Ministry of Higher Education and Scientific Research

Diyala University
Chemical Engineering Department

Unit Operations

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By

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Settling and sedimentation in particle-fluid separation:-

Application of settling and sedimentation include removal of solids from sewage waste, settling of crystals from mother liquid, separation of liquid- liquid mixture, settling of food particles from a liquid food, and settling of a slurry from a soybean leaching process. Particles could be solid particles or liquid drops. While fluid is liquid or gas and it may be at rest or in motion.

SETTLING of particles from suspension depends on:

Characteristics of the Particles		Concentration of Particles in Suspension	
DISCRETE PARTICLES particles whose size, shape and specific gravity do not change with time.	**FLOCCULATING PARTICLES** particles whose surface properties are such that they aggregate upon contact Thus, changing in size,shape, and perhaps specific gravity with each contact.	**DILUTE SUSPENSIONS** suspensions in which the concentration of particles is not sufficient to cause significant displacement of water as they settle or in which the particles will not be close enough for velocity field interference to occur	**CONCENTRATE DSUSPENSIONS** suspensions in which the concentration of particles is too great to meet the conditions mentioned for dilute suspensions

TYPE 1 (discrete particle settling)	· settling of discrete particles in dilute suspensions · particles have no tendency to flocculate · they settle as individual entities and there is no significant interaction with neighboring particles	Example: removal of grit and sand in wastewater treatment
TYPE 2 (flocculant settling)	· settling of flocculant particles in dilute suspensions · as particle settle and coalesce with other particles, the sizes of particles and their settling velocity increases	Examples: • removal of SS in primary sedimentation tanks of WWTP • settling of chemically coagulated waters
TYPE 3 (hindered settling) or (zone settling)	· settling of intermediate concentration of flocculant particles · particles are so close together that interparticle forces are able to hold them in fixed positions relative to each other and the mass of particles settles as a zone at a constant velocity	Example: biological floc removal in secondary settling basins
TYPE 4 (compression settling)	· settling of particles that are of such a high concentration that the particles touch each other and settling can occur only by compression which takes place from the weight of particles	Examples: · occurs in the bottom of deep secondary clarifiers · in sludge thickening facilities

1) **Free Settling:-**

if the particle is at a sufficient distance from the walls and from other particles so that its fall is not affected by them, ratio of particle diameter to container is less than (1/200) or if the particle concentration is less than 0.2 vol% in solution.

2) **Hindered Settling:-**

Particles are crowded, the settle at lower rate, the separation of a dilute slurry or suspension by gravity settling into a clear fluid and a slurry of higher solid content is called "Sedimentation" .

Theory of Particle Movement Through a Fluid:-

Number of forces will be act on a particle moving in a fluid:-

(1) Density difference is needed between particle and fluid(Buoyant Force)

(2) External force of gravity is needed to impart motion to the particle(gravitational Force)

(3) Resistance

For a rigid particle moving in a fluid, there are three forces acting on the body:-

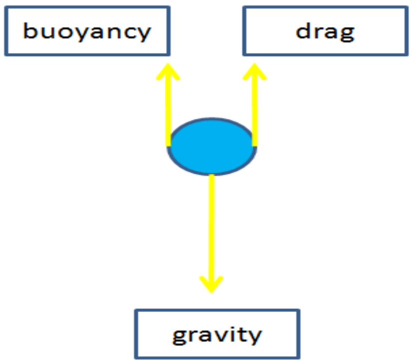
- Gravity acting downward; buoyant force acting upward; and resistance or drag force acting in opposite direction.

Assumption in Derivation:-

(1) Spherical, nonporous, in compressible.

(2) In compressible fluid, large diameter (to neglect wall effect).

(3) No friction between partricles.



*** Buoyant Force F_b in N on Particle is:-

$$F_b = \frac{m\rho g}{\rho_p} = V_p \rho g \qquad (1)$$

m:- mass of particle, kg (falling particle).

ρ_p:- Solid density, kg/m^3

ρ:- Liquid density kg/m^3

g:- Gravitational acceleration m/sec^2

*** Gravity Force F_g in N on a particle is:-

$$F_g = m.g \qquad (2)$$

*** Drag Force F_D in N on a particle is proportional to velocity head ($u^2/2$) of the Fluid displacement by moving body. This must be multiplied by density of fluid and by area :-

$$F_D = C_D \frac{u^2}{2} \rho A \qquad (3)$$

C_D:- drag coefficient (dimentionless).

A:- projected area of particle

$$\therefore F_g - F_b - F_D = Force\ \ due\ \ to\ \ Acceleration = m\frac{du}{dt}$$

$$\therefore m\frac{du}{dt} = m.g - \frac{m\rho g}{\rho_p} - \frac{C_D \rho A u^2}{2} \qquad (4)$$

The falling of particle consist of two periods; first one; accelerated fall,(very short about 1/10 sec.) then the second, constant velocity fall, the velocity is known as:-
"Free settling, or Terminal velocity (u_t)".

Take (du/dt)=0 in Eq.4 to get:-

$$\therefore u_t = \sqrt{\frac{2g(\rho_p - \rho)m}{A\rho_p C_D \rho}} \qquad (5)$$

For spherical particles; $m = \frac{\pi d_p^3 \rho_p}{6}$ and $A = \frac{\pi d_p^2}{4}$

$$\therefore u_t = \sqrt{\frac{4(\rho_p - \rho)d_p g}{3C_D \rho}} \qquad (6)$$

u_t is in m/sec; g= 9.81 m/sec2, dp in m.

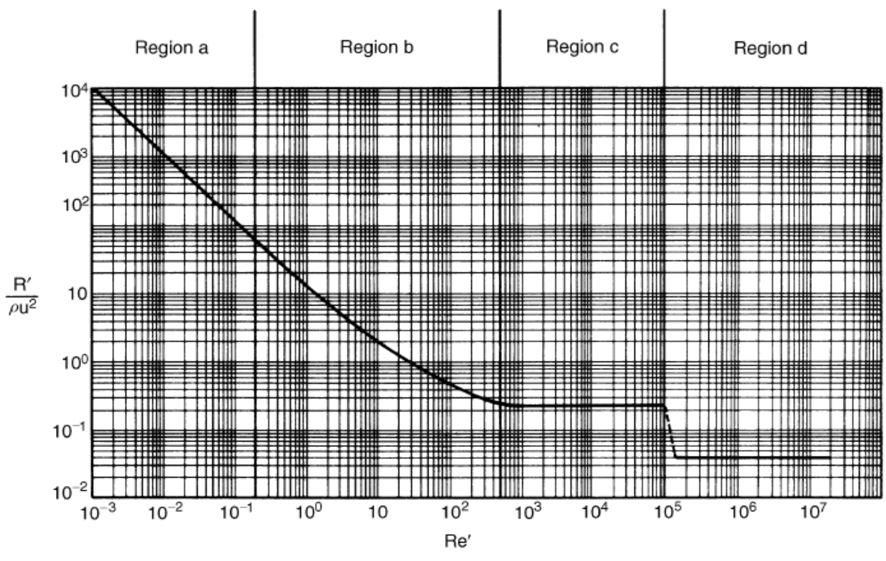


Figure 3.4. $R'/\rho u^2$ versus Re' for spherical particles

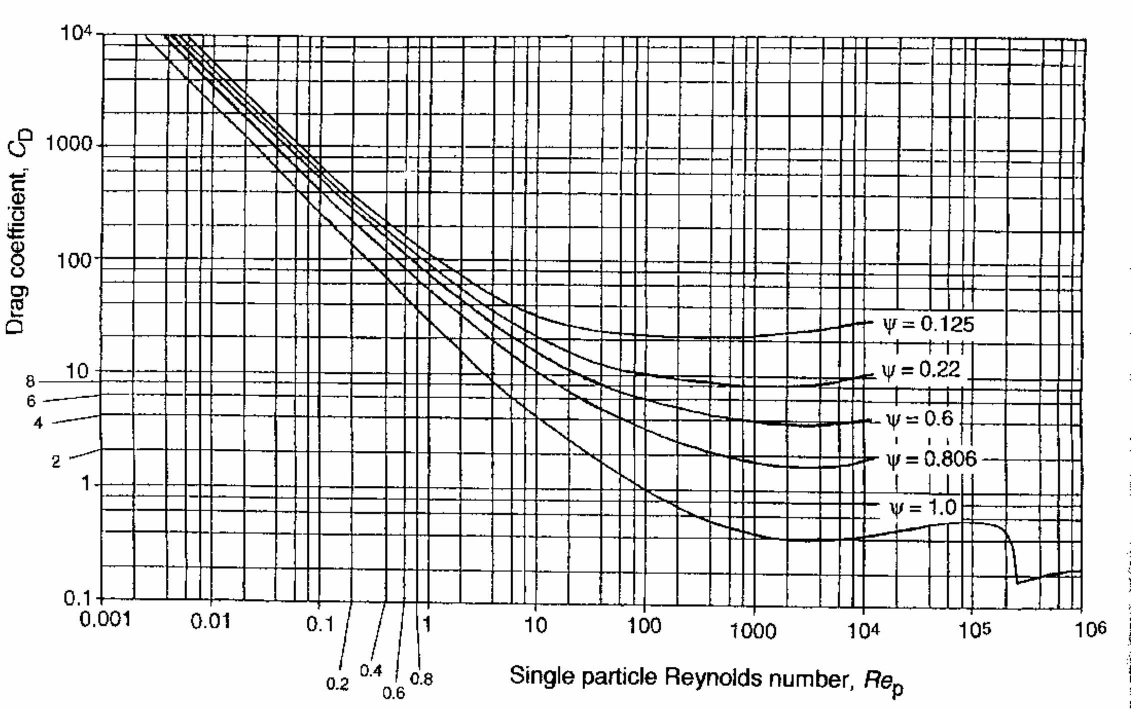


Figure 1.3 Drag coefficient C_D versus Reynolds number Re_p for particles of sphericity ψ ranging form 0.125 to 1.0 (note Re_p uses the equivalent volume diameter)

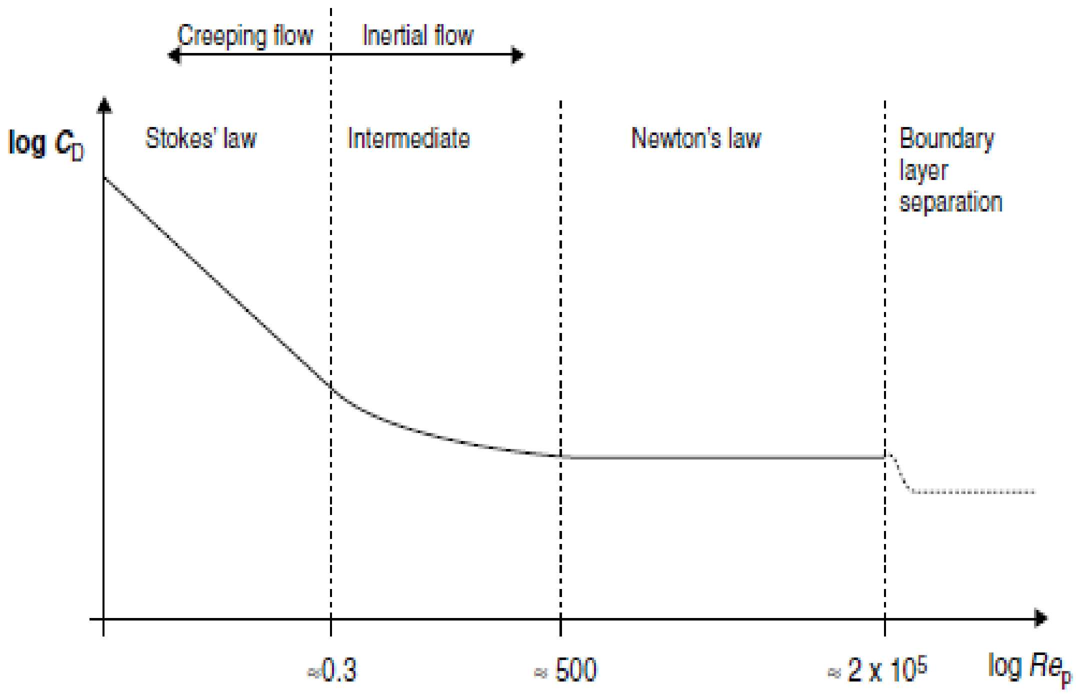


Figure 2.1 Standard drag curve for motion of a sphere in a fluid

Relation between $\frac{R'}{\rho u^2}$ and Re

The relation between $\frac{R'}{\rho u^2}$ and Re can be put in graphical form in which four regions can be distinguished:

Region (a) ($10^{-4} < Re' <$ 0.2-1.0): In this region, the relationship between $\frac{R'}{\rho u^2}$ and Re' is a straight line of slope −1.

And the drag coefficient is :-

$$C_D = \frac{24}{\mathrm{R\bar{e}}} \qquad (7) \quad Or \quad \frac{\bar{R}}{\rho u^2} = \frac{12}{\mathrm{R\bar{e}}}$$

And

$$\mathrm{R\bar{e}} = \frac{u_t \rho d_p}{\mu}$$

Eq.(6) becomes:-

$$\therefore u_t = \frac{g d_p^2 (\rho_p - \rho)}{18 \mu} \qquad (8) \quad Stokes\,Law$$

Region (b) (0.2-1.0 < Re' < 500-1000): In this region, the relationship between $\frac{R'}{\rho u^2}$ and Re' is:

$$\frac{R'}{\rho u^2} = \frac{12}{\mathrm{Re}'}\left(1 + 0.15\,\mathrm{Re}'^{0.687}\right)$$

Or

$$C_D = \frac{24}{\mathrm{Re}'}\left(1 + 0.15\,\mathrm{Re}'^{0.687}\right) \qquad (9)$$

Region (c) (1000 < Re' < 2×10^5): In this region, the value of $\frac{R'}{\rho u^2}$ is constant:

$$\frac{R'}{\rho u^2} = 0.22 \quad and \quad C_D = 0.44 \qquad (10)$$

Region (d) (Re' > 2×10^5): In this region, the flow changes to turbulent and separation of the BL occurs at the rear of the object. The value of $\frac{R'}{\rho u^2}$ is constant:

$$\frac{R'}{\rho u^2} = 0.05 \quad and \; C_D = 0.1 \qquad (11)$$

Drag Coefficient and Drag Force on a Particle

Region	Drag Coefficient	Drag Force
0.2<Re <Region (a)10^{-4}	$\frac{R'}{\rho u^2} = \frac{12}{Re'}$	$F = 3\pi\mu du$
10^3<Re <Region (b)0.2	$\frac{R'}{\rho u^2} = \frac{12}{Re'}(1+0.15\,Re'^{0.687})$	$F = 3\pi\mu du(1+0.15\,Re'^{0.687})$
2x10^5<Re <Region (c) 10^3	$\frac{R'}{\rho u^2} = 0.22$	$F = 0.055\pi d^2 \rho u^2$
2x10^5>Region (d) Re	$\frac{R'}{\rho u^2} = 0.05$	$F = 0.0125\pi d^2 \rho u^2$

The above equations were based on the assumption that the settling of the particle is not affected by other particles (free falling) and that there is no wall effects.

