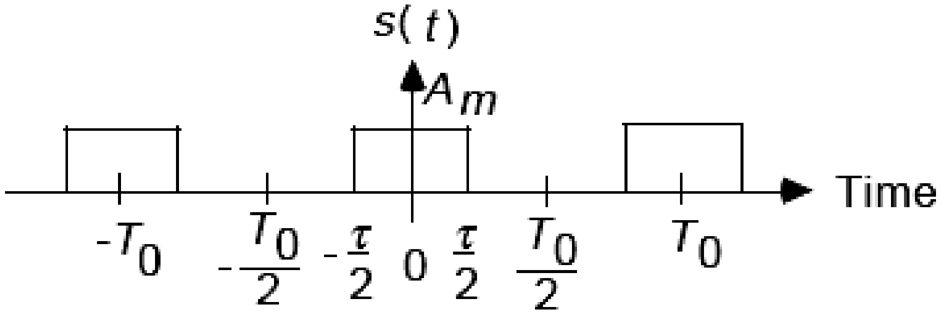
and plot its amplitude spectrum .

2-What is the F.S. expansion of the periodic signal whose definition in one period is :

$$s(t)=\begin{cases}0\text{............}-\pi \le \omega t\le 0 \\ \sin \omega t\text{.......}0\le \omega t\le \pi\end{cases}$$

Plot its amplitude spectrum .

3-What percentage of the total power is contained within the first zero crossing of the spectrum envelope for s(t) as shown in the following figure. Assume that Am= 1 v , T_0= 0.25 msec. , and τ = 0.05 msec



<u>2-6. Fourier Transform</u> :

In communication systems, we often deal with non-periodic signals. An extension of the time-frequency relationship to a non-periodic signal $s(t)$ requires the introduction of the Fourier Integral. A nonperiodic signal can be viewed as a limiting case of a periodic signal, where the period T_0 approaches infinity. As T_0 approaches infinity, the periodic signal will eventually become a single non-periodic signal. This is shown in Figure 2-7 .

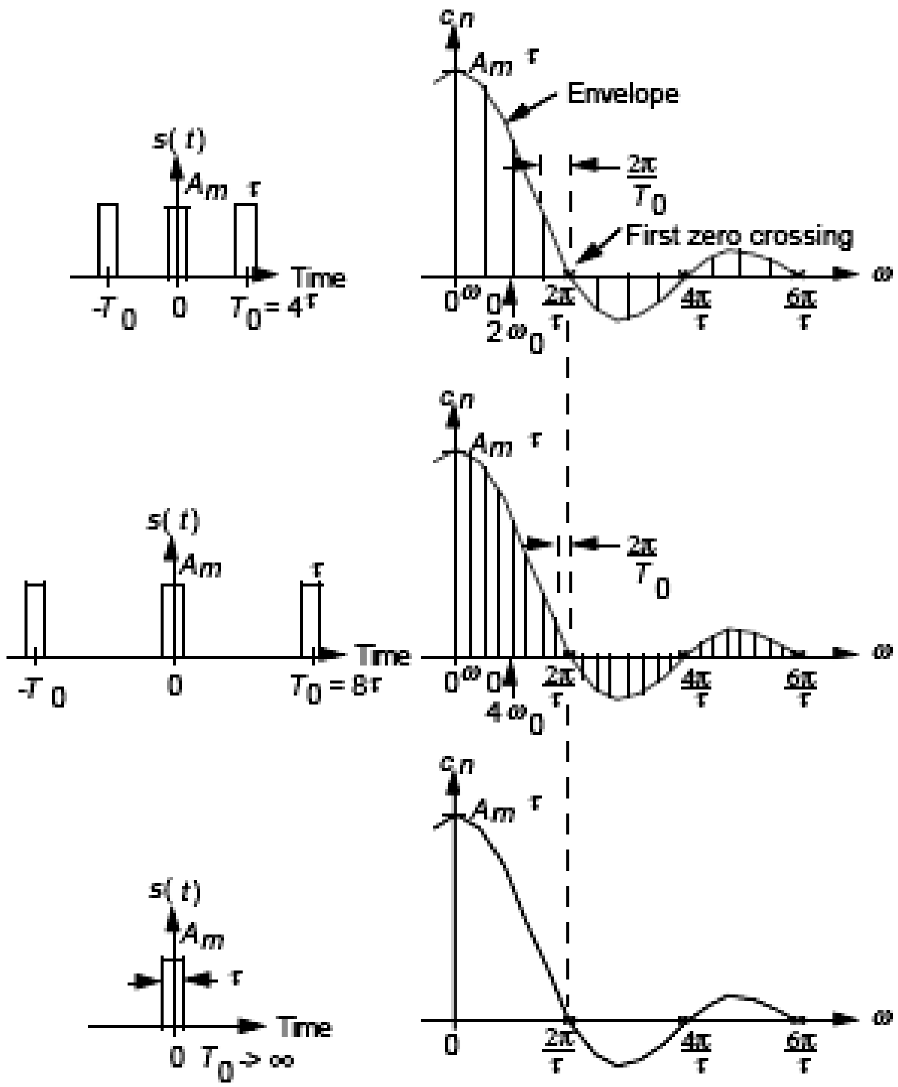


Figure 2.7 Effect on frequency spectrum of increasing period T_0 .

The normalized energy of the non-periodic signal becomes finite and its normalized power tends to zero.
Consider the amplitude spectrum of a periodic waveform as shown in Figure 2-8 .

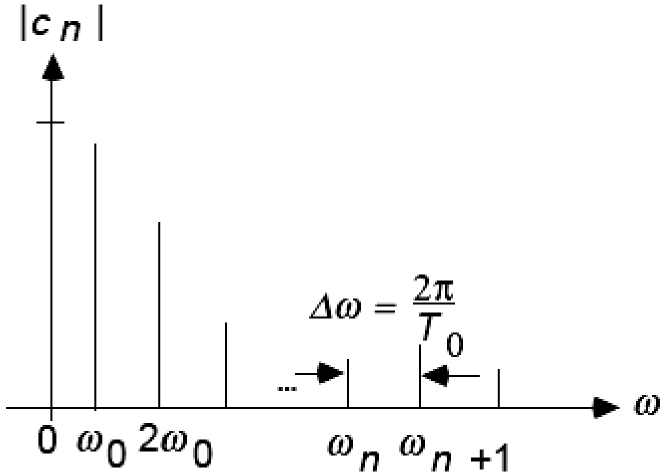


Figure 2.8 Amplitude spectrum of a periodic time function.

Let $\omega_n = n\omega_0$ and $\Delta\omega = \omega_{n+1} - \omega_n = 2\pi / T_0$. The Fourier series of a periodic waveform s(t) with period T_0 can be written as :

$$s(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \Delta\omega$$

and

$$c_n = \int_{a-T_0/2}^{a+T_0/2} s(t)\, e^{-jn\omega_0 t}\, dt$$

If T_0 approaches infinity, ω_0 goes to 0. The harmonics get closer and closer together. In the limit, the Fourier series summation representation of s(t) becomes an integral, c_n becomes a continuous function $S(\omega)$, and we have a continuous frequency spectrum. In summary, as $T_0 \longrightarrow \infty$, $\sum$ becomes $\int$, ω_n becomes ω, and $\Delta\omega$ becomes $d\omega$. We have :

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} c_n \, e^{j\omega t} \, d\omega$$

and

$$S(\omega) = F[s(t)] = \lim_{T_0 \to \infty} c_n = \int_{-\infty}^{\infty} s(t) \, e^{-j\omega t} \, dt$$

It is also very common to work in terms of frequency f, f = ω/2π, because spectrum analyzers are usually calibrated in hertz. Thus,

$$s(t) = F^{-1}[S(f)] = \int_{-\infty}^{\infty} S(f) \, e^{j2\pi ft} \, df$$

and

$$S(f) = F[s(t)] = \int_{-\infty}^{\infty} s(t) \, e^{-j2\pi ft} \, dt$$

The functions s(t) and S(f) are said to constitute a Fourier transform pair, where S(f) is the Fourier transform of a time function s(t), and s(t) is the Inverse Fourier transform (IFT) of a frequency-domain function S(f).
Shorthand notation expressed in terms of t and f :s(t) $\longleftrightarrow$ S(f)
Shorthand notation expressed in terms of t and ω : s(t) $\longleftrightarrow$ S(ω)
In general, S(f) is a complex function of frequency.

In two-dimensional Cartesian form, S(f) can be expressed as :

S(f) = X(f) + jY(f)

In polar form, S(f) can be expressed as :

$$S(f) = |S(f)| \, e^{j\theta(f)}$$

Where

$$|S(f)| = \sqrt{X^2(f) + Y^2(f)} \text{ and } \theta(f) = \tan^{-1}\frac{Y(f)}{X(f)}$$

|S(f)| represents the amplitude spectrum and θ(f) represents the phase spectrum of s(t) .

