**The control system**

A liquid stream at a temperature $T_i$ , enters an insulated, well-stirred tank at a constant flow rate w (mass/time). It is desired to maintain (or control) the temperature in the tank at $T_R$ by means of the controller. If the indicated (measured) tank temperature $T_m$ differs from the desired temperature $T_R$ , the controller senses the difference or ***error,*** E = $T_R$ - $T_m$

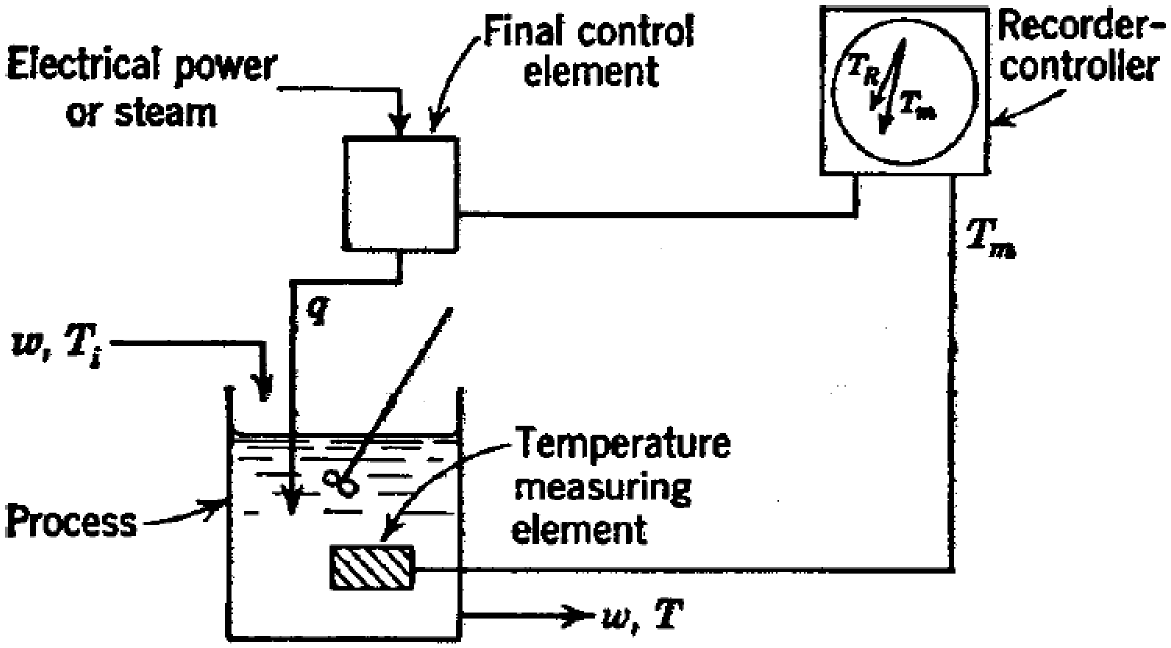



Figure (9-1) Control system for a stirred-tank heater.

There are two types of the control system:-

1) **Negative feedback control system**

   Negative feedback ensures that the difference between $T_R$ and $T_m$ is used to adjust the control element so that tendency is to reduce the error.

   $E = T_R - T_m$

2) **Positve feedback control system**

   If the signal to the compartos were obtained by adding $T_R$ and $T_m$ we would have a positive feedback systems which is inherently unstable. To see that this is true, again assume that be system is at steady state and that $T=T_R=T_i$.

   If $T_i$ were to increase, T and $T_m$ would increas which would cause the signal from the compartor to increase, with the result that the heat to the system would increse.

   At s.s. $T=T_R=T_{in}$

   $E = T_R + T_m$

## Servo Problem versus Regulator Problem

❖ **Servo Problem**

There is no change in load $T_i$, and that we are interested in changing the bath temperature (change in the desired value (set point) with no disturbance load).

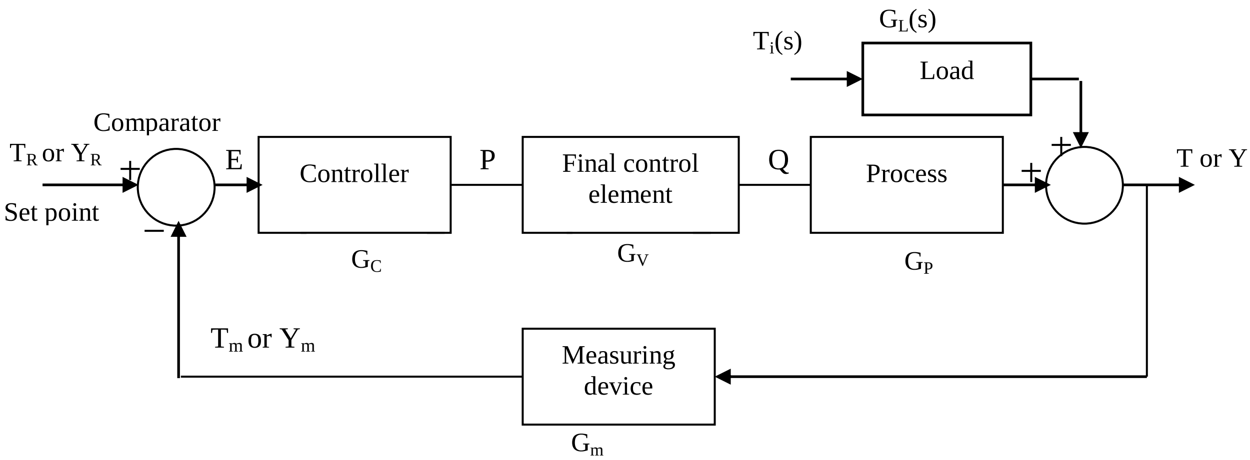
❖ **Requlating problem**

The desired value $T_R$ is to remain fixed and the purpose of the control system is to maintain the controlled variable $T_R$ in spite of change in load if there is a change in the input variable (disturbance load).

