C
ANSWERS TO SELECTED BASIC PROBLEMS

This appendix contains the answers to the first 20 basic problems in each chapter.

Answers to Basic Problems in Chapter 2

2.1. **(a)** Always (2), (3), (5). If $g[n]$ is bounded, (1).
(b) (3).
(c) Always (1), (3), (4). If $n_0 = 0$, (2) and (5).
(d) Always (1), (3), (4). If $n_0 \leq 0$, (2). If $n_0 = 0$, (5).
(e) (1), (2), (4), (5).
(f) Always (1), (2), (4), (5). If $b = 0$, (3).
(g) (1), (3).
(h) (1), (5).

2.2. **(a)** $N_4 = N_0 + N_2$, $N_5 = N_1 + N_3$.
(b) At most $N + M - 1$ nonzero points.

2.3.

$$y[n] = \begin{cases} \dfrac{a^{-n}}{1-a}, & n < 0, \\ \dfrac{1}{1-a}, & n \geq 0. \end{cases}$$

2.4. $y[n] = 8[(1/2)^n - (1/4)^n]u[n]$.

2.5. **(a)** $y_h[n] = A_1(2)^n + A_2(3)^n$.
(b) $h[n] = 2(3^n - 2^n)u[n]$.
(c) $s[n] = [-8(2)^{(n-1)} + 9(3)^{(n-1)} + 1]u[n]$.

2.6. (a)

$$H(e^{j\omega}) = \frac{1 + 2e^{-j\omega} + e^{-j2\omega}}{1 - \frac{1}{2}e^{-j\omega}}.$$

(b) $y[n] + \frac{1}{2}y[n-1] + \frac{3}{4}y[n-2] = x[n] - \frac{1}{2}x[n-1] + x[n-3]$.

2.7. (a) Periodic, $N = 6$.

(b) Periodic, $N = 8$.

(c) Not periodic.

(d) Not periodic.

2.8. $y[n] = 3(-1/2)^n u[n] + 2(1/3)^n u[n]$.

2.9. (a)

$$h[n] = 2\left[\left(\frac{1}{2}\right)^n - \left(\frac{1}{3}\right)^n\right]u[n],$$

$$H(e^{j\omega}) = \frac{\frac{1}{3}e^{-j\omega}}{1 - \frac{5}{6}e^{-j\omega} + \frac{1}{6}e^{-j2\omega}},$$

$$s[n] = \left[-2\left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n + 1\right]u[n].$$

(b) $y_h[n] = A_1(1/2)^n + A_2(1/3)^n$.

(c) $y[n] = 4(1/2)^n - 3(1/3)^n - 2(1/2)^n u[-n-1] + 2(1/3)^n u[-n-1]$.

2.10. (a)

$$y[n] = \begin{cases} a^{-1}/(1 - a^{-1}), & n \geq -1, \\ a^n/(1 - a^{-1}), & n \leq -2. \end{cases}$$

(b)

$$y[n] = \begin{cases} 1, & n \geq 3, \\ 2^{(n-3)}, & n \leq 2. \end{cases}$$

(c)

$$y[n] = \begin{cases} 1, & n \geq 0, \\ 2^n, & n \leq -1. \end{cases}$$

(d)

$$y[n] = \begin{cases} 0, & n \geq 9, \\ 1 - 2^{(n-9)}, & 8 \geq n \geq -1, \\ 2^{(n+1)} - 2^{(n-9)}, & -2 \geq n. \end{cases}$$

2.11. $y[n] = 2\sqrt{2}\sin(\pi(n-1)/4)$.

2.12. (a) $y[n] = n!u[n]$.

(b) The system is linear.

(c) The system is not time invariant.

2.13. **(a)**, **(b)**, and **(e)** are eigenfunctions of stable LTI systems.

2.14. **(a)** (iv).
(b) (i).
(c) (iii), $h[n] = (1/2)^n u[n]$.

2.15. **(a)** Not LTI. Inputs $\delta[n]$ and $\delta[n-1]$ violate TI.
(b) Not causal. Consider $x[n] = \delta[n-1]$.
(c) Stable.

2.16. **(a)** $y_h[n] = A_1(1/2)^n + A_2(-1/4)^n$.
(b) Causal: $h_c[n] = 2(1/2)^n u[n] + (-1/4)^n u[n]$.
Anticausal: $h_{ac}[n] = -2(1/2)^n u[-n-1] - (-1/4)^n u[-n-1]$.
(c) $h_c[n]$ is absolutely summable, $h_{ac}[n]$ is not.
(d) $y_p[n] = (1/3)(-1/4)^n u[n] + (2/3)(1/2)^n u[n] + 4(n+1)(1/2)^{(n+1)} u[n+1]$.

2.17. **(a)**

$$R(e^{j\omega}) = e^{-j\omega M/2} \frac{\sin\left(\omega\left(\frac{M+1}{2}\right)\right)}{\sin\left(\frac{\omega}{2}\right)}.$$

(b) $W(e^{j\omega}) = (1/2)R(e^{j\omega}) + (1/4)R(e^{j(\omega-2\pi/M)}) + (1/4)R(e^{j(\omega+2\pi/M)})$.

2.18. Systems (a) and (b) are causal.

2.19. Systems (b), (c), (e), and (f) are stable.

2.20. **(a)** $h[n] = (1/a)^{(n-1)} u[n-1]$.
(b) The system will be stable for $|a| > 1$.

Answers to Basic Problems in Chapter 3

3.1. **(a)** $\dfrac{1}{1-\frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}.$

(b) $\dfrac{1}{1-\frac{1}{2}z^{-1}}, \quad |z| < \frac{1}{2}.$

(c) $\dfrac{-\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}, \quad |z| < \frac{1}{2}.$

(d) $1, \quad$ all z.

(e) $z^{-1}, \quad z \neq 0.$

(f) $z, \quad |z| < \infty.$

(g) $\dfrac{1-\left(\frac{1}{2}\right)^{10} z^{-10}}{1-\frac{1}{2}z^{-1}}, \quad |z| \neq 0.$

3.2. $X(z) = \dfrac{z^{-1}(1-z^{-N})}{(1-z^{-1})^2}.$

3.3. **(a)** $X_a(z) = \dfrac{z^{-1}(\alpha - \alpha^{-1})}{(1-\alpha z^{-1})(1-\alpha^{-1}z^{-1})}$, ROC: $|\alpha| < |z| < |\alpha^{-1}|$.

(b) $X_b(z) = \dfrac{1-z^{-N}}{1-z^{-1}}$, ROC: $z \neq 0$.

(c) $X_c(z) = z^{-1}\dfrac{(1-z^{-N})^2}{(1-z^{-1})^2}$, ROC: $z \neq 0$.

3.4. **(a)** $(1/3) < |z| < 2$, two sided.

(b) Two sequences. $(1/3) < |z| < 2$ and $2 < |z| < 3$.