Example 2-4 : Find the spectrum of an exponential pulse

$$s(t) = \begin{cases} e^{-t}, & t > 0 \\ 0, & t < 0 \end{cases}$$

$$S(f) = \int_0^{\infty} e^{-t}\, e^{-j2\pi ft}\, dt$$

$$S(f) = \frac{1}{1+j\ 2\pi f}$$

$$|S(f)| = \sqrt{\frac{1}{1+(2\pi f)^2}}$$

$$\theta(f) = -\tan^{-1}(2\pi f)$$

Transforms of Some Useful Functions :

1. Dirac Delta Time Function :

$$F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\,2\pi ft}\, dt = 1$$

$$\boldsymbol{\delta(t) \longleftrightarrow 1}$$

Also, it can be shown that $\delta(t-t_0) \longleftrightarrow e^{-j2\pi ft_0}$

2. Dirac Delta Frequency-Domain Function :

$$F^{-1}[\delta(f)] = \int_{-\infty}^{\infty} \delta(f)\, e^{j\,2\pi ft}\, df = 1$$

$$\boldsymbol{1 \longleftrightarrow \delta(f)}$$

Also, it can be shown that $e^{j2\pi f_0 t} \longleftrightarrow \delta(f - f_0)$

Example 2.5 : Find the spectrum of a sinusoid :

$$v(t) = A \sin 2\pi f_0 t = A(e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}) / 2j$$

Since $e^{j2\pi f_0 t} \longleftrightarrow \delta(f - f_0)$, we have

$$V(f) = \frac{A}{2j}\delta(f - f_0) - \frac{A}{2j}\delta(f + f_0)$$

$$V(f) = -\frac{A}{2}j[\delta(f - f_0) - \delta(f + f_0)]$$

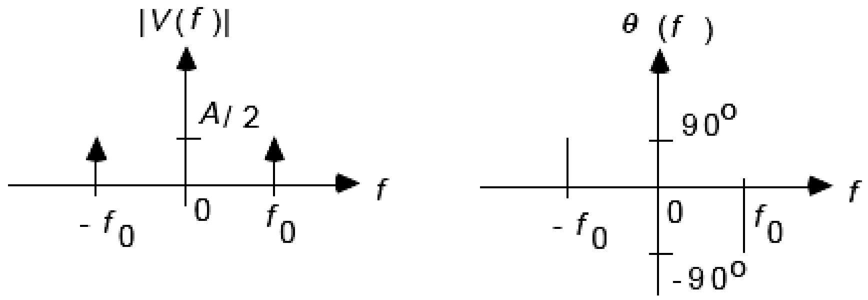
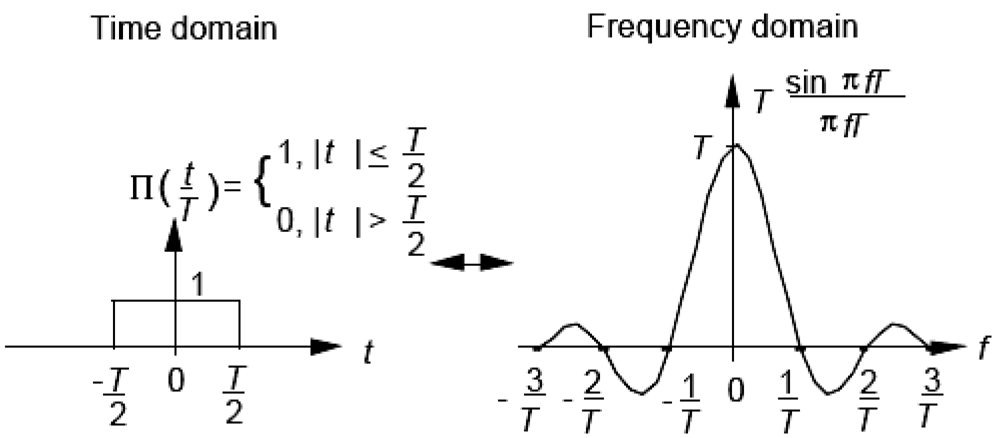
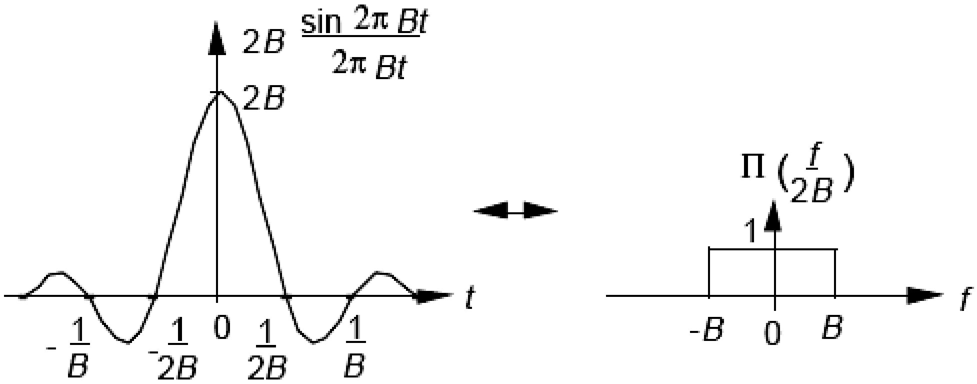


Figure 2.9 Spectrum of the periodic function $A \sin 2\pi f_0 t$

3. Rectangular, sin x/x, and Triangular Pulses :



(a)



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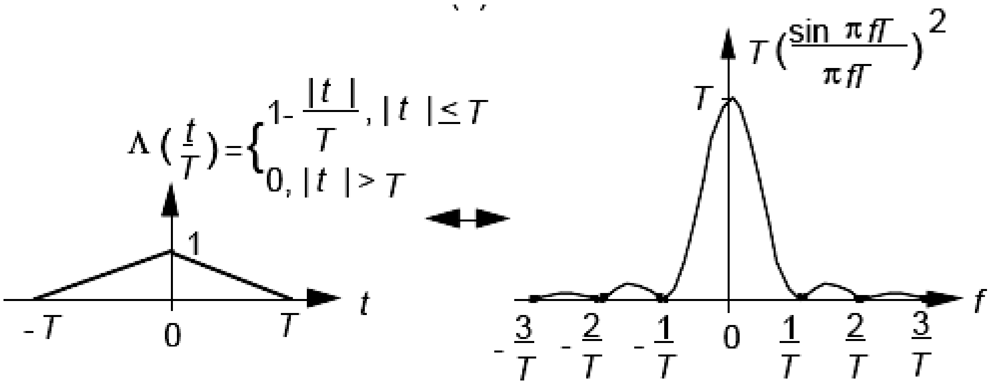


(c)

Figure 2.10 Spectra of (a) rectangular, (b) sin *x*/*x*, and (c) triangular pulses.

Observations:

1. Figure 2.10-a - Spectrum spreads out as the pulse width T decreases.
Bandwidth B = 1/T Hz and S(f) decreases as 1/f.

2. Figure 2.10c - spectrum spreads out as the pulse width T decreases.
Bandwidth B = 1/T Hz and S(f) decreases as $1/f^2$.

The smoother the time-domain function, the more rapidly the spectrum decreases with increasing frequency, packing more frequency contents into a specified bandwidth.

An inverse time-bandwidth relation always exists.

Bandwidth plays a significant role in determining transmission rate.

Properties of Fourier Transforms :

1. Symmetry (Duality) Property :

   $s(t) \longleftrightarrow S(-f)$

2. Scaling Property :

   $s(at) \longleftrightarrow \frac{1}{|a|} S(\frac{f}{a})$

3. Time Shifting (Time Delay) Property :

   $s(t - T_d) \longleftrightarrow S(f)\, e^{-j2\pi f T_d}$

4. Frequency Shifting Property :

   $s(t)\, e^{j2\pi f_c t} \longleftrightarrow S(f-f_c)$

5. Differentiation Property :

   $d^n\, s(t) / dt^n \longleftrightarrow (j2\pi f)^n\, S(f)$

Differentiation increases the high-frequency content of a signal. The derivative of an even function must be odd . Hence, the Fourier transform of the derivative of the function must be odd and imaginary.

6. Convolution Property :

   $s_1(t) * s_2(t) \longleftrightarrow S_1(f)\, S_2(f)$

7. If $s(t)$ is real, then :

   $S(-f) = S^*(f)$

8. If $s(t)$ is real, then :

   $|S(-f)| = |S(f)|$

   and

   $\theta(-f) = -\theta(f)$

The following table shows Fourier transform properties for various forms of s(t) .

If $s(t)$ is a:	Then $S(f)$ is a:
Real and even function of t	Real and even function of f
Real and odd	Imaginary and odd
Imaginary and even	Imaginary and even
Imaginary and odd	Real and odd
Complex and even	Complex and even
Complex and odd	Complex and odd

<u>Example 2.6:</u> Use the scaling and real-signal frequency-translation properties to find the Fourier transform of a damped sinusoid

$$s(t) = \begin{cases} e^{-t/T} \sin\omega_0 t, & t > 0, T > 0 \\ 0, & t < 0 \end{cases}$$

From Example 2.4 we have :

$$\begin{Bmatrix} e^{-t}, & t > 0 \\ 0, & t < 0 \end{Bmatrix} \longleftrightarrow \frac{T}{1+j\ 2\pi\ fT}$$

Using the scaling property with a = 1/T, we get :

$$\begin{Bmatrix} e^{-t/T}, & t > 0 \\ 0, & t < 0 \end{Bmatrix} \longleftrightarrow \frac{T}{1+j\ 2\pi\ fT}$$

Using the real-signal frequency-translation property with θ = -π/2, we get:

$$S(f) = \frac{1}{2}\left[e^{-j\pi/2}\frac{T}{1+j2\pi(f-f_0)T} + e^{j\pi/2}\frac{T}{1+j2\pi(f+f_0)T}\right]$$

The sin ω_0t factor causes the spectrum to move from f = 0 to f = ± f_0 .

