CHPTER SEVEN

Flow Measurement

7.1 Introduction

It is important to be able to measure and control the amount of material entering and leaving a chemical and other processing plants. Since many of the materials are in the form of fluids, they are flowing in pipes or conduits. Many different types of devices are used to measure the flow of fluids. The flow of fluids is most commonly measured using ***head flow meters.*** The operation of these flow meters is based on the Bernoulli's equation**.**

A construction in the flow path is used to increase in the lines flow velocity. This is accompanied by a decrease in pressure intensity or head *and since the resultant pressure drop is a function of the flow rate of fluid, the latter can be evaluated.*

7.2 Flow Measurement Apparatus

Head flow meters include orifice, venture meter, flow nozzles, Pitot tubes, and wiers. They consist of primary element, which causes the pressure or head loss and a secondary element, which measures it.

7.2.1 Pitot Tube

The Pitot tube is used to measure *the local velocity at a given point in the flow stream and not the average velocity in the pipe or conduit. In the Figures below a sketch of this simple device is shown. One tube*, ***the impact tube****, has its opening normal to the direction of flow* and **the static tube** *has its opening parallel to the direction of flow*.

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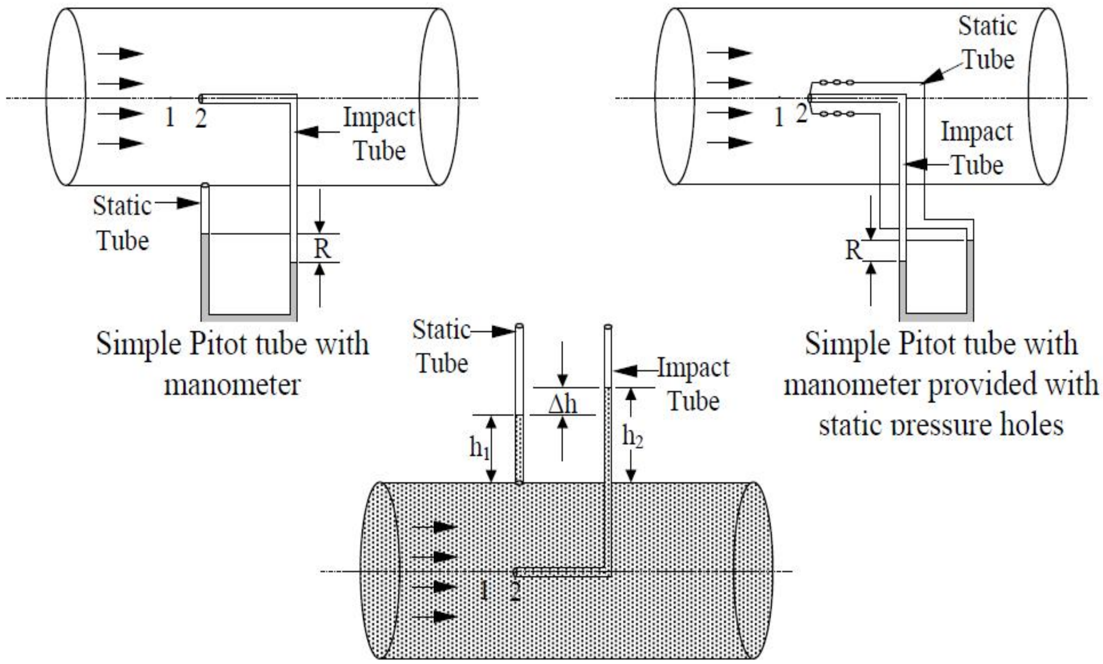


Simple Pitot tube with manometer

Simple Pitot tube with manometer provided with static pressure holes

Simple Pitot tube

Point 2 called ***stagnation point*** at which the impact pressure is be and $u_2 = 0$.

