Vector Value Function and Motion in Space

when a body travels through space, the three equations x=f(t), $y=g(t)$, and f(z) = $h(t)$ that give the body's coordinates as functions of time become parametric equations for the body's motion and path. With vector notation, we can condense these .three equations into a single- equation

$$\vec{r} = f(t)\,\mathrm{i} + g(t)\,\mathrm{j} + \mathrm{h}(t)\,\mathrm{k}$$

That gives the body's position as a vector function of t.

It will be shown how to use the calculus we know to differentiate and integrate vector functions and to study the paths, velocities, and accelerations of bodies in the plane and in space.

Curve in Space

When a particle moves through space as a function of time, each of its rectangular coordinates is a function of time. The particle path can be described with a triple of equations:

$$x = f(t) \quad , \qquad y = g(t) \quad , \quad z = \mathrm{h}(t)$$

The set of points *(x, y, z)* defined by these equations is called a **Curve in Space**. The equations are parametric equations for the curve, **t** being the **parameter** and **I** is the **parameter interval**, the point P$[f(t), g(t), \mathrm{h}(t)]$ is called the particle position at time t.

The vector

$$\vec{r} = \overrightarrow{OP} = f(t)\,\mathrm{i} + g(t)\,\mathrm{j} + \mathrm{h}(t)\,\mathrm{k}$$

Is called the particle position vector at time t. and **we think of particles' path as a curve traced by $\vec{r}$.**

Main Ref. 1) E. Kreyszig. "ADVANCED ENGINEERING MATHEMATICS "
2) L. Finny & B. Thomas, "CALCULAS".

If f, g and h are differentiable scalar functions of **t** and

$\vec{r} = f(t)\,\mathrm{i} + g(t)\,\mathrm{j} + \mathrm{h}(t)\,\mathrm{k}$, then the vector

$$r' = \frac{dr}{dt} = \frac{df}{dt}\,\mathrm{i} + \frac{dg}{dt}\,\mathrm{j} + \frac{dh}{dt}\,\mathrm{k}$$

Is the derivative of r with respect to t.

Velocity Vector

If the position vector of a particle moving in space is differentiable function of time t, then the velocity vector can be given by:-

$$\vec{V} = \frac{dr}{dt} = \frac{df}{dt}\,\mathrm{i} + \frac{dg}{dt}\,\mathrm{j} + \frac{dh}{dt}\,\mathrm{k}$$

Also, we define:

- Speed is the magnitude of the velocity = $|\vec{V}|$
- Acceleration is the derivative of velocity = $\frac{d\vec{V}}{dt}$
- The direction of motion at time t = $\frac{\vec{V}}{|\vec{V}|}$

Distance along the Curve (L)

In the plane, we define the length of a parameterized curve $x = f(t)\ y = g(t)$, a < t < b by the formula:-

$$\text{Length} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

But in space calculation, we have third coordinated, thus

$$\text{Length} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

Or

$$\text{Length} = \int_a^b |\vec{V}|$$

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The Unit Tangent Vector ($\vec{T}$)

The Unit Tangent Vector (T) is one of three unit vectors in a traveling reference frame that is used to describe the motion of space vehicles and other bodies moving in three dimensions.

$$\vec{T} = \frac{dr}{ds} = \frac{\vec{V}}{|\vec{V}|}$$

The Curvature of Plan Curve (k)

As we move along a differentiate curve in the plane, the unit tangent vector **T** turns as the curve bends.

We measure the rate at which **T** turns V measuring the change in the angle φ that **T** makes with **i** (Figure below).

At each point P, the absolute value of $\frac{d\varphi}{ds}$, stated in radians per unit length along the curve, is called the curvature at *P*.

- If $\frac{d\varphi}{ds}$ is large, T turns sharply as we pass through *P* and the curvature at *P* is large.
- If $\frac{d\varphi}{ds}$ is close to zero, T turns more slowly and the curvature at *P* is small.
- On circles and lines, the curvature is constant, the curvature can vary from place to place.

The tradition symbol for ***the curvature function*** is the Greek letter κ (kappa). $K = \frac{|\vec{V}\times\vec{a}|}{|\vec{V}|^3}$

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Home work

Show that the curvature of a circle of radius a is 1/a?

Principal Unit Normal Vector ($\vec{N}$)

$$\vec{N} = \frac{d\vec{T}/_{dt}}{\left|d\vec{T}/_{dt}\right|}$$

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