Parseval's Theorem for the Fourier Transform and Energy Spectral Density :

Parseval's Theorem for the Fourier transform states that if $s_1(t)$ and $s_2(t)$ are two complex energy signals, then :

$$\int_{-\infty}^{\infty} s_1(t)\, s^*_2(t)\, dt = \int_{-\infty}^{\infty} S_1(f)\, S^*_2(f)\, df$$

If $s_1(t) = s_2(t)$, then **Rayleigh's energy theorem** states that the normalised energy is :

$$E = \int_{-\infty}^{\infty} |s_1(t)|^2\, dt = \int_{-\infty}^{\infty} |S_1(f)|^2\, df$$

The energy spectral density (ESD) is defined for energy waveforms by:

$$E_{11}(f) = |S_1(f)|^2$$

Power Spectral Density (PSD) and Wiener-Khintchine Theorem :

PSD is a function showing the distribution of power in the signal as a function of frequency .
The Wiener-Khintchine theorem states that the power spectral density and the autocorrelation function are Fourier transform pairs.

$$R_{22}(\tau) \longleftrightarrow P_{22}(f)$$

Where $P_{22}(f)$ and $R_{22}(\tau)$ are the power spectral density and the autocorrelation of a power waveform $s_2(t)$ respectively .
Furthermore, the average normalized power is :

$$P = \langle s_2^2(t) \rangle = S^2_{2rms} = \int_{-\infty}^{\infty} P_{22}(f)\, df = R_{22}(0)$$

The average normalized power of a power waveform is now related to the power spectral density.
The power spectral density (PSD) for a power waveform $s_2(t)$ is :

$$P_{22}(f) = \lim_{T \to \infty} \left(\frac{|S_1(f)|^2}{T} \right)$$

where $S_1(f)$ is the Fourier transform of the truncated waveform $s_1(t)$ defined as :

$$s_1(t) = \begin{cases} s_2(t), & -T/2 < t < T/2 \\ 0, & elsewhere \end{cases} = s_2(t)\Pi\left(\frac{t}{T}\right)$$

and $s_1(t)$ is an energy waveform as long as T is finite.

The power spectral density is always a real nonnegative function of frequency. It is not sensitive to the phase spectrum of the truncated waveform s_1 (t). Thus, A sin $2\pi f_0 t$ and A cos $2\pi f_0 t$ have the same PSD because the phase has no effect on the power spectral density.

Fourier Transform of Periodic Signals :

So far we have used the Fourier series and the Fourier transform to represent periodic and nonperiodic signals, respectively. For periodic signals, we can use an impulse function in the frequency domain to represent discrete components of periodic signals using Fourier transforms. With this approach, both periodic and nonperiodic signals can be incorporated in a common Fourier-transform framework.

Recall:

$$A\delta(t) \longleftrightarrow A$$

$$A\delta(t - t_0) \longleftrightarrow Ae^{-j2\pi f\, t_0}$$

$$A \longleftrightarrow A\delta(f)$$

$$Ae^{j2\pi f_0 t} \longleftrightarrow A\delta(f - f_0)$$

The complex Fourier series of a periodic signal is given by :

$$s(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t}$$

and the Fourier transform of s(t) is :

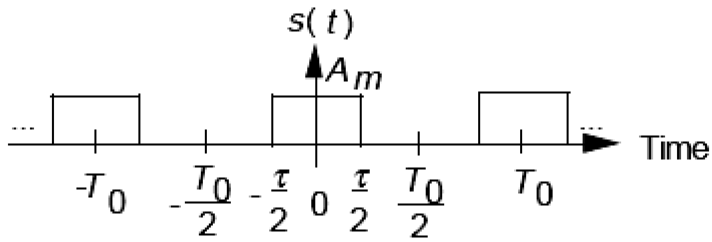
$$S(f) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_0)$$

<u>Example 2.7 :</u> The complex Fourier series of a periodic rectangular waveform s(t) is :

$$s(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t} <\!\!> \frac{1}{T_0} \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_0)$$

Where :

$$c_n = A_m \tau \left(\frac{\sin 2\pi\, n f_0 \tau/2}{2\pi\, n f_0 \tau/2} \right).$$



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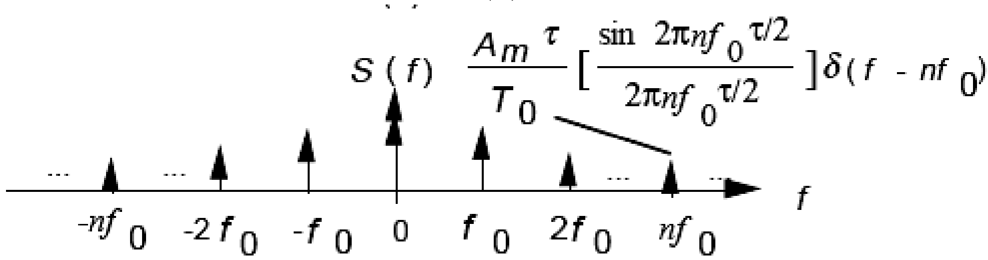
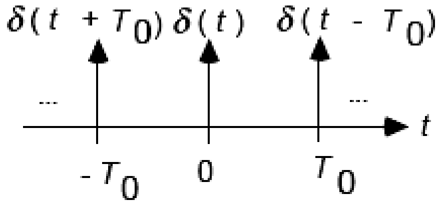


(b)

Figure 2.11 (a) A periodic rectangular waveform s(t), and (b) the Fourier transform spectrum of s(t) .

Example 2.8 : A periodic impulse s(t) is :

$$s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0) \longleftrightarrow f_0 \sum_{n=-\infty}^{\infty} \delta(f - nf_0).$$



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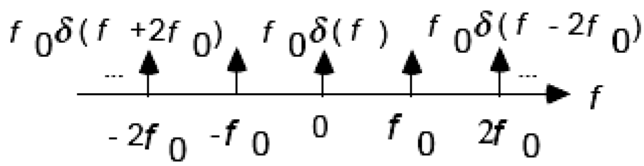

(b)

Figure 2.12 (a) Periodic impulse s(t), and (b) Fourier transform spectrum of s(t).

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[1] M. Schwartz, Information Transmission, Modulation, and Noise, 4/e, McGraw-Hill, 1990.
[2] J. D. Gibson, Modern Digital and Analog Communications, 2/e, Macmillan Publishing Company, 1993.
[3] L. W. Couch II, Digital and Analog Communication Systems, 5/e, Prentice Hall, 1997.
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3.21

2.7 Systems :

A system is a mathematical model that relates the output signal to the input signal of a physical process.

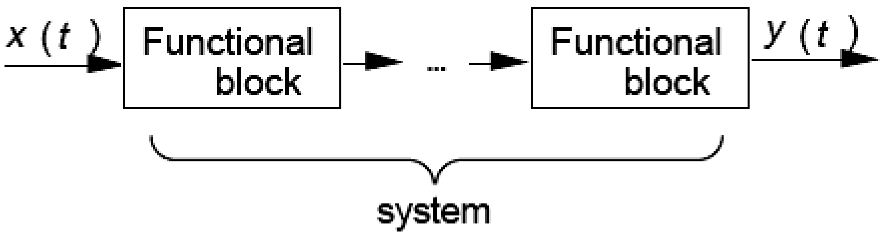

Figure 2.13 Representation of a system.

Classification of Systems :

1. Linear and non-linear systems

Let $x_i(t)$ and $y_i(t)$, $i > 1$, be input and output signals of a system, respectively. A system is called a linear system if the input $x_1(t) + x_2(t) + ... + x_i(t) + ...$ produces a response $y_1(t) + y_2(t) + ... + y_i(t) + ...$, and $ax_i(t)$ produces $ay_i(t)$ for all input signals $x_i(t)$ and scalar a. This is known as the superposition theorem and a linear system obeys this principle.

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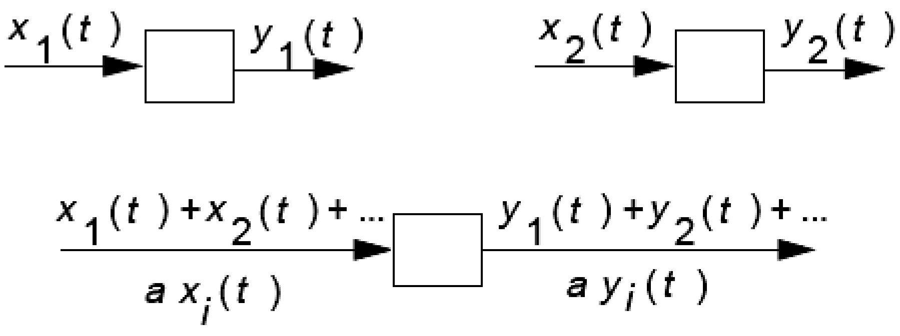

Figure 2.14 Linear system.

In practice, it may be found that a system is only linear over a limited range of input signals.

A non-linear system does not obey the superposition theorem.

