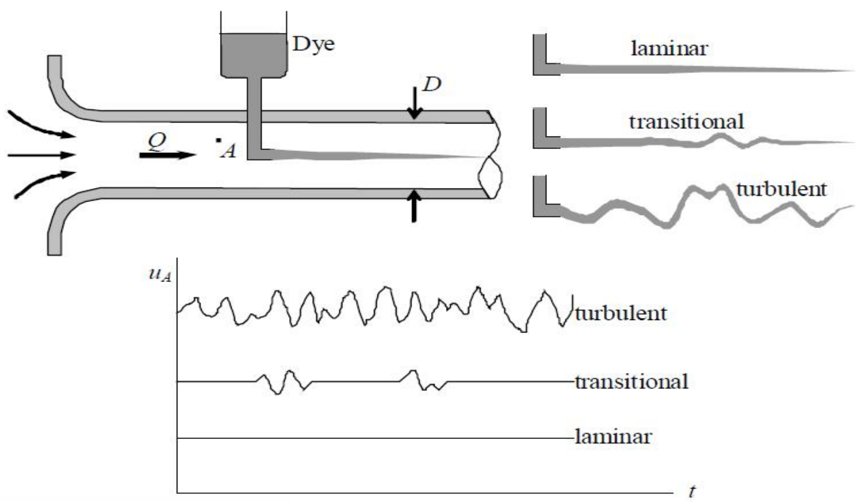
CHPTER FOUR

Fluid Dynamic

4.1 Introduction

In the process industries it is often necessary to pump fluids over long distances from storage to processing units, and there may be a substantial drop in pressure in both the pipeline and in individual units themselves. It is necessary, therefore, to consider the problems concerned with calculating the power requirements for pumping, with designing the most suitable flow system, with estimating the most economical sizes of pipes, with measuring the rate of flow, and frequently with controlling this flow at steady state rate.

It must be realized that when a fluid is flowing over a surface or through a pipe, the velocity at various points in a plane at right angles to the stream velocity is rarely uniform, and the rate change of velocity with distance from the surface will exert a vital influence on the resistance to flow and the rate of mass or heat transfer.



4.2 The Nature of Fluid Flow

When a fluid is flowing through a tube or over a surface, the pattern of flow will vary with the velocity, the physical properties of fluid, and the geometry of the surface. This problem was first examined by Reynolds in 1883. Reynolds has shown that when the velocity of the fluid is slow, the flow pattern is smooth. However, when the velocity is quite high, an unstable pattern is observed in which eddies or small packets of fluid particles are present moving in all directions and at all angles to the normal line of flow.

The first type of flow at low velocities where the layers of fluid seen to slide by one another without eddies or swirls being present is called "laminar flow" and Newton's law of viscosity holds.

The second type of flow at higher velocities where eddies are present giving the fluid a fluctuating nature is called "turbulent flow".

4.3 Reynolds Number (Re)

Studies have shown that the transition from laminar to turbulent flow in tubes is not only a function of velocity but also of density (ρ), dynamic viscosity (μ), and the diameter of tube. These variables are combining into the Reynolds number, which is dimensionless group.

$$Re = \frac{\rho u d}{\mu}$$

where u is the average velocity of fluid, which is defined as the volumetric flow rate divided by the cross-sectional area of the pipe.

$$u = \frac{Q}{A} = \frac{Q}{(\pi/4)d^2} \Rightarrow Re = \frac{4Q\rho}{\pi d\mu} = \frac{4\dot{m}}{\pi d\mu} = \frac{Gd}{\mu}$$

Where,

Q: volumetric flow rate m^3/s

m: mass flow rate kg/s &

G: mass flux or mass velocity $kg/m^2.s$

for a straight circular pipe when the value of Re is less than 2,100 the flow is always laminar. When the value is over 4,000 the flow be turbulent. In between, which is called the transition region the flow can be laminar or turbulent depending upon the apparatus details.

Example -4.1-

Water at 303 K is flowing at the rate of 10 gal/min in a pipe having an inside diameter I.D. of 2.067 in. calculate the Reynolds number using both English and S.I. units

Solution:

The volumetric flow rate (Q) = 10 gal/min (1.0 ft^3/7.481 gal) (min/60 s) = 0.0223 ft^3/s

Pipe diameter (d) = 2.067 in (ft/12 in) = 0.172 ft

Cross-sectional area (A) = π/4 d^2 = π/4 $(0.172)^2$ = 0.0233 ft^2

Average velocity (u) =Q/A = (0.0223 ft^3/s) / 0.0233 ft2 = 0.957 ft/s

At T = 303 K The density of water (ρ = 62.18 lb/ft^3),

The dynamic viscosity (μ = 5.38 x 10^{-4}) lb/ft.s

$$Re = \frac{\rho u d}{\mu} = \frac{62.18\ lb/ft^3 (0.957\ ft/s)(0.172\ ft)}{5.38 \times 10^{-4}\ lb/ft.s} = 1.902 \times 10^4 \quad (turbulent)$$

Using S.I. units

At T = 303 K The density of water (ρ = 996 kg/m^3),

The dynamic viscosity (μ = 8.007 x 10^{-4}) kg/m.s (or Pa.s)

Pipe diameter (d) = 0.172 ft (m/3.28 ft) = 0.0525m

Average velocity (u) =0.957 ft/s (m/3.28 ft) = 0.2917 m/s

$$Re = \frac{996\ kg/m^3 (0.2917\ m/s)(0.0525\ m)}{8.007 \times 10^{-4}\ kg/m.s} = 1.905 \times 10^4 \quad (turbulent)$$

4.4 Overall Mass Balance and Continuity Equation

In fluid dynamics, fluids are in motion. Generally, they are moved from place to place by means of mechanical devices such as pumps or blowers, by gravity head, or by pressure, and flow through systems of piping and/or process equipment.

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The first step in the solution of flow problems is generally to apply the principles of the conservation of mass to the whole system or any part of the system.

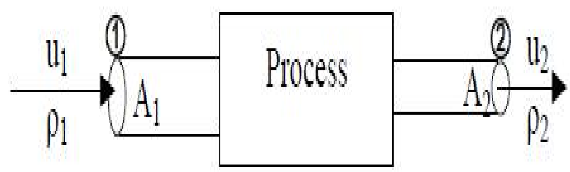
INPUT – OUTPUT = ACCUMULATION

At steady state, the rate of accumulation is zero

∴ INPUT = OUTPUT

In the following Figure a simple flow system is shown where fluid enters section (1) with an average velocity (u1) and density (ρ1) through the cross-sectional area (A1). The fluid leaves section(2) with an average velocity (u2) and density (ρ1) through the cross-sectional area (A2).

Thus, At steady state



$$\dot{m}_1 = \dot{m}_2$$
$$Q_1\,\rho_1 = Q_2\,\rho_2$$
$$u_1 A_1\,\rho_1 = u_2 A_2\,\rho_2$$

For incompressible fluids at the same temperature [$\rho_1 = \rho_2$]

$$\therefore u_1 A_1 = u_2 A_2$$

Example -4.2- **************

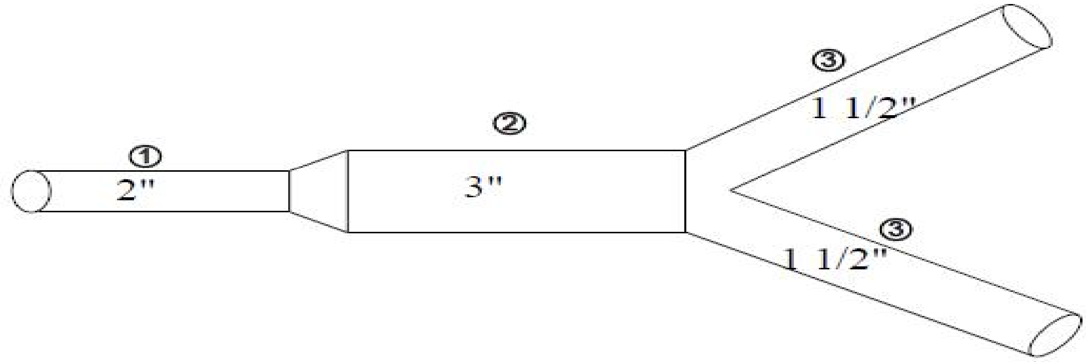
A petroleum crude oil having a density of 892 kg/m3 is flowing, through the piping arrangement shown in the below Figure, at total rate of 1.388 x 10^{-3} m^3/s entering pipe❶. The flow divides equally in each of pipes ❸. The steel pipes are schedule 40 pipe.

Calculate the following using SI units:

The total mass flow rate in pipe ❶ and pipes ❸.

The average velocity in pipe ❶ and pipes ❸.

The mass velocity in pipe ❶ .



Pipe ❶ I.D. = 0.0525 m, A1 = 21.65 x 10^{-4} m^2

Pipe ❷ I.D. = 0.07792 m, A2 = 47.69 x 10^{-4} m^2

Pipe ❸ I.D. = 0.04089 m, A3= 13.13 x 10^{-4} m^2

The total mass flow rate is the same through pipes ❶ and ❷ and is:-

$$\dot{m}_1 = Q_1\,\rho_1 = 1.388 \times 10^{-3}\,m^3/s\,(892\,kg/m^3) = 1.238\,kg/s$$

Since the flow divides equally in each pipes ❸

$$\dot{m}_3 = \frac{\dot{m}_1}{2} = (1.238/2) = 0.619\ kg/s$$

$$\dot{m}_1 = Q_1\,\rho = u_1 A_1 \rho \Rightarrow u_1 = \frac{\dot{m}_1}{A_1\rho} = \frac{1.238\,kg/s}{(21.65\,10^{-4} m^2)(892\,kg/m^3)} = 0.641\,\mathrm{m/s}$$

$$u_3 = \frac{\dot{m}_3}{A_3\rho} = \frac{0.619\,kg/s}{(13.13\,10^{-4} m^2)(892\,kg/m^3)} = 0.528\,\mathrm{m/s}$$

$$G_1 = u_1\,\rho_1 = 0.641\,\mathrm{m/s} * 892\,kg/m^3 = 572\ kg/m^2.s$$

$$or\ \ G = \frac{\dot{m}_1}{A_1} = \frac{1.238\,kg/s}{(21.65\,10^{-4} m^2)} = 572\ kg/m^2.s$$

4.5 Energy Relationships and Bernoulli’s Equation

The total energy of a fluid in motion consists of the following components: -

Internal Energy (U)

This is the energy associated with the physical state of fluid, i.e. the energy of atoms and molecules resulting from their motion and configuration. Internal energy is a function of temperature. It can be written as (U) energy per unit mass of fluid.

Potential Energy (PE)

This is the energy that a fluid has because of its position in the earth's field of gravity. The work required to raise a unit mass of fluid to a height (z) above a datum line is (zg), where (g) is gravitational acceleration. This work is equal to the potential energy per unit mass of fluid above the datum line.

Kinetic Energy (KE)

This is the energy associated with the physical state of fluid motion. The kinetic energy of unit mass of the fluid is ($u^2/2$), where (u) is the linear velocity of the fluid relative to some fixed body.

Pressure Energy (Prss.E)

This is the energy or work required to introduce the fluid into the system without a change in volume. If (P) is the pressure and (V) is the volume of a mass (m) of fluid, then ($PV/m \equiv P\upsilon$) is the pressure energy per unit mass of fluid. The ratio (V/m) is the fluid density (ρ).

The total energy (E) per unit mass of fluid is given by the equation: -

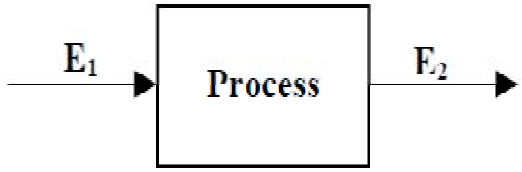
$$E = U + zg + P/\rho + u^2/2$$

where, each term has the dimension of force times distance per unit mass. In calculation, each term in the equation must be expressed in the same units, such as J/kg, Btu/lb or lbf.ft/lb. i.e.

$$(MLT^{-2})(L)(M^{-1}) = [L^2T^{-2}] \equiv \{m^2/s^2,\ ft^2/s^2\}.$$

A flowing fluid is required to do work in order to overcome viscous frictional forces that resist the flow.

The principle of the conservation of energy will be applied to a process of input and output streams for ideal fluid of constant density and without any pump present and no change in temperature.



$$E_1 = E_2$$

$$U_1 + z_1 g + P_1/\rho + u_1^2/2 = U_2 + z_2 g + P_2/\rho + u_2^2/2$$

$$U_1 = U_2 (no\ change\ in\ temperature)$$

$$\Rightarrow \mathrm{P}/\rho + \mathrm{u}^2/2 + \mathrm{zg} = \mathrm{constant}$$

$$P_1/\rho + u_1^2/2 + z_1 g = P_2/\rho + u_2^2/2 + z_2 g$$

$$\Rightarrow \Delta\mathrm{P}/\rho + \Delta\,\mathrm{u}^2/2 + \mathrm{zg} = 0 \qquad \text{(Bernollis equation)}$$

4.6 Equations of Motion

According to Newton's second law of motion, *the net force in x-direction (Fx) acting on a fluid element in x-direction is:* -

Fx = (mass) x (acceleration in x-direction)

Fx = (m) (ax)

In the fluid flow the following forces are present: -

1- F_g ---------force due to gravity

2- F_P ---------force due to pressure

3- F_V ---------force due to viscosity

4- F_t ---------force due to turbulence

5- F_c ---------force due to compressibility

6- F_σ ---------force due to surface tension

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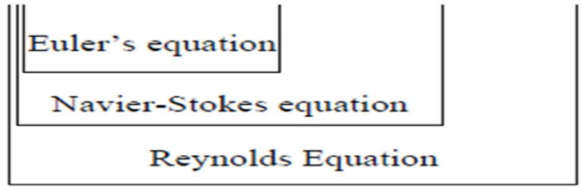
The net force is could be given by:-

$$Fx = (F_g)x + (F_P)x + (F_V)x + (F_t)x + (F_c)x + (F_\sigma)x$$



In most of the problems of fluid in motion the forces due to surface tension (Fσ), and the force due to compressibility (Fc) are neglected,

$$\Rightarrow Fx = (F_g)x + (F_P)x + (F_V)x + (F_t)x$$

This equation is called "Reynolds equation of motion" which is useful in the analysis of turbulent flow.

In laminar (viscous) flow, the turbulent force becomes insignificant and hence the equation of motion may be written as: -

$$Fx = (F_g)x + (F_P)x + (F_V)x$$

This equation is called "Navier-Stokes equation of motion" which is useful in the analysis of viscous flow.
If the flowing fluid is ideal and has very small viscosity, the viscous force and viscosity being almost insignificant and the equation will be: -

$$Fx = (F_g)x + (F_P)x$$

This equation is called "Euler's equation of motion".