(c) No. Causal sequence has $|z| > 3$, which does not include the unit circle.

3.5. $x[n] = 2\delta[n+1] + 5\delta[n] - 4\delta[n-1] - 3\delta[n-2]$.

3.6. **(a)** $x[n] = \left(-\frac{1}{2}\right)^n u[n]$, Fourier transform exists.

(b) $x[n] = -\left(-\frac{1}{2}\right)^n u[-n-1]$, Fourier transform does not exist.

(c) $x[n] = 4\left(-\frac{1}{2}\right)^n u[n] - 3\left(-\frac{1}{4}\right)^n u[n]$, Fourier transform exists.

(d) $x[n] = \left(-\frac{1}{2}\right)^n u[n]$, Fourier transform exists.

(e) $x[n] = -(a^{-(n+1)})u[n] + a^{-(n-1)}u[n-1]$, Fourier transform exists if $|a| > 1$.

3.7. **(a)** $H(z) = \dfrac{1-z^{-1}}{1+z^{-1}}$, $|z| > 1$.

(b) $\text{ROC}\{Y(z)\} = |z| > 1$.

(c) $y[n] = \left[-\frac{1}{3}\left(\frac{1}{2}\right)^n + \frac{1}{3}(-1)^n\right]u[n]$.

3.8. **(a)** $h[n] = \left(-\frac{3}{4}\right)^n u[n] - \left(-\frac{3}{4}\right)^{n-1} u[n-1]$.

(b) $y[n] = \frac{8}{13}\left(-\frac{3}{4}\right)^n u[n] - \frac{8}{13}\left(\frac{1}{3}\right)^n u[n]$.

(c) The system is stable.

3.9. **(a)** $|z| > (1/2)$.

(b) Yes. The ROC includes the unit circle.

(c) $X(z) = \dfrac{1-\frac{1}{2}z^{-1}}{1-2z^{-1}}$.

(d) $h[n] = 2\left(\frac{1}{2}\right)^n u[n] - \left(-\frac{1}{4}\right)^n u[n]$.

3.10. **(a)** $|z| > \frac{3}{4}$.

(b) $0 < |z| < \infty$.

(c) $|z| < 2$.

(d) $|z| > 1$.

(e) $|z| < \infty$.

(f) $\frac{1}{2} < |z| < \sqrt{13}$.

3.11. **(a)** Causal.

(b) Not causal.

(c) Causal.

(d) Not causal.

3.12. **(a)**

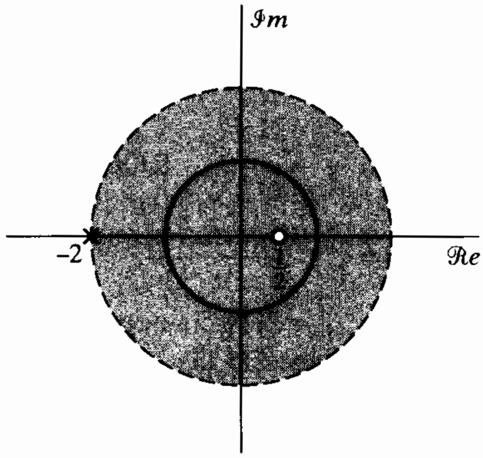

Figure P3.12-1

(b)

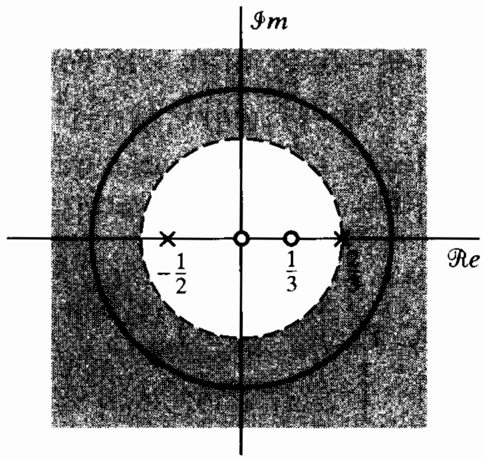

Figure P3.12-2

(c)

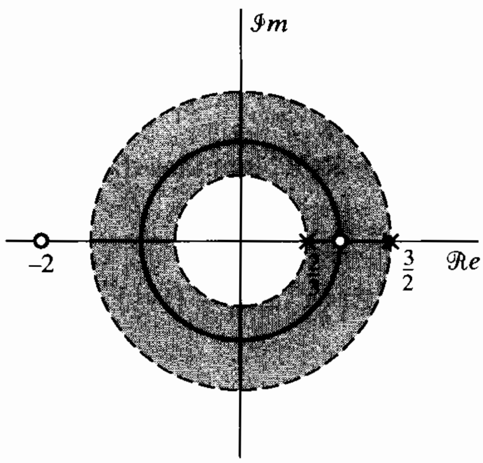

Figure P3.12-3

3.13. $g[11] = -\dfrac{1}{11!} + \dfrac{3}{9!} - \dfrac{2}{7!}.$

3.14. $A_1 = A_2 = 1/2, \quad \alpha_1 = -1/2, \quad \alpha_2 = 1/2.$

3.15. $h[n] = \left(\frac{1}{2}\right)^n (u[n] - u[n-10])$. The system is causal.

3.16. **(a)** $H(z) = \dfrac{1 - 2z^{-1}}{1 - \frac{2}{3}z^{-1}}, \quad |z| > \frac{2}{3}.$

(b) $h[n] = \left(\frac{2}{3}\right)^n u[n] - 2\left(\frac{2}{3}\right)^{(n-1)} u[n-1].$

(c) $y[n] - \frac{2}{3}y[n-1] = x[n] - 2x[n-1]$.
(d) The system is stable and causal.

3.17. $h[0]$ can be 0, 1/3, or 1. To be painstakingly literal, $h[0]$ can also be 2/3, due to the impulse response $h[n] = (2/3)(2)^n u[n] - (1/3)(1/2)^n u[-n-1]$, which satisfies the difference equation but has no ROC. This noncausal system with no ROC can be implemented as the parallel combination of its causal and anticausal components.

3.18. **(a)** $h[n] = -2\delta[n] + \frac{1}{3}(-\frac{1}{2})^n u[n] + \frac{8}{3}u[n]$.
(b) $y[n] = \left(\dfrac{2}{\frac{3}{2}+\frac{j}{2}}\right) e^{j(\pi/2)n}$.

3.19. **(a)** $|z| > 1/2$.
(b) $1/3 < |z| < 2$.
(c) $|z| > 1/3$.

3.20. **(a)** $|z| > 2/3$.
(b) $|z| > 1/6$.

Answers to Basic Problems in Chapter 4

4.1. $x[n] = \sin(\pi n/2)$.

4.2. $\Omega_0 = 250\pi, 1750\pi$.