2.Causal and non-causal systems :

Let x(t) and y(t) be the input and output signals of a system. A causal (physically realizable) system produces an output response at time t_1 for an input at time t_0, where

$$t_0 \geq 0 \text{ and } t_0 \leq t_1 .$$

In other words, a causal system is one whose response does not begin before the input signal is applied.
A non-causal system response will begin before the input signal is applied. It can be made realizable by introducing a positive time delay into the system .

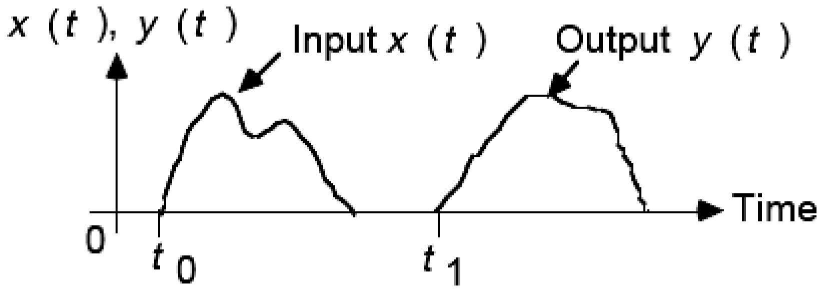

Figure 2.15 Signals associated with causal system.

3.Time-invariant and time-varying systems :
If the input $x(t - t_0)$ produces a response $y(t - t_0)$ where t_0 is any real constant, the system is called a time-invariant system.
If the above condition is not satisfied, the system is called a time-varying system.
A system is called a linear time-invariant (LTI) system if the system is linear and time-invariant.

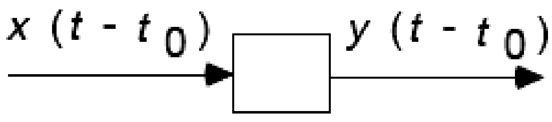

Figure 2.16 Time-invariant system.

Classification of signals and systems will help us in finding a suitable mathematical model for a given physical process that is to be analyzed.

Impulse Response :

The impulse response h(t) of an LTI system is defined as the response of the system when the input signal x(t) is a delta function δ(t). The output y(t) of an LTI system can be expressed as the convolution of the input signal x(t) and the impulse response h(t) of the system, i.e.,

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$
$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$

For a causal LTI system :

$$y(t) = \int_{0}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda = \int_{0}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$

Transfer Function :

In the frequency domain, the Fourier transform of y(t) = h(t) * x(t) is :

$$Y(f) = H(f)\, X(f)$$

where H(f) is the Fourier transform of h(t). H(f) is called the transfer function or frequency response of the LTI system.

Example 2.9 **:** An RC circuit is shown in Figure 2.17 , using Kirchhoff's voltage law, we get :

$$x(t) = i(t)\, R + y(t)$$

Since $i(t) = C\dfrac{dy}{dt}$, we can write

$$RC\frac{dy}{dt} + y(t) = x(t)$$

and

$$RC(j2\pi f)Y(f) + Y(f) = X(f)$$

The transfer function of the *RC* circuit is:

$$H(f) = \frac{1}{1 + j\ 2\pi\ RC\ f}$$

and the impulse response is :

$$h(t) = \begin{cases} \frac{1}{RC} e^{-\frac{t}{RC}}, & t \geq 0 \\ 0, & t < 0 \end{cases}.$$

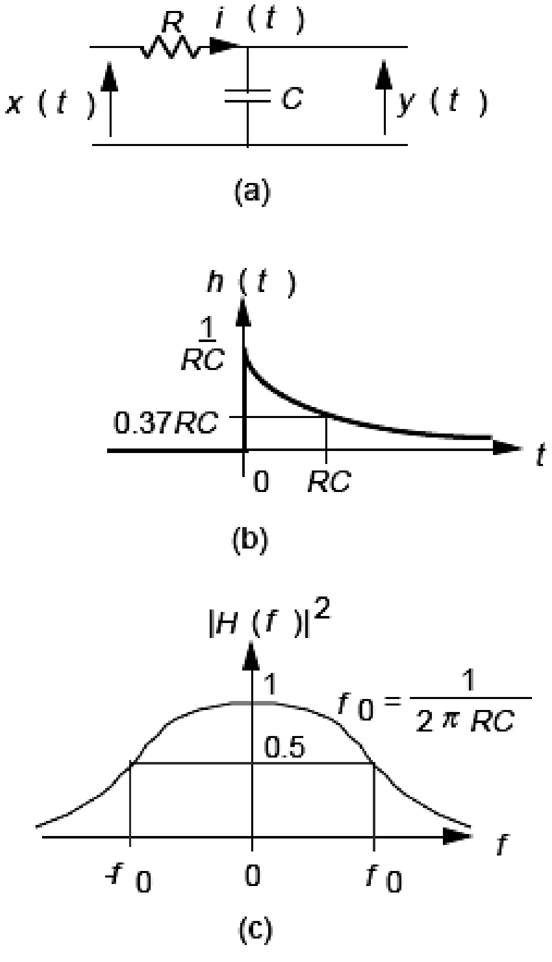

Figure 2.17 Characteristics of an *RC* circuit.

Consider the input signal $x(t) = \delta(t)$. The Fourier transform of $x(t)$ is

$$X(f) = F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi ft}\, dt = 1$$

and Y(f) = H(f).

X(f) = 1 implies equal amplitude at all frequencies. It is equivalent to exciting the system with all frequencies simultaneously.

Rather than applying a sinusoidal signal and varying its frequency continuously to obtain H(f), a technique to measure H(f) is as follows:

1. Excite the system with x(t) = δ(t).
2. Measure y(t) = h(t).
3. Find :

$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j2\pi ft}\, dt.$$

In general, H(f) is a complex function of frequency. In polar form, H(f) can be expressed as $H(f) = |H(f)|\ e^{j\theta(f)}$

From our earlier study of the properties of Fourier transform, we have seen that if h(t) is real,

H(-f) = H*(f)

|H(-f)| = |H(f)|

θ(-f) = -θ(f)

where H*(f) is the complex conjugate of H(f). If x(t) and y(t) are real and H(f) is the transfer function of a LTI system, we can obtain the following results.

Let x(t) and y(t) be the input and output energy signals of a LTI system. The energy spectral densities of x(t) and y(t) are $Exx(f) = |X(f)|^2$ and $Eyy(f) = |Y(f)|^2$, respectively.

Since Y(f) = H(f) X(f), we have

$Eyy(f) = |H(f)|^2\ Exx(f)$

Eyy(f) = H*(f)H(f) Exx(f)

Let x(t) and y(t) be the input and output power signals of a LTI system. Also, let $x_T(t)$ and $y_T(t)$ be the truncated signals of x(t) and y(t), respectively, where

$$x_T(t) = \begin{cases} x(t), & -T/2 < t < T/2 \\ 0, & elsewhere \end{cases} = x(t)\Pi\left(\frac{t}{T}\right)$$

and

$$y_T(t) = \begin{cases} y(t), & -T/2 < t < T/2 \\ 0, & elsewhere \end{cases} = y(t)\Pi\left(\frac{t}{T}\right)$$

$x_T(t)$ and $y_T(t)$ are energy signals as long as T is finite. The power spectral densities of x (t) and y (t) are :

$$P_{xx}(f) = \lim_{T \to \infty}\left(\frac{|X_T(f)|^2}{T}\right)$$

and

$$P_{yy}(f) = \lim_{T \to \infty}\left(\frac{|Y_T(f)|^2}{T}\right)$$

, respectively.

Since Y(f) = H(f) X(f), we have

$Pyy(f) = |H(f)|^2 Pxx(f)$

Pyy(f) = H*(f)H(f) Pxx(f)

We can also obtain the relationship between the input and output auto-correlation functions of a LTI system. Since the energy/power spectral density and the auto-correlation function are Fourier transform pairs, the inverse Fourier transform of Eyy(f) or Pyy(f) is :

$$R_{yy}(\tau) = h(-\tau) * h(\tau) * R_{xx}(\tau)$$

where Rxx(τ) and Ryy(τ) are the auto-correlation functions of x(t) and y(t) , respectively.

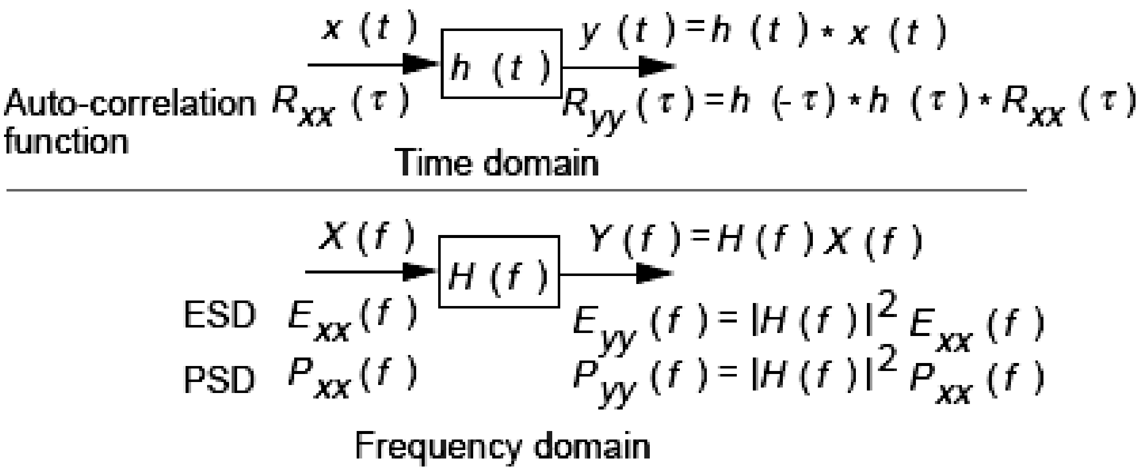

Figure 2.18 Input and output relationships of linear system.