4.6.1 <u>Euler's equation of motion</u>

The Euler's equation for steady state flow on an ideal fluid along a streamline is based on the Newton's second law of motion. The integration of the equation gives Bernoulli's equation in the form of energy per unit mass of the flowing fluid.

Consider a steady flow of an ideal fluid along a streamline. Now consider a small element of the flowing fluid as shown below,

Let:

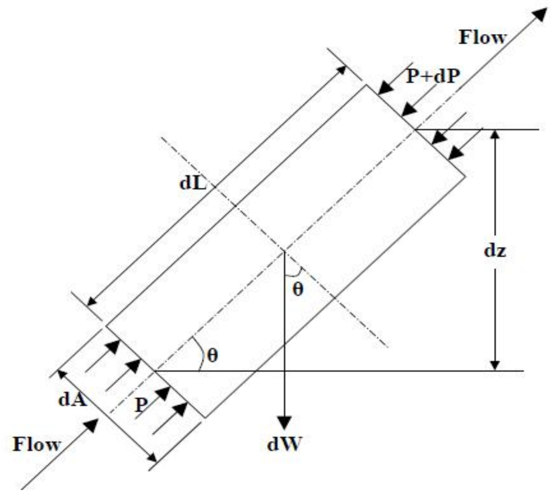
dA: cross-sectional area of the fluid element,

dL: Length of the fluid element'

dW: Weight of the fluid element'

u: Velocity of the fluid element'

P: Pressure of the fluid element'



The Euler's equation of motion is based

on the following assumption: -

1- The fluid is non-viscous (the frictional losses are zero).

2- The fluid is homogenous and Incompressible (the density of fluid

is constant).

3- The flow is continuous, steady, and along the streamline (laminar).

4- The velocity of flow is uniform over the section.

5- No energy or force except <u>gravity and pressure forces is involved in the flow.</u>

The forces on the cylindrical fluid element are,

1- Pressure force acting on the direction of flow (PdA)

2- Pressure force acting on the opposite direction of flow [(P+dP)dA]

3- A component of gravity force acting on the opposite direction of flow ($d_W \sin\theta$)

-The pressure force in the direction of flow :-

$$F_p = PdA - (\mathrm{P} + \mathrm{dP})\mathrm{dA} = -\mathrm{dPdA}$$

-The gravity force in the direction of flow :-

$$F_g = -dW \sin\theta \qquad \{W = m\,g = \rho dAdL\,g\}$$

$$= -\rho\,g\,dA\,dL\,\sin\theta \qquad \{\sin\theta = dz/dL\}$$

$$= -\rho\,g\,dA\,dz$$

-The net force in the direction of flow :-

$$F_x = m\,a \qquad \{m = \rho dAdL\}$$

$$= \rho dA\,dL\,a \qquad \left\{a = \frac{du}{dt} = \frac{du}{dL} * \frac{dL}{dt} = u\frac{du}{dL}\right\}$$

$$= \rho dA\,u\,du$$

We have $F_x = \left(F_g\right)_x + \left(F_p\right)_x$

$$\Rightarrow \rho dA\,u\,du = -\mathrm{dPdA} - \rho\,g\,dA\,dz \quad \{\div -\rho dA\}$$

$$\frac{dP}{\rho} + \frac{du^2}{2} + dz\,g = 0 \text{ ... Euler's equation of motion}$$

Bernoulli's equation could be obtain by integration the Euler's equation :-

$$\int \mathbf{dP}/\boldsymbol{\rho} + \int \mathbf{du^2/2} + \int \mathbf{dz\,g} = \mathbf{constant}$$

$$\Rightarrow \mathrm{P}/\boldsymbol{\rho} + \mathrm{u}^2/2 + \mathrm{zg} = \mathrm{constant}$$

$$\Rightarrow \Delta\mathrm{P}/\boldsymbol{\rho} + \Delta\,\mathrm{u}^2/2 + \Delta\mathrm{z}\,\mathrm{g} = 0 \text{ ... ... (Bernollis equation)}$$

4.7 Modification of Bernoulli's Equation

1- Correction of the kinetic energy term

The velocity in kinetic energy term is the mean linear velocity in the pipe. To account the effect of the velocity distribution across the pipe [(α) dimensionless correction factor] is used.

For a circular cross sectional pipe:

- $\alpha = 0.5$ for laminar flow

- $\alpha = 1.0$ for turbulent flow

2- Modification for real fluid

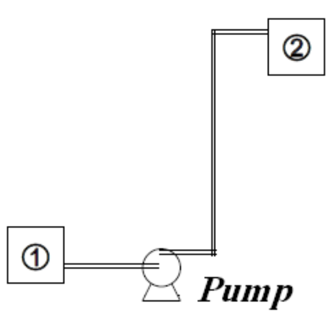
The real fluids are viscous and hence offer resistance to flow. Friction appears wherever the fluid flow is surrounding by solid boundary. Friction can be defined as the amount of mechanical energy irreversibly converted into heat in a flow in stream. As a result of that the total energy is always decrease in the flow direction i.e. (E2 < E1). Therefore E1 = E2 + F, where F is the energy losses due to friction.

Thus the modified Bernoulli's equation becomes,

$P_1/\rho + u_1^2/2 + z_1 g = P_2/\rho + u_2^2/2 + z_2 g + F$ ------($J/kg \equiv m^2/s^2$)

3- Pump work in Bernoulli's equation

A pump is used in a flow system to increase *the mechanical energy of the fluid. The increase being used to maintain flow of the fluid. Assume a pump is installed between the stations ❶ and ❷ as shown in Figure. The work supplied to the pump is shaft work (–Ws), the negative sign is due to work added to fluid.*



Frictions occurring within the pump are: -

a-Friction by fluid

b-Mechanical friction

Since the shaft work must be discounted by these frictional force (losses) to give net mechanical energy as actually delivered to the fluid by pump (Wp).

Thus, Wp = η Ws where η, is the efficiency of the pump.

Thus the modified Bernoulli's equation for present of pump between the two selected points ❶ and ❷ becomes,

$$\frac{P_1}{\rho} + \frac{u_1^2}{2\alpha_1} + g\,z_1 + \eta\,Ws = \frac{P_2}{\rho} + \frac{u_2^2}{2\alpha_2} + g\,z_2 + F \quad \text{---------} (\text{J/kg} \equiv \text{m}^2/\text{s}^2)$$

By dividing each term of this equation by (g), each term will have a length units, and the equation will be: -

$$\frac{P_1}{\rho\,g} + \frac{u_1^2}{2\alpha_1 g} + z_1 + \frac{\eta\,Ws}{g} = \frac{P_2}{\rho\,g} + \frac{u_2^2}{2\alpha_2 g} + z_2 + h_F \quad \text{---------(m)}$$

where $h_F = F/g \equiv$ head losses due to friction.

<u>4.8 Friction in Pipes</u>

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of energy of fluid is lost. This loss of energy is classified on: -

ENERGY LOSS

Major energy losses
(Skin friction)
Due to surface skin of the pipe

Minor energy losses
(Form friction)
- sudden expantion or contraction pipe
- Bends, valves and fittings
- An obstruction in the pipe

4.8.1 Relation between Skin Friction and Wall Shear Stress

For the flow of a fluid in short length of pipe (dL) of diameter (d), the total frictional force at the wall is the product of shear stress (τ_{rx}) and the surface area of the pipe (π d dL). This frictional force causes a drop in pressure ($-dP_{fs}$).

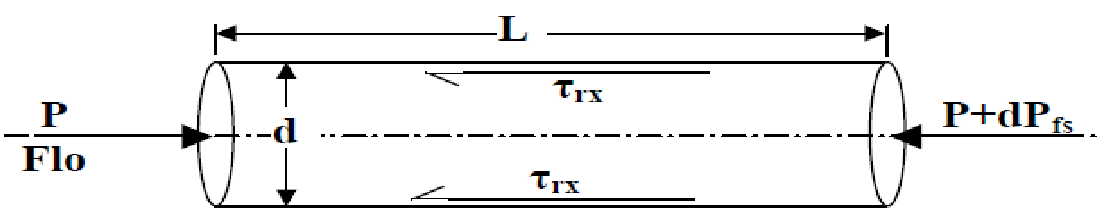
Consider a horizontal pipe as shown in Figure;

Force balance on element (dL)

$\tau(\pi\ d\ dL) = [P - (P + dP_{fs})]\ (\pi/4\ d^2)$ $\qquad * \ \rho u_x^2 / \rho u_x^2$

$\Rightarrow -dP_{fs} = 4(\tau\ dL/d) = 4\ (\tau/\rho u_x^2)\ (dL/d)\ \rho u_x^2$ ------(*)

where, $(\tau/\rho u_x^2) = \Phi = J_f = f/2 = f'/2$



Φ(or J_f): Basic friction Factor

f: Fanning (or Darcy) friction Factor

f′: Moody friction Factor.

For incompressible fluid flowing in a pipe of constant cross-sectional area, (u) is not a function of pressure or length and equation (*) can be integrated over a length (L) to give the equation of pressure drop due to skin friction:

$$-\Delta P_{fs} = 4f\,(L/d)\,(\rho u^2/2) \text{ ---------------------} (Pa)$$

The energy lost per unit mass Fs is then given by:

$$Fs = (-\Delta P_{fs}/\rho) = 4f\,(L/d)\,(u^2/2) \text{ -----------------} (J/kg) \text{ or } (m^2/s^2)$$

The head loss due to skin friction (hFs) is given by:

$$h_{Fs} = F_s/g = (-\Delta P_{fs}/\rho g) = 4f\,(L/d)\,(u^2/2g) \text{ ---------------} (m)$$

<u>Note: -</u>

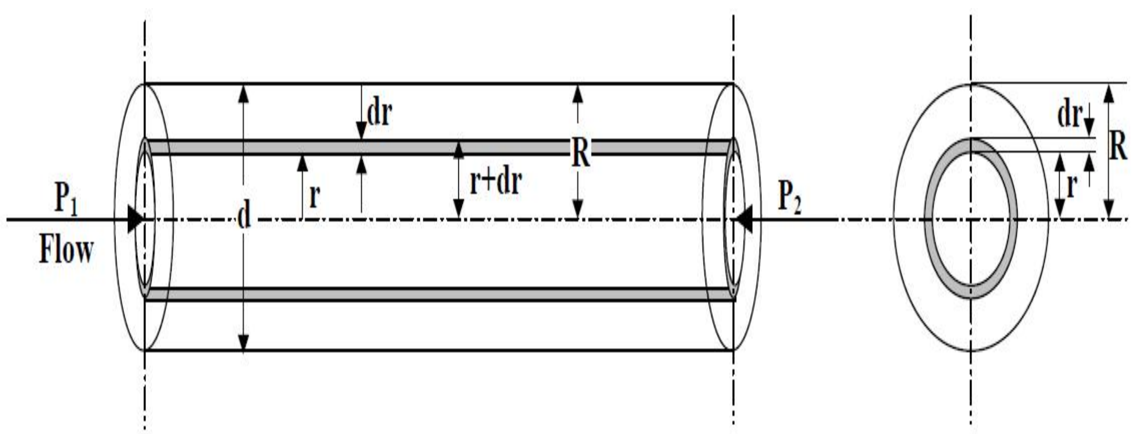
All the above equations could be used for laminar and turbulent flow.

$\Delta P_{fs} = P2 - P1 \Rightarrow -\Delta P_{fs} = P1 - P2$ (+ve value)

4.8.2 <u>Evaluation of Friction Factor in Straight Pipes</u>

<u>1-Velocity distribution in laminar flow</u>

Consider a horizontal circular pipe of a uniform diameter in which a Newtonian, incompressible fluid flowing as shown in Figure:



Consider the cylinder of radius (r) sliding in a cylinder of radius (r+dr). Force balance on cylinder of radius (r)

$\tau_{rx}\,(2\pi\, r\, L) = (P_1 - P_2)\,(\pi\, r^2)$

for laminar flow $\tau_{rx} = -\,\mu\,(du_x/dr)$

$\Rightarrow r(P_1 - P_2) = -\,\mu(du_x/dr)2L \Rightarrow [(P_2 - P_1)/(2L\,\mu)]rdr = du_x$

$\Rightarrow [\Delta P_{fs}/(2L\,\mu)]\, r^2/2 = u_x + C$

- **<u>Boundary Condition (1) (for evaluation of C)</u>**

at r = R $\quad u_x = 0 \quad \Rightarrow C = [(\Delta P_{fs}\, R^2)/(4L\,\mu)]$

$\Rightarrow [(\Delta P_{fs}\, r^2)/(4L\,\mu)] = u_x + [(\Delta P_{fs}\, R^2)/(4L\,\mu)]$

$\Rightarrow u_x = [(-\Delta P_{fs}\, R^2)/(4L\,\mu)][1 - (r/R)^2]$

velocity distribution (profile) in laminar flow

- **<u>Boundary Condition (2) (for evaluation of u_{max})</u>**

at r = o $\quad u_x = u_{max} \quad \Rightarrow u_{max} = [(-\Delta Pfs\, R^2)/(4\, L\,\mu)]$

$\Rightarrow u_{max} = [(-\Delta P_{fs}\, d^2)/(16\, L\,\mu)]$

----------centerline velocity in laminar flow----------

$\therefore u_x / u_{max} = [1 - (r/R)^2]$

---velocity distribution (profile)in laminar flow----

2-Average (mean) linear velocity in laminar flow

Q = u A------------------------ (1)

Where, (u) is the average velocity and (A) is the cross-sectional area = (πR^2)

$d_Q = u_x\, dA$ where $u_x = u_{max}[1-(r/R)^2]$, and $d_A = 2\pi\, r\, dr$

$\Rightarrow dQ = u_{max}[1-(r/R)^2]\, 2\pi\, r\, dr$

$$\int_0^Q dQ = 2\pi\, u_{max} \int_0^R \left(r - \frac{r^3}{R^2}\right) dr = 2\pi\, u_{max} \left| \frac{r^2}{2} - \frac{r^4}{4R^2} \right|_0^R$$

$\Rightarrow Q = u_{max}/2\ (\pi R^2)$ ----------------------- (2)

By equalization of equations (1) and (2)

$\Rightarrow u = u_{max}/2 = [(-\Delta P_{fs}\, R^2)/(8L\, \mu)] = [(-\Delta P_{fs}\, d^2)/(32\, L\, \mu)]$

$\therefore -\Delta P_{fs} = (32\, L\, \mu\, u) / d^2$ Hagen–Poiseuille equation

3-Friction factor in laminar flow

We have $-\Delta P_{fs} = 4f\, (L/d)\, (\rho u^2/2)$----------------------(3)

and also $-\Delta P_{fs} = (32\, L\, \mu\, u) / d^2$ ----------------------(4)

By equalization of these equations [i.e. eqs. (3) and (4)]

$\Rightarrow (32\, L\, \mu\, u) / d^2 = 4f\, (L/d)\, (\rho u^2/2) \Rightarrow f = 16\, \mu / (\rho\, u\, d)$

$\therefore ((\ f = 16 / Re))$

Fanning or Darcy friction factor in laminar flow.