4.3. **(a)** $T = 1/12{,}000$. **(b)** Not unique. $T = 5/12{,}000$.

4.4. **(a)** $T = 1/100$. **(b)** Not unique. $T = 11/100$.

4.5. **(a)** $T \le 1/10{,}000$. **(b)** 625 Hz. **(c)** 1250 Hz.

4.6. **(a)** $H_c(j\Omega) = 1/(a + j\Omega)$.
(b) $H_d(e^{j\omega}) = T/(1 - e^{-aT}e^{-j\omega})$.

4.7. **(a)**

$$X_c(j\Omega) = S_c(j\Omega)(1 + \alpha e^{-j\Omega\tau_d}),$$

$$X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} S_c\left(j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right) + \frac{\alpha e^{-j\omega\tau_d/T}}{T}\sum_{k=-\infty}^{\infty} S_c\left(j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right)$$

(b) $H(e^{j\omega}) = 1 + \alpha e^{-j\omega\tau_d/T}$.
(c) (i) $h[n] = \delta[n] + \alpha\delta[n-1]$.
(ii) $h[n] = \delta[n] + \alpha\frac{\sin(\pi(n-1/2))}{\pi(n-1/2)}$.

4.8. **(a)** $T \le 1/20{,}000$.
(b) $h[n] = Tu[n]$.
(c) $X(e^{j\omega})|_{\omega=0}$.
(d) $T \le 1/10{,}000$.

4.9. **(a)** $X\left(e^{j(\omega+\pi)}\right) = X\left(e^{j(\omega+\pi-\pi)}\right) = X\left(e^{j\omega}\right)$.
(b) $x[3] = 0$.
(c) $x[n] = \begin{cases} y[n/2], & n \text{ even}, \\ 0, & n \text{ odd}. \end{cases}$

4.10. **(a)** $x[n] = \cos(2\pi n/3)$.
(b) $x[n] = -\sin(2\pi n/3)$.
(c) $x[n] = \sin(2\pi n/5)/(\pi n/5000)$.

4.11. **(a)** $T = 1/40$, $T = 9/40$.
(b) $t = 1/20$, unique.

4.12. **(a)** (i) $y_c(t) = -6\pi \sin(6\pi t)$.
(ii) $y_c(t) = -6\pi \sin(6\pi t)$.
(b) (i) Yes.
(ii) No.

4.13. **(a)** $y[n] = \sin\left(\frac{\pi n}{2} - \frac{\pi}{4}\right)$.
(b) Same $y[n]$.
(c) $h_c(t)$ has no effect on T.

4.14. **(a)** No.
(b) Yes.
(c) No.
(d) Yes.
(e) Yes. (No information is lost; however, the signal cannot be recovered by the system in Figure 3.21.)

4.15. **(a)** Yes.
(b) No.
(c) Yes.

4.16. **(a)** $M/L = 5/2$, unique.
(b) $M/L = 2/3$; also, $M/L = 7/3$.

4.17. **(a)** $\tilde{x}_d[n] = (4/3)\sin(\pi n/2)/(\pi n)$.
(b) $\tilde{x}_d[n] = -\sin(3\pi n/4)$.

4.18. **(a)** $\omega_0 = 2\pi/3$.
(b) $\omega_0 = 3\pi/5$.
(c) $\omega_0 = \pi$.

4.19. $T \le \pi/\Omega_0$.

4.20. **(a)** $F_s \ge 2000$ Hz.
(b) $F_s \ge 4000$ Hz.

Answers to Basic Problems in Chapter 5

5.1. $x[n] = y[n]$, $\omega_c = \pi$.

5.2. **(a)** Poles: $z = 3, 1/3$, Zeros: $z = 0, \infty$.
(b) $h[n] = -(3/8)(1/3)^n u[n] - (3/8)3^n u[-n-1]$.

5.3. **(a)**, **(d)** are the impulse responses.

5.4. **(a)** $H(z) = \dfrac{1 - 2z^{-1}}{1 - \frac{3}{4}z^{-1}}$, $|z| > 3/4$.
(b) $h[n] = (3/4)^n u[n] - 2(3/4)^n u[n-1]$.
(c) $y[n] - (3/4)y[n-1] = x[n] - 2x[n-1]$.

(d) Stable and causal.

5.5. (a) $y[n]-(7/12)y[n-1]+(1/12)y[n-2] = 3x[n]-(19/6)x[n-1]+(2/3)x[n-2]$.
(b) $h[n] = 3\delta[n] - (2/3)(1/3)^{n-1}u[n-1] - (3/4)(1/4)^{n-1}u[n-1]$.
(c) Stable.

5.6. (a) $X(z) = \dfrac{1}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}, \quad \dfrac{1}{2} < |z| < 2.$
(b) $\frac{1}{2} < |z| < 2$.
(c) $h[n] = \delta[n] - \delta[n-2]$.

5.7. (a) $H(z) = \dfrac{1-z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{3}{4}z^{-1})}, \quad |z| > \dfrac{3}{4}.$
(b) $h[n] = -(2/5)(1/2)^n u[n] + (7/5)(-3/4)^n u[n]$.
(c) $y[n] + (1/4)y[n-1] - (3/8)y[n-2] = x[n] - x[n-1]$.

5.8. (a) $H(z) = \dfrac{z^{-1}}{1-\frac{3}{2}z^{-1} - z^{-2}}, \quad |z| > 2.$
(b) $h[n] = -(2/5)(-1/2)^n u[n] + (2/5)(2)^n u[n]$.
(c) $h[n] = -(2/5)(-1/2)^n u[n] - (2/5)(2)^n u[-n-1]$.

5.9.

$$h[n] = \left[-\frac{4}{3}(2)^{n-1} + \frac{1}{3}\left(\frac{1}{2}\right)^{n-1}\right]u[-n], \qquad |z| < \frac{1}{2},$$

$$h[n] = -\frac{4}{3}(2)^{n-1}u[-n] - \frac{1}{3}\left(\frac{1}{2}\right)^{n-1}u[n-1], \qquad \frac{1}{2} < |z| < 2,$$

$$h[n] = \frac{4}{3}(2)^{n-1}u[n-1] - \frac{1}{3}\left(\frac{1}{2}\right)^{n-1}u[n-1], \qquad |z| > 2.$$

5.10. $H_i(z)$ cannot be causal and stable. The zero of a $H(z)$ at $z = \infty$ is a pole of $H_i(z)$. The existence of a pole at $z = \infty$ implies that the system is not causal.

5.11. (a) Cannot be determined.
(b) Cannot be determined.
(c) True.
(d) False.

5.12. (a) Stable.
(b)

$$H_1(z) = -9\frac{(1+0.2z^{-1})\left(1-\frac{1}{3}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)}{(1-j0.9z^{-1})(1+j0.9z^{-1})},$$

$$H_{ap}(z) = \frac{\left(z^{-1}-\frac{1}{3}\right)\left(z^{-1}+\frac{1}{3}\right)}{\left(1-\frac{1}{3}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)}.$$

5.13. $H_1(z)$, $H_3(z)$, and $H_4(z)$ are all-pass systems.