Distortionless Transmission :

In communication systems, a distortionless transmission is often desired. This implies that the output signal $y(t)$ is given by :

$y(t) = K\, x(t - t_d)$

where K is a constant and t_d is a time delay. The Fourier transform of $y(t)$ is :

$$\begin{aligned} Y(f) &= K\, e^{-j2\pi f t_d} X(f) \\ &= H(f)\, X(f) \end{aligned}$$

where

$$H(f) = K\, e^{-j2\pi f t_d}.$$

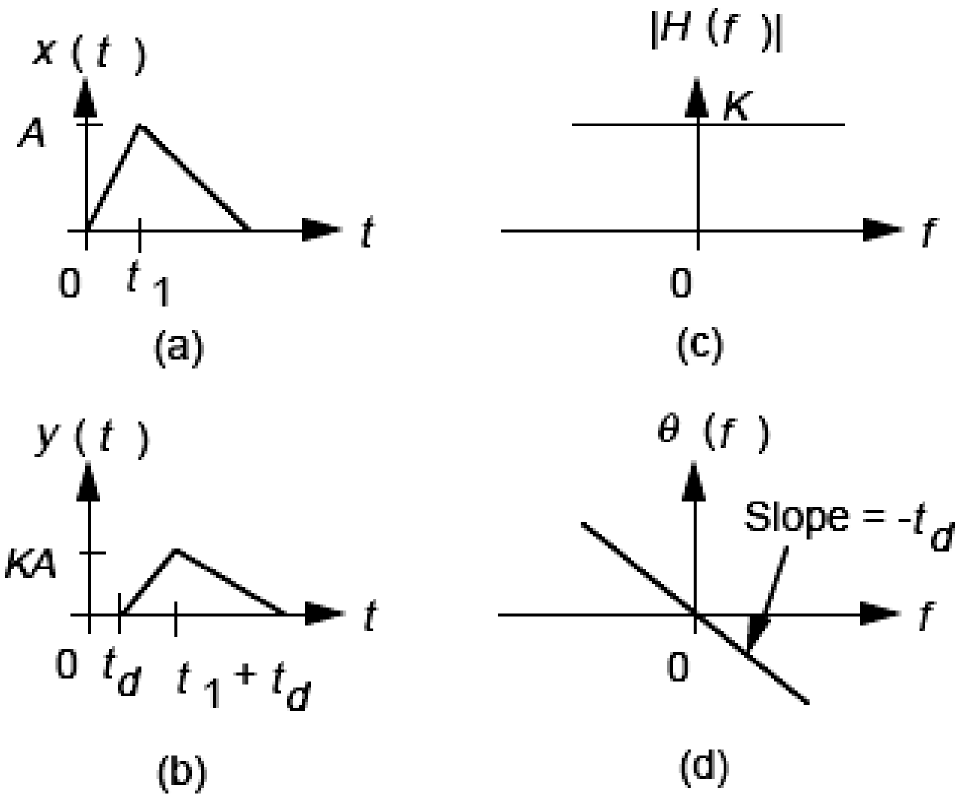

Figure 2.18 Waveforms and spectra associated with distortionless transmission.

Classification of Filters

1. Ideal Low-Pass Filter.

The transfer function of an ideal low-pass filter is defined by:

$$H_{LPF}(f) = \begin{cases} e^{-j\,2\pi f t_d} & \text{for } |f| \le f_c \\ 0 & \text{elsewhere} \end{cases}$$

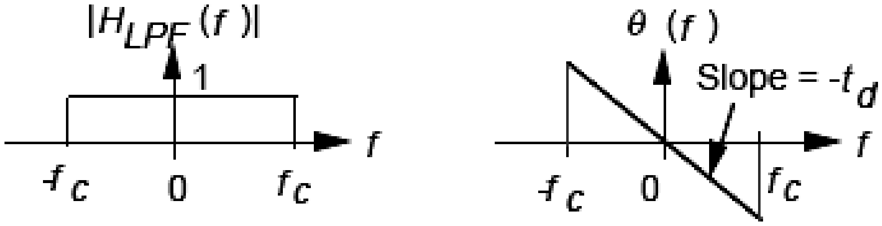

Figure 2.19 Frequency response of an ideal LPF.

The bandwidth of an ideal low-pass filter is equal to f_c.

2. Ideal High-Pass Filter.

The transfer function of an ideal high-pass filter is defined by :

$$H_{HPF}(f) = \begin{cases} e^{-j\,2\pi f t_d} & \text{for } |f| \ge f_c \\ 0 & \text{elsewhere} \end{cases}$$

or

$$H_{HPF}(f) = e^{-j2\pi f t_d} - H_{LPF}(f)$$

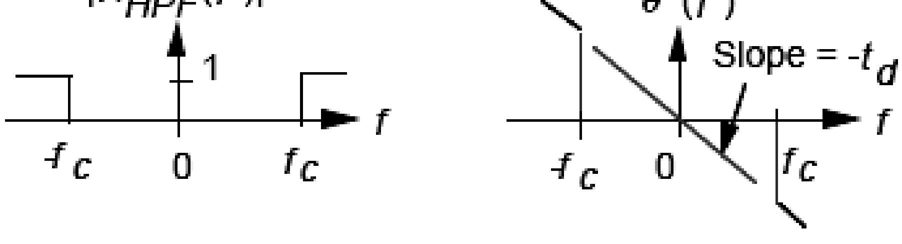

Figure 2.20 Frequency response of an ideal HPF.

The bandwidth of an ideal high-pass filter is not defined, or infinite.

3. Ideal Bandpass Filter.
The transfer function of an ideal bandpass filter is defined by:

$$H_{BPF}(f) = \begin{cases} e^{-j\,2\pi f t_d} & \text{for } f_{c1} \le |f| \le f_{c2} \\ 0 & \textit{elsewhere} \end{cases}$$

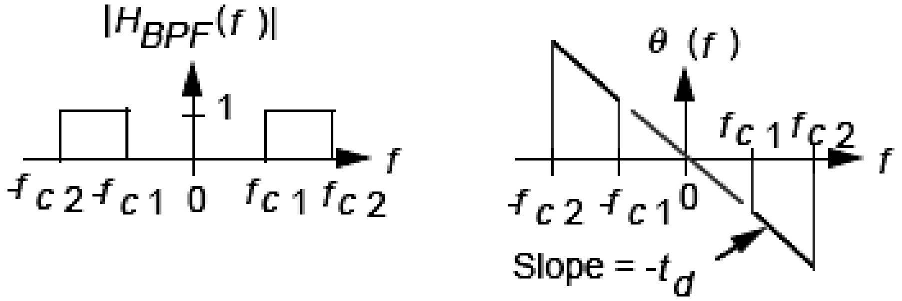

Figure 2.21 Frequency response of an ideal BPF.
The bandwidth of an ideal bandpass filter is equal to $f_{c2} - f_{c1}$.

4. Ideal Bandstop Filter.
The transfer function of an ideal bandstop filter is defined by:

$$H_{BSF}(f) = \begin{cases} e^{-j\,2\pi f t_d} & \text{for } 0 \le |f| \le f_{c1} \text{ and } |f| \ge f_{c2} \\ 0 & \textit{elsewhere} \end{cases}$$

or

$$H_{BSF}(f) = e^{-j2\pi f t_d} - H_{BPF}(f)$$

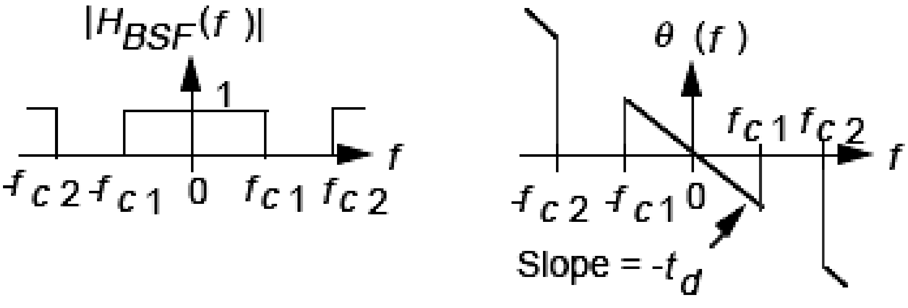

Figure 2.22 Frequency response of an ideal BSF.
The bandwidth of an ideal bandstop filter is not defined, or infinite.

Concept of Non-Ideal or Practical Filter (System) Bandwidth

A common definition of a non-ideal filter's bandwidth is the 3dB bandwidth. For low-pass filters, the bandwidth is defined as the positive frequency at which the amplitude spectrum |H(f)| drops to a value equal to $|H(f)|/\sqrt{2}$. For bandpass filters, the bandwidth is defined as the difference between the frequencies at which the amplitude spectrum |H(f)| drops to a value equal to $|H(f_0)|/\sqrt{2}$, where $H(f_0)$ is the peak value of |H(f)|.