Example 1

A particle of 2 mm in diameter and density of 2500 kg/m^3 is settling in a stagnant fluid in the Stokes' flow regime.

a) Calculate the viscosity of the fluid if the fluid density is 1000 kg/m3 and the particle falls at a terminal velocity of 4 mm/s.

b) What is the drag force on the particle at these conditions?

c) What is the particle drag coefficient at these conditions?

c) What is the particle acceleration at these conditions?

d) What is the apparent weight of the particle?

SOLUTION TO EXERCISE 2.9

In the Stokes law region: $U_T = \frac{x^2(\rho_p - \rho_f)g}{18\mu}$ (EQ1

Hence, with $U_T = 4 \times 10^{-3}$ m/s, $\rho_f = 1000$ kg/m^3, $\rho_p = 2500$ kg/m^3 and x = 2 x 10

(a) Viscosity, $\mu = \frac{(2\times10^{-3})^2(2500-1000)\times9.81}{18\times4\times10^{-3}} = 0.8175$ Pa.s

(b) Drag force, $F_D = 3\pi\mu Ux$ (EQ 1.3)

So $F_D = 3\pi\times0.8175\times(4\times10^{-3})\times(2\times10^{-3}) = 6.164\times10^{-5}$ N

(c) Drag coefficient $C_D = 24/Re_p$

So: $C_D = \frac{24\mu}{U\rho_f x} = \frac{24\times0.8175}{(4\times10^{-3})\times1000\times(2\times10^{-3})} = 2452.5$

(d) At terminal velocity, acceleration is zero.

(e) Apparent weight: at terminal velocity apparent weight = drag force = 6.164 x 10^{-5} N

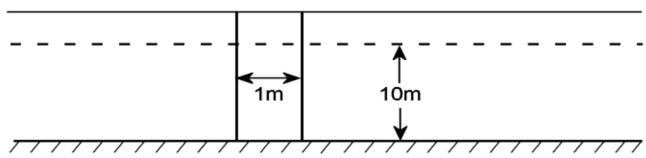
As a check, we calculate the apparent weight $= \frac{\pi x^3}{6}(\rho_p - \rho_f)g = 6.164\times10^{-5}$ N

Example 2

A cylindrical bridge pier 1 meter in diameter is submerged to a depth of 10m in a river at 20°C. Water is flowing past at a velocity of 1.2 m/s. Calculate the force in Newton on the pier.

$$\rho_{water} = 998.2\ kg/m^3$$

$$\mu_{water} = 1.005\times10^{-3}\ kg/ms$$



$$F_k = \frac{C_d A_P \rho u^2}{2}$$

$$Re = \frac{\rho u D}{\mu} = \frac{998.2\times1.2\times1}{1.005\times10^{-3}} = 1192\times10^6$$

From the figure, $C_d \approx 0.35$

Projected Area = DL = 10 m^2

$$F_k = \frac{0.35}{2}\times10\times998.2\times1.2^2 = 2{,}515\ N$$

2. Hindered Settling:-

Settling velocity is less than free settling for Stokes Law, drag force is greater in the suspension because of the interface of other particles. This higher effective viscosity of mixture(μ_m) is equal to the actual viscosity of the liquid itself (μ) divided by an empirical correction factor(ψ_p), which depends upon (ε), the volume fraction of slurry mixture occupied by liquid.

$$\mu_m = \frac{\mu}{\psi_p} \qquad (10)$$

And

$$\psi = \frac{1}{10^{1.83(1-\varepsilon)}} \qquad (11)$$

It is dimensionless, and density becomes:-

$$\rho_m = \varepsilon\rho - (1-\varepsilon)\rho_p \qquad (12)$$

ρ_m is density of slurry in (kg solid+liq./m^3).

$$\therefore \rho_p - \rho_m = \rho_p - \left[\varepsilon_p + (1-\varepsilon)\rho_p\right] = \varepsilon\left(\rho_p - \rho\right) \qquad (13)$$

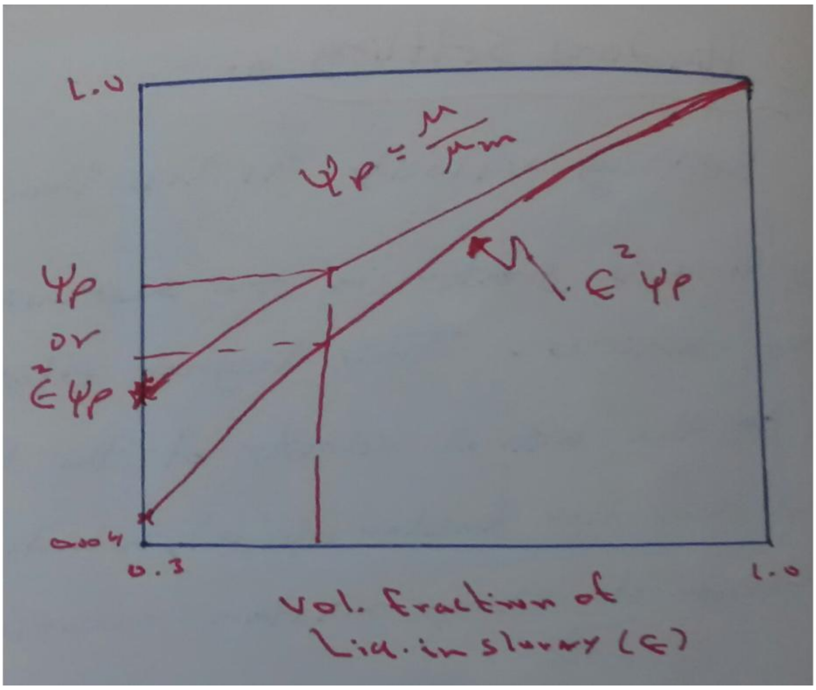
The settling velocity u_t with respect to the apparatus is ε times the velocity calculated by Stokes Law for laminar flow:-

$$\therefore \bar{u}_t = \frac{g\,d_p^2\,(\rho_p - \rho_m)}{18\mu}\,\varepsilon \qquad (14)$$

$$\therefore \bar{u}_t = \frac{g\, d_p^2\, (\rho_p - \rho)}{18\,\mu} \left(\varepsilon^2 \psi_p\right) \qquad (15)$$

And

$$\mathrm{R\bar{e}} = \frac{d_p \bar{u}_t \rho_m}{\mu_m\, \varepsilon} = \frac{g\, d_p^2\, (\rho_p - \rho)\rho_m\, \varepsilon \psi_p^2}{18\,\mu^2} \qquad (16)$$



Ex.3

Oil droplet having a diameter of 0.02 mm are to be settled from air at an air temperature of 37.8°C at 101.3 kpa and density of oil 1s 900 kg/m^3. Assume the drop is rigid sphere. Calculate terminal velocity of the drop given that viscosity of air is 1.9 810^{-5} pa.sec?

Sol.

$$\rho_{air} = \frac{P.M.Wt}{RT} = 1.137\, kg/m^3$$

$$\mathrm{R\bar{e}} = \frac{d_p \bar{u}_t \rho_m}{\mu} = 1.197 \bar{u}_t \qquad (1)$$

$$\therefore u_t = \sqrt{\frac{4\,(\rho_p - \rho)\, d_p\, g}{3 C_D\, \rho}} = \sqrt{\frac{4\,(900 - 1.137) * 9.81 (2 * 10^{-5})}{3 C_D\, (1.137)}}$$

$$\Rightarrow C_D = \frac{0.0267}{u_t^2}$$

By Trial and Error

Assume u_t=0.305, C_D= 2.22

u_t	Re Eq1	C_D Eq.2

0.305	**0.365**	**2.22**
0.0305	**0.0365**	**222**
0.00305	**0.00365**	**22200**

Plot on the figure then from computed line intercept with original line, get Ut=0.01 m/sec; Re less than 1

Ex.4

Glass sphere having diameter of $1.554*10^{-4}$ m settled in water at 293 k. the slurry contain 60% weight solids. The density of glass sphere is 2467 kg/m^3 given density of water is 998 kg/m^3, viscosity of water is $1.005*10^{-3}$ pa.sec.?

Sol.

$$\varepsilon = \frac{40/998}{[(40/998)+(60/2460)]} = 0.622$$

Bulk density of slurry ρ_m is:-

$$\rho_m = \varepsilon\rho - (1-\varepsilon)\rho_p$$
$$= 0.622*(998) + (1-0.622)\ (2467)$$
$$= 1553\, kg/m^3$$

$$\psi = \frac{1}{10^{1.83(1-\varepsilon)}} = 0.205$$

$$\therefore \bar{u}_t = \frac{g\, d_p^2\ (\rho_p - \rho)}{18\,\mu}\left(\varepsilon^2 \psi_p\right)$$

$$= \frac{9.81*(1.554*10^{-4})^2\ (2467-998)(0.662)^2(0205)}{18*(1.005*10^{-3})} = 1.525*10^{-3}\, m/\sec$$

$$\mathrm{R\bar{e}} = \frac{d_p \bar{u}_t \rho_m}{\mu_m \varepsilon} = \frac{d_p * \bar{u}_t * \rho_m}{(\mu/\psi_p) * \varepsilon} = 0.121$$

Criteria of Equal Settling Velocity for Different Particles

For a particle of material A of diameter dA and density ρA, Stokes' law is applicable, and then theterminal falling velocity u0A is given by:

$$u_A = \frac{d_A^2(\rho_A - \rho)g}{18\mu}$$

For a particle of material B:

$$u_B = \frac{d_B^2(\rho_B - \rho)g}{18\mu}$$

The condition for the two terminal velocities to be equal is then:

$$\frac{d_B}{d_A}=\left(\frac{\rho_A-\rho}{\rho_B-\rho}\right)^{1/2} \qquad (17)$$

If Newton's law is applicabl-e:

$$u_A^2=\frac{3gd_A(\rho_A-\rho)}{\rho}$$

$$u_B^2=\frac{3gd_B(\rho_B-\rho)}{\rho}$$

For equal settling velocity

$$\frac{d_B}{d_A}=\left(\frac{\rho_A-\rho}{\rho_B-\rho}\right) \qquad (18)$$

In general, the relationship for equal settling velocities is:

$$\frac{d_B}{d_A}=\left(\frac{\rho_A-\rho}{\rho_B-\rho}\right)^S \qquad (19)$$

where S = ½ for the Stokes' law region, S = 1 for Newton's law and, as an approximation,1/2 < S < 1 for the intermediate region.

Problems with Unknown Terminal Velocity

In problems at which terminal velocity is not known, Reynolds number cannot be found to decide the settling region, and therefore the drag coefficient cannot be found.
The drag force at the equilibrium conditions can be written as:

$$R'_o\frac{\pi}{4}d^2=\frac{\pi}{6}d^3(\rho_s-\rho)g$$

$$R'_o=\frac{2}{3}d(\rho_s-\rho)g$$

$$\frac{R'_o}{\rho u_o^2}=\frac{2}{3\rho u_o^2}d(\rho_s-\rho)g \qquad (20)$$

In this case, we use the dimensionless group $\frac{R'}{\rho u_0^2}\mathrm{Re}'^2$ which does not involve u_o since:

$$\frac{R'}{\rho u_o^2}\frac{\rho^2d^2u_o^2}{\mu^2}=\frac{2}{3\rho u_o^2}d(\rho_s-\rho)g\frac{\rho^2d^2u_o^2}{\mu^2}=\frac{2d^3(\rho_s-\rho)\rho g}{3\mu^2} \qquad (21)$$

The dimensionless group $\frac{d^3(\rho_s-\rho)\rho g}{\mu^2}$ is called Galileo number (Ga) and that

$$\frac{R'}{\rho u_o^2}\mathrm{Re}'^2=\frac{2}{3}Ga \qquad (22)$$

The group $\frac{R'}{\rho u_0^2}\mathrm{Re}'^2$ was plotted versus Re' to be used in the case that terminal velocity is not known.

Example 5

Calculate the terminal velocity of a steel ball, 2 mm diameter and of density 7870 kg/m^3 in an oil of density 900 kg/m^3 and viscosity 50 mNs/m^2

Solution

For a sphere:

$$(R'_0/\rho u_0^2)Re_0'^2 = (2d^3/3\mu^2)\rho(\rho_s - \rho)g$$
$$= (2 \times 0.002^3/3 \times 0.05^2)900(7870 - 900)9.81$$
$$= 131.3$$
$$\log_{10} 131.3 = 2.118$$

From Table 3.4: $\log_{10} Re'_0 = 0.833$

or: $Re'_0 = 6.80$

Thus: $u_0 = (6.80 \times 0.05)/(900 \times 0.002) = \underline{\underline{0.189 \text{ m/s}}}$

Example 6

Estimate the terminal velocity of limestone particles (d = 0.15 mm, ρ = 2800 kg/m^3) in water at 20°C.