5.14. (a) 5.
(b) $\frac{1}{2}$.

5.15. **(a)** $\alpha = 1, \beta = 0,\ A(e^{j\omega}) = 1 + 4\cos(\omega n)$. The system is a generalized linear-phase system but not a linear-phase system, because $A(e^{j\omega})$ is not nonnegative for all ω.

(b) Not a generalized linear-phase or a linear-phase system.

(c) $\alpha = 1, \beta = 0,\ A(e^{j\omega}) = 3 + \cos(\omega n)$. Linear phase, since $|H(e^{j\omega})| = A(e^{j\omega}) \geq 0$ for all ω.

(d) $\alpha = 1/2, \beta = 0,\ A(e^{j\omega}) = 2\cos(\omega n/2)$. Generalized linear phase, because $A(e^{j\omega})$ is not nonnegative at all ω.

(e) $\alpha = 1, \beta = \pi/2,\ A(e^{j\omega}) = 2\sin(\omega n)$. Generalized linear phase, because $\beta \neq 0$.

5.16. $h[n]$ is not necessarily causal. Both $h[n] = \delta[n-\alpha]$ and $h[n] = \delta[n+1] + \delta[n-(2\alpha+1)]$ will have this phase.

5.17. $H_2(z)$ and $H_3(z)$ are linear-phase systems.

5.18. **(a)** $H_{\min}(z) = \dfrac{2\left(1 - \frac{1}{2}z^{-1}\right)}{1 + \frac{1}{3}z^{-1}}$.

(b) $H_{\min}(z) = 3\left(1 - \dfrac{1}{2}z^{-1}\right)$.

(c) $H_{\min}(z) = \dfrac{9}{4}\dfrac{\left(1 - \frac{1}{3}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}{\left(1 - \frac{3}{4}z^{-1}\right)^2}$.

5.19. $h_1[n] : 2,\ h_2[n] : 3/2,\ h_3[n] : 2,\ h_4[n] : 3,\ h_5[n] : 3,\ h_6[n] : 7/2$.

5.20. Systems $H_1(z)$ and $H_3(z)$ have a linear phase and can be implemented by a real-valued difference equation.

Answers to Basic Problems in Chapter 6

6.1. **Network 1:**

$$H(z) = \frac{1}{1 - 2r\cos\theta z^{-1} + r^2 z^{-2}}.$$

Network 2:

$$H(z) = \frac{r\sin\theta z^{-1}}{1 - 2r\cos\theta z^{-1} + r^2 z^{-2}}.$$

Both systems have the same denominators and thus the same poles.

6.2. $y[n] - 3y[n-1] - y[n-2] - y[n-3] = x[n] - 2x[n-1] + x[n-2]$.

6.3. The system in Part (d) is the same as that in Part (a).

6.4. **(a)**

$$H(z) = \frac{2 + \frac{1}{4}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}.$$

(b)

$$y[n] + \frac{1}{4}y[n-1] - \frac{3}{8}y[n-2] = 2x[n] + \frac{1}{4}x[n-1].$$

6.5. (a)

$$y[n] - 4y[n-1] + 7y[n-3] + 2y[n-4] = x[n].$$

(b)

$$H(z) = \frac{1}{1 - 4z^{-1} + 7z^{-3} + 2z^{-4}}.$$

(c) Two multiplications and four additions.
(d) No. It requires at least four delays to implement a fourth-order system.

6.6.

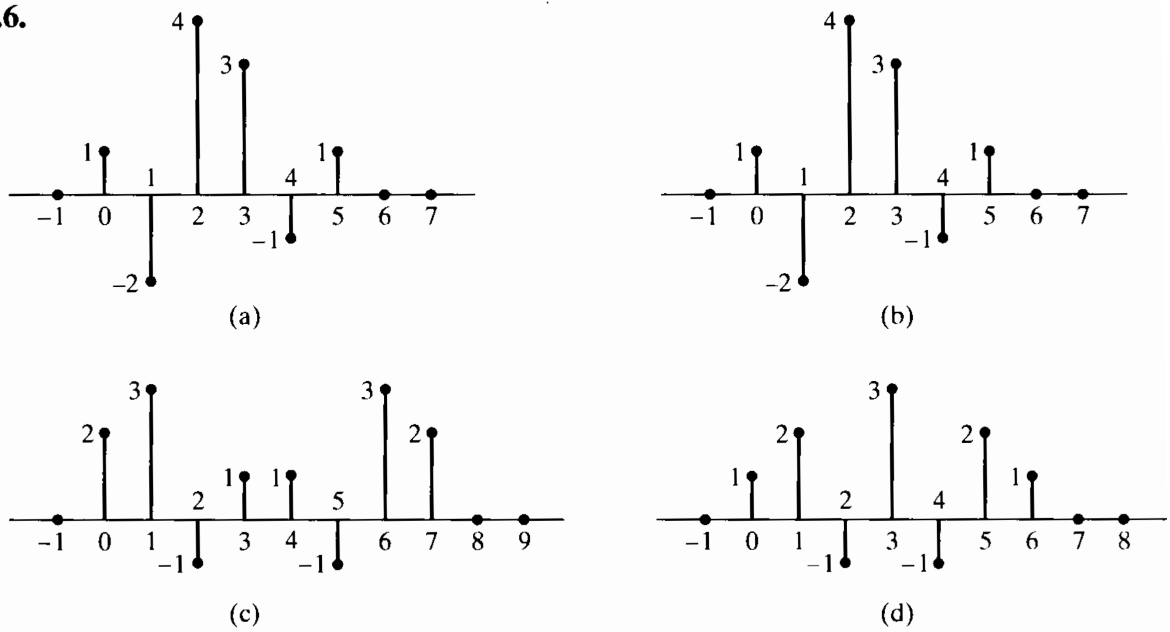

Figure P6.6-1

6.7.

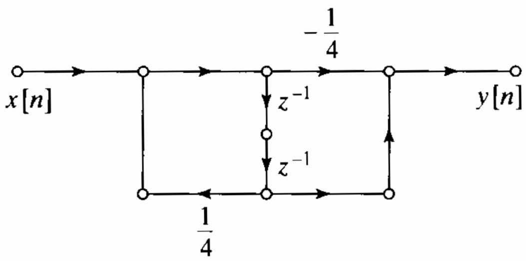

Figure P6.7-1

6.8. $y[n] - 2y[n-2] = 3x[n-1] + x[n-2]$.

6.9. (a) $h[1] = 2$.
(b) $y[n] + y[n-1] - 8y[n-2] = x[n] + 3x[n-1] + x[n-2] - 8x[n-3]$.