By applying Bernoulli's equation between points 1 and 2

$$\frac{P_1}{\rho g} + \frac{u_1^2}{2\alpha_1 g} + z_1 = \frac{P_2}{\rho g} + \frac{u_2^2}{2\alpha_2 g} + z_2$$

$$u_1 = \sqrt{\frac{2(-\Delta P)}{\rho}} = \sqrt{2g\Delta h} = \sqrt{\frac{2R(\rho_m - \rho)g}{\rho}}$$

where, $\Delta P = R(\rho_m - \rho)g$

The fluid flows into the opening at point 2, pressure builds up, and then remains stationary at this point, called "**Stagnation Point". The difference in the stagnation pressure (impact pressure) at this point (2) and the static pressure measured by the static tube represents the pressure rise associated with the direction of the fluid.**

Impact pressure head = Static pressure head + kinetic energy head

Since Bernoulli's equation is used for ideal fluids, therefore for real fluids the last equations of local velocity become:

$$u_x = CP\sqrt{\frac{2(-\Delta P)}{\rho}} = CP\sqrt{2g\Delta h} = CP\sqrt{\frac{2R(\rho_m - \rho)g}{\rho}}$$

where, Cp: dimensionless coefficient to take into account deviations from Bernoulli's equation and general varies between about 0.98 to 1.0.

Since the Pitot tube measures velocity at one point only in the flow, several methods can be used to obtain the average velocity in the pipe;

The first method, the velocity is measured at the exact center of the tube to obtain u_{max}. then by using the Figure, the average velocity can be obtained.

The second method, readings are taken at several known positions in the pipe cross section and then a graphical or numerical integration is performed to obtain the average velocity, from the following equation;

$$u = \frac{\iint u_x dA}{A}$$ (see Problem 5.16 Vol.I)

Example -7.1-

Find the local velocity of the flow of an oil of sp.gr. =0.8 through a pipe, when the difference of mercury level in differential U-tube manometer connected to the two tapping of the Pitot tube is 10 cm Hg. Take Cp = 0.98.

Solution:

$$u_x = CP\sqrt{\frac{2R(\rho_m - \rho)g}{\rho}} \qquad u_x = 0.98\sqrt{\frac{2(0.1)(13600 - 1000)9.81}{800}} = 5.49\ {}^{m}/_{s}$$

Example -7.2-

A Pitot tube is placed at a center of a 30 cm I.D. pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.84 of the center velocity (i.e. u/u_x =0.94). Find the discharge through the pipe if: -

i-The fluid flow through the pipe is water and the pressure difference between orifice is 6 cm H_2O.

ii-The fluid flow through the pipe is oil of sp.gr. = 0.78 and the reading manometer is 6 cm H_2O. Take Cp = 0.98.

Solution:

i- $u_x = CP\sqrt{2g\Delta h} = 0.98\sqrt{2(9.81)(0.06)} = 1.063\ m/s$

u = 0.84 (1.063) = 0.893 m/s, Q = A.u = $\pi/4(0.3)^2$ (0.893) = 0.063 m^3/s

ii- $u_x = CP\sqrt{\frac{2R(\rho_m - \rho)g}{\rho}} = 0.98\sqrt{\frac{2(0.06)(13600 - 780)9.81}{780}} = 0.565\ m/s$

u = 0.84 (0.565) = 0.475 m/s, Q = A.u = $\pi/4(0.3)^2$ (0.475) = 0.0335 m^3/s

Example -7.3-

A Pitot tube is inserted in the pipe of 30 cm I.D. The static pressure head is 10 cm Hg vacuum, and the stagnation pressure at center of the pipe is 0.981 N/cm2 gauge. Calculate the discharge of water through the pipe if u/umax = 0.85. Take Cp = 0.98.