Solution

$$\frac{\tau'}{\rho u_o^2}Re'^2 = \frac{2d^3(\rho_s - \rho)\rho g}{3\mu^2}$$

$$= \frac{2 \times (1.5 \times 10^{-4})^3 \times (2800 - 998.2) \times 998.2 \times 9.81}{3 \times (1.005 \times 10^{-3})^2} = 39.3$$

From the figure Re = 2.6

$$Re'_o = 2.6 = \frac{\rho u_o d}{\mu} = \frac{998.2 \times u_o \times 1.5 \times 10^{-4}}{1.005 \times 10^{-3}} \Rightarrow u_o = 0.0174\ m/s$$

Problems with Unknown Particle Size

For unknown size of a particle that have a given terminal velocity we use:

$$\frac{R'}{\rho u_o^2}Re'^{-1} = \frac{2\mu g}{3\rho^2 u_o^3}(\rho_s - \rho) \qquad (27)$$

Table 3.5 or Fig. 3.6 can be used to solve such problems.

Example 7

A spherical particle of density 1500 kg/m^3 has a terminal velocity of 1 cm/s in a fluid of density 800 kg/m^3 and viscosity 0.001 Pas. Estimate the diameter of the particle.

Solution

SOLUTION TO EXERCISE 2.12:

When U_T is known and x unknown, we first calculate the dimensionless group:

$$\frac{C_D}{Re_p} = \frac{4}{3}\frac{g\mu(\rho_p - \rho_f)}{U_T^3 \rho_f^2}$$

So, $$\frac{C_D}{Re_p} = \frac{4}{3} \times \frac{9.81 \times 0.001 \times (1500 - 800)}{(0.01)^3 \times 800^2} = 14.306$$

Plotted on the drag curve (C_D versus Re_p) this gives a straight line of slope +1 (see Figure 2E12.1)

This line intersects the curve for spherical particles (ψ = 1.0) at a Re_p value of 1.4.

Hence:

$$Re_p = \frac{U_T \rho_f x}{\mu} = 1.4,$$

giving particle size, x = 175 μm

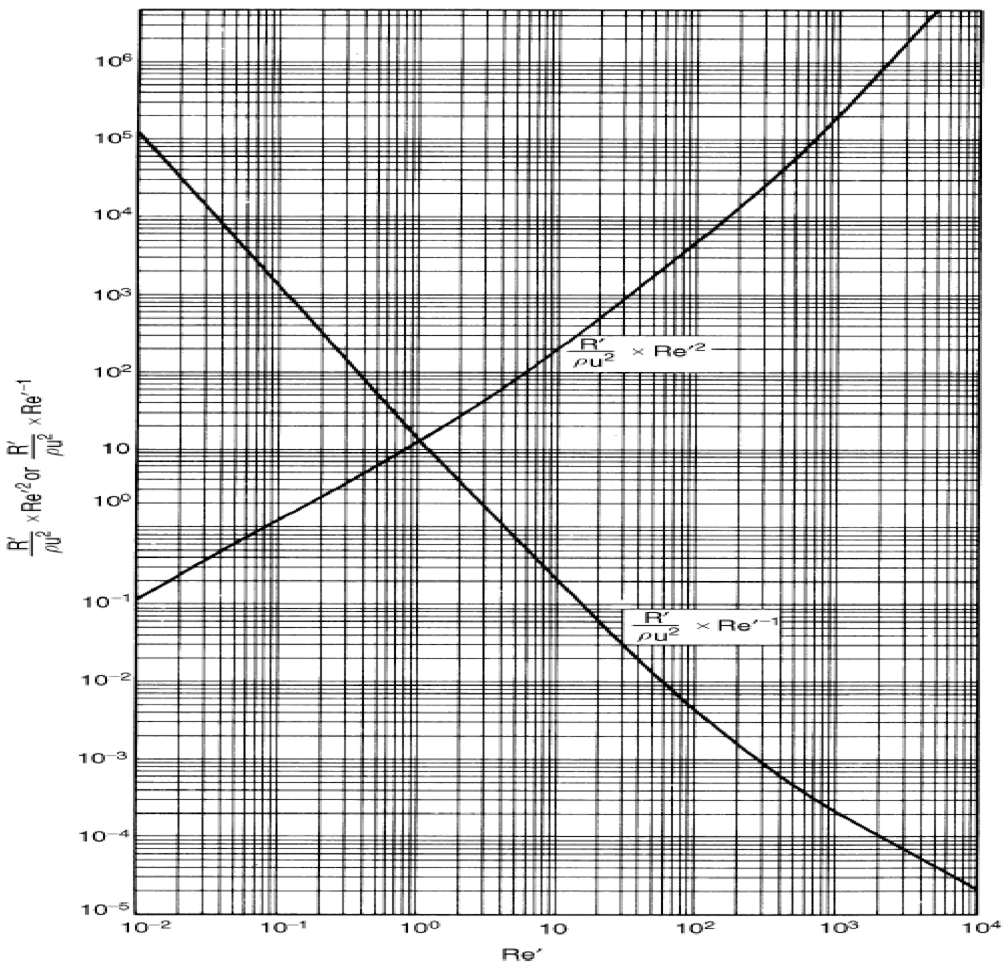


Figure 3.6. $(R'/\rho u^2)Re'^2$ and $(R'/\rho u^2)Re'^{-1}$ versus Re' for spherical particles

Non-Spherical Particles
Effect of particle shape and orientation on drag

A spherical particle is unique in that it presents the same area to the oncoming fluid whatever its orientation. For non-spherical particles, the orientation must be specified before the drag force can be calculated.
For non-spherical particles, the orientation must be specified before the drag force can be calculated. The experimental data for the drag can be correlated in the same way as for the sphere, by plotting the dimensionless group $\frac{R'}{\rho u^2}$ against the Reynolds number, $Re' = \frac{\rho u d'}{\mu}$, d' is defined as the diameter of the circle havingthe same area as the projected area of the particle and is therefore a function of theorientation, as well as the shape, of the particle.
The curve for $\frac{R'}{\rho u^2}$ against Re' may be divided into four regions, (a), (b), (c)and (d), as before. In region *(a)* the flow is entirely streamline and, although no theoretical expressions have been developed for the drag on the particle, the practical data suggest that a law of the form: $\frac{R'}{\rho u^2} = \frac{K}{Re'}$ is applicable. The constant *K* varies somewhat according to the shape and orientation of the particle although it always has a value of about 12.

Typical values of $\frac{R'}{\rho u^2}$ for non-spherical particles in region (c)are given in Table 3.6.

Terminal falling velocities

If $d_p$is the mean projected diameter, the mean projected area is $\frac{\pi d_p^2}{4}$ and the volume is $k' d_p^3$, where k' is a constant whose value depends on the shape of the particle.

- Fora spherical particle, k' is equal to $\pi/6$.
  - For rounded isometric particles, that is particlesin which the dimension in three mutually perpendicular directions is approximately thesame, k' is about 0.5.
- For angular particles, k' is about 0.5.
- For most minerals, k' lies between 0.2 and 0.5.

To find the terminal velocity of the non-spherical particle:

Drag force $F = R_o{}' \cdot \frac{\pi d_p^2}{4} = (\rho_s - \rho) g k' d_p^3$

$$\frac{R_o{}'}{\rho u_o^2} = \frac{4k' d_p g(\rho_s - \rho)}{\pi \rho u_o^2}$$

$$\frac{R_o{}'}{\rho u_o^2} Re_o'^2 = \frac{4k' \rho d_p^3 g(\rho_s - \rho)}{\mu^2 \pi}$$

and $\frac{R_o{}'}{\rho u_o^2} Re_o'^{-1} = \frac{4k' \mu g(\rho_s - \rho)}{\pi \rho^2 u_o^3}$

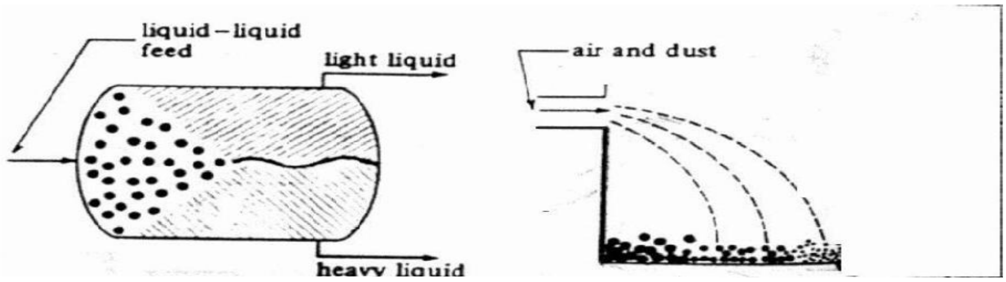
1. Evaluate $\frac{R_o{}'}{\rho u_o^2} Re_o'^2$ using $d_p$as the characteristic linear dimension.
2. The corresponding value of Re_o' is then found from Table 3.4 or from Figure 3.6 (for spherical particle).
3. Correct the value of $\log Re_o{}'$ to account for non sphericity using Table 3.7.
4. A similar procedure is adopted for calculating the size of a particle of given terminal velocity, using Tables 3.5 and 3.8.

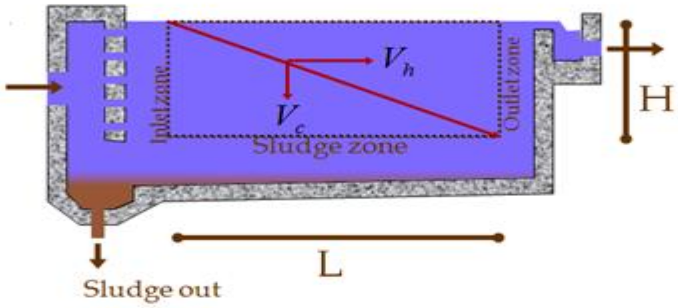
Provided k' is known, the appropriate dimensionless group may be evaluated and the terminal falling velocity, or diameter, calculated.

Equipment for settling and sedimentation

- (1) Simple Gravity Settling Tank
- (2) Equipment for Classification
- (3) Spitzkasten Classifier
- (4) Sedimentation Thickener

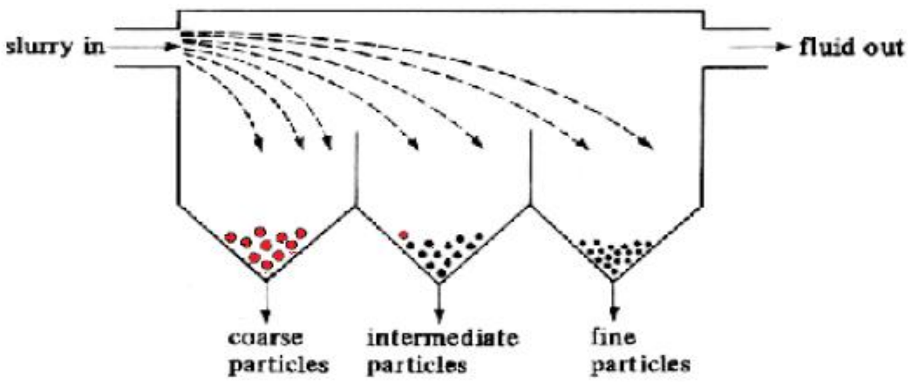
(1) Simple Gravity Settling Tank:-





(2) Equipment For Classification:-

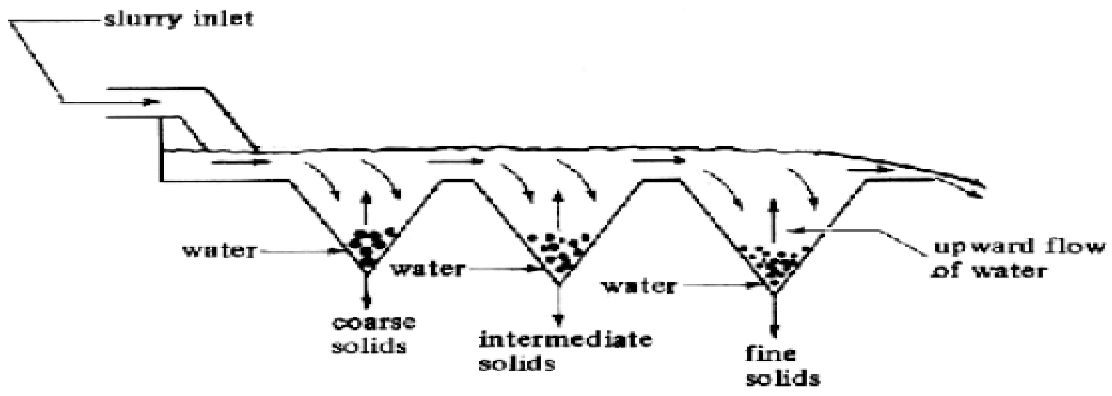
The simplest type of classifier is one in which a large tank is subdivided into several sections.



Note:-

Separation of solid particles into several fractions based on their terminal velocities is called Classifier or Classification

(3) Spitzkasten Classifier:-



Sedimentation:-

Separation of dilute slurry by gravity settling in to clear fluid and a slurry of higher solids concentration is called Sedimentation.

Sedimentation Tanks

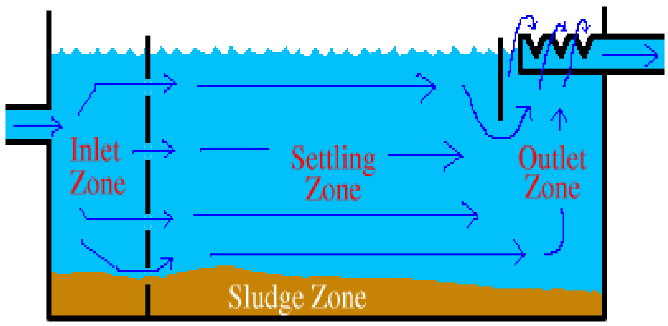
Sedimentation tanks can be divided into 4 different functional zones:

1. Inlet zone

2. Settling zone

3. Sludge zone

4. Outlet zone



INLET ZONE

•should disseminate influent energy

•distribute the flow

•mitigate density currents

•minimize sludge blanket disturbance

Inlet structures are designed to uniformly distribute the influent suspension over the cross section of the settling zone.

For Rectangular Basins

full width inlet channels - effective spreading of flow introduce a vertical velocity component into sludge happen that may resuspend sludge.