6.10. (a)

$$y[n] = x[n] + v[n-1].$$

$$v[n] = 2x[n] + \frac{1}{2}y[n] + w[n-1].$$

$$w[n] = x[n] + \frac{1}{2}y[n].$$

(b)

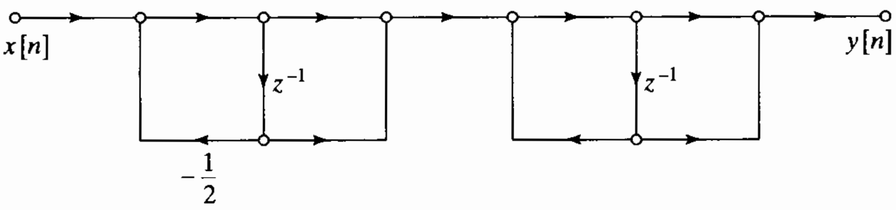

Figure P6.10-1

(c) The poles are at $z = -1/2$ and $z = 1$. Since the second pole is on the unit circle, the system is not stable.

6.11. (a)

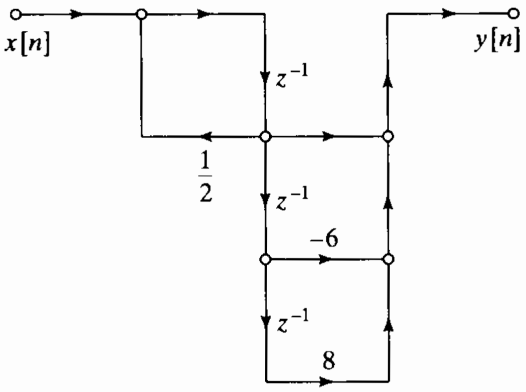

Figure P6.11-1

(b)

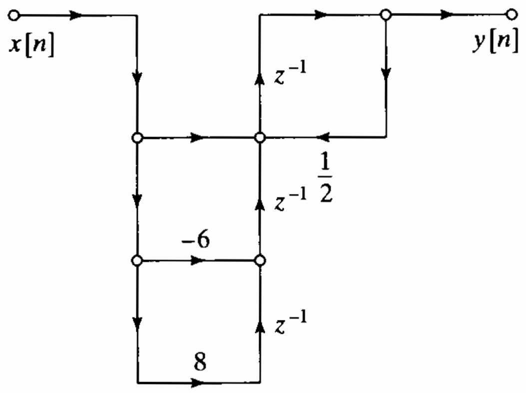

Figure P6.11-2

6.12. $y[n] - 8y[n-1] = -2x[n] + 6x[n-1] + 2x[n-2]$.

6.13.

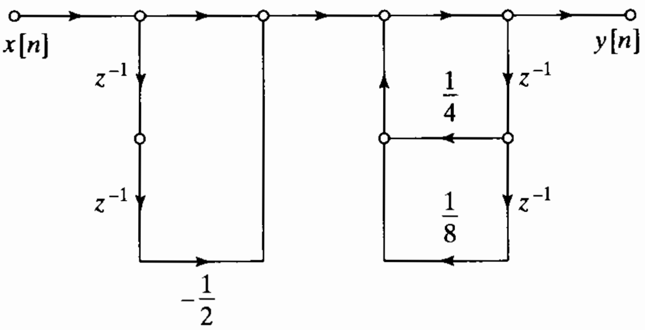

Figure P6.13-1

6.14.

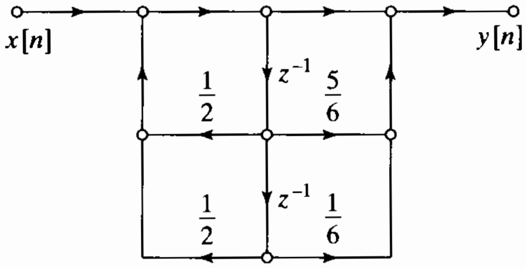

Figure P6.14-1

6.15.

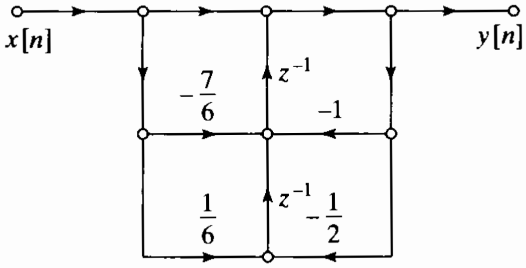

Figure P6.15-1

6.16. (a)

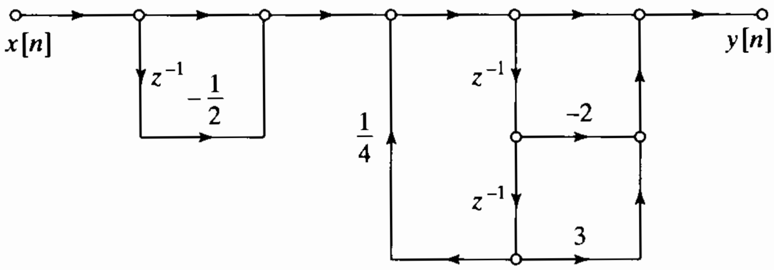

Figure P6.16-1

(b) Both systems have the system function

$$H(z) = \frac{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - 2z^{-1} + 3z^{-2}\right)}{1 - \frac{1}{4}z^{-2}}.$$

6.17. (a)

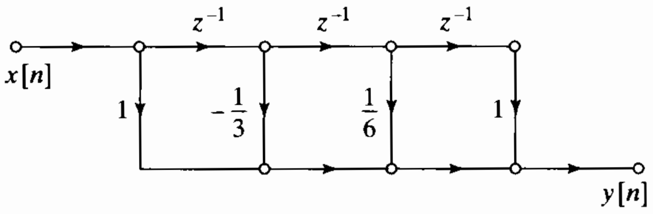

Figure P6.17-1

(b)

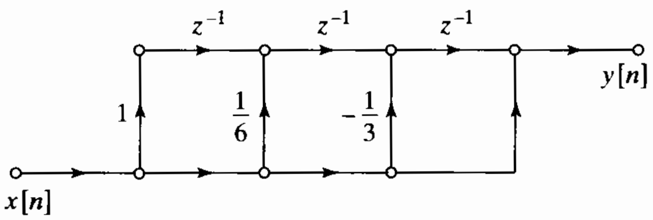

Figure P6.17-2

6.18. If $a = 2/3$, the overall system function is

$$H(z) = \frac{1 + 2z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}.$$

If $a = -2$, the overall system function is

$$H(z) = \frac{1 - \frac{2}{3}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}.$$

6.19.