4-Velocity distribution in turbulent flow

The velocity, at any point in the cross-section of cylindrical pipe, in turbulent flow is proportional to the one-seventh power of the distance from the wall. This may be expressed as follows: -

$u_x / u_{max} = [1-(r/R)]^{1/7}$

Prandtl one-seventh law equation.
velocity distribution (profile)in laminar flow

5-Average (mean) linear velocity in Turbulent flow

Q = u A----------------------- (1)

$d_Q = u_x\, dA$ where $u_x = u_{max} [1-(r/R)]^{1/7}$, and $dA = 2\pi r\, dr$

$\Rightarrow dQ = u_{max} [1- (r/R)]^{1/7} 2\pi r\, dr$

$$\int_0^Q dQ = u_{max} 2\pi \int_0^R r\left(1-\frac{r}{R}\right)^{\frac{1}{7}} dr$$

Let $M = (1- r/R)$ $dM = (-1/R)\, dr$

or $r = R(1 - M)$ $dr = - R\, dM$

at r = 0 M=1

at r = R M=0

Rearranging the integration

$$Q = u_{max} 2\pi R \int_1^0 (1-M)M^{\frac{1}{7}}(-dM)$$

$$= u_{max} 2\pi R^2 \int_0^1 \left(M^{\frac{1}{7}} - M^{\frac{8}{7}}\frac{r}{R}\right) d\,M$$

$\Rightarrow Q = 49/60\ u_{max}\ (\pi R^2)$ ----------------------- (5)

By equalization of equations (1) and (5)

$\therefore\ u = 49/60\ u_{max} \approx 0.82\ u_{max}$

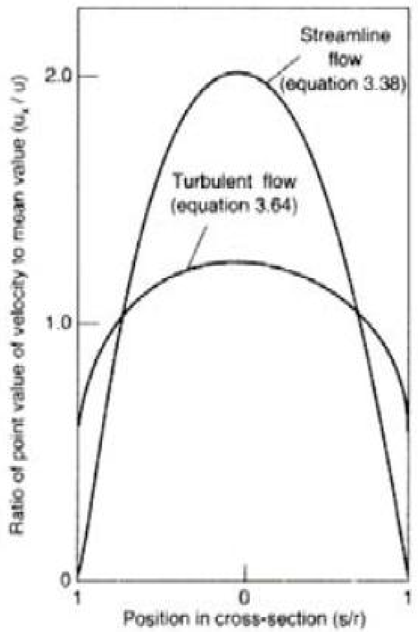


Figure of the shape of velocity profiles for streamline and turbulent flow

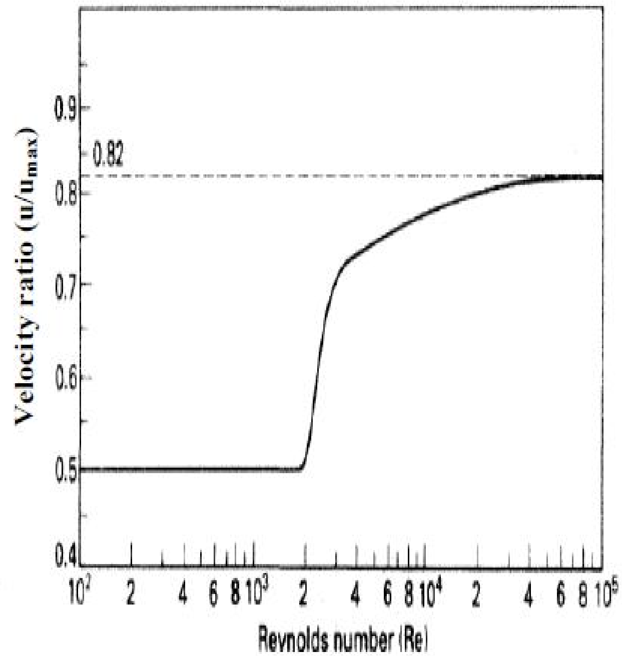


Figure of the Variation of (u/u_{max}) with Reynolds number in a pipe

Friction factor in Turbulent flow

A number of expressions have been proposed for calculating friction factor in terms of or function of (Re). Some of these expressions are given here: -

$$f = \frac{0.079}{Re^{0.25}}$$ *for 2,500 < Re <100,000*

and, $$f^{-0.5} = 4\,log\left(Ref^{0.5}\right) - 0.4$$ for 2,500 < Re <10,000,000

These equations are for *smooth pipes in turbulent flow. For rough pipes, the ratio of (e/d) acts an important role in evaluating the friction factor in turbulent flow as shown in the following equation:-*

$$\left({}^{f}/_{2}\right)^{-0.5} = -2.5\,ln[0.27\frac{e}{d} + 0.885Re^{-1}({}^{f}/_{2})^{-0.5}]$$

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Table of the roughness values *e.*

Surface type	ft	mm
Planed wood or finished concrete	0.00015	0.046
Unplaned wood	0.00024	0.073
Unfinished concrete	0.00037	0.11
Cast iron	0.00056	0.17
Brick	0.00082	0.25
Riveted steel	0.0017	0.51
Corrugated metal	0.0055	1.68
Rubble	0.012	3.66

7-Graphical evaluation of friction factor

As with the results of Reynolds number the curves are in three regions (Figure 3.7 vol.I). At low values of Re (Re < 2,000), the friction factor is independent of the surface roughness, but at high values of Re (Re > 2,500) the friction factor vary with the surface roughness. At very high Re, the friction factor become independent of Re and a function of the surface roughness only. Over the transition region of Re from 2,000 to 2,500 the friction factor increased rapidly showing the great increase in friction factor as soon as turbulent motion established.

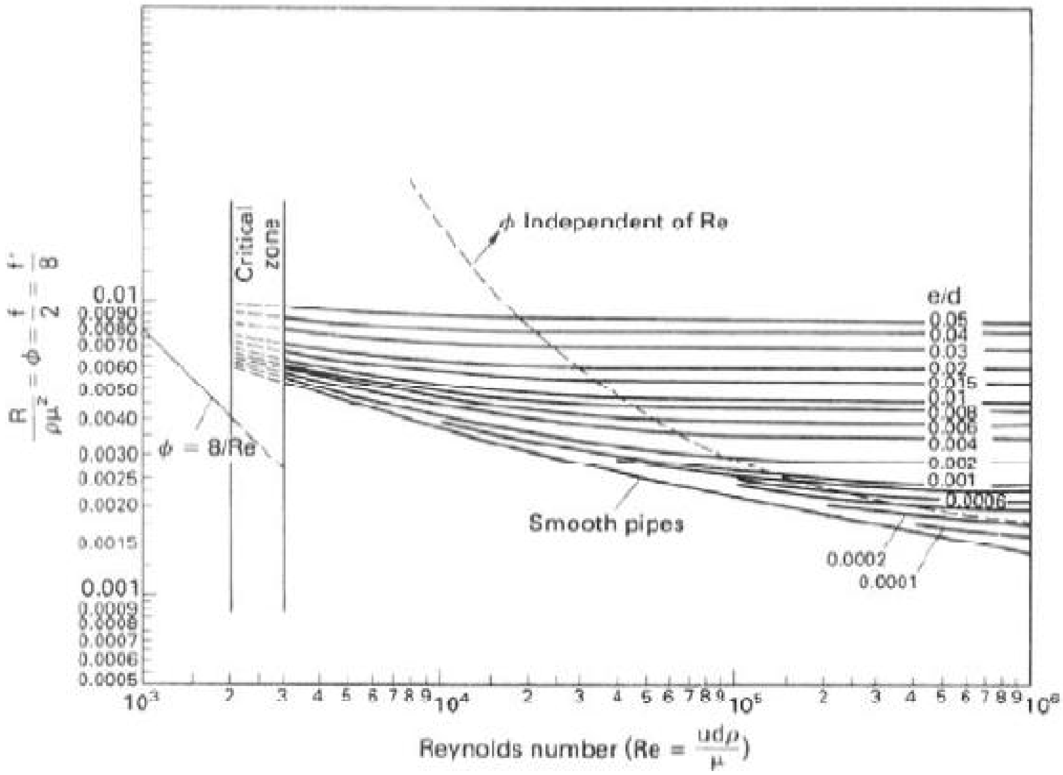
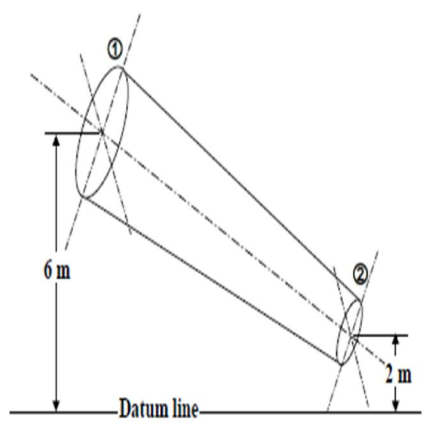


Figure (3.7) Pipe friction chart Φ versus Re

Example -4.3-

Water flowing through a pipe of 20 cm I.D. at section ❶ and 10 cm at section ❷. The discharge through the pipe is 35 lit/s. The section ❶ is 6 m above the datum line and section ❷ is 2 m above it. If the pressure at section ❶ is 245 kPa, find the intensity of pressure at section ❷. Given that ρ = 1000 kg/m3, μ= 1.0 mPa.s.



Solution:

Q = 35 lit/s = 0.035 m^3/s

u = Q/A ⇒ u_1= (0.035 m^3/s) / (0.2^2 π/4) m^2 = 1.114 m/s

⇒ u_2= (0.035 m^3/s) / (0.1^2 π/4) m^2 = 4.456 m/s

Re = ρud / μ ⇒ Re1= (1000 kg/m^3 x 1.114 m/s x 0.2 m) / (0.001Pa.s) = 222,800

Re = ρud / μ ⇒ Re2= (1000 kg/m^3 x 4.456 m/s x 0.1 m) / (0.001Pa.s) = 445,600

The flow is turbulent along the tube (i.e. α1 = α2 =1.0)

$$\frac{P_1}{\rho}+\frac{u_1^2}{2\alpha_1}+g\,z_1+\cancel{\eta W_s}=\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2}+g\,z_2+\cancel{F}$$

$$P_2=\rho\left[\frac{P_1}{\rho}+g(z_1-z_2+(\frac{u_1^2}{2\alpha_1}-\frac{u_2^2}{2\alpha_2})\right]=253.3\ kP_a$$

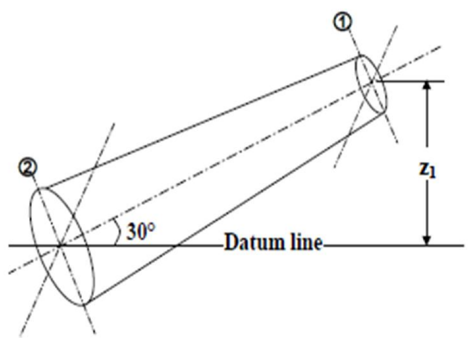
H.W.

If the pipe is smooth and its length is 20 m, find P_2.

Ans. P_2 = 246.06 kPa

Example -4.4-

A conical tube of 4 m length is fixed at an inclined angle of 30° with the horizontal-line and its small diameter upwards. The velocity at smaller end is (u_1 = 5 m/s), while (u_2 = 2 m/s) at other end. The head losses in the tub is [0.35 $(u_1-u_2)^2/2g$]. Determine the pressure head at lower end if the flow takes place in down direction and the pressure head at smaller end is 2 m of liquid.



Solution:

No information of the fluid properties.

Then assume the flow is turbulent, (i.e. α1 = α2 =1.0); z_1 = L sinθ=4 sin 30 =2 m

$$\frac{P_1}{\rho g}+\frac{u_1^2}{2\alpha_1 g}+z_1+\cancel{\frac{\eta W_s}{g}}=\frac{P_2}{\rho g}+\frac{u_2^2}{2\alpha_2 g}+z_2+\cancel{h_f}$$

$$\frac{P_2}{\rho g}=\frac{P_1}{\rho g}+z_1+\left(\frac{u_1^2-u_2^2}{2g}\right)\ \Rightarrow\ =2.0+2.0+(\frac{25-4}{2*9.81})-\frac{0.35(5-2)^2}{(2*9.81)}=4.9\ m$$

Example -4.5-

Water with density ρ = 998 kg/m³, is flowing at steady mass flow rate through a uniform-diameter pipe. The entrance pressure of the fluid is 68.9 kP_a in the pipe, which connects to a pump, which actually supplies 155.4 J/kg of fluid flowing in the pipe. The exit pipe from the pump is the same diameter as the inlet pipe. The exit section of the pipe is 3.05 m higher than the entrance, and the exit pressure is 137.8 kP_a. The Reynolds number in the pipe is above 4,000 in this system. Calculate the frictional loss (F) in the pipe system.

Solution:

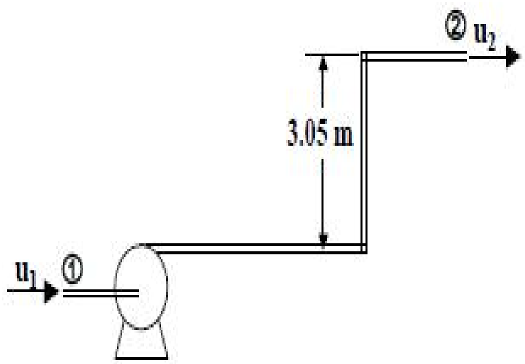
Setting the datum line at z_1 thus, $z_1 = 0$, $z_2 = 3.05$ m

$$\frac{P_1}{\rho}+\frac{u_1^2}{2\alpha_1}+g\,z_1+\eta\,W_s=\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2}+g\,z_2+F$$

$$\Rightarrow F=\left[\frac{P_1-P_2}{\rho}+\eta\,Ws-g\,z_2\right]$$

= (68.9 - 137.8) x 1000/998 +155.4 - 9.81(3.05)

= 56.5 J/kg or m^2/s^2



Example -4.6

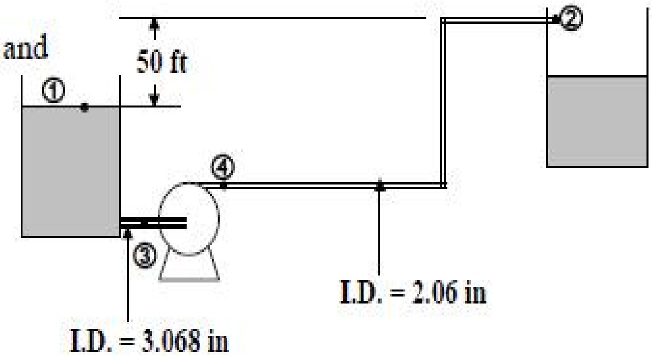
A pump draws 69.1 gal/min of liquid solution having a density of 114.8 lb/ft³ from an open storage feed tank of large cross-sectional area through a 3.068″I.D. suction pipe. The pump discharges its flow through a 2.067″I.D. line to an open over head tank. The end of the discharge line is 50′ above the level of the liquid in the feed tank. The friction losses in the piping system are F = 10 ft lbf/lb. what pressure must the pump develop and what is the horsepower of the pump if its efficiency is η=0.65.