"Settleable" doesn't necessarily mean that these particles will settle easily by gravity. In many cases they must be coaxed out of suspension or "solution" by the addition of chemicals or increased gravity (centrifugation or filtration).

Because of the high volumetric flow rates associated with water and wastewater treatment systems, gravity sedimentation is the only practical, economical method to remove these solids. i.e., processes such as centrifugation are not economical, in most cases.

Gravity separation can obviously be applied only to those particles which have density greater than water. But this density must be significantly greater than that of water due to particle surface effects and turbulence in the sedimentation tanks.

Goals of gravity sedimentation:

1) Produce a clarified (free of suspended solids) effluent.

2) Produce a highly concentrated solid sludge stream.

Review of Type I and II sedimentation:-

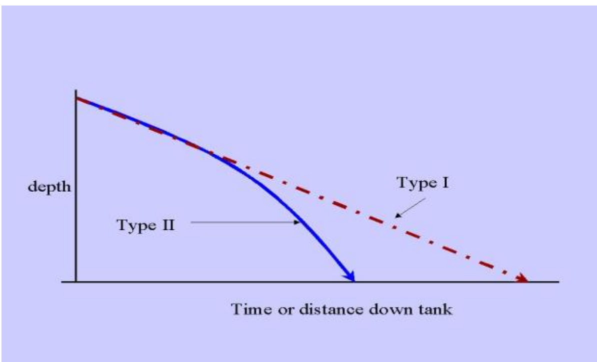
Type I (Discrete sedimentation):

- **Occurs in dilute suspensions, particles which have very little interaction with each other as they settle.**
- **Particles settle according to Stokes law**
- **Design parameter is surface overflow rate (Q/A_s)**

Type II (flocculent sedimentation):-

- **Particles flocculate as they settle**
- **Floc particle velocity increase with time**
- **Design parameters:**
  (1) Surface overflow rate (2) Depth of tank or, (3) Hydraulic retention Time

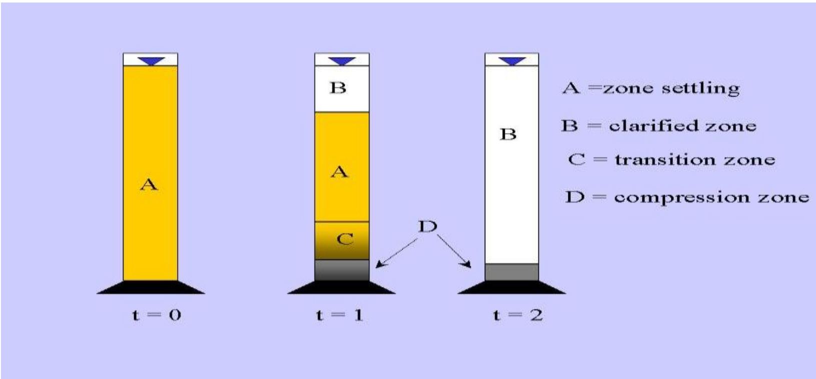
Comparison of Type I and II sedimentation



Zone Settling &Compression (Type III and IV):-

Zone settling occurs when a flocculent suspensions with high initial concentration (on the order of 500 mg/L) settles by gravity. Flocculant forces between particles causes settling as a matrix (particles remain in a fixed position relative to each other as they settle). When matrix sedimentation is constrained from the bottom the matrix begins to compress. Such a situation occurs when the matrix encounters the bottom of tank in which it is settling. This is called compression (Type IV) settling.

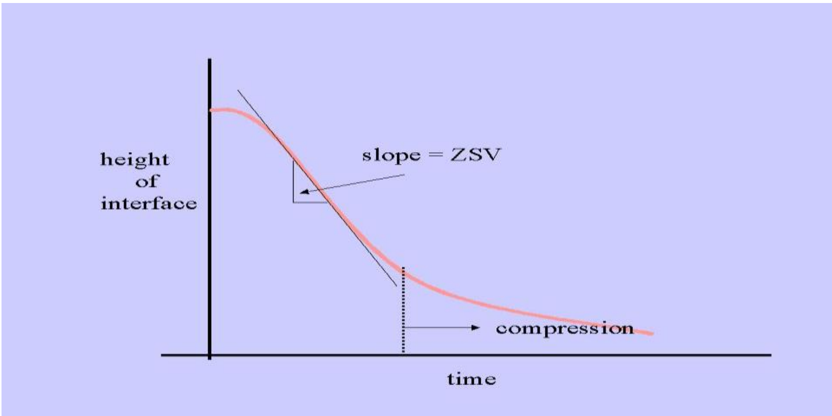
These settling types are demonstrated in a batch settling test as illustrated below:



Sludge time settling:-

At t=0 , uniform concentration

At t>0 , A, B, c, and D region will be formed .

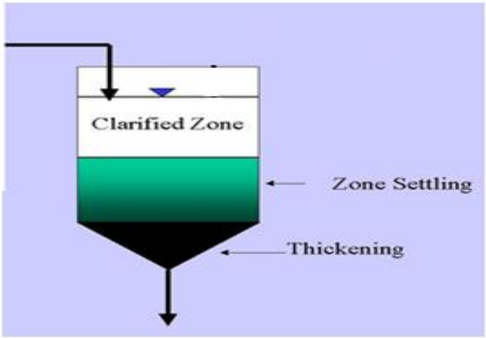


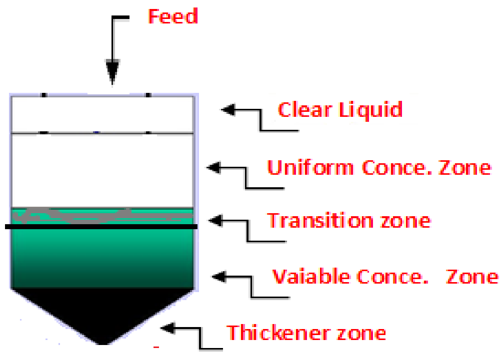
Factors affecting zone settling velocity:

1. Suspended solids concentration
2. Depth of settling column (or tank)
3. Stirring (0.5 – 2 rpm to prevent “arching”)
4. Temperature
5. Polymer addition (affects matrix structure)

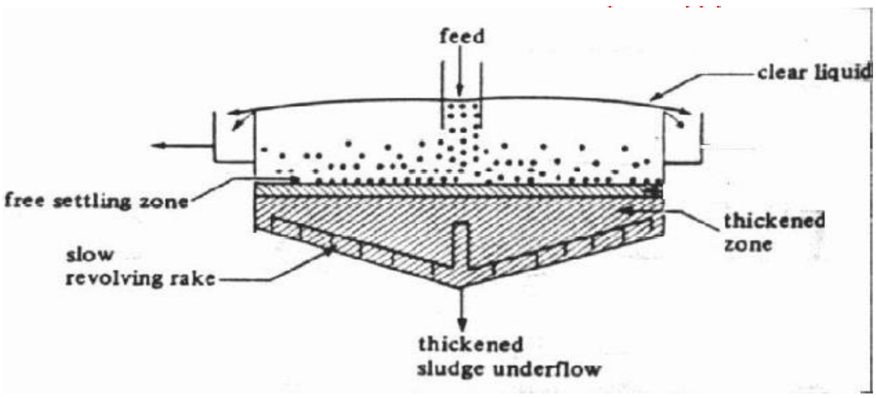
Thickener :-

Batch and continuous industrial equipment used for sedimentation is called thickener .





Batch Thickener



Continuous Thickener Design:-

From batch experiment we can determine settling characteristic. Slope of the curve at any point represent settling velocities of separation at that instant. The height of the interface (between the clarified zone and the zone settling zone) versus time is plotted in the figure below to determine the "zone settling velocity" (ZSV). Velocity of this interface is steady after some induction period but changes with time as compression begins. The slope of the steady interface subsidence rate represents zone settling velocity.

Rate Laminar Layer :-

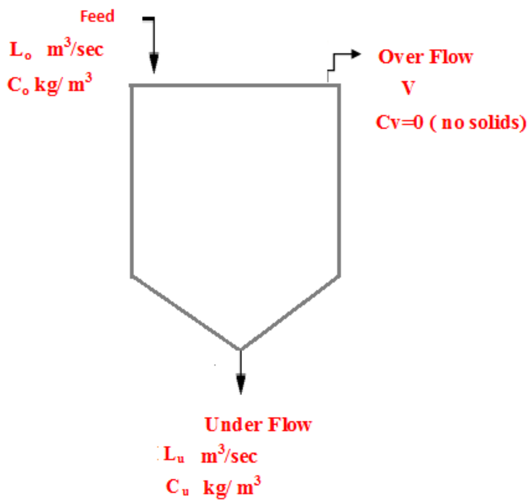
Concentration of layer having lowest capacity for the passage of solids through it under operating conditions.

C_o:- Initial concentration kg/m^3

Z_o:- Height of suspended solid in batch test.

S:- Cross-sectional area of cylinder

The limiting layer form from the bottom and moved upward to the clear liquid interface.



If the concentration of the limiting layer is C_L

And time for it to reach interface is θ_L.

● the total weight of solids in the slurry= $C_o\ Z_o\ S$-----(1)

● the quantity of solids passing through the rate limiting layer is:-

$$= C_L S \theta_L (U_L + \overline{U}_L) \qquad (2)$$

Settling velocity

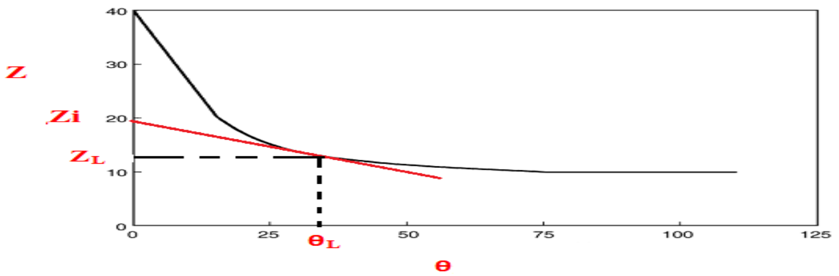
Upward velocity of L.L

$$\therefore C_0 Z_0 S = C_L S \theta_L \left(U_L + \overline{U}_L\right) \qquad (3)$$

$$\overline{U}_L = \frac{Z_L}{\theta_L} \qquad (4)$$

It is constant at Z_L (height of interface), and at θ_L

$$\therefore C_L = \frac{C_0 Z_0}{Z_L + U_L \theta_L} \qquad (5)$$



From figure

$$\frac{Z_i - Z_L}{\theta_L} = U_L \qquad (6)$$

Or

$$Z_i = Z_L + \theta_L U_L \qquad (7)$$

Combine Eq.(5) and (7)

$$C_L Z_i = C_0 Z_0 \Rightarrow C_L = \frac{C_0 Z_0}{Z_i} \qquad (8)$$

Zi:- Height which the slurry would occupy if all the solids present at C_L
CL:- Minimum conce. At which boundary layer interface.

Ex1.

A single batch-settling test was made on a limestone slurry. The interface between clear liquid and suspension solids was observed as a function of time and the results are tabulated bellow. The test was using 236 gm limestone per liter of slurry. Prepare a curve showing the relation between settling rate and solid concentration?

Time,θ, hr	0.00	0.25	0.50	1.00	1.75	3.00	4.75	12.00	20.00
Height of interface, mm	36	32.4	28.6	21.0	14.7	12.3	11.55	9.8	8.8

Sol of Ex1.

Plot relation of time θ with Z

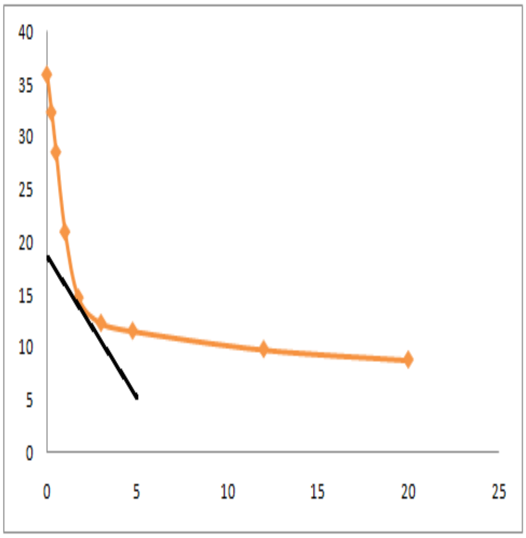
$$C_L Z_i = C_0 Z_0$$

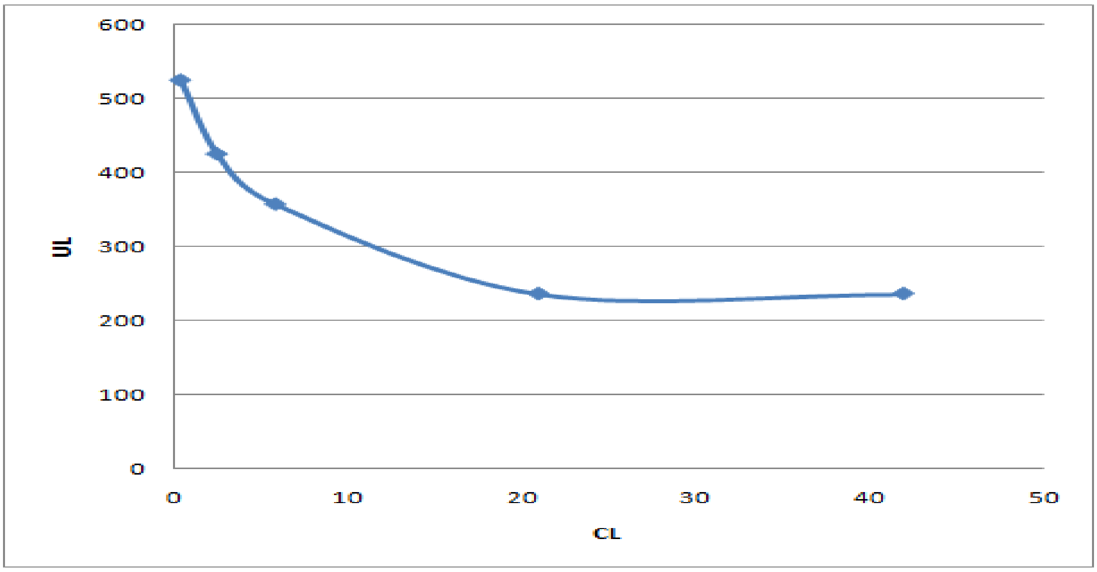
At θ=2, Slope of curve is dz/dθ

$$slope = \frac{Z_i - Z_L}{\theta} = u_{:L} \Rightarrow u_{:L} = \frac{20-15}{2} = 2.5c\,cm/\sec$$

$$\therefore C_L = \frac{C_0 Z_0}{Z_i} = \frac{236-36}{20} = 425gm/L$$

Θ (hr)	Zi(cm)	uL(cm/sec)	CL(gm/L)
0.5	36		
1.0	36		
1.5	23.8		
2.0	20		
3.0	16.2		
3.4	14.2		
4.0	10.4		
8.0	11.9		