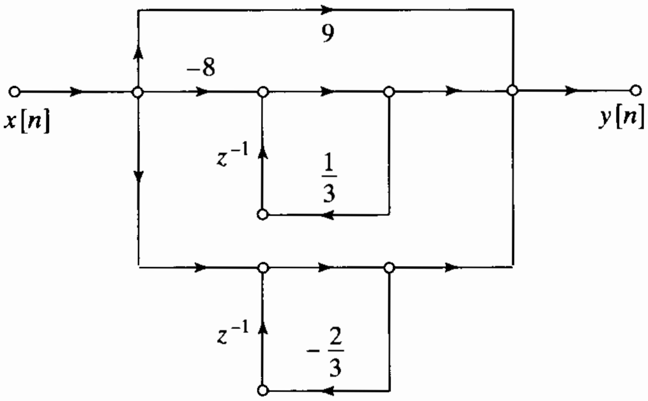

Figure P6.19-1

6.20.

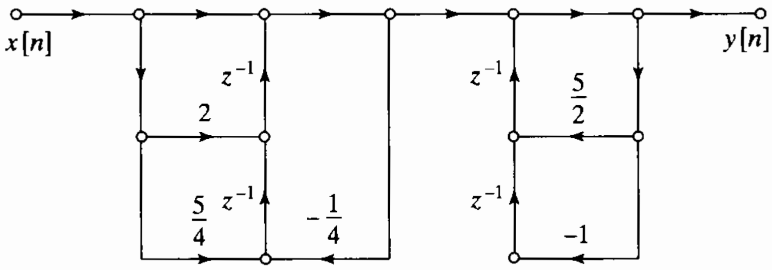

Figure P6.20-1

Answers to Basic Problems in Chapter 7

7.1. (a)

$$H_1(z) = \frac{1 - e^{-aT}\cos(bT)z^{-1}}{1 - 2e^{-aT}\cos(bT)z^{-1} + e^{-2aT}z^{-2}}.$$

(b)

$H_2(z) = (1 - z^{-1})S_2(z)$, where

$$S_2(z) = \frac{a}{a^2+b^2}\frac{1}{1-z^{-1}} - \frac{1}{2(a+jb)}\frac{1}{1-e^{-(a+jb)T}z^{-1}} - \frac{1}{2(a-jb)}\frac{1}{1-e^{-(a-jb)T}z^{-1}}$$

(c) They are not equal.

7.2. (a)

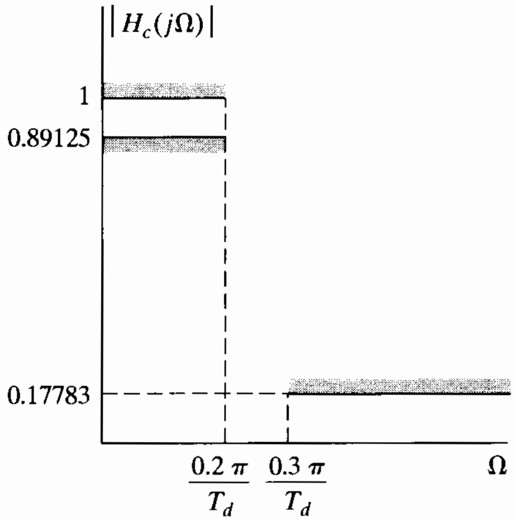

Figure P7.2-1

(b) $N = 6, \Omega_c T_d = 0.7032$.

(c) The poles in the s-plane are on a circle of radius $R = 0.7032/T_d$. They map to poles in the z-plane at $z = e^{s_k T_d}$. The factors of T_d cancel out, leaving the pole locations in the z-plane for $H(z)$ independent of T_d.

7.3. **(a)** $\hat{\delta}_2 = \delta_2/(1+\delta_1)$.

(b)

$$\delta_2 = 0.18806$$

$$H(z) = \frac{0.3036 - 0.4723z^{-1}}{1 - 1.2971z^{-1} + 0.6949z^{-2}} + \frac{-2.2660 + 1.2114z^{-1}}{1 - 1.0691z^{-1} + 0.3699z^{-2}} + \frac{1.9624 - 0.6665z^{-1}}{1 - 0.9972z^{-1} + 0.2570z^{-2}}$$

(c) Use the same δ_2.

$$\frac{0.0007802(1+z^{-1})^6}{(1 - 1.2686z^{-1} + 0.7051z^{-2})(1 - 1.0106z^{-1} + 0.3583z^{-2})(1 - 0.9044z^{-1} + 0.2155z^{-2})}.$$

7.4. **(a)**

$$H_c(s) = \frac{1}{s+0.1} - \frac{0.5}{s+0.2}.$$

The answer is not unique. Another possibility is

$$H_c(s) = \frac{1}{s+0.1+j2\pi} - \frac{0.5}{s+0.2+j2\pi}.$$

(b)

$$H_c(s) = \frac{2(1+s)}{0.1813 + 1.8187s} - \frac{1+s}{0.3297 + 1.6703s}.$$

This answer is unique.

7.5. **(a)** $M + 1 = 91$.

(b) $M/2 = 45$.

(c) $h_d[n] = \dfrac{\sin[0.625\pi(n-45)]}{\pi(n-45)} - \dfrac{\sin[0.3\pi(n-45)]}{\pi(n-45)}$.

7.6. **(a)** $\delta = 0.03, \beta = 2.181$.

(b) $\Delta\omega = 0.05\pi, M = 50$.

7.7.

$$0.99 \le |H(e^{j\omega})| \le 1.01, \qquad |\omega| \le 0.2\pi,$$

$$|H(e^{j\omega})| \le 0.01, \qquad 0.22\pi \le |\omega| \le \pi$$

7.8. **(a)** Six alternations. $L = 5$, so this does not satisfy the alternation theorem and is not optimal.

(b) Seven alternations, which satisfies the alternation theorem for $L = 5$.

7.9. $\omega_c = 0.4\pi$.

7.10. $\omega_c = 2.3842$ rad.

7.11. $\Omega_c = 2\pi(1250)$ rad/sec.

7.12. $\Omega_c = 2000$ rad/sec.

7.13. $T = 50\,\mu$s. This T is unique.

7.14. $T = 1.46$ ms. This T is unique.

7.15. The Hamming, Hanning, and Blackman windows may be used.

7.16. $\beta = 2.6524$, $M = 181$.

7.17.

$$\begin{aligned} |H_c(j\Omega)| &< 0.02, && |\Omega| \le 2\pi(20) \text{ rad/sec}, \\ 0.95 < |H_c(j\Omega)| &< 1.05, && 2\pi(30) \le |\Omega| \le 2\pi(70) \text{ rad/sec} \\ |H_c(j\Omega)| &< 0.001, && 2\pi(75) \text{ rad/sec} \le |\Omega|. \end{aligned}$$

7.18.

$$\begin{aligned} |H_c(j\Omega)| &< 0.04, && |\Omega| \le 726.5 \text{ rad/sec} \\ 0.995 < |H_c(j\Omega)| &< 1.005, && |\Omega| \ge 1376.4\text{rad/sec}. \end{aligned}$$

7.19. $T = 0.41667$ ms. This T is unique.