Solution:

P_1 = –10 cm Hg (13600) 9.81 (m / 100 cm) = –13.3416 kPa

P_2 = 0.981 N/cm^2 (m / 100 cm)^2 = 9.81 kPa

$\Delta P = P_2 - P_1$ = 9.81- (-13.3416) = 23.1516 kPa

$$u_x - CP\sqrt{\frac{2(-\Delta P)}{\rho}} \quad = 0.98\sqrt{\frac{2 * 23.1516 * 10^3}{1000}} = 6.67\, m/s$$

u = 0.85 (6.67) = 5.67 m/s, Q = A.u = π/4(0.3)^2 (5.67) = 0.4 m^3/s

Example -7.4-

A Pitot tube is used to measure the air flow rate in a circular duct 60 cm I.D. The flowing air temperature is 65.5ºC. The Pitot tube is placed at the center of the duct and the reading R on the manometer is 10.7 mm of water. A static pressure measurement obtained at the Pitot tube position is 205 mm of water above atmospheric. Take Cp = 0.98, μ = 2.03 x 10^{-5} Pa.s

Calculate the velocity at the center and the average velocity.

Calculate the volumetric flow rate of the flowing air in the duct.

Solution: a-

P1 ≡ the static pressure

P1(gauge) = 0.205 (1000) 9.81 = 2011 kPa

P1(abs) = 2011 + 1.01325 x 105 Pa = 1.03336 x 10^5 Pa

ρ_{air} = M_{wt}.P/(R.T)=29(1.03336 x 10^5)/[(8314 Pa.m^3/kmol.K)(65.5+273.15)]

= 1.064 kg/m^3

$$u_x = CP\sqrt{\frac{2R(\rho_m - \rho)g}{\rho}} \quad = 0.98\sqrt{\frac{2(0.0107)(1000 - 1.065)9.81}{1.064}} = 14.04\frac{m}{s} = u_{max}$$

$Re_{max} = \rho u_{max} d/\mu = 1.064(14.04)0.6/2.03 \times 10^{-5} = 4.415 \times 10^{5}$

From Figure $u/u_{max} = 0.85 \Rightarrow u = 0.85\ (14.04) = 11.934$

b- $Q = A.u = \pi/4(0.6)^2\ (11.934) = 3.374\ m^3/s$

H.W. Problem 5.17 Vol.I

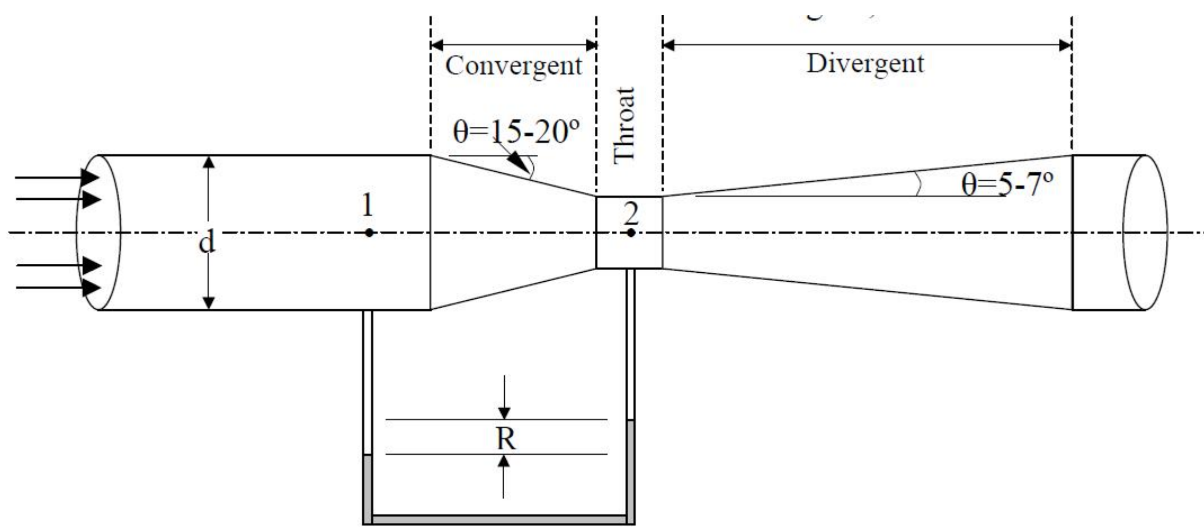
7.2.2 Measurement by Flow Through a Constriction

In measuring devices where the fluid is accelerated by causing it to flow through a constriction, the kinetic energy is thereby increased and the pressure energy therefore decreases. The flow rate is obtained by measuring the pressure difference between the inlet of the meter and a point of reduced pressure.