Thickener Area:-

M.B on solids is:-

$L_oC_o = L_uC_u$ (9) $L_u = L_oC_o / C_u$ (10)

Over all Liquid material balance gives:-

$$L_o\rho_o - L_oC_o = V\rho_w + L_u\rho_u - L_uC_u \qquad (11)$$

ρ_o:- Density of slurry in; ρ_u:- Density of slurry in; ρ_w:- Density of water

$$\left[L_oC_o\left(\frac{\rho_o}{C_o}-1\right) = V\rho_w + L_uC_u\left(\frac{\rho_u}{C_u}-1\right) \qquad (12)\right]$$

Sub Eq.10 into Eq. 11 and $V\rho_w = L_oC_o$

$$V\rho_w = L_oC_o\left(\frac{\rho_o}{C_o}-1\right) - \frac{L_oC_oC_u}{C_u}\left(\frac{\rho_u}{C_u}-1\right) \qquad (13)$$

$$V = L_oC_o\left[\frac{\rho_o}{C_o} - \frac{\rho_u}{C_u}\right]\frac{1}{\rho_w} \qquad /S$$

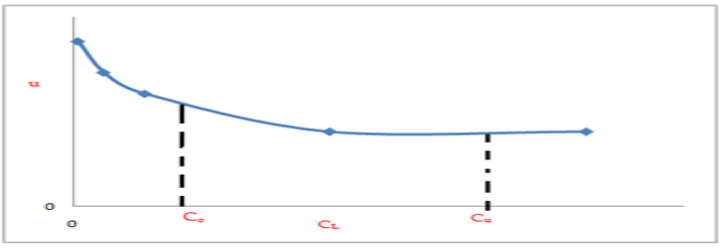
$$\frac{V}{S} = \frac{L_oC_o}{S}\left[\frac{1}{C_o} - \frac{1}{C_u}\right]\frac{\rho_{av}}{\rho_w} \qquad (14)$$

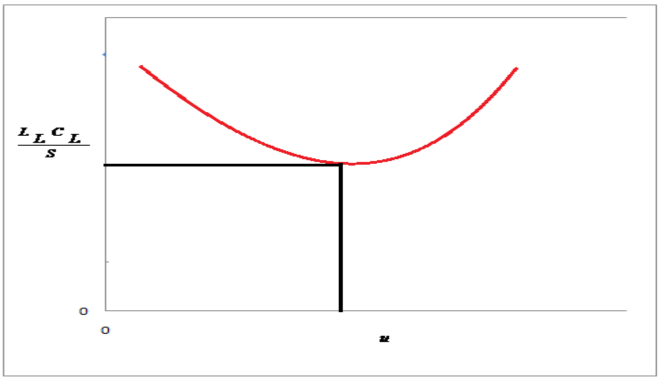
$$\&\ \rho_{av} = \frac{\rho_o + \rho_u}{2}$$

V/S:- Upward velocity and it is equal or less than settling velocity. So it may be replaced by u. At any time and u; $L_oC_o = L_LC_L$

$$\frac{L_LC_L}{S} = \frac{u}{\left[\frac{1}{C_L} - \frac{1}{C_u}\right]\frac{\rho_{av}}{\rho_w}} \qquad (15)$$

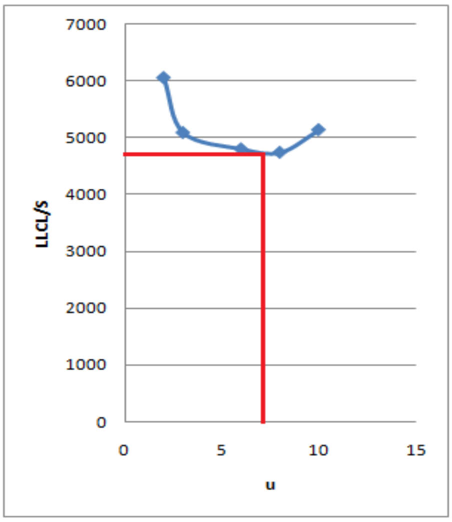
u	1/C_L	1/C_u	Eq.15





$(L_L C_L/S)$=value then $S=(L_L C_L)$/value.
$S=(\pi/4)*d^2$; then get diameter.
Ex2.
a limestone slurry equivalent to that in Ex1.; is fed to a thickener at a rate of 50 ton dry solid/hr, producing thickened sludge of 550 gm limestone/L. for an initial slurry concentration of 263 gm/L. specify the thickener area required? Water density is 1, solid density is 2.09 gm/cm³
Sol/
From Eq.15 and fig. in Ex1.

U cm/sec	C_L(gm/L)	$(1/C_L)*10^5$	$10^5 * \left(\frac{1}{C_L}-\frac{1}{C_u}\right)$	Eq.15
10	265	377	195	5140
8	285	351	169	4740
6	325	307	125	4800
3	415	241	59	5090
2	465	213	33	6060
1	550	182	0	∞



From figure EX1. (C_L)min=310 gm/L
Umin=6.9 cm/hr
$L_L C_L=50*10^6$ gm/hr
$S= 50*10^6/4730=1.0571\ m^2$
Diameter= 1.16 m

Thickener Depth:-

Depth of the thickening zone increased as the under flow rate was decreased. To calculate the depth of thickener, it is important to determine the volume of the compression zone, where the compression zone depends on the retention(residence) time, where the time is calculated from laboratory batch test data.

The shape of the compression zone is given by:-

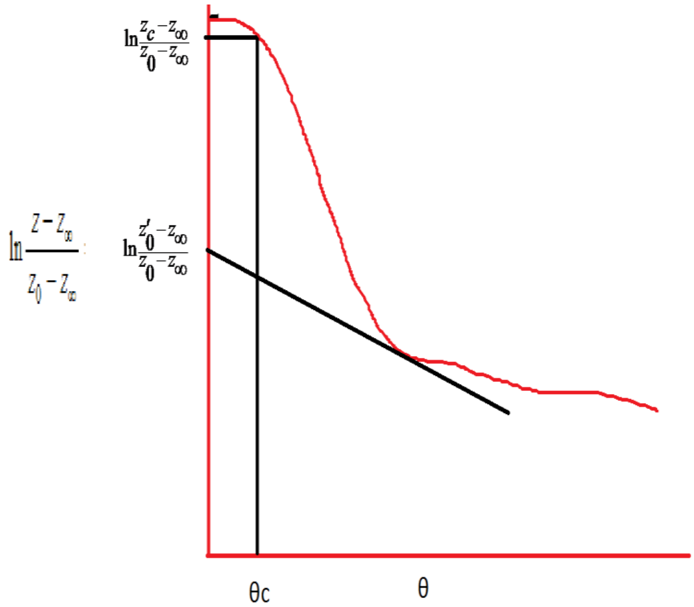
$$\frac{dz}{d\theta} = k(z_\infty - z_\theta) \qquad (16)$$

Z:-Hight of the compression zone at time θ
Z∞:-Hight of the compression zone at infinite time.
K:- constant for a particular system.
Integrating Eq.(16):-

$$\frac{dz}{d\theta} = k(z - z_\infty) \qquad (16)$$

$$\int_{z}^{z_0} \frac{dz}{z - z_\infty} = \int_{0}^{\theta} d\theta \Rightarrow\Rightarrow -\int_{z_0}^{z} \frac{dz}{z - z_\infty} = \int_0^\theta d\theta$$

$$\therefore \ln\frac{z - z_\infty}{z_0 - z_\infty} = -k\theta \qquad (17)$$

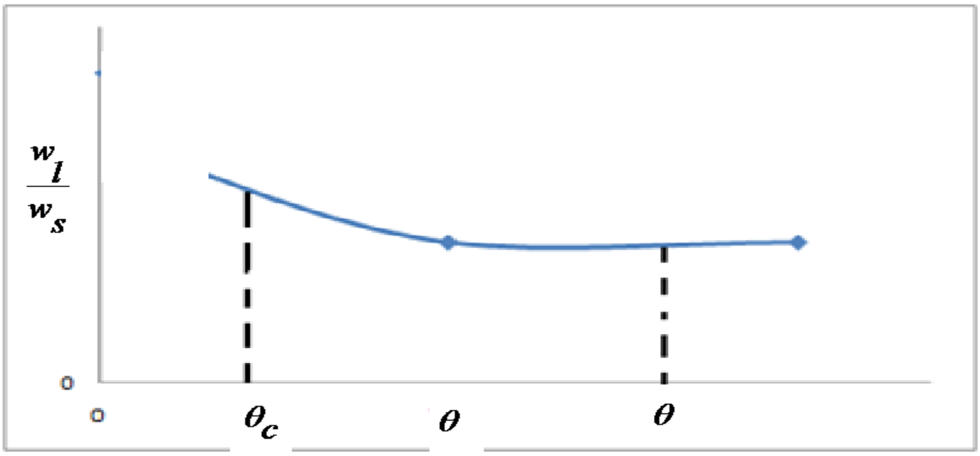


$$\frac{z_0 + z'_\theta}{2} \; find\ new\ z_c$$

1) Z_0 known, initial value
2) Z_∞ known, and it is = z_u (final height).
3) Tangent of the curve of slope(-k) cuts y-axis at $\ln\frac{z'_0 - z_\infty}{z_0 - z_\infty}$
4) Then calculate z'_0
5) Determine z_c ,compression zone height or critical height $\frac{z_0 + z'_\theta}{2}$
6) Calculate $\ln\frac{z_c - z_\infty}{z_0 - z_\infty}$ then find θ_c from fig.

Required volume for the compression zone= volume occupied by solid + volume occupied by associated liquid.

$$v = \frac{L_0 C_0}{\rho_s}(\theta - \theta_c) + \frac{L_0 C_0}{\rho}\int_0^\theta \frac{w_l}{w_s} d\theta \qquad \theta = final\ \ time = \theta_u$$



$\theta - \theta_c$:– Compression zone retention time, hr.

w_l :– Mass of liquid in compression zone. lb.

w_s :– Mass of solid in compression zone. lb.

$\frac{w_l}{w_s}$:- Liquid conce, Depth= $\frac{V}{S}$

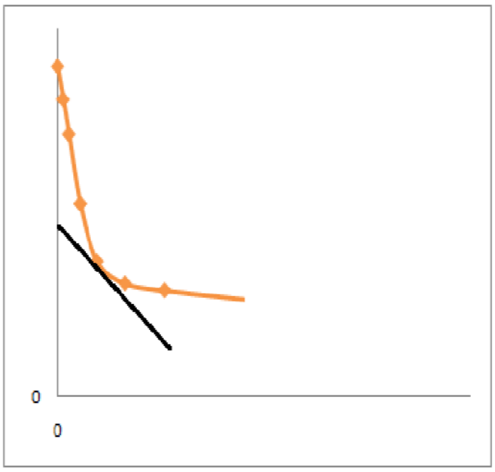
$$\begin{pmatrix}\text{Total depth of}\\ \text{thickener}\end{pmatrix} = \begin{pmatrix}\text{Depth of}\\ \text{compression zone}\end{pmatrix} + \begin{pmatrix}\text{Bottom pitch}\\ (1-2\text{ ft})\end{pmatrix} + \begin{pmatrix}\text{storage capacity}\\ (1-2\text{ ft})\end{pmatrix} + \begin{pmatrix}\text{submergence of feed}\\ (1-3\text{ ft})\end{pmatrix}$$

If Z_∞ unknown, then by trial and error calculation is consider to plot Eq.17, assume Z_∞ , then plot Eq.17.
If resulting line of Eq.17 is not straight line, then assumed another value to get a straight line of slop(-k).

Ex2.
Estimate depth of thickener required to perform operation of Ex2. The batch settling test indicate a value of Z_∞ =7.7 cm. The specific gravity of limestone is 2.09.
Sol/
From Ex2:-

Time,θ, hr	0.00	0.25	0.50	1.00	1.75	3.00	4.75	12.00	20.00	∞
$Z-Z_\infty$	28.3								1.1	0.00
$(Z-Z_\infty)/(Z_0-Z_\infty)$	1.0								0.0389	0.00

(1) Plot $\ln\dfrac{z-z_\infty}{z_0-z_\infty}$ vis. θ.



(2) Then plot tangent line of slop(-k)

$$\ln\frac{z_0'-z_\infty}{z_0-z_\infty}=-1.7$$

$$\frac{z_0'-z_\infty}{z_0-z_\infty}=0.2\Rightarrow z_0=13.36\,cm$$

$$z_c\ \frac{z_0'+z_0}{2}=24.68\,cm$$

$$\ln\frac{z_c-z_\infty}{z_0-z_\infty}=\ln\frac{24.68-7.7}{28.3}=-0.5108$$

$$\frac{z_c-z_\infty}{z_0-z_\infty}=0.6$$

(3) Specify θ_c from fig.= 0.8 hr.

From Ex2. Final concentration is 550 gm/L and time is 3.4 hr from Ex1.