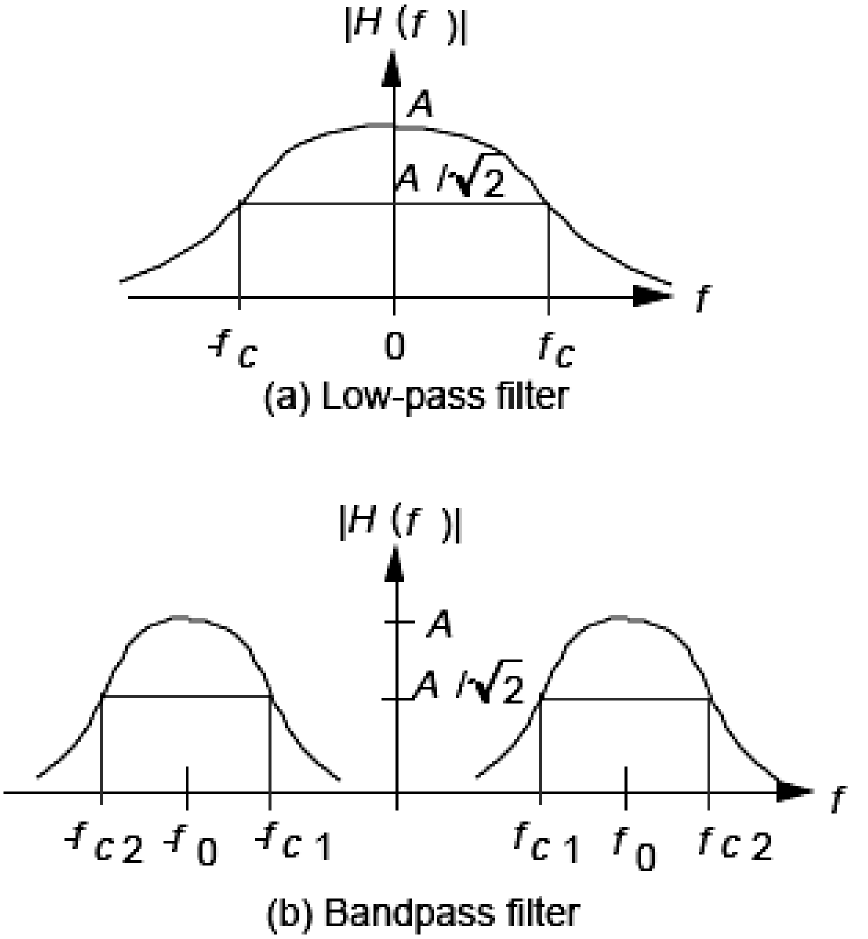

Figure 2.23 Non-ideal filter bandwidth.

MODULATION

Modulation is a process by which some parameter (amplitude, frequency or phase) of a carrier signal is varied in accordance with a message signal. The message signal is called a modulating signal.
A carrier signal, in analog modulation, usually a simple sine wave, contains no information in itself. This gives us three possibilities:

- Amplitude modulation (AM), where the amplitude or strength of the carrier is varied.

- Frequency modulation (FM), where the frequency of the carrier is varied.

- Phase modulation (PM), where the phase of the carrier is varied.
  It actually turns out that FM and PM are very close relatives.

Reasons of modulation:

♦ Match the signal to the channel characteristics and increase the efficiency of information transmission.
♦ Shift the frequency band occupied by the message signal.
♦ Reduce the relative bandwidth.
♦Separate the signals in the frequency domain.
♦ Less sensitivity to channel distortion.

Demodulation:

It is the process of separating the original message (base-band) signal from the noisy and distorted received modulated carrier signal .

3. Amplitude Modulation

It is defined as the process of varying the amplitude of a sinusoidal carrier wave in synchronism with , and in direct proportion , to the amplitude of a modulating signal .

The unmodulated carrier signal is given as :

$$s(t)= A \cos 2\pi f_c t$$

A sinusoidal modulated (band-pass) carrier signal is represented by:

$$s_c(t) = A(t) \cos \theta(t)$$

where $A(t)$ is the envelope and $\theta(t) = \omega_c t + \phi(t) = 2\pi f_c t + \phi(t)$. $\phi(t)$ is called the instantaneous phase deviation of $s_c(t)$ and f_c is the carrier frequency. For amplitude modulation, we can write

$$s_c(t) = A(t) \cos 2\pi f_c t$$

where $A(t)$ is linearly related to the modulating signal m(t). $A(t)$ is called the instantaneous amplitude of $s_c(t)$ and amplitude modulation is also referred to as linear modulation. Depending on the relationship between $m(t)$ and $A(t)$, we have the following types of amplitude modulation schemes:

- Normal amplitude modulation (AM) .
- Double-sideband (DSB) modulation .
- Single-sideband (SSB) modulation .
- Vestigial-sideband (VSB) modulation .

Normal Amplitude Modulation (AM) :

A normal amplitude-modulated signal is given by:

$$s_C(t) = [A + m(t)] \cos 2\pi f_C t$$

$$s_C(t) = \underbrace{A \cos 2\pi f_C t}_{carrier} + \underbrace{m(t) \cos 2\pi f_C t}_{sidebands}$$

where A is a constant and $m(t)$ is the modulating signal.

The modulation index m is defined as the ratio of the amplitude of the modulating signal to that of the unmodulated carrier. It is a value between 0 and 1 which describes the "degree of modulation" of the carrier. If $m = 0$ there is no modulation, while $m = 1$ is the maximum modulation that can occur without distortion.

Figure 3.1 shows normal AM signals for various values of modulation index. Clearly, the envelope of the modulated signals has the same shape as m(t) when $m < 1$. When $m > 1$, the carrier signal is said to be over- modulated and the envelope is distorted.

If $m(t) = E_m \cos\omega_m t$, and $\omega_m = 2\pi f_m$.

$$A(t) = A + E_m \cos\omega_m t$$

$$s_c(t) = A\,(1 + m \cos\omega_m t)\cos\omega_c t$$

$$= A \cos\omega_c t + m A \cos\omega_m t \cos\omega_c t$$

$$= A \cos\omega_c t + 0.5\, mA \cos(\omega_c - \omega_m)t + 0.5\, mA \cos(\omega_c + \omega_m)t$$

and $m = E_m / A$.

The total average power of normal AM signal is:

$$P_t = P_c + 0.25\, m^2 P_c + 0.25\, m^2 P_c$$

$$= P_c\,(1 + 0.5\, m^2)$$

where $P_c = (A_{rms})^2 / R$ is the power of the unmodulated carrier wave &

$P_s = 0.5\, m^2 P_c$ is the power of the upper and lower sidebands.

For *100% modulation* , $m = 1$, $P_t = 1.5\, P_c$

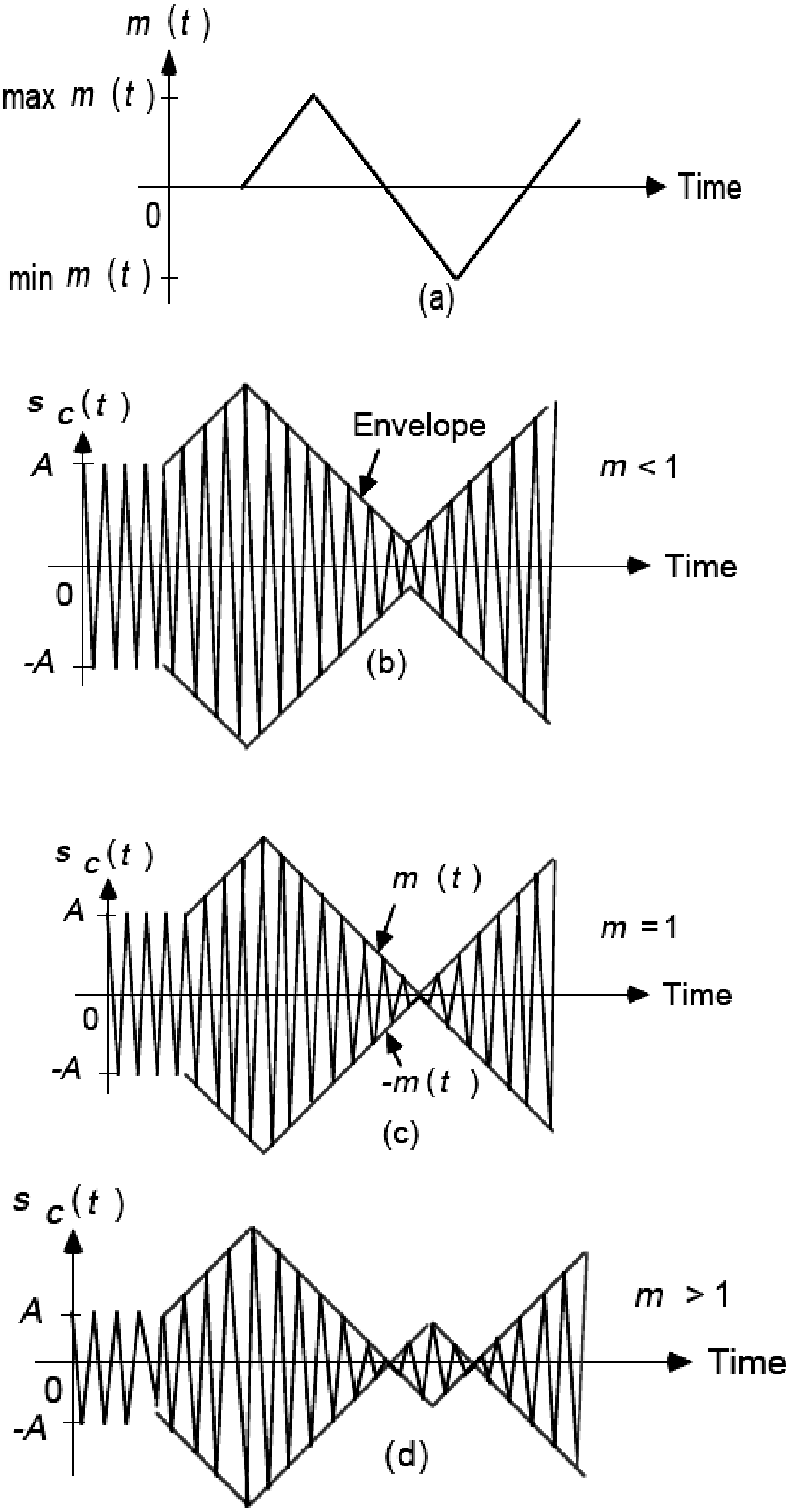

Figure 3.1 Normal AM signals for various values of modulation index

Spectrum of Normal AM Signals :