Venturi meters, orifice meters, and flow nozzles measure the volumetric flow rate Q or average (mean linear) velocity u. In contrast the Pitot tube measures a point (local) velocity ux.

7.2.2.1 Venturi Meter

Venturi meters consist of three sections as shown in Figure;



- From continuity equation $A_1u_1 = A_2u_2 \Rightarrow u_1 = (A_2/A_1)\, u_2$

- From Bernoulli's equation between points 1 and 2

$$\frac{P_1}{\rho g}+\frac{u_1^2}{2\alpha_1 g}+z_1=\frac{P_2}{\rho g}+\frac{u_2^2}{2\alpha_2 g}+z_2 \Rightarrow \frac{P_1-P_2}{\rho g}=\frac{u_2^2-u_1^2}{2g}=\frac{u_2^2}{2g}\left[1-\left(\frac{A_2}{A_1}\right)^2\right]=\frac{u_2^2}{2g}\left[\frac{A_1^2 A_2^2}{A_1^2}\right]$$

$$\Rightarrow u_2 = \sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)\left[\frac{1}{1-(A_2/A_1)^2}\right]} = \sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)}\,\frac{A_1}{\sqrt{A_1^2-A_2^2}}$$

$$or\ u_2 = \sqrt{2g\Delta h\left[\frac{1}{1-(A_2/A_1)^2}\right]} = \sqrt{2g\Delta h}\,\frac{A_1}{\sqrt{A_1^2-A_2^2}}$$

$$or\ u_2 = \sqrt{\frac{2R(\rho_m-\rho)g}{\rho}\left[\frac{1}{1-(A_2/A_1)^2}\right]} = \sqrt{\frac{2R(\rho_m-\rho)g}{\rho}}\,\frac{A_1}{\sqrt{A_1^2-A_2^2}}$$

All these equation of velocity at throat u2, which derived from Bernoulli's equation are for ideal fluids. Using a coefficient of discharge Cd to take account of the frictional losses in the meter and of the parameters of kinetic energy correction α_1 and α_2. Thus the volumetric flow rate will be obtained by: -

$$Q = u_2A_2 = Cd\sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)\left[\frac{1}{1-(A_2/A_1)^2}\right]} = Cd\sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)}\,\frac{A_1}{\sqrt{A_1^2-A_2^2}}$$

$$Or\ Q = u_2A_2 = Cd\sqrt{2g\Delta h\left[\frac{1}{1-(A_2/A_1)^2}\right]} = Cd\sqrt{2g\Delta h}\,\frac{A_1}{\sqrt{A_1^2-A_2^2}}$$

$$Q = u_2A_2 = Cd\sqrt{\frac{2R(\rho_m-\rho)g}{\rho}\left[\frac{1}{1\quad(A_2/A_1)^2}\right]} = Cd\sqrt{\frac{2R(\rho_m-\rho)g}{\rho}}\,\frac{A_1}{\sqrt{A_1^2-A_2^2}}$$

$$\dot{m} = Q\rho, \qquad G = u\rho = \frac{\dot{m}}{A}$$

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For many meters and for $Re > 10^4$ at point 1

Cd = 0.98 for d1 < 20 cm

Cd = 0.99 for d1 > 20 cm

Example -7.5-

A horizontal Venturi meter with d1 = 20 cm, and d2 = 10 cm, is used to measure the flow rate of oil of sp.gr. = 0.8, the discharge through venture meter is 60 lit/s. find the reading of (oil-Hg) differential Take Cd = 0.98.