Solution:

No information of the type of fluid and then its viscosity, therefore assume the flow is turbulent.

$P_1 = P_2 =$ atmospheric press.

$u_1 \approx 0$ large area of the tank



$$\cancel{\frac{P_1}{\rho}} + \cancel{\frac{u_1^2}{2\alpha_1 g_c}} + \cancel{\frac{z_1 g}{g_c}} + \eta W_s = \cancel{\frac{P_2}{\rho}} + \frac{u_2^2}{2\alpha_2 g_c} + \frac{z_2 g}{g_c} + F$$

$$\Rightarrow \eta Ws = \left[\frac{g z_2}{g_c} + \frac{u_2^2}{2 g_c} + F\right]$$

Q = 69.1 gal/min (ft^3/7.48 gal)(min/60 s) = 0.154 ft^3/s

A_3 (area of suction line) = π/4 (3.068 in)^2 (ft/12 in)^2 = 0.0513 ft^2

$A_4 = A_2$ (area of discharge line) = π/4 (2.067 in)^2 (ft/12 in)^2 = 0.0235 ft^2

$u_2 = Q / A_2$ = (0.154 ft^3/s) / (0.0235 ft^2) = 6.55 ft/s

$u_3 = Q / A_3$ = (0.154 ft^3/s) / (0.0513 ft^2) = 3.0 ft/s

$$\Rightarrow \eta Ws = \frac{32.174 ft/s^2 \times 50\ ft}{32.174\ lb\ ft/lb_f\ s^2} + \frac{(6.55\ ft/s)^2}{2 \times 32.174 lb\ ft/lb_f\ s^2} + 10 ft\ lb_f/lb = 60.655 \text{ ft lb}_f\text{/lb}$$

Ws = ηWs/η = 60.655/0.65 = 93.3 lb_f ft/lb

Mass flow rate $\dot{m} = Q\rho$ = 0.1539 ft^3/s (114.8 lb/ft^3) = 17.65 lb/s

Power required for pump = $\dot{m}\ Ws$ = 17.65 lb/s (93.3 ft lb_f/lb)(hp/550 ft lb_f/s)

= 3.0 hP

To calculate the pressure that must be developed by the pump, Energy Balance equation must be applied over the pump itself (points ③ and ④)

$u_4 = u_2$ = 6.55 ft / s and u_3 = 3 ft / s

$$\boxed{\frac{P_3}{\rho}+\frac{u_3^2}{2\alpha_1 g_c}+\cancel{\frac{z_3 g}{g_c}}+\eta W_s=\frac{P_4}{\rho}+\frac{u_4^2}{2\alpha_2 g_c}+\cancel{\frac{z_4 g}{g_c}}+\cancel{F}}$$

$$\Rightarrow \frac{P_4-P_3}{\rho}=\eta\ Ws+\frac{u_3^2-u_4^2}{2 g_c}=60.655+(-0.527)=60.13\ \text{ft lb}_f/\text{lb}$$

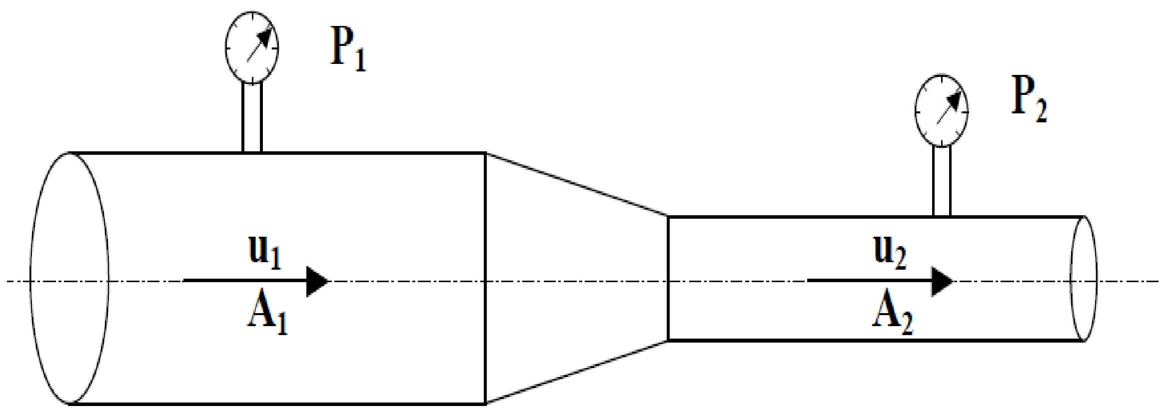
$$\Rightarrow \Delta P=60.13\ \text{ft lb}_f/\text{lb}\ (114.8)\ \text{lb/ft}^3=69.03\ \text{lb}_f/\text{ft}^2$$

$$=47.94\ \text{psi}$$

$$=3.26\ \text{bar}$$

Example -4.7-

A liquid with a constant density (ρ) is flowing at an unknown velocity (u_1) through a horizontal pipe of cross-sectional area (A_1) at a pressure (P_i), and then it passes to a section of the pipe in which the area is reduced gradually to (A_2) and the pressure (P_2). Assume no friction losses, find the velocities (u_1) and (u_2) if the pressure difference (P_1-P_2) is measured.



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Solution:

From continuity equation $\dot{m} = \dot{m}_1 = \dot{m}_2 = \rho_1 Q_1 = \rho_2 Q_2$

And for constant density $\Rightarrow Q = Q_1 = Q_2$

$\Rightarrow u\,A = u_1\,A_1 = u_2\,A_2$

$\Rightarrow u_2 = u_1\,A_1/A_2$

$$\frac{P_1}{\rho} + \frac{u_1^2}{2\alpha_1} + \cancel{z_1 g} + \cancel{\eta W_s} = \frac{P_2}{\rho} + \frac{u_2^2}{2\alpha_2} + \cancel{z_2 g} + \cancel{F}$$

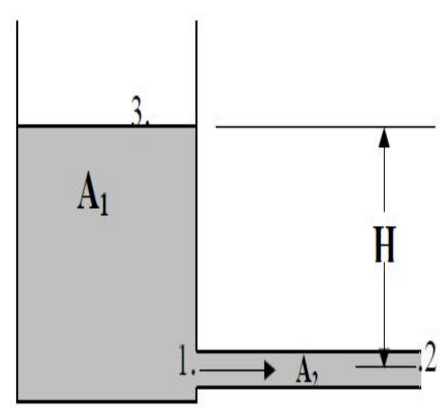
Assume the flow is turbulent ($\alpha_1 = \alpha_2$)

$$u_1 = \sqrt{\left(\frac{P_1 - P_2}{\rho}\right)\frac{2}{\left(\frac{A_1}{A_2}\right)^2 - 1}} \qquad u_2 = \sqrt{\left(\frac{P_1 - P_2}{\rho}\right)\frac{2}{1 - \left(\frac{A_1}{A_2}\right)^2}}$$

Example -4.8-

A nozzle of cross-sectional area (A_2) is discharging to the atmosphere and is located in the side of a large tank, in which the open surface of liquid in the tank is (H) above the centerline of the nozzle. Calculate the velocity (u_2) in the nozzle and the volumetric rate of discharge if no friction losses are assumed and the flow is turbulent.

Solution:



Since A_1 is very large compared to A_2 ($\Rightarrow u_1 \approx 0$).

The pressure P_1 is greater than atmosphere pressure by the head of fluid H.

The pressure P_2 which is at nozzle exit, is at atmospheric pressure .

$$\frac{P_1}{\rho}+\frac{u_1^2}{2\alpha_1}+z_1g+\eta W_s=\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2}+z_2g+F$$

$$\rightarrow u_2=\sqrt{\frac{2(P_1-P_2)}{\rho}}$$

But we have:-

$$(P_1-P_3)=H\rho g\rightarrow u_2=\sqrt{2Hg}\rightarrow Q_2=A_2\sqrt{2Hg}$$

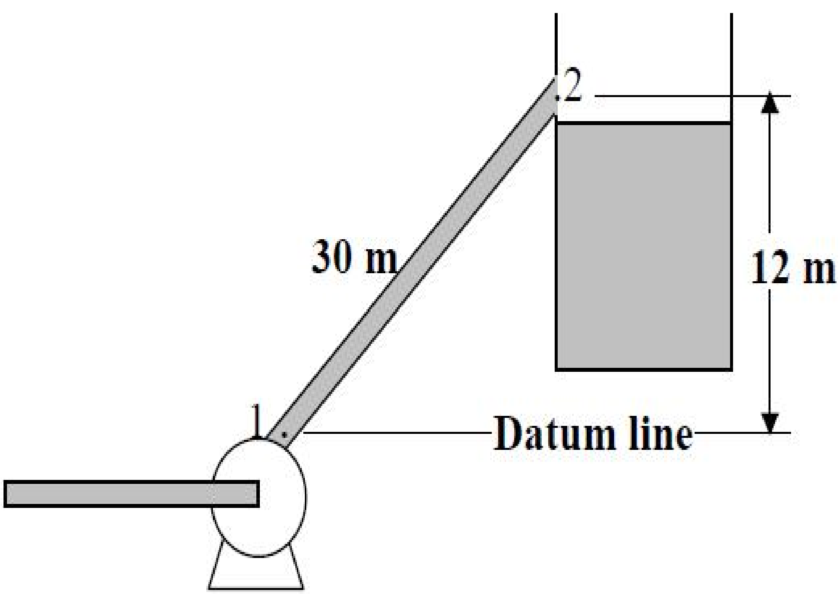
Or by :-

$$\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2}+z_2g+F=\frac{P_3}{\rho}+\frac{u_3^2}{2\alpha_3}+z_3g+F$$

$$\rightarrow u_2=\sqrt{2Hg}$$

Example -4.9-

98% H_2So_4 is pumped at 1.25 kg/s through a 25 mm inside diameter pipe, 30 m long, to a reservoir 12 m higher than the feed point. Calculate the pressure drop in the pipeline. Take that ρ = 1840 kg/m^3, μ= 25 mPa.s, e = 0.05 mm.



Solution:

$$\dot{m} = Q\rho = uA\rho \rightarrow u = \frac{\dot{m}}{A\rho}$$

⇒u =(1.25 kg/s) / (π/4 0.0252)(1840 kg/m³)

⇒u = 1.38 m/s

Re = (1840 x 1.38 x 0.025)/0.025 = 2546

e/d = 0.05 x 10^{-3} /0.025 = 0.002

From Figure (3.7)- Vol.I

Φ = 0.006 ⇒*f = 2 Φ = 0.012*

Fs = (–ΔP_{fs}/ρ) = 4f (L/d) (u^2/2)

Fs = 4(0.012) (30/0.025)(1.382/2) = 54.84 m^2/s^2

$$\frac{P_1}{\rho} + \frac{u_1^2}{2\alpha_1} + \cancel{z_1 g} + \cancel{\eta W_s} = \frac{P_2}{\rho} + \frac{u_2^2}{2\alpha_2} + z_2 g + F$$

If the kinetic energy at point 2 is neglected

$$\frac{P_1 - P_2}{\rho} = z_2 g + F \frac{u_1^2}{2\alpha_1}$$

F = Fs = 54.84 m^2/s^2

gz_2 = 9.81(12) = 117.72 m^2/s^2

$u_1^2/2\alpha_1$ = 1.382/2 = 0.952 m^2/s^2

⇒(P_1-P_2) = 1840 [117.72+54.84-0.952] = 315.762 kPa

4.8.3 Figure (3.8)- Vol.I

For turbulent flow, it is not possible to determine directly the fluid flow rate through a pipe from Figure (3.7) –Vol.I. For a known pressure drop, the method of solution to this problem is as follows:

$\Phi = Jf = \tau/\rho u^2 \Rightarrow \tau = \Phi\ \rho u^2$ ------------------------------(1)

But from force balance for fluid flow through horizontal pipe

$\tau\ \pi\ dL = -\Delta P_{fs}\ (\pi/4\ d^2)$

$\Rightarrow \tau = -\Delta P_{fs}/L\ (d/4)$---(2)

By equalization of equations (1) and (2)

$\Rightarrow \Phi\rho u_2 = -\Delta P_{fs}/L\ (d/4)$ -------------- x $\rho d^2/\mu^2$

$\Rightarrow \Phi\rho^2\ u^2\ d^2\ /\mu^2 = -\Delta Pfs/L\ (d3\rho\ /4\ \mu^2)$

or $\Phi\ Re^2 = -\Delta P_{fs}/L\ (\rho\ d^3\ /4\ \mu^2)$ -----------------------(3)

This equation dose not contains the mean linear velocity (u) of fluid. This can be determine through using Figure (3.8)- Vol.I as follows:

1- Calculate the value of ΦRe^2 from equation (3) of (ΔP_{fs}, ρ, d, L, and μ).

2- Read the corresponding value of Re from Figure (3.8) for a known value of (e/d).

3- Calculate U from the extracted value of Re.

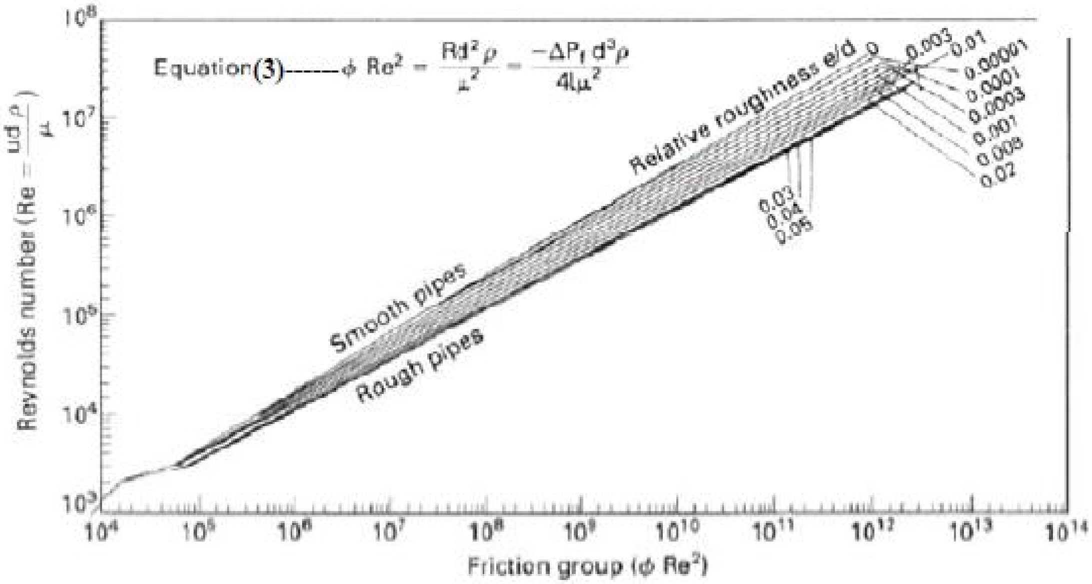


Figure (3.8) Pipe friction chart ΦRe2 versus Re for various values of e/d

Example -4.10-

Calculate the pressure drop in Pa for a fluid flowing through a 30.48 m long commercial steel pipe of I.D. 0.0526 m and a pipe roughness (e = 0.045 mm). The fluid flows at steady transfer rate of 9.085 m^3/h. Take that $\rho = 1200\ kg/m^3$, $\mu = 0.01$ Pa.s.