## Control system elements

Control system elements are:-

1) Process
2) Measuring element
3) Controller
4) Final Control Element




Closed Loop Feedback control

# Development of block Diagram

**Process**

The procedure for developing the transfer function remain the same.

An unsteady-state energy balance around the heating tank gives.

$$W\,Cp(T_i - T_o) + q - W\,Cp(T - T_o) = \rho Cp V \frac{dT}{dt}$$

Where $T_o$ is the reference temperature

At steady state, $\frac{dT}{dt} = 0$

$$W\,Cp(T_{is} - T_o) + q_s - W\,Cp(T_s - T_o) = \rho Cp V \frac{dT}{dt} = 0$$

By substracting both equations

$$W\,Cp((T_i - T_{is}) - (T - T_s)) + q - q_s = \rho Cp V \frac{d(T - T_s)}{dt}$$

Note that the refernece temperature $T_o$ cancels in the subtraction. If we introduce the deviation variables.

$$\overline{T}_i = T_i - T_{is}$$
$$\overline{T} = T - T_s$$
$$Q = q - q_s$$

$$W\,Cp(\overline{T}_i - \overline{T}) + Q = \rho Cp V \frac{d\overline{T}}{dt}$$

Taking the laplace transform gives

$$W\,Cp(\overline{T}_i(s) - \overline{T}(s)) + Q(s) = \rho Cp V s \overline{T} \qquad \div W\,Cp$$

$$\frac{\rho V}{W} s\overline{T} + \overline{T}(s) = \frac{Q(s)}{WCp} + \overline{T}_i(s)$$

The last expression can be written as

$$\boxed{\overline{T}(s) = \frac{1}{(\tau s + 1)} \frac{Q(s)}{WCp} + \frac{\overline{T}_i(s)}{\tau s + 1}}$$

Where

$$\tau = \frac{\rho V}{W}$$

$\overline{T}(s)$ or $Y(s)$ = controlled variable

$Q(s)$ or $m(s)$ = manipulated variable

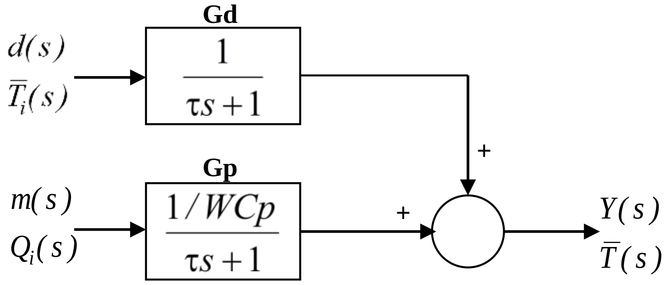
$\overline{T}_i(s)$ or $d(s)$ = disturbance variable

If there is a change in Q(t) only then $\overline{T}_i(t) = 0$ and the transfer function relating $\overline{T}_i$ to $Q$ is

$$\boxed{\frac{\overline{T}(s)}{Q(s)} = \frac{1}{(\tau s + 1)} \frac{1}{WCp}}$$

If there is a change in $\overline{T}_i(s)$ only then Q(t)=0 and the transfer function relating $\overline{T}$ to $\overline{T}_i$ is

$$\frac{\overline{T}(s)}{\overline{T}_i(s)} = \frac{1}{(\tau s + 1)}$$




Block Diagram for process

$$Y(s) = G_p \cdot m(s) + G_d \cdot d(s)$$

**Measuring Element**

The T.F. of the temperature-measuring element is a first order system

$$\frac{\overline{T}_m(s)}{\overline{T}(s)} = \frac{k_m}{\tau_m s + 1} \qquad \Rightarrow \overline{T}_m(s) = G_m \overline{T}(s)$$

$$G_m = \frac{k_m}{\tau_m s + 1}$$

Where $\overline{T}$ and $\overline{T}_m$ are deviation variables defined as

$\overline{T} = T - T_s$

$\overline{T}_m = T_m - T_{m_s}$

$$K_m = steady\ state\ gain = \frac{\Delta Output}{\Delta input}$$

*$\tau_m$=time lag (time constant)=(1-9) sec*

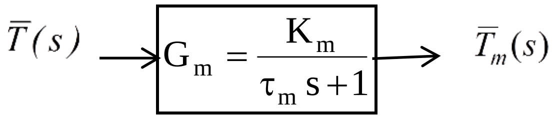
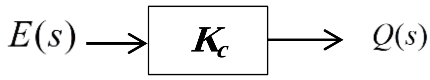



***Figure Block diagram of measuring element***

**Controller and final control element**

The relationship for proportional controller is

$$\frac{P(s)}{G(s)} = G_c(s)$$




$$Q(s) = K_C E(s)$$

Where

$$P = \overline{P} - \overline{P}_s$$
$$E = \overline{T}_R - \overline{T}_m$$

G(s) for propertional controller $G_c(s) = K_C$

$\overline{T}_R = \overline{T}_m = \overline{T}$ at steady state