For normal amplitude modulation,

$$s_c\,(t) = [A + m(t)]\cos 2\pi f_c t$$
$$= A\cos 2\pi f_c t + m(t)\cos 2\pi f_c$$

The Fourier transform of $s_c(t)$ is :

$$S_c(f) = 0.5\,A[\delta(f\text{-}f_c) + \delta(f\text{+}f_c)] + 0.5\,[M(f\text{-}f_c) + M(f\text{+}f_c)]$$

Normal amplitude modulation simply shifts the spectrum of $m(t)$ to the carrier frequency f_c. The bandwidth of the modulated signal is $2\,f_m$ Hz, where f_m is the bandwidth of the modulating signal $m(t)$.

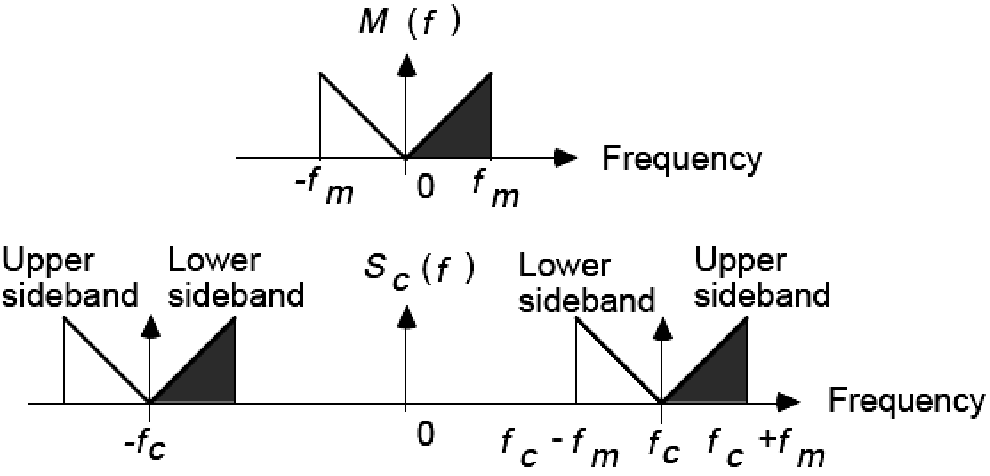

Figure 3.2 Spectrum of normal AM signal.

The efficiency η of a normal AM signal is defined as :

$$\eta = \frac{P_s}{P_t}\ \text{x}\ 100\%$$

where P_s is the power carried by the sidebands and P_t is the total power of the normal AM signal.

Generation of Normal AM Signals :

A process of generating a normal AM signal is shown in Figure 3.3. This type of modulation can be achieved by using a non-linear device, such as a diode .This is shown in Figure 3.4 .

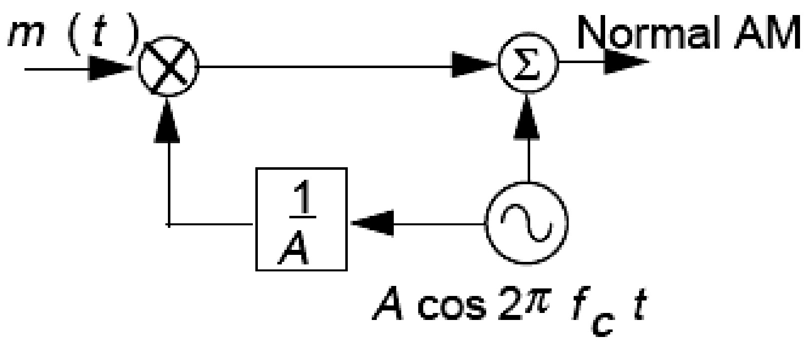

Figure 3.3 Generation of normal AM signal.

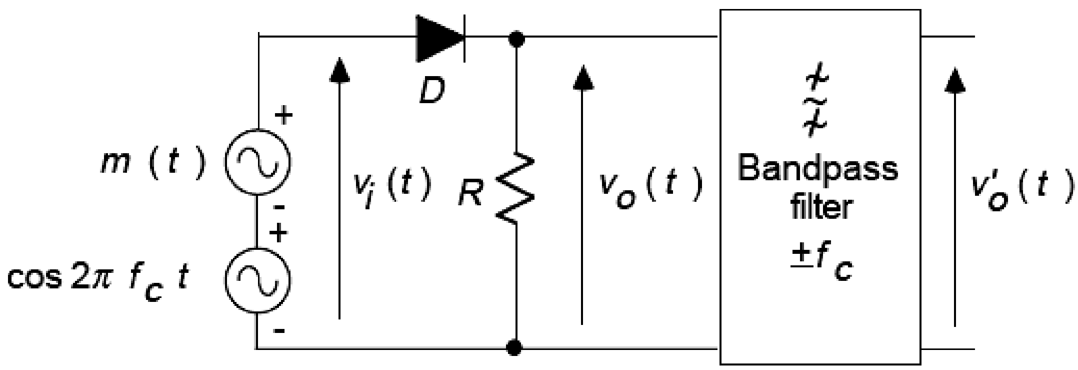

Figure 3.4 Amplitude modulator using a diode.

Let the input-output characteristic of a diode be approximated by a power series :

$$v_o(t) = a\, v_i(t) + b\, v_i^2(t)$$

where a & b are constants and

$$v_i(t) = \cos 2\pi f_c t + m(t)$$

Hence :

$$v_o(t) = a[\cos 2\pi f_c t + m(t)] + b[\cos 2\pi f_c t + m(t)]^2$$
$$= a\, m(t) + b \cos^2 2\pi f_c t + b\, m(t)^2 +$$
$$a \cos 2\pi f_c t + 2b\, m(t) \cos 2\pi f_c t$$

If we pass the signal $v_o(t)$ through a band- pass filter centered at $\pm f_c$, we obtain

$$v'_o(t) = [a + 2b\, m(t)] \cos 2\pi f_c t$$
$$= 2b[A + m(t)] \cos 2\pi f_c t$$

where A = a /2b . We generate a normal AM signal.

A normal amplitude-modulated signal can also be obtained by multiplying $m(t)$ by a periodic signal $s(t)$. The modulator is called a *switching modulator*. If we take a periodic rectangular waveform $s(t)$ of period $T_c = 1/f_c$, amplitude A_m , and pulse width τ , the trigonometric Fourier series of $s(t)$ is :

$$s(t) = \frac{A_m \tau}{T_c} + \frac{2}{T_c} \sum_{n=1}^{\infty} \left(A_m \tau \frac{\sin 2\pi n f_c \tau /2}{2\pi n f_c \tau /2}\right) \cos 2\pi n f_c t$$

$$s(t) = \frac{c_0}{T_c} + \frac{2}{T_c} \sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t$$

where $c_o = A_m \tau$ and

$$c_n = A_m \tau \frac{\sin 2\pi n f_c \tau /2}{2\pi n f_c \tau /2}$$

The corresponding complex Fourier series is :

$$s(t) = \frac{1}{T_c} \sum_{n=-\infty}^{\infty} \left(A_m \tau \frac{\sin 2\pi n f_c \tau /2}{2\pi n f_c \tau /2}\right) e^{j 2\pi n f_c t}$$