Solution:

$Q = 9.085\ m^3/h \times h/3600s = 2.524 \times 10^{-3}\ m^3/s$

$u = Q/A = (2.524 \times 10^{-3}\ m^3/s) / (\pi/4 \times 0.0526^2) = 1.16\ m/s$

Re = (1200 x1.16 x 0.0526)/0.01 = 7322

e/d =0.000045/0.0526 = 0.000856

Figure (3.7) $\Phi = 0.0042 \Rightarrow f = 2\ \Phi = 0.084$

$$\frac{\Delta P}{\rho} + g\Delta z + \frac{\Delta u^2}{2\alpha} - \eta W_s + F = 0$$

$\Rightarrow -\Delta P_{fs} = \rho\ Fs = 4\ f\ (L/d)(\ \rho u^2/2) = 4\ (0.0084)\ (30.48/0.0526)\ (1200 \times 1.162/2)$
= 15719 Pa

Example -4.11-

Repeat the previous example with the following conditions, the volumetric flow rate (i.e. the velocity) is unknown and the pressure drop in the pipe is 15.72 kPa.

Solution:

$\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2)$

$=[(15720)/(30.48)][(1200)(0.0526)^3/(4)(0.01)^2]$

$= 2.252 \times 10^5$

e/d = 0.000856 ⇒Figure (3.8) Re = 7200

⇒u = 7200(0.01)/(1200 x 0.0526)= 1.141m/s

Example -4.12-

Repeat the previous example with the following conditions, the diameter of the pipe is unknown and the pressure drop in the pipe is 15.72 kPa and the velocity of the liquid is 1.15 m/s. Estimate the diameter of the pipe.

Solution:

$-\Delta P_{fs} = \rho\, Fs = 4\, f\, (L/d)(\rho u^2/2) \Rightarrow d = (4\, \rho\, f\, L\, u^2/2)/ -\Delta P_{fs}$

⇒d = 6.154 f ---------------------------------(1)

Re = ρud/μ = 138000 d ----------------------(2)

e/d = 0.000045/d ----------------------------(3)

Φ Figure (3.7)	*f=2Φ* *f*	Eq.(1) d	Eq.(2) Re	Eq.(3) e/d
assumed →	0.001	0.006145	849	0.0073
0.01	0.02	0.123	16985	0.00036
0.0037	0.007	0.045	6284	0.00099
0.0045	0.009	0.0554	7643	0.0008
0.0043	0.0086	0.0529	7303	0.00085
0.0043	0.0086			

⇒d = 0.0529 m

Example -4.13-

Sulfuric acid is pumped at 3 kg/s through a 60 m length of smooth 25 mm pipe. Calculate the drop in pressure. If the pressure drop falls to one half, what will new flow rate be? Take that ρ = 1840 kg/m^3, μ= 25 mPa.s.

Solution:

$$u = \frac{Q}{A} = \frac{\dot{m}}{A\rho} = \frac{3\ kg/s}{\left(1840\frac{kg}{m^3}\right)\left(\frac{\pi}{4} * 0.025^2\right)m^2} = 3.32\ m/s$$

Re = (1840 x3.32 x 0.025)/0.025 = 6111

Figure (3.7) for smooth pipe Φ = 0.0043 ⇒ $f = 0.0086$

$-\Delta P_{fs} = \rho\ Fs = 4\ f\ (L/d)(\ \rho u^2/2)$

$= 4(0.0086)(60/0.025)(1840 \times 3.32^2)/2 = 837.209\ kPa$

The pressure drop falls to one half (i.e. -ΔPfs = 837.209 kPa/2 = 418.604kPa

$\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2) = [(418604)/(60)][(1840)(0.025)^3/(4)(0.025)^2]$

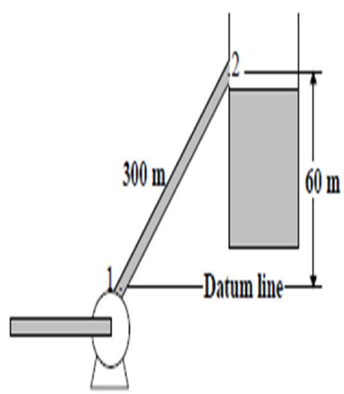
$= 8.02 \times 10^4$

From Figure (3.8) for smooth pipe Re = 3800 ⇒u = 2.06m/s

$$\dot{m} = \rho u A = 1.865\ kg/s$$

Example -4.14-

A pump developing a pressure of 800 kPa is used to pump water through a 150 mm pipe, 300 m long to a reservoir 60 m higher. With the valves fully open, the flow rate obtained is 0.05 m^3/s. As a result of corrosion and scalling the effective absolute roughness of the pipe surface increases by a factor of 10 by what percentage is the flow rate reduced.
μ= 1 mPa.s



Solution:

The total head of pump developing =(ΔP/ρg)
= 800,000/(1000 x 9.81)= 81.55 mH$_2$o

The head of potential energy = 60 m

Neglecting the kinetic energy losses (same diameter)

$$\frac{\Delta P}{\rho} + g\Delta z + \frac{\Delta u^2}{2\alpha} - \eta W_s + F = 0$$

⇒$\Delta P/\rho g + \Delta z + hF = 0$

⇒ $hF = -\Delta P/\rho g - \Delta z = 81.55 - 60 = 21.55$ m

$u = Q/A = (0.05\ m^3/s)/((\pi/4)\ 0.15^2) = 2.83$ m/s

$hFs = (-\Delta P_{fs}/\rho g) = 4f\ (L/d)\ (u^2/2g)$

⇒$f = h_{Fs}\ d\ 2g/(4Lu^2) = (21.55)\ (0.15)(9.81)/(2 \times 300 \times 2.83^2) = 0.0066$

$\Phi = 0.0033$, $Re = (1000 \times 2.83 \times 0.15)/0.001 = 4.23 \times 10^5$

From Figure (3.7) e/d = 0.003

Due to corrosion and scalling the roughness increase by factor 10

i.e. (e/d)new = 10 (e/d)old = 0.03

The pump head that supplied is the same

$(-\Delta P_{fs}) = h_{Fs}\ \rho g = 21.55\ (1000)\ 9.81 = 211.41$ kPa

$\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2) =$

$[(211410)/(300)][(1000)(0.15)^3/(4)(0.01)^2] = 6 \times 10^8$

From Figure (3.8) Re = 2.95 x105 ⇒u = 1.97m/s

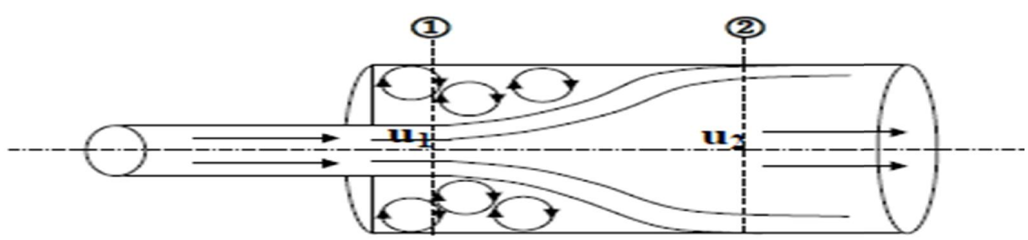
The percentage reduced in flow rate = (2.83 - 1.97)/ 2.83 x 100 % = 30.1 %.

4.8.4 Form Friction

Skin friction loss in flow straight pipe is calculated by using the Fanning friction factor (f). However, if the velocity of the fluid is changed in *direction or magnitude*, additional friction losses occur. This results from additional turbulence, which develops because of vertices and other factors.

1- <u>Sudden Expansion (Enlargement) Losses</u>

If the cross section of a pipe enlarges gradually, very little or no extra losses are incurred. If the change is sudden, as that in Figure, it results in additional losses due to eddies formed by the jet expanding in the enlarged suction. This friction loss can be calculated by the following for laminar or turbulent flow in both sections, as:



Continuity equation $u_1A_1 = u_2A_2 \Rightarrow u_2 = u_1(A_1/A_2)$

Momentum balance $$F_e = \frac{u_1^2}{2\alpha_1} - \frac{u_2^2}{2\alpha_2} + u_2^2 - u_1u_2$$

For fully turbulent flow in both sections

$$F_e = \frac{(u_1 - u_2)^2}{2} = \frac{u_1^2}{2}\left[1 - \left(\frac{A_1}{A_2}\right)\right]^2$$

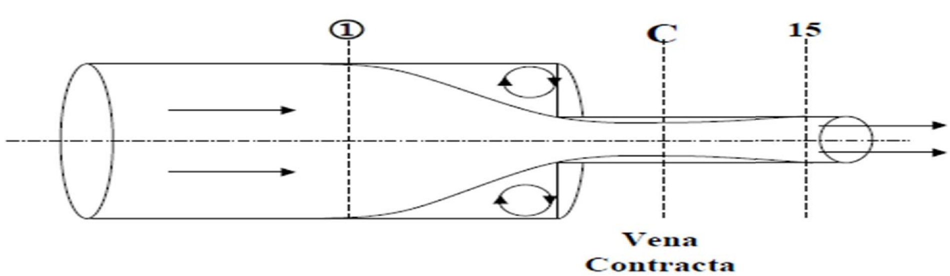
$$\therefore \; F_e = K_e \frac{u_1^2}{2}$$

Where

$$K_e = \left[1 - \left(\frac{A_1}{A_2}\right)\right]^2$$

2- <u>Sudden Contraction Losses</u>

The effective area for the flow gradually decreases as the sudden contraction is approached and then continues to decrease, for a short distance, to what is known as the "Vena Contacta". After the Vena Contracta the flow area gradually approaches that of the smaller pipe, as shown in Figure. When the cross section of the pipe is suddenly reduced, the stream cannot follow around the sharp corner, and additional loses due to eddies occur.



$$F_c = K_c \frac{u_2^2}{2} \quad where \; K_c = 0.55\left[1 - \frac{A_2}{A_1}\right]$$

3- Losses in Fittings and Valves

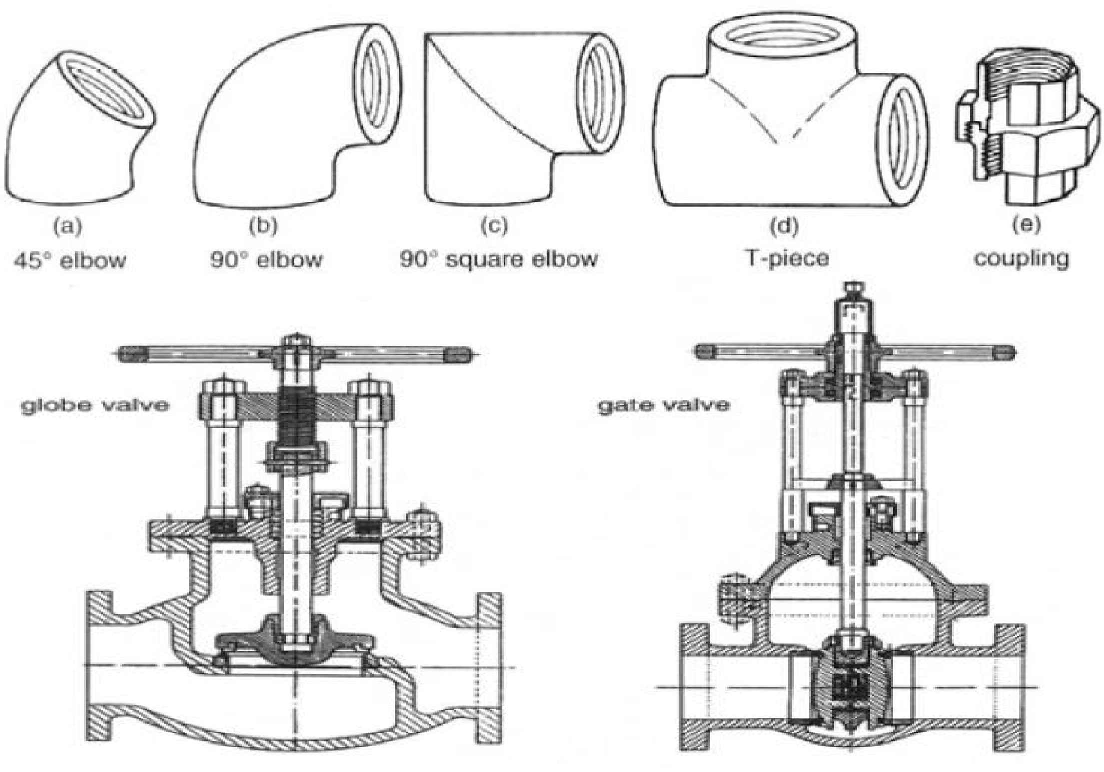
Pipe fittings and valves also disturb the normal flow lines in a pipe and cause additional friction losses. In a short pipe with many fittings, the friction losses from these fittings could be greater than in the straight pipe. The friction loss for fittings and valves is:

$$F_f = K_f \frac{u^2}{2}; \quad F_f = 4f \frac{L_e}{d} \frac{u^2}{2} \qquad where\ K_f\ , as\ in\ table\ below$$

Table of the Friction losses in pipe fittings

Fittings	K_f	Le/d
45° elbows (a)*	15	0.3
90° elbows (standard radius) (b)	30-40	0.6-0.8
90° square elbows (c)	60	1.2
Entry from leg of T-piece (d)	60	1.2
Entry into leg of T-piece (d)	90	1.8
Unions and couplings (e)	Very small	Very small
Globe valves fully open	60-300	1.2-6.0
Gate valves: fully open	7	0.15
3/4 open	40	1
1/2 open	200	4
1/4 open	800	16

* See Figure below



Figures of standard pipe fittings and standard valves

4.8.5 Total Friction Losses

The frictional losses from the friction in the straight pipe (skin friction), enlargement losses, contraction losses, and losses in fittings and valves are all incorporated in F term in mechanical energy balance equation (modified Bernoulli's equation), so that,

$$F_e = 4f\frac{l}{d}\frac{u_1}{2} + K_e\frac{u_1^2}{2} + K_c\frac{u_2^2}{2} + K_f\frac{u^2}{2}$$

If all the velocity *u*, u_1, *and* u_2 *are the same, then this equation becomes, for this special case;*

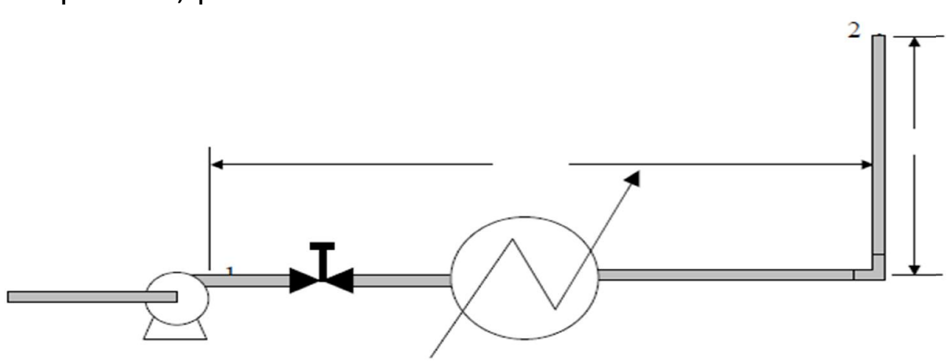
$$F = \left[4f\frac{l}{d} + K_e + K_c + K_f\right]\frac{u^2}{2}$$

If equivalent length of the straight pipe for the losses in fittings and/or valves, then this equation becomes;

$$F = \left[4f\left(\frac{l}{d} + \sum\frac{l}{d}\right) + K_e + K_c\right]\frac{u^2}{2}$$

Example -4.15-

630 cm^3/s water at 320 K is pumped in a 40 mm I.D. pipe through a length of 150 m in horizontal direction and up through a vertical height of 10 m. In the pipe there is a control valve which may be taken as equivalent to 200 pipe diameters and also other fittings equivalent to 60 pipe diameters. Also other pipe fittings equivalent to 60 pipe diameters. Also in the line there is a heat exchanger across which there is a loss in head of 1.5 m H_2o. If the main pipe has a roughness of 0.0002 m, what power must supplied to the pump if η = 60%, μ= 0.65 mPa.s.