Θ= 3.4 hr, θ_c=0.8hr

$$v=\frac{L_0C_0}{\rho_s}(\theta-\theta_c)+\frac{L_0C_0}{\rho}\int_0^\theta\frac{w_l}{w_s}d\theta$$

L_0C_0 :- 100,000 lb/hr, ρ_s = 2.09*62.3=130 lb/ft^3

Θ- θ_c=2.6 hr

$C_L=\dfrac{w_l}{w_s}$ (gm solid/gm water)	$\dfrac{w_s}{w_l}=\dfrac{1}{C_L}$
236	0.00424
236	0.00424
236	0.00424
236	0.00424
392	0.00255
525	0.00190
644	0.00155
850	0.00118

$$\int_{0.8}^{3.4}\frac{w_l}{w_s}d\theta=area\ \ under\ the\ curve=6.89\,hr$$

$$v=\frac{100000}{130}(2.6)+\frac{100000}{62.3}(6.89)=13000\ ft^3$$

From Ex1. S= 10329 ft^2

Depth of thickener= $(13000/10329)_{\text{compression zone}}+(2\text{ ft})_{\text{bottom pitch}}+(2\text{ ft})_{\text{storage capacity}}+(2\text{ ft})_{\text{submergence feed}}$= 7.3 ft

Flow of Fluids through Packed Columns

Chemical engineering operations commonly involve the use of packed fixed beds. This is equipment in which a large surface area for contact between a liquid and a gas (absorption, distillation) or a solid and a gas or liquid (adsorption, catalysis) is obtained for achieving rapid mass and heat transfer.

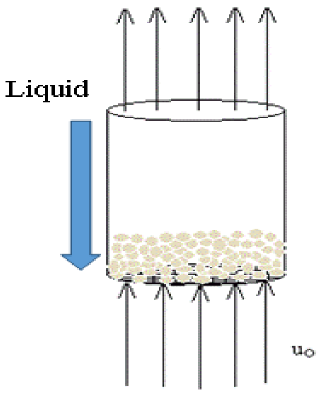
A typical packed bed is a cylindrical column that is filled with a suitable packing material. The liquid is distributed as uniformly as possible at the top of the column and flows downward, wetting the packing material. A gas is admitted at the bottom, and flows upward, contacting the liquid in a countercurrent fashion. Expressions are needed to predict pressure drop(Δp) across the bed due to the resistance caused by the presence of particles.

Flow of Single Fluids through a Packed Columns:-

Darcy Law & Permeability:-

Darcy (1830) showed that the average velocity as measured over the whole area of the bed was directly proportional to the driving pressure and inversely proportional to thickness of bed. This relation, often termed Darcy Law , has subsequently been confirmed by a number of workers & can be written as :-

$$u = k\frac{(-\Delta p)}{l} \qquad (1)$$



u:- average velocity of flow through bed.

(-Δp):- pressure drop across bed.

l:- thickness of bed.

K:- Constant depending on physical properties of the bed & fluid

$$u = \frac{1}{A}\frac{dv}{dt}$$

V:- volume of fluid flowing in time t

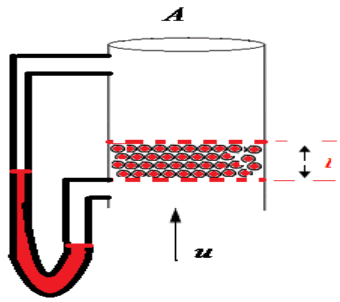
A:- Total cross-sectional area of fluid

Since both velocity and width of channels are normally small, the resistance to the flow then arises mainly from viscous drag. Eq.1 can be represent as:-

$$u = k\frac{(-\Delta p)}{l} = \frac{B}{\mu}\left[\frac{(-\Delta p)}{l}\right] \qquad (2)$$

μ:- viscosity of fluid.

B:- permeability coefficient for the bed, depends only on bed properties, and its apply only for laminar regions and some values of it for various packing are shown in table 4.1 p.127 Vol. 2



Specific Surface & Voidage (porosity):-

Specific surface area (S_B) is the area presented to the fluid per unit volume of bed when the particles are packed in a bed. Its units are length^{-1}

$$S_B = \frac{\text{surface area presented to the fluid}}{\text{volume of bed}}$$

The void fraction, ε, (porosity) in a packed bed is defined as the volume of the bed not occupied by solid particles. It is a dimensionless. Thus the fractional volume of the bed occupied by solid is (1- ε).

$$\varepsilon = \frac{total\ \ volume\ of\ column - volume\ \ of\ particles}{ttotal\ \ volume\ of\ column\ (voids\ plus\ solids)}$$

The specific surface of a particle (S) is the surface area of a particl divided by its volume. Its unit are length^{-1}. :

$$S = \frac{\text{surface area of particle}}{\text{volume of particle}} = \frac{S_p}{V_p}$$

Where: S_p is the surface area of a particle and V_p its volume. For a **spherical particle**: $S_p = \pi d^2$, $V_p = \frac{\pi}{6} d^3$, then $S = \frac{6}{d}$ (3)

For a packed bed of non-spherical particles, the effective particle diameter **d** is:

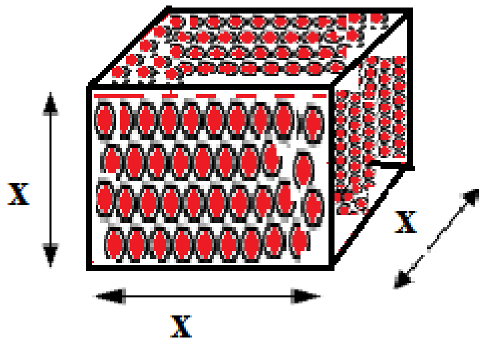
$$d = \frac{6}{S}$$

The volume fraction of the particles in the bed is (1- ε) and, thus, **the ratio of the total surface area in the bed to total volume of the bed**, S_B:

$$S_B = S(1-\varepsilon) = \frac{6}{d}(1-\varepsilon) \qquad (4)$$

Some value of S, ε for different beds of particles are listed in Table 4.1. Vol. 2. P 127

- **volume of free space =ε x²**
- **total volume of packed column = x³**
- **length of channel = x**



Mean cross-sectional area of flow is:-

$$\frac{\text{volume of free space}}{\text{length of channel}} = \frac{\varepsilon\, x^3}{x} = \varepsilon\, x^2$$

Volume flow rate through this cube without particles = $u\,.x^2$

Volume flow rate through this cube with particles = $u_1\,.\varepsilon.x^2$

$u\,.x^2 = u_1\,.\varepsilon.x^2$

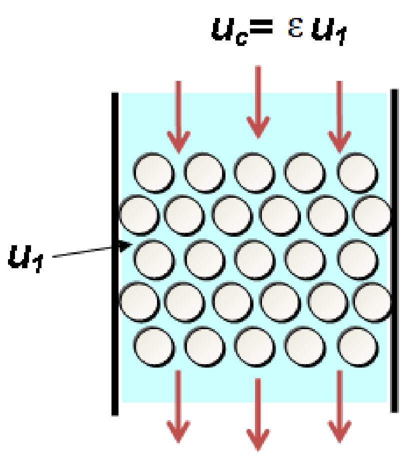
$$u_1 = \frac{u\,x^2}{u_1\,x^2\varepsilon} = \frac{u}{\varepsilon} \qquad (7)$$

The equivalent diameter for the flow in pores d'_m is modified to be:

$$d'_m = \frac{\textit{Cross sectional area available for flow}}{\textit{Wetted perimeter}}$$

$$= \frac{\text{Void volume available for flow}}{\text{Total wetted surface of solids}}$$

$$= \frac{\text{Volume of voids / Volume of bed}}{\text{Wetted of surface/ Volume of bed}}$$



Or

$$d'_m = \frac{\varepsilon}{S_B} = \frac{\varepsilon}{(1-\varepsilon)} \qquad (8)$$

Combining equations 5 and 8 gives

$S_B = S(1-\varepsilon) = \frac{6}{d}(1-\varepsilon)$ and $\quad d'_m = \frac{\varepsilon}{S_B}$:

$$d'_m = \frac{\varepsilon}{6(1-\varepsilon)} d$$

Since the hydraulic mean diameter for a channel (d_h) is:-

$$d_h = 4 \times \frac{Cross\ sectional\ area}{Wetted\ \ perimeter}$$

It is then seen that: $\quad d'_m = \frac{\varepsilon}{S_B} = \frac{1}{4} d_h$

For laminar flow, using $u_1 = \frac{u_c}{\varepsilon}$, and $l' \propto l$, the superficial velocity in the packed bed can be related to the pressure gradient as:

$$u_C = \frac{1}{K''} \frac{\varepsilon^3}{S_B^2} \frac{1}{\mu} \frac{(-\Delta p)}{l} \qquad (9)$$

And K'' = 5 = Kozeny constant

Comparison with Eq.2 shows that B, the permeability coefficient is given by:-

$$B = \frac{1}{K''} \frac{\varepsilon^3}{S_B^2} \frac{1}{(1-\varepsilon)} \qquad (10)$$

Using a value of 5 for K'' and 6/d for S, we get

$$u_c = 0.0055 \frac{e^3}{(1-e)^2} \frac{d^2}{\mu} \frac{(-\Delta p)}{l} \qquad (10a)$$

Laminar and Turbulent Flow:-

Equation (9) applies to laminar flow condition, but Carman extended the analogy with pipe flow to cover both laminar and turbulent flow conditions through packed beds. In this treatment a modified Reynolds No.(Re1) is plotted against a modified factor ($\frac{R_1}{\rho u_1^2}$) for flow through a pipe.

Then Re1 is obtain by:-

$$Re_1 = \frac{d'_m u_1 \rho}{\mu} = \frac{\varepsilon}{S(1-\varepsilon)} \frac{u_c}{\varepsilon} \frac{\rho}{\mu}$$

Or $$Re_1 = \frac{u_c \rho}{S(1-\varepsilon)\mu} \qquad (11)$$

The friction factor, which is plotted against Re_1 is $\left(\frac{R_1}{\rho u_1^2}\right)$, where R1 is the component of the drag force per unit area of particle surface in the direction of motion. R_1 can be related to the properties of the bed & pressure gradient(Δp) as follows:-

Consider:--
- Cross-sectional area of the bed= A=1.0
- Thickness of the bed = l
- Volume of particles in the bed= l. (1-ε)A= l. (1-ε)
- Total surface of packing = S. l. (1-ε)
- Resistance force (frictional force) = $R_1.A$

F=R_1.S. l. (1-ε)

F=Δp.a ; a= ε .l/l= ε[free cross-sectional area of fluid is volume/height]

F=Δp. ε

Force balance:-

(-Δp). ε = R_1.S. l. (1-ε)

$$R1 = \frac{\varepsilon}{(1-\varepsilon)} \cdot \frac{(-\Delta p)}{l} \qquad (12) \qquad / \rho u_1^2; \ and \ \ u_1 = u/\varepsilon$$

$$\frac{R1}{\rho u_1^2} = \frac{\varepsilon^3}{S.(1-\varepsilon)} \cdot \frac{(-\Delta p)}{l} \cdot \frac{1}{\rho u_1^2} \qquad (13)$$

Carmen found the relation between Re_1 and $\left(\frac{R_1}{\rho u_1^2}\right)$ for the flow through randomly packed bed of solids particles:-

The modified friction factor was plotted versus the modifier Reynolds number as shown in the figure below:

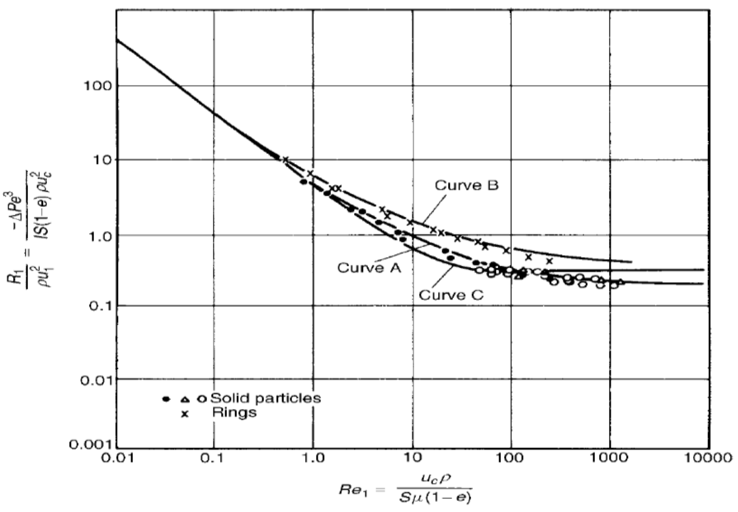


Figure 4.1. Carman's graph of $R_1/\rho u_1^2$ against Re_1

The use of the curves in the graph depends on the type of the packing as follows:

Curve A is used for the flow through randomly packed beds of solid particles.
Curve fitting gives the following relation for curve A:

$$\frac{\tau_1}{\rho u_1^2} = 5\,\mathrm{Re}_1^{-1} + 0.4\,\mathrm{Re}_1^{-0.1} \qquad (14)$$

For low value of Reynolds number, $\mathrm{Re}_1 < 2$, the above equation can be used as:

$$\frac{\tau_1}{\rho u_1^2} = 5\,\mathrm{Re}_1^{-1} \qquad (15)$$ (The second term is negligible)

This equation can be obtained from Carman Kozeny equation when using the value of K'' equals to 5

Curve B is used for flow of fluids through beds of Raschig rings and hollow packings. Curve fitting gives the following relation for curve B:

$$\frac{\tau_1}{\rho u_1^2} = 5\,\mathrm{Re}_1^{-1} + \mathrm{Re}_1^{-0.1} \qquad (16)$$

Curve C is fitted by a semi-empirical relation proposed by Ergun and used for flow of fluids through beds of solid particles.

$$\frac{-\Delta p}{l} = 150\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu u_c}{d^2} + 1.75\frac{(1-\varepsilon)}{\varepsilon^3}\frac{\rho u_c^2}{d} \qquad (17)$$

Writing $\mathrm{d} = \frac{6}{\mathrm{s}}$, we get:

$$\frac{-\Delta p}{Sl\rho u_c^2}\frac{\varepsilon^3}{(1-\varepsilon)} = 4.17\frac{\mu S(1-\varepsilon)}{\rho u_c} + 0.29 \qquad (18)$$

Or $\frac{\tau_1}{\rho u_1^2} = 4.17\,\mathrm{Re}_1^{-1} + 0.29$ (19)

Ergun equation can be used for the range of $\mathrm{Re}_1/(1-\varepsilon)$ from 1 to 2000.