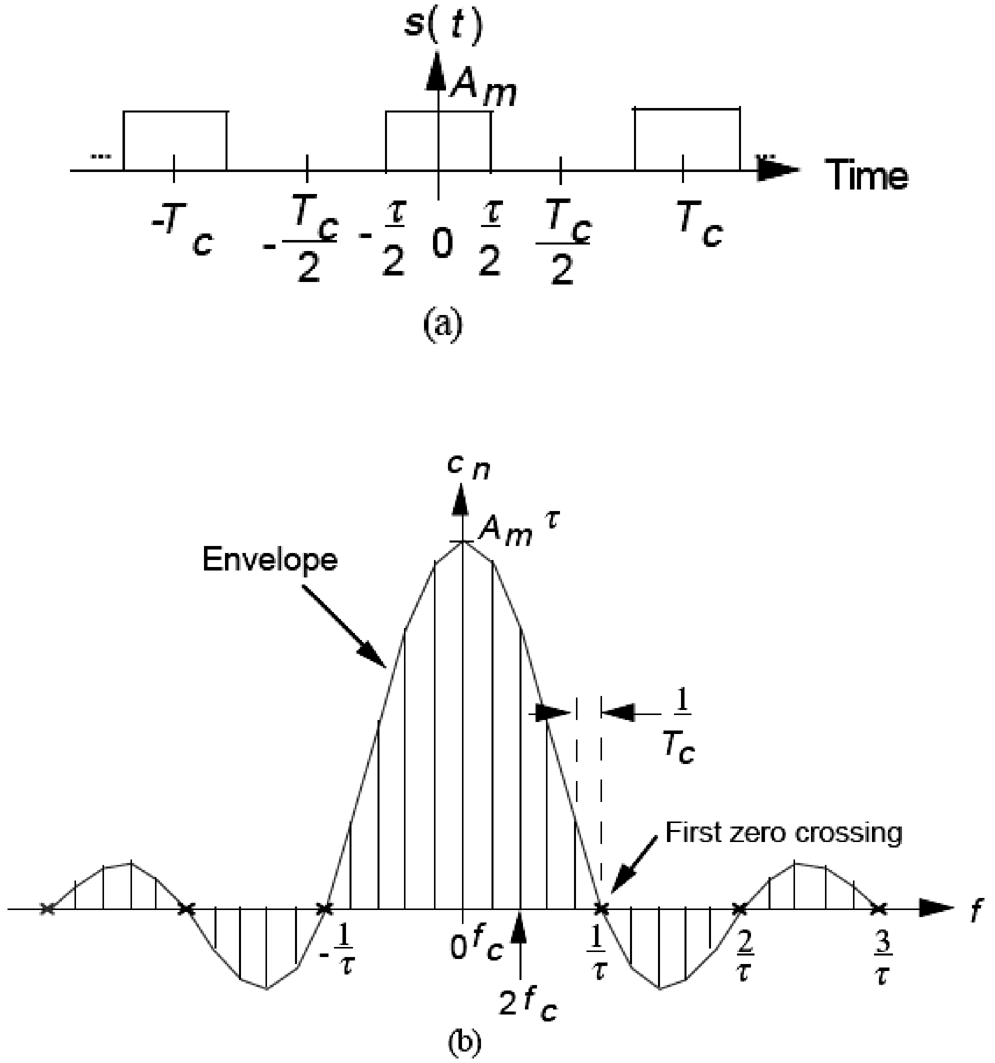

Figure 3.5 (a) A periodic rectangular waveform, and (b) its line spectrum.

If the input signal is :

$$v_i(t) = \cos 2\pi f_c t + m(t),$$

the output of a switching modulator is :

$$v_o(t) = v_i(t)\, s(t)$$

$$= [\cos 2\pi f_c t + m(t)]\, s(t)$$

$$= [\cos 2\pi f_c t + m(t)] \left(\frac{c_0}{T_c} + \frac{2}{T_c} \sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t\right)$$

$$= \left[\frac{2}{T_c} \cos 2\pi f_c t \sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t\right] + \frac{c_0}{T_c}\, m(t) +$$

$$\frac{c_0}{T_c} \cos 2\pi f_c t + m(t) \frac{2}{T_c} \sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t$$

$v_o(t)$ consists of a dc term, the component $m(t)$, and an infinite number of normal AM signals at carrier frequencies f_c, $2f_c$, $3f_c$, ... If we pass the signal $v_o(t)$ through a band-pass filter centered at $\pm f_c$, the filtered signal is :

$$v'_o(t) = \frac{c_0}{T_c}\cos 2\pi f_c t + \frac{2}{T_c} m(t) c_1 \cos 2\pi f_c t + \frac{c_2}{T_c}\cos 2\pi f_c t$$
$$= \frac{2c_1}{T_c}[A + m(t)]\cos 2\pi f_c t$$

where $A = (c_0 + c_2) / 2c_1$. That is, we obtain a normal AM signal.

Demodulation of Normal AM Signals:

The process of recovering the message signal from the modulated signal is called ***demodulation*** **or** ***detection***. Two basic methods are available :

1.Envelope Detection **:**

In this method, an envelope detector is used to recover the message signal. An envelope detector consists of a diode and a resistor-capacitor combination. This is shown in Figure 3.6.

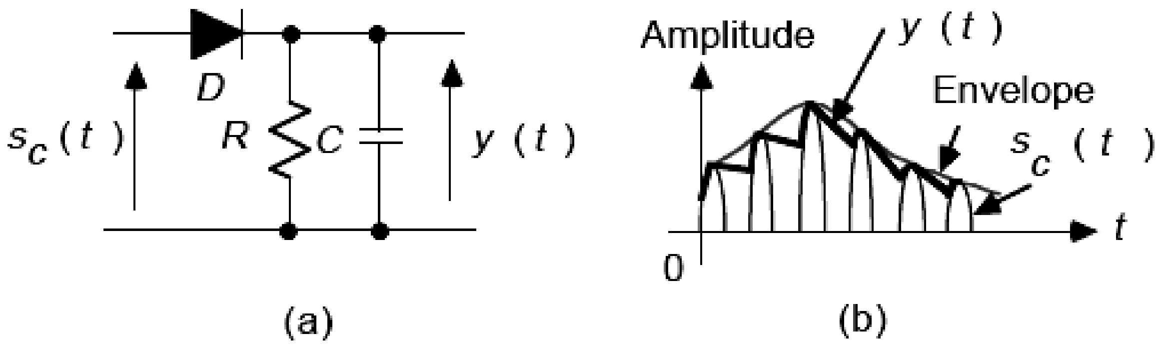

Figure 3.6 Envelope detector.

During the positive half-cycle peaks of the modulated signal, the diode is forward biased, and the capacitor charges up to the peak value of the modulated signal. As the modulated signal falls from its maximum, the diode turns off and the capacitor discharges through the resistor. The process repeats in this way. For proper operation, the discharge time constant *RC* must be chosen properly.

2.Synchronous (Coherent) Detection :

Here, a product detector is used to convert the band-pass signal to base-band. This is shown in Figure3.7.

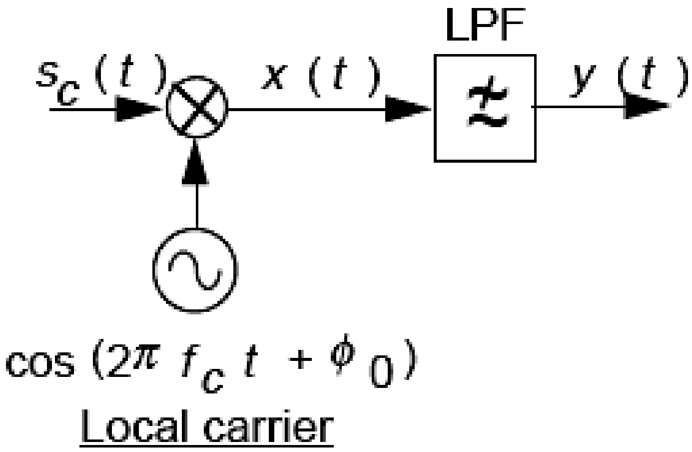

Figure 3.7 Synchronous detector.

At the receiving end, the band-pass signal is multiplied by a locally generated carrier signal $\cos(2\pi f_c t + \phi_0)$,
where ϕ_0 is an initial phase. The output of the multiplier is

$$\begin{aligned} x(t) &= [A + m(t)] \cos 2\pi f_c t \cos(2\pi f_c t + \phi_0) \\ &= 0.5[A + m(t)] [\cos \phi_0 + \cos(4\pi f_c t + \phi_0)] \\ &= 0.5[A + m(t)] \cos(4\pi f_c t + \phi_0) + 0.5A\cos \phi_0 + 0.5m(t)\cos \phi_0 \end{aligned}$$

If we suppress the first term by a low-pass filter, we get

$$y(t) = 0.5A\cos \phi_0 + 0.5\, m(t) \cos \phi_0$$

It can be seen that we can recover the component *m*(*t*) if the initial phase ϕ_0 is constant and small.
Suppose that the local carrier signal is :

$$\cos[2\pi (f_c + \Delta f)t],$$

The multiplier output becomes :

$$\begin{aligned} x(t) &= [A + m(t)] \cos 2\pi f_c t \cos[2\pi(f_c + \Delta f)t] \\ &= 0.5[A + m(t)] [\cos 2\pi\Delta f\, t + \cos 2\pi(2f_c + \Delta f)t] \\ &= 0.5[A + m(t)] \cos 2\pi(2f_c + \Delta f)t + \\ &\quad 0.5A\cos 2\pi\Delta f\, t + 0.5m(t)\cos 2\pi\Delta f\, t \end{aligned}$$

If we suppress the first term by a low-pass filter, we get

$$y(t) = 0.5A\cos 2\pi\Delta f\, t + 0.5m(t)\cos 2\pi\Delta f\, t$$

We cannot recover the component *m*(*t*) unless the frequency drift Δf is zero. Therefore, the local carrier must not only be of the same frequency but must be synchronized in phase with the carrier signal. If the carrier shifts in frequency or phase, the resultant signal is distorted or attenuated. Synchronous detection is sometimes called ***coherent detection.***