Solution:

Q = 630 cc/s (m/100 cm)^3 = 6.3 x 10^{-4} m^3/s

u = (6.3 x 10^{-4} m^3/s)/(π/4* 0.04^2) = 0.5 m/s

Re = (1000 x 0.5 x 0.04)/0.00065 = 30,770

e/d = 0.0002/0.04 = 0.005

From Figure (3.7) Φ = 0.004, ⇒*f = 0.008*

L = 150 m + 10 m = 160 m

$$\frac{\Delta P}{\rho g} + \Delta z + \frac{\Delta u^2}{2\alpha g} - \frac{\eta W_s}{g} + h_f + (h)_{H.Ex} = 0$$

$$h_f = \left[4f\left(\frac{l}{d} + \sum \frac{l}{d}\right)\right]\frac{u^2}{2g}$$

= 4 (0.008) (160/0.04 + 200 + 60) x 0.5^2/(2 x 9.81) = 1.74 m

⇒(-ΔP/ρg) = Δh =10 + 1.74 + 1.5 = 13.24 m

⇒ The head required (that must be supplied to water by the pump) is Δh =13.24 m

and the power required for the water is ηWs = Q ΔP = Q (Δh ρg)

⇒ ηWs = 6.3 x 10^{-4} m^3/s (13.24 x 1000 x 9.81) = 81.8 (N.m/s ≡ J/s ≡ W)

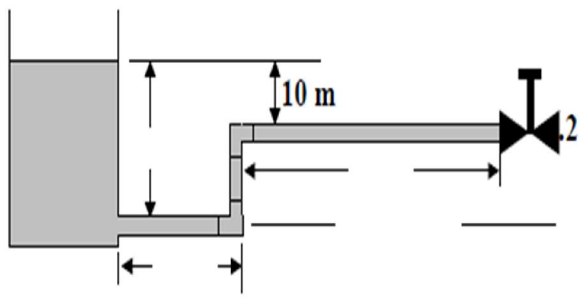
The power required for the pump is (Ws) = ηWs / η = 81.8 / 0.6 =136.4 W.

Example -4.16-

Water in a tank flows through an outlet 25 m below the water level into a 0.15 m I.D. horizontal pipe 30 m long, with 90° elbow at the end leading to vertical pipe of the same diameter 15 m long. This is connected to a second 90° elbow which leads to a horizontal pipe of the same diameter, 60 m long, containing a fully open globe valve and discharge to atmosphere 10 m below the level of the water in the tank. Calculate *the initial rate. Take that μ= 1 mPa.s, e/d =0.01*

Solution:

L = 30 + 15 + 60 = 105 m



$$\frac{P_1}{\rho g} + \frac{u_1^2}{2\alpha_1 g} + z_1 + \frac{\eta W_s}{g} = \frac{P_2}{\rho g} + \frac{u_2^2}{2g\alpha_2} + z_2 + h_f$$

$$\rightarrow z_1 - z_2 = \frac{u_2^2}{2g\alpha_2} + h_f \qquad h_f = \left[4f\left(\frac{l}{d} + \sum \frac{l}{d}\right)\right]\frac{u^2}{2g}$$

$$h_f = 210f\, u_2^2$$

Assume turbulent flow (α_2 =1.0) $\rightarrow u_2 = \sqrt{\frac{10}{0.05 + 210f}}$ - - - - - (*)

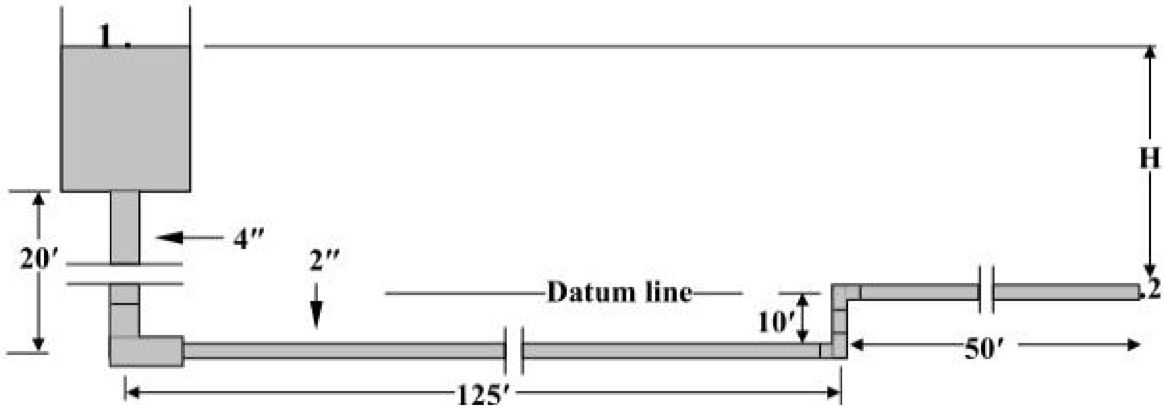
This equation solved by trial and error

Eq.(*)			Figure (3.7)
f	u_2	Re	Φ
0.01	2.16	3.235 x10^5	0.0046
0.0092	2.246	3.37 x10^5	0.0046

⇒ u_2(t=0) = 2.246 m/s, Re = 3.37 x10^5 (turbulent) ⇒Q=0.04 m^3/s ; m = 40 kg/s

Example -4.17-

An elevated storage tank contains water at 82.2°C as shown in Figure below. It is desired to have a discharge rate at point 2 of 0.223 ft^3/s. What must be the height H in ft of the surface of the water in the tank relative to discharge point? The pipe is schedule 40, e = 1.5 x10^{-4} ft. Take that ρ = 60.52 lb/ft^3, μ= 2.33 x10^{-4} lb/ft.s.



Solution:

$$\frac{P_1}{\rho}+\frac{u_1^2}{2\alpha_1 g_c}+\frac{g\,z_1}{g_c}+\eta W_s=\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2 g_c}+\frac{g\,z_2}{g_c}+F$$

$$\Rightarrow \frac{g}{g_c}z_1=\frac{u_2^2}{2\alpha_2 g_c}+F$$, where z_1 = H

for schedule 40

$d_{4''}$ = 4.026/12 = 0.3353 ft, $A_{4''}$ = 0.0884 ft^2,

$d_{2''}$ = 2.067/12 = 0.1722 ft, $A_{4''}$ = 0.0233 ft^2,

$u_{4''}$ = (0.223 ft^3/s) / (0.0884 ft^2) = 2.523 ft, $u_{2''}$ = (0.223 ft^3/s) / (0.0233 ft^2) = 9.57 ft,

The F-term for friction losses in the system includes the followings:

1- Contraction losses at tank exit. 2-Friction in 4″ straight pipe.

3- Friction in 4″ elbow. 4-Contraction losses in 4″ to 2″ pipe.

5- Friction in 2″ straight pipe. 6-Friction in the two 2″ elbows.

1- Contraction losses at tank exit. (let tank area =A_1, 4″ pipe area = A_3)

$$F_c=K_c\frac{u_2^2}{2};\text{ where } K_c=0.55\left[1-\frac{A_2}{A_1}\right]\approx 0.5$$

$\Rightarrow$ F_c = 0.55 (2.523^2 /2 x 32.174) = 0.054 ft.lb_f/lb.

2- Friction in 4″ straight pipe.

Re = (60.52 x 2.523 x 0.3353)/ 2.33 x10^{-4} = 2.193 x10^5

e/d = 0.000448 ⇒ Figure (3.7) f = 0.0047

$F_{Fs} = 4f\frac{L}{d}\frac{u^2}{2g_c}$ =4 (0.0047) (20/0.3353) x 2.523^2/(2 x 32.174) = 0.111 ft.lb_f/lb.

3- Friction in 4″ elbow.

$F_f = K_f \frac{u_2}{2}$; where K_f = 0.75 ⇒ F_f = 0.074 ft.lb_f/lb.

4- Contraction losses in 4″ to 2″ pipe.

$F_c = K_c \frac{u_2^2}{2}$; where $K_c = 0.55\left[1 - \frac{A_2}{A_1}\right]$ = 0.55 (1- 0.0233/0.0884) = 0.405

⇒ F_c = 0.405 (9.57^2 /2 x 32.174) = 0.575 ft.lb_f/lb.

5- Friction in 2″ straight pipe.

Re = (60.52 x 9.57 x 0.1722)/ 2.33 x10^{-4} = 4.28 x10^5

e/d = 0.00087 ⇒ Figure (3.7) f = 0.0048

$F_{Fs} = 4f\frac{L}{d}\frac{u^2}{2g_c}$ =4 (0.0048) (185/0.3353) x 9.57^2/(2 x 32.174) = 29.4 ft.lb_f/lb.

6- Friction in the two 2″ elbow.

$F_f = 2(K_f \frac{u_2}{2})$; where K_f = 0.75 ⇒ F_f = 2.136 ft.lb_f/lb.

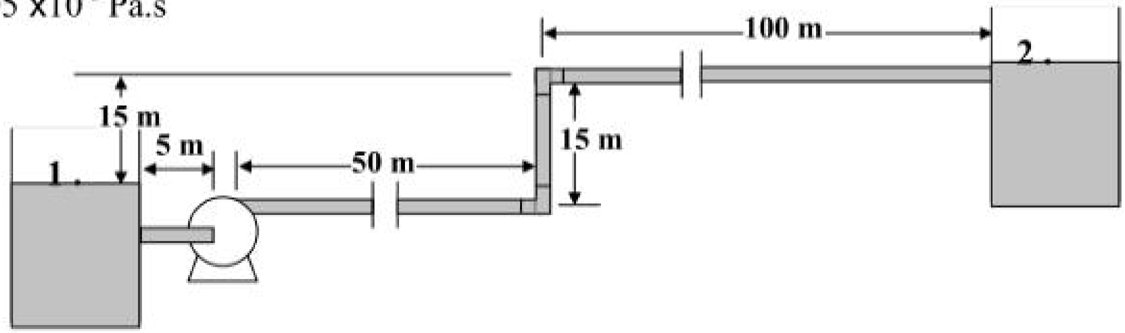
F (total frictional losses) = 0.054 + 0.111 + 0.575 + 29.4 + 2.136 =32.35 ft.lb_f/lb

⇒ H g/g_c = (9.57^2 /2 x 32.174) + 32.35= 33.77 ft.lb_f/lb

H = 33.77 ft ≈ 10.3m (height of water level above the discharge outlet)

Example -4.18-

Water at 20°C being pumped from a tank to an elevated tank at the rate of 0.005 m^3/s. All the piping in the Figure below is 4″ Schedule 40 pipe. The pump has an efficiency of η= 0.65. calculate the kW power needed for the pump. e = 4.6 x10^{-5} m ρ = 998.2 kg/m^3, μ= 1.005 x10^{-3} Pa.s



Solution:

For 4″ Schedule 40 pipe d = 0.1023 m, A = 8.219 x10^{-3} m^2

u = Q/A = (5 x10^{-3} m^3/s)/ 8.219 x10^{-3} m^2 = 0.6083 m/s

$$\frac{\Delta P}{\rho} + g\Delta z + \frac{\Delta u^2}{2\alpha} - \eta\, Ws + F = 0 \Rightarrow \eta Ws = F + g\Delta z$$

The F-term for friction losses in the system includes the followings:

1- Contraction losses at tank exit. 2-Friction loss in straight pipe.

3- Friction in the two elbows. 4- Expansion loss at the tank entrance.

1- Contraction losses at tank exit.

$$F_c = K_c \frac{u_2^2}{2};\ \text{where}\ K_c = 0.55\left[1 - \frac{A_2}{A_1}\right] \approx 0.5$$

⇒ F_c = 0.55 (0.6083^2 /2) = 0.102 J/kg or m^2/s^2.

2- Friction loss in straight pipe.

Re = (998.2 x 0.6083 x 0.1023)/ 1.005x10^{-3} = 6.181 x10^4

e/d = 0.00045 ⇒ Figure (3.7) f = 0.0051

L = 5+50+15+100 = 170 m

$F_{Fs} = 4f\frac{L}{d}\frac{u^2}{2}$ =4 (0.0051) (170/0.1023) x (0.6083^2/2) = 6.272 J/kg or m^2/s^2.

3- Friction in the two elbows.

$F_f = 2(K_f\frac{u_2}{2})$; where K_f = 0.75 ⇒ F_f = 0.278 J/kg or m^2/s^2.

4- Expansion loss at the tank entrance.

$F_e = K_e\frac{u_1^2}{2}$; where $K_e = \left[1-\left(\frac{A_1}{A_2}\right)\right]^2 \approx 1.0$ ⇒ F_e = 0.185 J/kg or m^2/s^2.

F (total frictional losses) = 0.102 + 6.272 + 0.278 + 0.185 = 6.837 J/kg or m^2/s^2.