7.20. True.

Answers to Basic Problems in Chapter 8

8.1. **(a)** $x[n]$ is periodic with period $N = 6$.

(b) T will not avoid aliasing.

(c)

$$\tilde{X}[k] = 2\pi \begin{cases} a_0 + a_6 + a_{-6}, & k = 0, \\ a_1 + a_7 + a_{-5}, & k = 1, \\ a_2 + a_8 + a_{-4}, & k = 2, \\ a_3 + a_9 + a_{-3} + a_{-9}, & k = 3, \\ a_4 + a_{-2} + a_{-8}, & k = 4, \\ a_5 + a_{-1} + a_{-7}, & k = 5. \end{cases}$$

8.2. **(a)**

$$\tilde{X}_3 = \begin{cases} 3X[k/3], & \text{for } k = 3\ell, \\ 0, & \text{otherwise.} \end{cases}$$

(b)

$$\tilde{X}[k] = \begin{cases} 3, & k = 0, \\ -1, & k = 1. \end{cases}$$

$$\tilde{X}_3[k] = \begin{cases} 9, & k = 0, \\ 0, & k = 1, 2, 4, 5, \\ -3, & k = 3. \end{cases}$$

8.3. **(a)** $\tilde{x}_2[n]$.

(b) None of the sequences.

(c) $\tilde{x}_1[n]$ and $\tilde{x}_3[n]$.

8.4. **(a)**

$$X(e^{j\omega}) = \frac{1}{1 - ae^{-j\omega}}.$$

(b)

$$\tilde{X}[k] = \frac{1}{1 - ae^{-j(2\pi/N)k}}.$$

(c)

$$\tilde{X}[k] = X(e^{j\omega})|_{\omega=(2\pi k/N)}.$$

8.5. **(a)** $X[k] = 1.$
(b) $X[k] = W_N^{kn_0}.$
(c)

$$X[k] = \begin{cases} N/2, & k = 0, N/2, \\ 0, & \text{otherwise.} \end{cases}$$

(d)

$$X[k] = \begin{cases} N/2, & k = 0, \\ e^{-j(\pi k/N)(N/2-1)}(-1)^{(k-1)/2}\frac{1}{\sin(k\pi/N)}, & k \text{ odd}, \\ 0, & \text{otherwise.} \end{cases}$$

(e)

$$X[k] = \frac{1-a}{1-aW_N^k}.$$

8.6. **(a)**

$$X(e^{j\omega}) = \frac{1 - e^{j(\omega_0-\omega)N}}{1 - e^{j(\omega_0-\omega)}}.$$

(b)

$$X[k] = \frac{1 - e^{j\omega_0 N}}{1 - e^{j\omega_0}W_N^k}.$$

(c)

$$X[k] = \begin{cases} N, & k = k_0 \\ 0, & \text{otherwise.} \end{cases}$$

8.7.

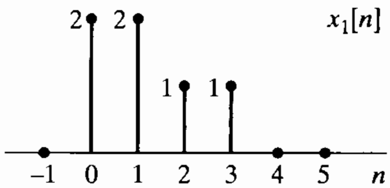

Figure P8.7-1

8.8.

$$y[n] = \begin{cases} \frac{1024}{1023}\left(\frac{1}{2}\right)^n, & 0 \le n \le 9, \\ 0, & \text{otherwise.} \end{cases}$$

8.9. **(a)** **1.** Let $x_1[n] = \sum_m x[n+5m]$.
2. Let $X_1[k]$ be the five-point FFT of $x_1[n]$. $M = 5$.
3. $X_1[2]$ is $X(e^{j\omega})$ at $\omega = 4\pi/5$.

(b) **1.** Let $x_2[n]$ be $x[n]$ followed by seven zeros.
2. Let $X_2[k]$ be the 27-point FFT of $x_2[n]$. $L = 27$.
3. $X_2[5]$ is $X(e^{j\omega})$ at $\omega = 10\pi/27$.

8.10. $X_2[k] = (-1)^k X_1[k]$.

8.11.

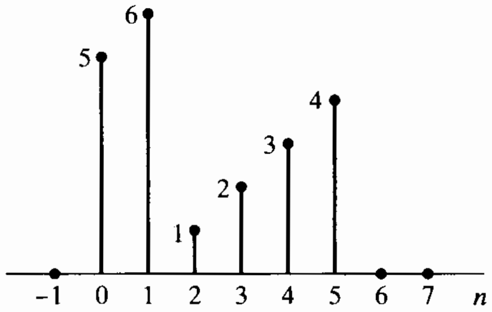

Figure P8.11-1

8.12. **(a)**

$$X[k] = \begin{cases} 2, & k = 1, 3, \\ 0, & k = 2, 4. \end{cases}$$

(b)

$$H[k] = \begin{cases} 15, & k = 0, \\ -3 + j6, & k = 1, \\ -5, & k = 2, \\ -3 - j6, & k = 3. \end{cases}$$

(c) $y[n] = -3\delta[n] - 6\delta[n-1] + 3\delta[n-2] + 6\delta[n-3]$.
(d) $y[n] = -3\delta[n] - 6\delta[n-1] + 3\delta[n-2] + 6\delta[n-3]$.

8.13.

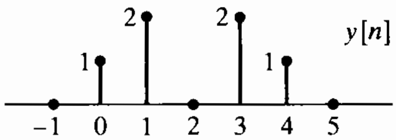

Figure P8.13-1

8.14. $x_3[2] = 9$.

8.15. $a = -1$. This is unique.

8.16. $b = 3$. This is unique.

8.17. $N = 9$.

8.18. $c = 2$.

8.19. $m = 2$. This is not unique. Any $m = 2 + 6\ell$ for integer ℓ works.

8.20. $N = 5$. This is unique.

Answers to Basic Problems in Chapter 9

9.1. If the input is $(1/N)X[((-n))_N]$, the output of the DFT program will be $x[n]$, the IDFT of $X[k]$.

9.2. **(a)** The gain is $-W_N^2$.
(b) There is one path. In general, there is only one path from any input sample to any output sample.
(c) By tracing paths, we see

$$X[2] = x[0] \cdot 1 + x[1]W_8^2 - x[2] - x[3]W_8^2 + \dots$$
$$x[4] + x[5]W_8^2 - x[6] - x[7]W_8^2.$$

9.3. **(a)** Store $x[n]$ in $A[\cdot]$ in bit-reversed order, and $D[\cdot]$ will contain $X[k]$ in sequential (normal) order.

(b)

$$D[r] = \begin{cases} N, & r = 3, \\ 0, & \text{otherwise.} \end{cases}$$

(c)

$$C[r] = \begin{cases} 1, & r = 0, 1, 2, 3, \\ 0, & \text{otherwise.} \end{cases}$$

9.4. **(a)** $N/2$ butterflies with $2^{(m-1)}$ different coefficients.