The change from completestreamline flow to complete turbulent flow is very gradual because flow conditions are not the same in all the pores. Thus, the flow starts to become turbulent in the largerpores, and subsequently in successively smaller pores as the value of Re_1 increases. It is probable that the flow never becomes completely turbulent since some of the passages may be so small that streamline conditions prevail even at high flow rates.

From the equations above it can be seen that the main factors affecting the pressure gradient in the packed beds are the specific surface area (S) and bed void fraction (ε). In addition, many other factors can affect the pressure gradient such as: particle size distribution, the particle shape, wall effect, and the nature of the bed support.Table 4.3 gives the design data for different particles.

Dependence of $\overline{\overline{K}}$ on Bed structure:-

(1)Tortuosity:-

Carman has shown that :-

$$K'' = \left(\frac{l'}{l}\right)^2 k_0 \qquad (20)$$

$\frac{l'}{l}$:- is tortuoisty and it is measure of the fluid path length through the bed compared to actual depth of the bed.

K_0:- factor depend on the shape of cross-section of channel through which fluid is passing. It is equal to 2 for circular pipe, 2.65 for laminar flow through rectangular.

Note:-

It is obvious that if k_0 is constant then $k^{\#\#}$ increased with increasing tortuosity, it is about 5 since change in tortuosity will be compensate by k_o changes.

The value of K'' was plotted against bed void fraction (ε) in Figure 4.2 for different types of packing.

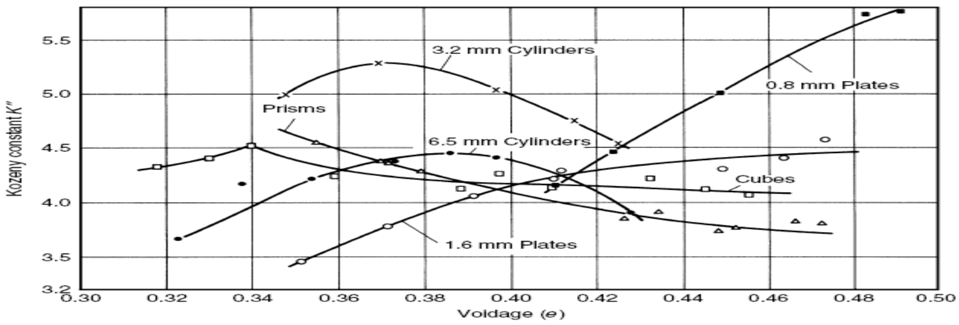


Figure 4.2. Variation of Kozeny's constant K'' with voidage for various shapes

(2) Wall Effects:-

In packed bed, particles will not packed as closely in the region near the wall as in the centre of bed, so that the actual resistance to flow in a bed of small diameter is less than in an infinite container for same flow rate per unit area of cross-section. Correction factor for this effect(f_w) has been determined by Coulson as: -

$$f_w = \left(1 + \frac{1}{2}\frac{S_c}{S}\right)^2 \qquad (21)$$

Where S_c is the surface of the container per unit volume of bed.

Then Carman-Kozeney equation becomes:

$$u_c = \frac{1}{K''}\frac{\varepsilon^3}{S^2(1-\varepsilon)^2}\frac{1}{\mu}\frac{(-\Delta p)}{l} f_w \qquad (22)$$

(3) Non Spherical Particles:-

$K^{\#\#}$ is about 3-6 according to shape and size with extreme value only occurring with thin plates.

(4) Spherical Particles:-

Equation (22) has been tested with spherical particles over the wide range of size and values of $K^{\#\#}$ is about 4.8 ∓ 0.3 .

(5) Beds with high porosity:-

Normal values of bed void fractions are in the range of 0.3 t0 0.6 and Kozeny remain active. For beds packed with fibers and some ring pickings, the values of void fractions reaches a value near unity. For these high values of void fractions, K'' rises rapidly and Kozeny will not work. Some values are given in Table 4.2.

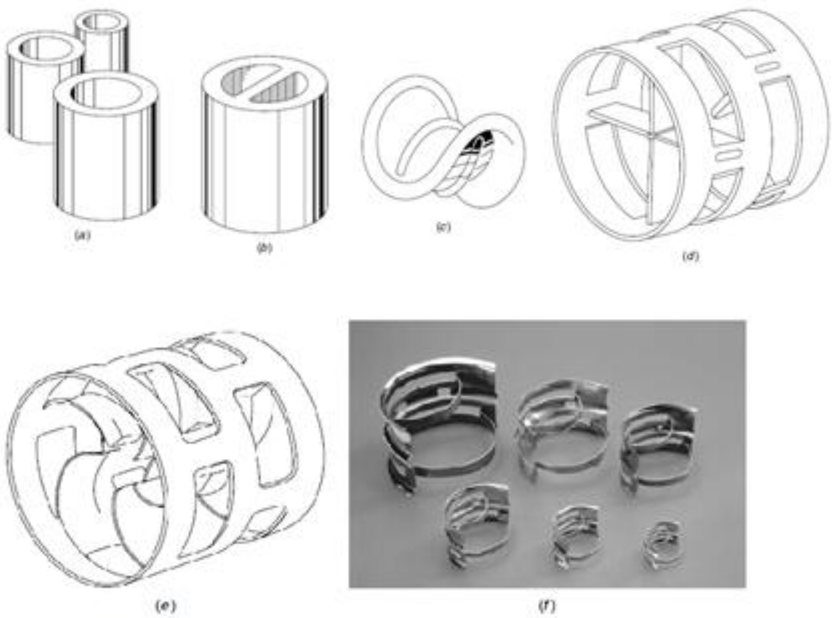


Figure 4.13. (a) Ceramic Raschig rings; (b) Ceramic Lessing ring; (c) Ceramic Berl saddle; (d) Pall ring (plastic); (e) Pall ring (metal); (f) Metal Nutter rings; (g) Plastic Nutter ring

Table 4.2. Experimental values of K'' for beds of high porosity

Voidage e	Experimental value of K'' BRINKMAN[3]	DAVIES[21]	Silk fibres LORD[20]
0.5	5.5		
0.6	4.3		
0.8	5.4	6.7	5.35
0.9	8.8	9.7	6.8
0.95	15.2	15.3	9.2
0.98	32.8	27.6	15.3

Example

Air (ρ = 1.22 kg/m^3, μ = 1.9 X 10^{-5} pa.s) is flowing in a fixed bed of a diameter 0.5 m and height 2.5 m. The bed is packed with spherical particles of diameter 10 mm. The void fraction is 0.38. The air mass flow rate is 0.5 kg/s. Calculate the pressure drop across the bed of particles.

Solution

Volumetric flow rate, $Q=\dfrac{0.5}{1.22}=0.41\,m^3/s$

$$A=\frac{\pi}{4}D^2=\frac{\pi}{4}(0.5)^2=0.1963\,m^2$$

$$u_c=\frac{Q}{A}=\frac{0.41}{0.1963}=2.1\,m/s$$

$$Re_1=\frac{u_c\rho}{S(1-e)\mu}=\frac{u_c\rho d}{6(1-e)\mu}=\frac{2.1\times1.22\times0.01}{6\times(1-0.38)\times1.9\times10^{-5}}=362$$

For solid particles curve A equation is applied

$$\frac{\tau_1}{\rho u_1^2}=5\,Re_1^{-1}+0.4\,Re_1^{-0.1}=\frac{5}{362}+\frac{0.4}{(362)^{0.1}}=0.0138+0.2219=0.2357$$

Since $\dfrac{\tau_1}{\rho u_1^2}=\dfrac{e^3}{S(1-e)}\dfrac{(-\Delta p)}{l}\dfrac{1}{\rho u_c^2}$

$$\therefore 0.2357=\frac{(0.38)^3}{(6/0.01)\times(1-0.38)}\frac{(-\Delta p)}{(2.5)}\frac{1}{1.22\times(1.2)^2}$$

$$\therefore(-\Delta p)=7017\,N/m$$

Example

Gas of density 1.25 kg/m^3 and dynamic viscosity (μ = 1.5 X 10^{-5} kg/m.sec) flows steadily through a bed of spherical particles 0.005 m diameter. The bed has height of 3 m and a voidage 0.333. the laminar approach velocity is 0.03 m/sec. Calculate Re_1, Δp in N/m^2 over the bed?

Solution

$$\mathrm{Re}_1 = \frac{u_c \rho}{S(1-e)\mu}$$

For spherical, S=6/d= 1200m^{-1}

$$\mathrm{Re}_1 = \frac{1.25 * 0.03 * 0.005}{6(1.5 * 10^{-5})(1-(1/3))} = 3.125$$

From Eq.9

$$\Delta p = K'' \frac{S^2(1-\varepsilon)^2}{\varepsilon^3} .\mu.l.u$$
$$= 5\left(\frac{6}{d}\right)^2 (1-\varepsilon)^2 \frac{1}{\varepsilon^3} .\mu.l.u = 180.\mu.l\left[\frac{(1-\varepsilon)^2}{\varepsilon^3}\right]u$$
$$= 116.6\ N/m^2$$

From Eq.(13)

$$\frac{\mathrm{R1}}{\rho \mathrm{u}_1^2} = \frac{\varepsilon^3}{\mathrm{S.}(1-\varepsilon)} . \frac{(-\Delta p)}{l} . \frac{1}{\rho \mathrm{u}_1^2} \quad \Rightarrow\Rightarrow \Delta p = \left(\frac{\mathrm{R1}}{\rho \mathrm{u}_1^2}\right)\left(\frac{\mathrm{S.}(1-\varepsilon)}{\varepsilon^3}\right)\rho \mathrm{u}_1^2 .l$$

$$\frac{\tau_1}{\rho u_1^2} = 5\,\mathrm{Re}_1^{-1} + 0.4\,\mathrm{Re}_1^{-0.1}$$

$$\Delta p = 142.6\, N/m^2$$

By Ergun Eq. it will be found to be 118.42N/m^2

Real Packed Columns

In real packed columns which are used in many industrial applications, two main difficulties arise:

1. The size of the packing elements in the column is much larger and the Reynolds number is normally turbulent.
2. The packing have a large internal surface which will offer ahigher flow resistance than their external surface.

Fluid flow in packed columns

Pressure Drop

$$\frac{\Delta p}{l} = k\,u^{n} \qquad (*)$$

Same as Darcy Equation, where k,n are constant depends on packings.

Type of Packings	N		k	
	Furnace and Bellinges	Mach	Furnace	Mach
3/8 in rings	1.8	1.8	0.82	1.02
1 in. rings	1.9	1.88	0.27	0.22
1 in. saddle rings	1.9	1.83	0.125	0.16

When we used Eq. (*) , we must take average values of n,k:-

$$\left[n = \frac{n_{Furnace} + n_{Mach}}{2} \right]$$

Due to particles placed randomly , some of them broken, the values are different and we have to take the average values.

All these equations are used for one fluid, if more than one, the volume of packing increased due to liquid covered it , decreasing porosity and increasing pressure drop.

K values depends on flow rate, and Leva equation as:-

$$\Delta p = \alpha \rho_g u_g^2 (10)^{\beta L}$$

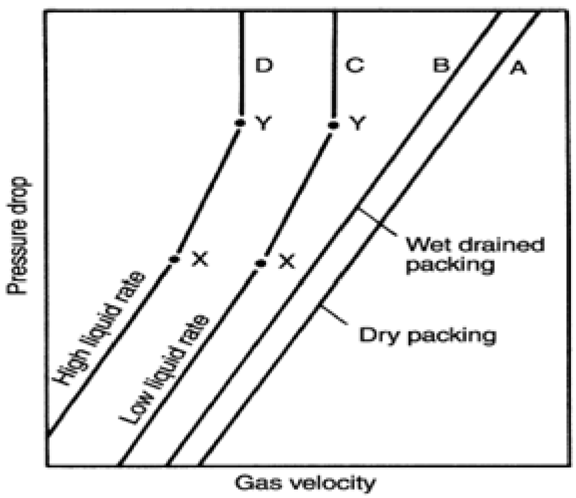
α , β :- constants depends on packings as follows:-

Type of Packings	α	β
3/4 in rings	0.82	0.72
1 in. rings	0.8	0.69
1 in. saddle rings	0.4	0.47

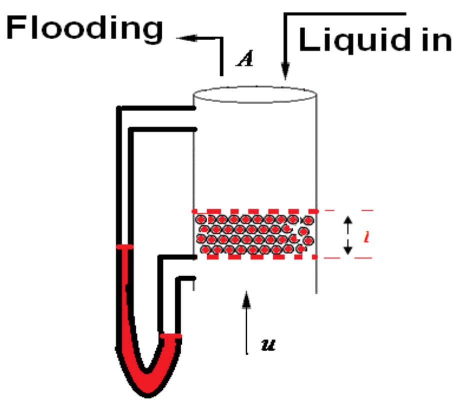
$$\Delta p = \alpha (10)^{\beta L} \frac{G^2}{\rho_g}, \quad G = gas\ flowrate$$

Loading and Flooding:-

To find the pressure drop in packed column with two fluids; liquid and gas, same general form of approach is usefully adopted for the flow of two fluids through packed columns. Fig. 4.16 shows the pressure drop in packed column.



Pressure drops in wet packings (logarithmic axes)



- The gas flow is mostly turbulent and the general form of the relation between the drop in pressure −ΔP and the volumetric gas flow rate per unit area of column u_G is shown on curve A of Figure 4.16 and $\Delta P \propto u_G^{1.8}$.

- If, liquid flows down the tower, the passage of the gas is not significantly affected at low liquid rates and the pressure drop line is similar to line A.
- When the gas rate reaches a certain value, the pressure drop then rises very much more quickly and is $\Delta P \propto u_G^{2.5}$ (XY on curve C).
- At gas flows beyond Y, $-\Delta P$ rises very steeply and the liquid is held up in the column.
- The point X is known as the loading point, and point Y as the flooding point for the given liquid flow.
- If the flow rate of liquid is increased, a similar plot D is obtained in which the loading point is achieved at a lower gas rate though at a similar value of $-\Delta P$.
- It is not practicable to operate under flooding conditions, and columns are best operated over the section XY.