⇒ ηWs = 6.837 + 9.81(15) = 153.93 J/kg or m^2/s^2

The power required for pump (Ws) = ηWs/Ws = 153.93/0.65 = 236.8 J/kg or m^2/s^2

The total power required for pump ($\dot{m}$ Ws) = Q ρ Ws

= (5 x10^{-3} m^3/s) 998.2 kg/m^3 (236.8 J/kg) = 1.182 kW.

Example -4.19-

Water at 4.4°C is to flow through a horizontal commercial steel pipe having a length of 305 m at the rate of 150 gal/min. A head of water of 6.1 m is available to overcome the skin friction losses (h_{Fs}). Calculate the pipe diameter. e = 4.6 x10^{-5} m ρ = 1000 kg/m^3, μ= 1.55 x10^{-3} Pa.s.

Solution:

$$h_{Fs} = \left[4f\left(\frac{L}{d}\right)\right]\frac{u^2}{2g} = 6.1 \text{ m}$$

Q = 150 gal/min (ft^3/ 7.481gal)(min/60s) (m/3.28 ft)^3 = 9.64 x10^{-3} m^3/s

u = Q/A = (9.64 x10^{-3} m^3/s)/(π/4 d^2) ⇒ u = 0.01204 d^{-2}.

⇒ 6.1 = 4 f (305/d)(0.01204 d^{-2})/(2 x9.81)

⇒ f = 676.73 d^5 ⇒ d = (f /676.73)$^{1/5}$ -----------------------(1)

Re = (1000 x (0.01204 d^{-2}) x d)/ 1.55x10^{-3} = 7769.74 d^{-1} -----------------------(2)

e/d = 4.6 x10^{-5} d^{-1} -----------------------(3)

solution by trial and error

		Eq.(1)	Eq.(2)	Fq. (3)	Figure (3.7)
	f	d	Re	e/d	f = 2 Φ
Assumed →	0.00378	0.089	8.73 x10^4	0.00052	0.0052
	0.0052	0.095	8.176 x10^4	0.000484	0.0051
	0.0051	0.0945	8.22 x10^4	0.00049	0.0051

⇒ d = 0.0945 m.

Example -4.20-

A petroleum fraction is pumped 2 km from a distillation plant to storage take through a mild steel pipeline, 150 mm I.D. at 0.04 m^3/s rate. What is the pressure drop along the pipe and the power supplied to the pumping unit if it has an efficiency of 50%. The pump impeller is eroded and the pressure at its delivery falls to one half. By how much is the flow rate reduced? Take that: sp.gr. = 0.705, μ= 0.5 m Pa.s e = 0.004 mm.

Solution:

$u = Q/A = (0.04\ m^3/s)/(\pi/4 \times 0.15^2) \Rightarrow u = 2.26\ m/s$

$Re = (705 \times 2.26 \times 0.15)/\ 0.5\times10^{-3} = 4.78\times10^5$

$e/d = 0.000027 \Rightarrow$ Figure (3.7) $f = 2\ \Phi \Rightarrow f = 0.0033$

$$-\Delta P_{Fs} = \left[4f\left(\frac{L}{d}\right)\right]\frac{\rho u^2}{2} = 4\ (0.0033)\ (2000/0.15)\ (705 \times 2.26^2/2) = 316876\ Pa.$$

$$\text{Power} = \frac{Q(-\Delta P)}{\eta} = (0.04\ m^3/s)(316876\ Pa)/0.5 = 25.35\ kW$$

Due to impeller erosion $(-\Delta P)_{new} = (-\Delta P)_{old}/2 = 316876\ Pa/2 = 158438\ Pa$

$\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2) = [(158438)/(2000)][(1000)(0.15)^3/(4)(0.5\times10^{-3})^2] = 1.885\times10^8$

$e/d = 0.000027 \Rightarrow$ From Figure (3.8) $Re = 3\times10^5 \Rightarrow u = 1.42 m/s$

The new volumetric flow rate is now $Q = 1.42\ (\pi/4 \times 0.15^2) = 0.025\ m^3/s$.

4.9 Friction Losses in Noncircular Conduits

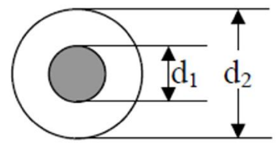
The friction loss in long straight channels or conduits of noncircular cross-section can be estimated by using the same equations employed for circular pipes if the diameter in the Reynolds number and in the friction factor equation is taken as equivalent diameter. The equivalent diameter De or hydraulic diameter defined as four times the cross-sectional area divided by the wetted perimeter of the conduit.

$$De = 4\,\frac{\text{cross} - \text{sectional area of channel}}{\text{wetted perimeter of channel}}$$

For circular cross section.

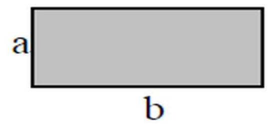
$De = 4\ (\pi/4 \times d^2)/\ \pi\ d = d$

For an annular space with outside diameter d_1 and inside d_2.



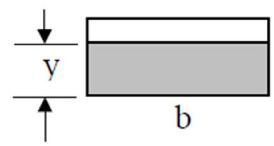
$De = 4\ [\pi/4 \times (d_1^2 - d_2^2)]/\ \pi\ (d_1 + d_2) = d_1 + d_2$

For a rectangular duct of sides a and b.



$De = 4\ (a.b)\ /\ 2(a + b) = 2ab\ /\ (a + b)$

For open channels and pattly filled ducts of y-liquid depth and b width



$De = 4\ (b.y)\ /\ (b + 2y)$

4.10 <u>Selection of Pipe Sizes</u>

In large or complex piping systems, the optimum size of pipe to use for a specific situation depends upon *the relative costs of capital investment, power, maintenance, and so on. Charts are available for determining these optimum sizes. However, for small* installations approximations are usually sufficient accurate. Representative values of ranges of velocity in pipes is shown in the following table: -

Type of fluid	Type of flow	Velocity	
		ft/s	m/s
Nonviscous liquid	Inlet to pump	2 - 3	0.6 – 0.9
	Process line or Pump discharge	5 - 8	1.5 – 2.5
Viscous liquid	Inlet to pump	0.2 – 0.8	0.06 – 0.25
	Process line or Pump discharge	0.5 - 2	0.15 – 0.6
Gas		30 - 120	9 – 36
Steam		30 - 75	9 – 23

4.11 The Boundary Layer

When a fluid flow over a surface, that part of the stream, which is close to the surface, suffers a significant retardation, and a velocity profile develops in the fluid. In the bulk of the fluid away from the boundary layer the flow can be adequately described by the theory of ideal fluids with zero viscosity (μ =0). However in the thin boundary layer, viscosity is important.

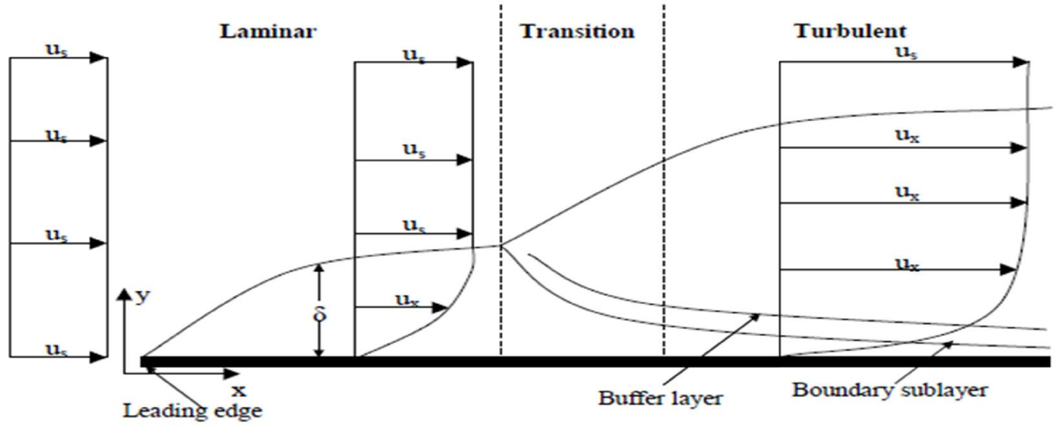


Figure of boundary layer for flow past a flat plate

If the velocity profile of the entrance region of a tube is flat, a certain length of the tube is necessary for the velocity profile to be fully established (developed). This length for the establishment of fully developed flow is called “entrance length”.

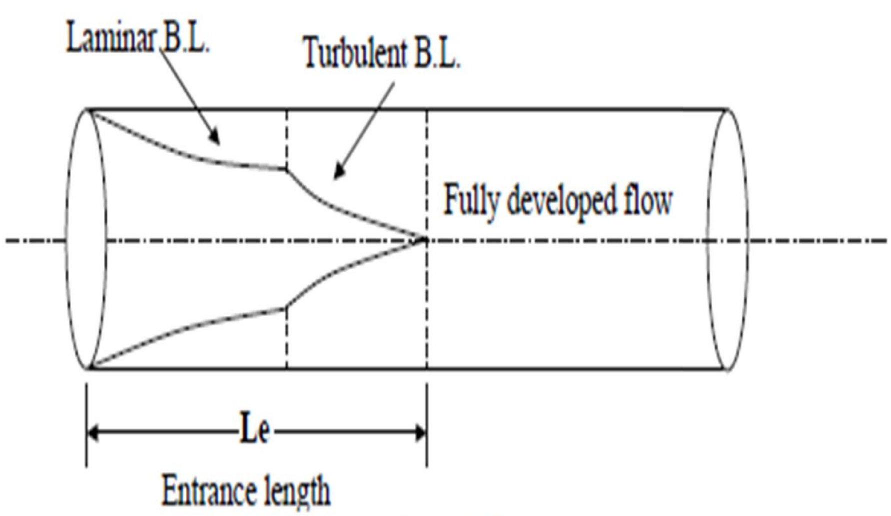


Figure of conditions at entry to pipe.

At the entrance the velocity profile is flat; i.e. the velocity is the same at all positions. As the fluid progresses down the tube, the boundary layer thickness increases until finally they meet at the centerline of the pipe.

For fully developed velocity profile to be formed in laminar flow, the approximate entry length (Le) of pipe having diameter d, is: -

Le/d = 0.0575 Re --------------------laminar

For fully developed velocity profile to be formed in turbulent flow, no relation is available to predict the entry length. As an approximation the entry length (Le) is after 50 diameters downstream of pipe. Thus;

Le/d = 50 ------------------turbulent

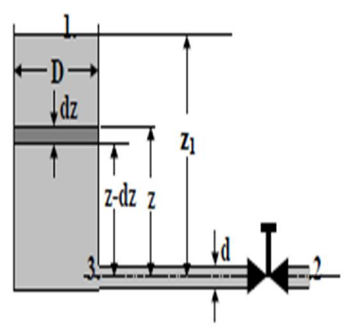
4.12 <u>Unsteady State Problems</u>

<u>Example -4.21-</u>

A cylindrical tank, 5 m in diameter, discharges through a horizontal mild steel pipe 100 m long and 225 mm diameter connected to the base of the tank. Find the time taken for the water level in the tank to drop from 3 m to 0.3 m above the bottom. The viscosity of water is 1.0 mNs/m^2, e = 0.05 mm.

<u>Solution:</u>

$$\cancel{\frac{\Delta P}{\rho g}} + \Delta z + \frac{\Delta u^2}{2\alpha g} - \cancel{\frac{\eta W_s}{g}} + h_f = 0$$



$u_1 \approx 0$, $z_2 = 0$ (datum line)

at time = 0 ⇒ $z = z_1$

at time = t ⇒ z = z

$$z_1 = \frac{u_2^2}{2\alpha_2} + h_f \; at \; t = 0$$

$$h_{FS} = \left[4f\left(\frac{L}{d}\right)\right]\frac{u_2}{2g} = 90.61\, f\, u_2^2$$

at time = 0 ⇒z_1 = (0.051 + 90.61 *f*) u_2^2 $\Rightarrow\ u_2 = \sqrt{\dfrac{z_1}{0.05 + 90.61\ f}}$

at time = 0 ⇒z_1 = (0.051 + 90.61 *f*) u_2^2 $\Rightarrow\ u_2 = \sqrt{\dfrac{z}{0.051 + 90.61\ f}}$ ---(1)

Let the level of liquid in the tank at time (t) is (z)
and the level of liquid in the tank at time (t+dt) is (z–dz)
The volume of liquid discharge during (time =t) to (time = t+dt) is (– dV)
= (π/4 D^2) [z– (z-dz)] = (19.63 dz) m^3
Q = dV/dt = –19.63 (dz/dt) m^3/s --(2)
But Q = A u_2 = (π/4 d^2) u_2 = (0.04 m^2) u_2 --------------------------(3)
Substitute eq.(1) into eq.(3) to give;

$$Q = 0.04\sqrt{\frac{z}{0.051 + 90.61\ f}} - - - - - - - - (4)$$

The equalization between eq.(2) and eq.(4) gives;

$$Q = -19.63\frac{d_z}{d_t} = 0.04\sqrt{\frac{z}{0.051 + 90.61\ f}} \rightarrow \int_0^t d_t = \int_3^{0.3} -490.75\sqrt{\frac{z}{0.051 + 90.61\ f}}\ z^{-1/2} d_z$$

$$\rightarrow t = 490.75\sqrt{0.051 + 90.61\ f}\int_{0.3}^{3} z^{-\frac{1}{2}} d_z$$

$$= 490.75\sqrt{0.051 + 90.61\ f}\ \frac{z^{1/2}}{1/2}\Big|_{0.3}^{3}$$

$$T = 1169.4\sqrt{0.051 + 90.61\ f}$$

$P_3 = P_o + z\rho g$, and $P_2 = P_o$

$\Rightarrow (P_3 - P_2) = (-\Delta PFs)$ the pressure drop along the pipe due to friction

From applied the modified Bernoulli's equation between 3 and 2

$\Rightarrow (-\Delta PFs) = z\rho g$

But $\Phi Re^2 = (-\Delta Pfs/L)(\rho d^3/4\mu^2) = [(z\rho g)/(L)][(\rho d^3/4\mu^2)] = 2.79x108z$

at $z = 3.0$ m $\Rightarrow \Phi Re^2 = 8.79 \times 10^8$

at $z = 0.3$ m $\Rightarrow \Phi Re^2 = 8.38 \times 10^7$

$e/d = 0.0002 \Rightarrow$ From Figure (3.8)

$\Rightarrow$at $z = 3.0$ m $Re = 7.0 \times 10^5$
$\Rightarrow$at $z = 0.3$ m $Re = 2.2 \times 10^5$ —— Turbulent

$e/d = 0.0002 \Rightarrow$ From Figure (3.7)

$\Rightarrow$at $z = 3.0$ m $Re = 7.0 \times 10^5 \Rightarrow f = 0.0038$

$\Rightarrow$at $z = 0.3$ m $Re = 2.2 \times 10^5 \Rightarrow f = 0.004$

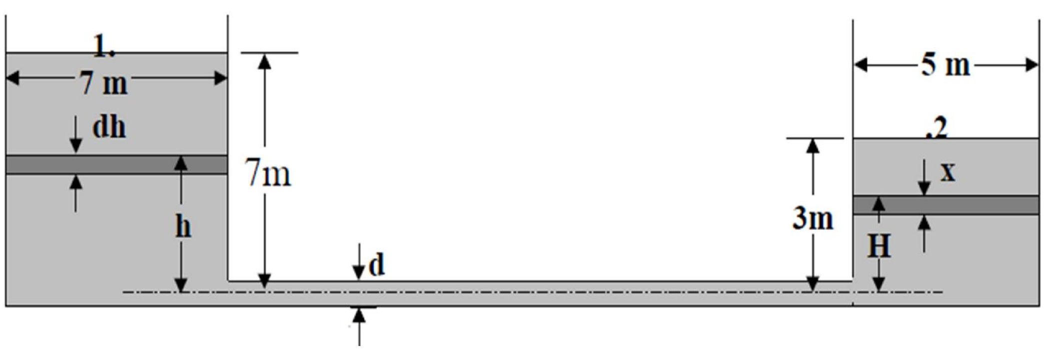
taking a value of *f = 0.004, and assume it constant*

$\therefore T = 752$ s

<u>Example -4.22-</u>

Two tanks, the bottoms of which are at the same level, are connected with one another by a horizontal pipe 75 mm diameter and 300 m long. The pipe is bell-mouthed at each end so that losses on entry and exit are negligible. One tank is 7 m diameter and contains water to a depth of 7 m. The other tank is 5 m diameter and contains water to a depth of 3 m. If the tanks are connected to each other by means of the pipe, how long will it take before the water level in the larger tank has fallen to 6 m? Take e =0.05 mm.