(b) $y[n] = W_N^{2^{\nu-m}} y[n-1] + x[n]$.

(c) Period: 2^m, Frequency: $2\pi 2^{-m}$.

9.5.

$$X = AD - BD + CA - DA = AC - BD$$

$$Y = AD - BD + BC + BD = BC + AD.$$

9.6. Statement 1.

9.7. $\omega_k = 7\pi/16$.

9.8.

$$y[n] = X(e^{j\omega})|_{\omega=(2\pi/7)+(2\pi/21)(n-19)}.$$

9.9. **(a)** 2^{m-1}.

(b) 2^m.

9.10. $r[n] = e^{-j(2\pi/19)} W^{n^2/2}$ where $W = e^{-j(2\pi/10)}$.

9.11. $x[0], x[8], x[4], x[12], x[2], x[10], x[6], x[14], x[1], x[9], x[5], x[13], x[3], x[11], x[7], x[15]$.

9.12. False.

9.13. $m = 1$.

9.14.

$$r = \begin{cases} 0, & m = 0, \\ 0, 4, & m = 1, \\ 0, 2, 4, 6, & m = 2, \\ 0, 1, 2, 3, 4, 5, 6, 7, & m = 3. \end{cases}$$

9.15. $N = 64$.

9.16. $m = 3$ or 4.

9.17. Decimation-in-time.

9.18. 1021 is prime, so the program must implement the full DFT equations and cannot exploit any FFT algorithm. The computation time goes as N^2. Contrastingly, 1024 is a power of 2 and can exploit the $N \log N$ computation time of the FFT.

9.19.

$$a = -\sqrt{2}$$

$$b = -e^{-j(6\pi/8)}.$$

9.20.

$$y[n] = e^{j(2\pi/32)7} X^*(e^{j(7\pi/16)}).$$

Answers to Basic Problems in Chapter 10

10.1. **(a)** $f = 1500$ Hz.
(b) $f = -2000$ Hz.

10.2. $N = 2048$ and $10000 \text{ Hz} < f < 10240$ Hz.

10.3. **(a)** 320 samples.
(b) 400 DFT/second.
(c) $N = 64$.
(d) 250 Hz.

10.4. **(a)** $X[200] = 1 - j$.
(b)

$$X(j2\pi(4000)) = 5 \times 10^{-5}(1 - j)$$

$$X(-j2\pi(4000)) = 5 \times 10^{-5}(1 + j).$$

10.5. **(a)** $T = 2\pi k_0/(N\Omega_0)$.
(b) Not unique. $T = (2\pi/\Omega_0)(1 - k_0/N)$.

10.6.

$$X_c(j2\pi(4200)) = 5 \times 10^{-4}$$

$$X_c(-j2\pi(4200)) = 5 \times 10^{-4}$$

$$X_c(j2\pi(1000)) = 10^{-4}$$

$$X_c(-j2\pi(1000)) = 10^{-4}$$

10.7. $L = 1024$.

10.8. Rectangular, Hanning, Hamming, and Bartlett windows work.

10.9. $x_2[n]$ will have two distinct peaks.

10.10. $T > 1/1024$ sec.

10.11. $\Delta\Omega = 2\pi(2.44)$ rad/sec.

10.12. $N \geq 1600$.

10.13.

$$X_0[k] = \begin{cases} 18, & k = 3, 33, \\ 0, & \text{otherwise.} \end{cases}$$

$$X_1[k] = \begin{cases} 18, & k = 9, 27, \\ 0, & \text{otherwise.} \end{cases}$$

10.14. $x_2[n]$, $x_3[n]$, $x_6[n]$.

10.15. $\omega_0 = 0.25\pi$ rad/sample, $\lambda = \pi/80000$ rad/sample^2.

10.16. $\Delta f = 9.77$ Hz.

10.17. Methods 2 and 5 will improve the resolution.

10.18. The peaks will not have the same height. The peak from the rectangular window will be bigger.

10.19. $L = M + 1 = 124$.

10.20. **(a)** $A = 44.68$ dB.
(b) Weak components will be visible if their amplitude exceeds 0.0058.

Answers to Basic Problems in Chapter 11

11.1. $\mathcal{J}m\{X(e^{j\omega})\} = 2a\sin\omega$.

11.2. $x[n] = (5/4)\delta[n] - \delta[n-1]$.

11.3.

$$x_1[n] = \delta[n] - \frac{1}{2}\delta[n-1]$$

$$x_2[n] = \delta[n] - \frac{1}{2}\delta[n+1].$$

11.4. $\mathcal{R}e\{X(e^{j\omega})\} = 1 - \cos(2\omega)$, $\mathcal{J}m\{X(e^{j\omega})\} = 0$.

11.5. **(a)** $x_i[n] = \sin\omega_0 n$.
(b) $x_i[n] = -\cos\omega_0 n$.
(c) $x_i[n] = (1 - \cos\omega_0 n)/(\pi n)$.

11.6. $x[n] = 5\delta[n] - 2\delta[n-1] + 3\delta[n-4]$.

11.7. **(a)** $x[n] = -\delta[n-1] - 2\delta[n-2]$.
(b) Not unique. $x[n] = \delta[n] - \delta[n-1] - 2\delta[n-2]$ also satisfies the information given.

11.8. $X_{R2}(e^{j\omega})$ and $X_{R3}(e^{j\omega})$ are the answers.

11.9. $x[n] = -\delta[n] - 3\delta[n-1] - \delta[n-3]$ is the unique sequence satisfying the information given.

11.10. $h[n] = \pm(1/2)\left\{(-1/2)^n u[n] - (1/2)(-1/2)^{n-1}u[n-1]\right\}$.

11.11.

$$\mathcal{R}e\{X(e^{j\omega})\} = \begin{cases} 16\sin 3\omega, & 0 \le \omega \le \pi, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{J}m\{X(e^{j\omega})\} = 0.$$

11.12. **(a)** $h[n] = \delta[n] - (1/3)\delta[n-1]$.
(b) $h[n] = (1/3)\delta[n] - \delta[n-1]$.

11.13. $X_I(e^{j\omega}) = \cos\omega - \sin\omega - \cos 2\omega$.

11.14. $X_I(e^{j\omega}) = \sum_{k=0}^{\infty}(1/2)^k \sin k\omega$.

11.15. $x[n] = 4\delta[n] - \delta[n-1]$.

11.16. The facts are not consistent.

11.17. $x[n] = -\delta[n] + 3\delta[n-1]$ is the unique sequence satisfying the information given.

11.18. Two choices are $x[n] = 7\delta[n] + 2\delta[n-1]$ or $x[n] = 7\delta[n] + 2\delta[n-2]$.

11.19. $jX_I[k] = -j\delta[k-1] + j\delta[k-3]$.

11.20. $x_2[n]$ and $x_3[n]$ are consistent with the information given.