For wet drained column packed with Raschig rings:

$$-\Delta P_w = -\Delta P_d\left(1+\frac{3.3}{dn}\right)$$

Where:

$-\Delta P_w$: is the pressure drop across the wet drained column

$-\Delta P_d$: is the pressure drop across the dry column

dn: is the nominal size of the Raschig rings in mm.

Example 3

A column is packed with 2 in.(50 mm) ceramic Rasching. The tower is to be operate with 15000 lb/hr.ft^2(20.3 kg/m^2.sec)of water and 1000 lb/hr.ft^2(1.35 kg/m^2.sec) of air. Calculate the pressure drop through the packing, density of air is 0.075 lb/ ft^3. α , β is 0.24, 0.14 respectively.

Solution:-

$$\Delta p = \alpha \, \rho_g u_g^2 (10)^{\beta L} = \alpha (10)^{\beta L} \frac{G^2}{\rho_g}$$

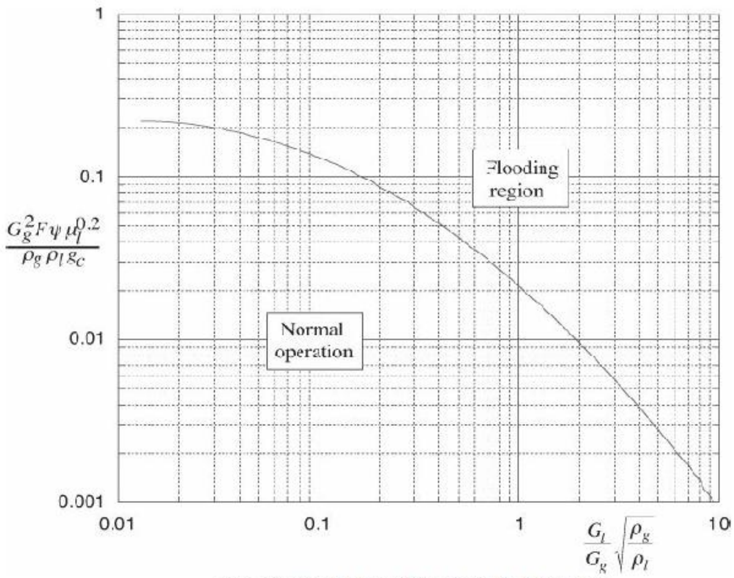
ρ_g = 0.075 lb/ft^3

L= 15000/3600= 4.16 lb/ft^2.sec

G= 1000/3600= 0.278 lb/ft^2.sec

$$\Delta p = 0.24 (10)^{0.14*4.16} \frac{0.278^2}{0.075} = 0.95 \, in / \, ft = 79 mm / m$$

Sherwood et.al. found a correlation for flooding rates for random packing as shown in figure below:-



Correlation for flooding in packed columns.

Where:-

u_g:- gas velocity at flooding point.

S_B:- S(1-ε)

g:- Acceleration due to gravity.

L,G:- Mass rate of flow of liquid and gas, kg/m².sec

μ_w:-viscosity of water at 293 k= 1.0 mN.sec/m²

ρ_g, ρ_L:- gas , and liquid density.

Area inside curve represent possible conditions of operations. In the above expressions, ratio $\frac{\rho_g}{\rho_L}$ & $\frac{\mu_L}{\mu_w}$ have been introduced so that the relationship can be applied for a wide range of liquid and gasses.

$$y \Rightarrow ug \Rightarrow u = 0.7g$$

$$G' = \rho.g.u.A^* = \rho'.g.A^*(0.7u'g), A^* = \frac{\pi}{4}d^2, \ d = column \ diameter.$$

Bain & Hongen found that:-

$$\log\left[\frac{u_g^2 S_B}{g\,\varepsilon^3}\left(\frac{\rho_g}{\rho_L}\right)\left(\frac{\mu_L}{\mu_w}\right)^{0.16}\right] = b - 1.75\left(\frac{L}{G}\right)^{0.25}\left(\frac{\rho_g}{\rho_L}\right)^{1/8}$$

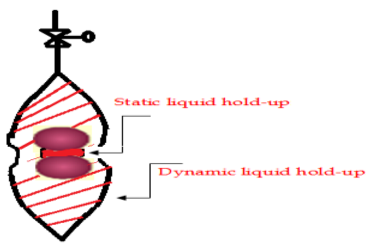
b= 0.022 For Rasching rings, b =0.26 saddle packing.

Velocity of Loading & Flooding could be find:-

Packing	Loading	Flooding
½ in. Saddle	$G = 915\left(1 - \frac{0.042L^{0.54}}{10.65}\right)$	$G = 1268\left(1 - \frac{0.042L^{0.54}}{14.59}\right)$
½ in. Rings	$G = 610\left(1 - \frac{0.031L^{0.631}}{15.05}\right)$	$G = 1019\left(1 - \frac{0.031L^{0.631}}{14.59}\right)$
1 in. Rings	$G = 1620\left(1 - \frac{0.025L^{0.631}}{13.61}\right)$	$G = 1010\left(1 - \frac{0.027L^{0.631}}{27}\right)$

Liquid Hold-up on Packing:-

It is important to identify liquid amounts remaining on particles and supports, which needed to knowing weight of

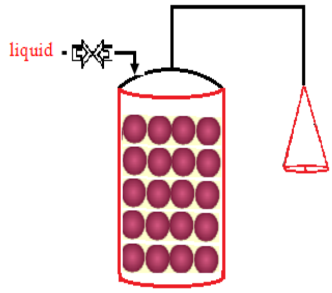


column(liquid and particles) & velocity of wins in the area of construction.

Liquid hold-up divided into two parts:-

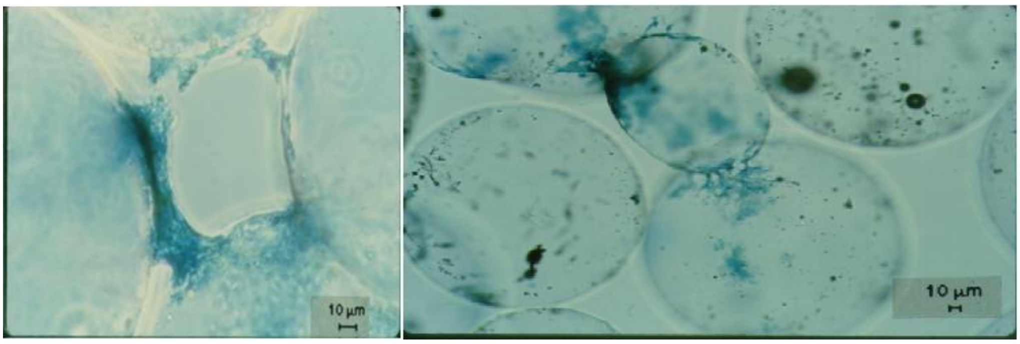
(1) Static Liquid hold-up.
(2) Dynamic Liquid hold-up.
(3) Total Liquid hold-up.

- Static liquid hold-up:- Liquid remain on particles after closing flow rate for five minutes(static). W/W or V/V.
  $h_s=aD^{-b}$
- Dynamic liquid hold-up:- Liquid remain on particles during flow rate. W/W or V/V.
- Total liquid hold-up:- Total of the static and dynamic liquid remain on particles.
  $h_t= c\ D^2\ L^d$ and $d=r^n$
  c,r,a,n,& b are constants



To find liquid hold-up:-

Weight of column+ weight of packing=W_1.
Weight of column+ weight of packing+ weight of liquid=W_2.
weight of liquid hold-up=W_2-W_1.
Weight of liquid static + column+ packing=W_3.
weight of static liquid hold-up=W_3-W_1.
weight of dynamic liquid hold-up= (W_2-W_1)-(W_3-W_1).



constants	Porcelain Ring	Carbon Ring	Porcelain Saddle
c	$2.25*10^{-5}$	$7.9*10^{-5}$	$2.5*10^{-5}$
r	0.965	0.76	0.965
a	0.00104	0.0025	0.00032
n	0.376	0.0475	0.376
b	1.21	1.21	1.56

If the flow rate with water, we can use Shulman Eq.:-

$$h_0 = \alpha L^{0.57} \mu^{\beta} \left(\frac{1}{\rho}\right)^{0.84} \left[\frac{\delta}{73}\right]^{\gamma - 0.262 \log L}$$

For computing h_s for all liquid.

$$h_s = c\,\mu^{m} \left(\frac{1}{\rho}\right)^{0.37} \delta^{n}$$

c,m,n, α, γ, and β are constant

Example 4

A packed bed of solids of density 2000 kg/m^3 occupies a depth of 0.6 m in a cylindrical vessel of inside diameter 0.1 m. The mass of solids in the bed is 5 kg and the mean diameter of the particles is 300 μm. Water (density 1000 kg/m^3 and viscosity 0.001 Pas) flows upwards through the bed.

a) What is the voidage of the packed bed?

b) Calculate the superficial liquid velocity at which the frictional pressure drop across the bed is 4130 Pa.

Solution

(a) Knowing the mass of particles in the bed, the density of the particles and the volume of the bed, the bed voidage may be calculated:

mass of bed, $M = AH(1-\varepsilon)\rho_p$

giving voidage, $\varepsilon = 1 - \dfrac{5}{2000 \times \frac{\pi}{4}(0.1)^2 \times 0.6} = 0.4695$

Use the Ergun equation (Text-Equation 6.15) to estimate the relationship between pressure drop across the bed and superficial liquid velocity:

$$\frac{(-\Delta p)}{H} = 150\frac{\mu U}{x_{sv}^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + 1.75\frac{\rho_f U^2}{x_{sv}}\frac{(1-\varepsilon)}{\varepsilon^3}$$

With $\mu = 0.001$ Pa.s, $\rho_f = 1000$ kg/m^3, $x_{sv} = 300$ μm and $H = 0.6$ m,

$(-\Delta p) = 2.72 \times 10^6 \times U^2 + 17.9 \times 10^6 U$

With $(-\Delta p) = 4130$ Pa, we solve the quadratic for U:
$U = 1.5 \times 10^{-3}$ m/s, i.e. 1.5 mm/s (positive root)

Example 5

A packed bed of solid particles of density 2500 kg/m^3, occupies a depth of 1 m in a vessel of cross-sectional area 0.04 m^2. The mass of solids in the bed is 50 kg and the mean diameter of the particles is 1 mm. A liquid of density 800 kg/m^3 and viscosity 0.002 Pas flows upwards through the bed, which is restrained at its upper surface.

(a) Calculate the voidage (volume fraction occupied by voids) of the bed.

(b) Calculate the pressure drop across the bed when the volume flow rate of liquid is 1.44 m^3/h.

Check by the value of Reynolds number to make sure that you use the suitable equation in your calculation.

SOLUTION

(a) Knowing the mass of particles in the bed, the density of the particles and the volume of the bed, the bed voidage may be calculated:

mass of bed, M = $M = AH(1-\varepsilon)\rho_p$

giving voidage, $\varepsilon = 1 - \dfrac{50}{2500 \times 0.04 \times 1} = 0.5$

(b) With a liquid flow rate of 1.44 m^3/h, the superficial liquid velocity through the bed, U is given by:

$$U = \frac{1.44}{3600 \times A} = 0.01\ m/s$$

Use the Ergun equation (Text-Equation 6.15) to estimate the pressure drop across the bed at this flow rate:

$$\frac{(-\Delta p)}{H} = 150\frac{\mu U}{x_{sv}^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + 1.75\frac{\rho_f U^2}{x_{sv}}\frac{(1-\varepsilon)}{\varepsilon^3}$$

With $\mu = 0.002$ Pa.s, $\rho_f = 800$ kg/m^3, $x_{sv} = 1$ mm and H = 1 m,

$$(-\Delta p) = 600 \times 10^3 U + 5.6 \times 10^6 U^2 = 6560 \text{ Pa}$$

Checking the Reynolds number, $Re' = \dfrac{U\rho_f x_{sv}}{\mu(1-\varepsilon)} = 8$ (Text-Equation 6.12). Since the Reynolds number is less than 10, we might estimate the pressure drop using the Carman Kozeny equation (Text-Equation 6.16):

$$\frac{(-\Delta p)}{H} = 180\frac{\mu U}{x_{sv}^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} = 7200 \text{ Pa.}$$

Alternatively we could use the laminar part of the Ergun equation, which gives, (-Δp) = 6000 Pa.

Example 6

A gas of density ρ = 1.25 kg/m^3 and dynamic viscosity μ= 1.5 X 10^{-5} Pa s flows steadily through a bed of spherical particles of diameter *d* = 0.005 m. The bed has a height of 3.0 m and a voidage of (1/3). The superficial velocity *u* = 0.03 m/s. Calculate the Reynolds number and the frictional pressure drop over the bed.

Solution:

Calculations

$$\text{Reynolds number } Re'_b = \frac{\rho u d_p}{6\mu(1-\varepsilon)}$$

Substituting the given values

$$Re'_b = \frac{(1.25 \text{ kg/m}^3)(0.03 \text{ m/s})(0.005 \text{ m})(3)}{(6)(1.50 \times 10^{-5} \text{ Pa s})(2)}$$

$$= 3.125$$

The frictional pressure drop is given by

$$\Delta P_f = (180\mu L)\left[\frac{(1-\varepsilon)^2}{\varepsilon^3 d_p^2}\right] u$$

Given that

$$(1-\varepsilon)^2 = \tfrac{4}{9}$$

$$\frac{(1-\varepsilon)^2}{\varepsilon^3} = 12$$

$$d_p^2 = 2.5 \times 10^{-5} \text{ m}^2$$

$$u = 0.03 \text{ m/s}$$

$$\mu = 1.5 \times 10^{-5} \text{ Pa s}$$

$$L = 3.0 \text{ m}$$

it follows that

$$\Delta P_f = (180)(1.5 \times 10^{-5} \text{ Pa s}) \frac{(3.0 \text{ m})(12)(0.03 \text{ m/s})}{(2.5 \times 10^{-5} \text{ m}^2)}$$

$$= \underline{116.6 \text{ Pa}}$$