<u>Solution:</u>



At any time (t) the depth in the larger tank is (h) and the depth in the smaller tank is (H)

$$\frac{\Delta P}{\rho g} + \Delta z + \frac{\Delta u^2}{2\alpha g} - \frac{\eta W_s}{g} + h_F = 0$$

$$\Rightarrow \Delta z = h - H = h_{Fs} \text{ ------------------------------------(1)}$$

When the level in the large tank fall to (h), the level in the small tank will rise by a height (x) by increasing to reach a height (H).
The volume of the liquid in large tank that discharged to small tank is;

$= \pi/4 \times 7^2 (7–h) = 38.48 (7–h) \ m^3$

and is equal to $= \pi/4 \times 5^2 (x) = 38.48 (7–h)$

$\Rightarrow x = 13.72 – 1.96 h$ ---------------------------(2)

$H = 3 + x = 3 + 13.72 – 1.96 h$

$\Rightarrow H = 16.72 – 1.96 h$ ---------------------------(3)

Substitute eq.(3) into eq.(1), to give,

$$h - (16.72 - 1.96\, h) = h_{Fs} = \left[4f\left(\frac{L}{d}\right)\right]\frac{u^2}{2g}$$

$$2.96\, h - 16.72 = 815.5\, f u^2 \quad \Rightarrow u = \sqrt{\frac{0.00363\, h - 0.02}{f}}$$

The level of water in the large at (t = 0) = 7 m
The level of water in the large at (t = t) = h m
The level of water in the large at (t = t+dt) = (h–dh) m
The discharge of liquid during the timed (dt) is,

$Q = dV / dt = \pi/4 \ 7^2 [h–(h–dh)] / dt = \pi/4 \ 7^2 (dh / dt)$ -----------(4)

$$\text{But } Q = A\,u = \pi/4\, d^2 \Rightarrow Q = \frac{\pi}{4}(0.075)^2\sqrt{\frac{0.00363\, h - 0.02}{f}} \text{ ------------------------(5)}$$

$$\text{By equalization between eq.(4) and eq.(5)} \quad \Rightarrow dt = -8711.11\frac{dh}{\sqrt{\dfrac{0.00363 - 0.02}{f}}}$$

e/d = 0.05/75 = 0.00067 assume *f = 0.004*

$$\int_0^{T} dt = -8711.11\int_7^6 (0.9h-5)^{-0.5}\,dh = 8711.11\left(\frac{1}{0.9}\right)\left.\frac{(0.9h-5)^{0.5}}{0.5}\right|_6^7 = 19358(1.3^{0.5}-0.4^{0.5})$$

⇒ T = 9777.67 s

Q = [π/4 7^2 (7 – 6)] / 9777.67 = 0.00393 m^3/s average volumetric flow rate

u = Q / A = (0.00393 m^3/s) / (π/4 x 0.075^2) = 0.89 m/s

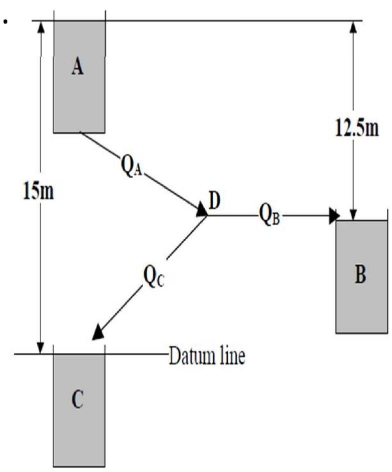
⇒ Re = 6.6552 $x10^4$

e/d = 0.00067 ⇒ From Figure (3.7) ⇒ *f =0.006*

Repeat the integration based on the new value of (*f = 0.006) ⇒ T = 9777.67 s*

Example -4.23-

Water is being discharged, from a reservoir, through a pipe 4 km long and 50 cm I.D. to another reservoir having water level 12.5 m below the first reservoir. It is required to feed a third reservoir, whose level is 15 m below the first reservoir, through a pipe line 1.5 km long to be connected to the pipe at distance of 1.0 km from its entrance. Find the diameter of this new pipe, so that the flow into both the reservoirs may be the same.



Solution:

AD + DB = 4,000 m, its i.d = 50 cm

AD = 1,000 m ⇒ DB = 3,000 m

DC = 1,500 m

$Q_A = Q_B + Q_C$

$Q_B = Q_C = Q_A/ 2$-----------------(1)

A-D

$$-\frac{P_D}{\rho}+(z_D-15)g+4f(\frac{L}{d})_A\frac{u_A^2}{2}=0$$

$$u=\frac{4Q}{\pi\ d^2}$$

$$-\frac{P_D}{\rho}+z_Dg-15g+4f(\frac{1,000}{0.5})(\frac{16Q_A^2}{2\pi^2d_A^4})=0$$

$$-\frac{P_D}{\rho}+z_Dg-147.1+830\ Q_A^2=0 \text{ ------------------------------(2)}$$

A-B

$$-\frac{P_D}{\rho}-(z_D-15)g+4f(\frac{3,000}{0.5})(\frac{16Q_B^2}{2\pi^2d_B^4})=0$$

$$-\frac{P_D}{\rho}-z_Dg+24.5+2,490\ Q_B^2=0 \text{ ----------------------------(3)}$$

A-B

$$-\frac{P_D}{\rho}-z_Dg+4f(\frac{1,500}{d_C})(\frac{16Q_C^2}{2\pi^2d_C^4})=0$$

$$-\frac{P_D}{\rho}-z_Dg+38.9\ \frac{Q_C^2}{d_C^5}=0 \text{ -------------------------------------(4)}$$

Substitute eq.(1) into eqs.(3) and (4)

Equation (2) $-\dfrac{P_D}{\rho}+z_D g-147.1+830\,Q_A^2=0$ ------------------------(2)

Equation (3) $-\dfrac{P_D}{\rho}-z_D g+24.5+622.5\,Q_A^2=0$ ------------------------(5)

Equation (4) $-\dfrac{P_D}{\rho}-z_D g+9.72\,\dfrac{Q_A^2}{d_C^5}=0$ ------------------------------(6)

eq.(2) + eq.(5) $\Rightarrow -122.6+1452.5\,Q_A^2=0 \Rightarrow Q_A=0.29\ \text{m}^3/\text{s}$

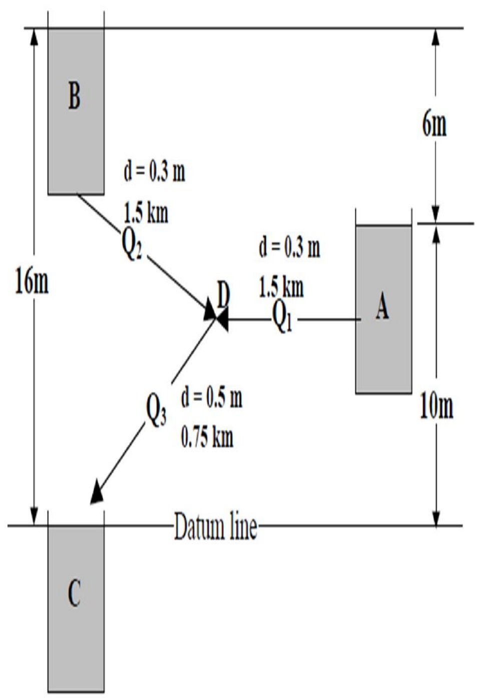
$\Rightarrow Q_B=Q_C=(0.29\ \text{m}^3/\text{s})/2=0.145\ \text{m}^3/\text{s}$

eq.(5) – eq.(6) $\Rightarrow\ 24.5+622.5(0.29)^2-9.72(0.29)^2/d_C^5=0\ \Rightarrow\ d_C^5=0.0106\ \text{m}^5$

$\Rightarrow\ d_C=0.4\ \text{m}=40\ \text{cm.}$

Example -4.24-

Two storage tanks, A and B, containing a petroleum product, discharge through pipes each 0.3 m in diameter and 1.5 km long to a junction at D, as shown in Figure. From D the liquid is passed through a 0.5 m diameter pipe to a third storage tank C, 0.75 km away. The surface of the liquid in A is initially 10 m above that in C and the liquid level in B is 6 m higher than that in A. Calculate the initial rate of discharge of liquid into tank C assuming the pipes are of mild steel. The density and viscosity of the liquid are 870 kg/m^3 and 0.7 m Pa.s respectively.



Solution:

Because the pipes are long, the kinetic energy of the fluid and minor losses at the entry to the pipes may be neglected. It may be assumed, as a first approximation, that *f is the same in each pipe and that the velocities in pipes AD, BD, and DC are u1, u2, and u3 respectively, if the pressure at D is taken as PD and point D is zD m above the datum for the calculation of potential energy, the liquid level in C.*

Then applying the energy balance equation between D and the liquid level in each of the tanks gives:

A-D $$-\frac{P_D}{\rho}+(z_D-10)g+4f\left(\frac{1{,}500}{0.3}\right)\frac{u_1^2}{2\alpha_1}=0$$

B-D $$-\frac{P_D}{\rho}+(z_D-16)g+4f\left(\frac{1{,}500}{0.3}\right)\frac{u_2^2}{2\alpha_2}=0$$

D-C $$-\frac{P_D}{\rho}-(z_D)g+4f\left(\frac{750}{0.5}\right)\frac{u_3^2}{2\alpha_3}=0$$

Assume turbulent flow in all pipes

$\Rightarrow$ **A-D** $-\frac{P_D}{\rho}+z_D g-98.1+10{,}000 f\ u_1^2=0$ ------------------------------(1)

B-D $-\frac{P_D}{\rho}+z_D g-156.96+10{,}000 f\ u_2^2=0$ ----------------------------(2)

D-C $-\frac{P_D}{\rho}-z_D g+3{,}000 f\ u_3^2=0$ ------------------------------(3)

eq.(1) – eq.(2) $\Rightarrow 58.86 + 10{,}000 f(u_1^2 - u_2^2) = 0$ ---------------------(4)

eq.(2) – eq.(3) $\Rightarrow -156.96 + 10{,}000 f(u_2^2 + 0.3u_3^2) = 0$ ---------------(5)

$Q_1 + Q_2 = Q_3 \Rightarrow [(\pi/4\ 0.3^2)\ u_1] + [(\pi/4\ 0.3^2)\ u_2] = [(\pi/4\ 0.5^2)\ u_3]$

$\Rightarrow u_1 + u_2 = 2.78\ u_3$ -----------------------------------(6)

equations (4), (5), and (6) are three equations with 4 unkowns. As first approximation

for e/d = 0.0001 to 0.00017 $\Rightarrow f = 0.004$

$\Rightarrow$ eq.(4) become $58.86 + 40\ (u_1^2 - u_2^2) = 0$ ----------------------------(7)

$\Rightarrow$ eq.(5) become $-156.96 + 40\ (u_2^2 - 0.3u_3^2) = 0$ ---------------------(8)

From eq.(7) $u_1^2 = u_2^2 - 1.47$ ------------------------------(9)

$u_3 = (u_1 + u_2) \Rightarrow u_3^2 = (1/2.78)^2\ (u_1^2 + 2u_1\,u_2 + u_2^2)$

$\Rightarrow u_3^2 = (1/2.78)^2\ [u_2^2 + (u_2^2 - 1.47) + 2u_2(u_2^2 - 1.47)^{0.5}]$

$\Rightarrow u_3^2 = (1/2.78)^2\ [2u_2^2 - 1.47 + 2u_2(u_2^2 - 1.47)^{0.5}]$ ----------------(10)

Substitute eq.(10) into (8)

$\Rightarrow -156.96 + 40\ \{u_2^2 + 0.3(1/2.78)^2\ [2u_2^2 - 1.47 + 2u_2(u_2^2 - 1.47)^{0.5}]\} = 0$

$\Rightarrow u_2(u_2^2 - 1.47)^{0.5} = (159.24 - 43.2\, u_2^2)/\, 3.2$ squaring the two limits

$\Rightarrow u_2^2(u_2^2 - 1.47)^{0.5} = (49.8 - 13.5\, u_2^2)^2$

$\Rightarrow u_2^4 - 7.41 u_2^2 + 13.68 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow$ either $u_2^2 = 3.922$ or $u_2^2 = 3.488$

$\Rightarrow u_2 = 1.98$ m/s or $u_2 = 1.87$ m/s

Substituting u_2 into eq.(9)

$\Rightarrow u_1 = 1.56$ m/s or $u_1 = 1.42$ m/s

Substituting u_2, and u_2 into eq.(6)

$\Rightarrow u_3 = 1.3$ m/s or $u_3 = 1.18$ m/s

The lower set values satisfies equation (8)

$\Rightarrow u_1 = 1.42$ m/s, $u_2 = 1.87$ m/s, and $u_3 = 1.18$ m/s

$\Rightarrow Re_1 = 5.3 \times 10^5$, $Re_2 = 6.9 \times 10^5$, and $Re_3 = 7.3 \times 10^5$

From Figure (3.7) $\Rightarrow f_1 = 0.0043$, $f_2 = 0.0036$, and $f_3 = 0.0038$

$\Rightarrow$ the assumption <u>$f = 0.004$ is ok.</u>

$Q_3 = (\pi/4\ 0.5^2)\, u_3 = (\pi/4\ 0.5^2)\ 1.18 = 0.23\ m^3/s$