Solution:

$Q = u_2 A_2 = 60$ lit/s (m^3/1000lit) = 0.06 m^3/s

$$Q = u_2 A_2 = 0.06 = Cd \sqrt{\frac{2R(\rho_m - \rho)g}{\rho}} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$= 0.98 \sqrt{\frac{2R(1366 - 800)g}{800}} \frac{(\pi/4)^2 (0.1)^2 (0.2)^2}{\sqrt{(\pi/4)^2[(0.2)^4 - (0.1)^4]}}$$

$\Rightarrow R = 0.1815$ m Hg = 18.15 cm Hg

Example -7.6-

A horizontal Venturi meter is used to measure the flow rate of water through the piping system of 20 cm I.D, where the diameter of throat in the meter is d_2 = 10 cm. The pressure at inlet is 17.658 N/cm^2 gauge and the vacuum pressure of 35 cm Hg at throat. Find the discharge of water. Take Cd = 0.98.

Solution:

$P_1 = 17.658\ N/cm^2\ (100\ cm / m)^2 = 176580$ Pa

P_2 = –35 mm Hg (m / 100 cm) 9.81 (13600) = –46695.6 Pa

$P_1 - P_2$ = 176580–(–46695.6) = 223275.6 Pa

$$Q = u_2 A_2 = Cd\sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)}\ \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$= 0.98\sqrt{\frac{2(223275.6)}{1000}}\ \frac{0.2^2[\pi/4](0.1)^2}{\sqrt{[(0.2)^4 - (0.1)^4]}}$$

$$= 0.168\ m^3/s$$

Example -7.7-

A Venturi meter is to be fitted to a 25 cm diameter pipe, in which the maximum flow is 7200 lit/min and the pressure head is 6 m of water. What is the maximum diameter of throat, so that there is non-negative head on it?

Solution:

Since the pressure head at the throat is not to be negative, or maximum it can be zero (i.e. h_2 = zero). Therefore;

$\Delta h = h_1 - h_2 = 6 - 0 = 6$ m H_2O

$Q = u_2 A_2$ = 7200 lit/min (m^3/1000lit) (min / 60 s) = 0.12 m^3/s

$$0.12 = Cd\sqrt{2g\Delta h}\ \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} = 1.0\sqrt{2(9.81)(6)}\ \frac{0.25^2[\pi/4](d_2)^2}{\sqrt{[(0.25)^4 - (d_2)^4]}}$$

$$0.225 = \frac{d_2^2}{\sqrt{(0.25)^4 - (d_2)^4}} \Rightarrow 0.0507 = \frac{d_2^4}{(0.25)^4 - (d_2)^4}$$

$$d_2^4 + 0.507 d_2^4 - 1.983 * 10^{-4} = 0 \Longrightarrow d_2 = 0.1172\ m = 11.72\ cm$$

Note: -

In case of using vertical or inclined Venturi meter instead of horizontal one, the same equations for estimation the actual velocity are used.

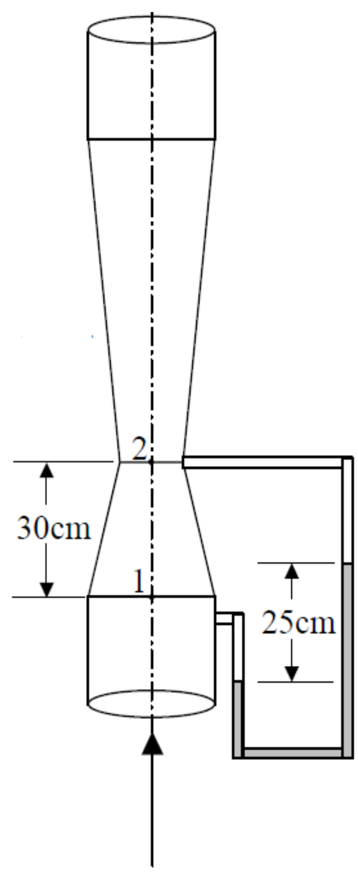
Example -7.8-

A (30cm x 15cm) Venturi meter is provided in a vertical pipe-line carrying oil of sp.gr. = 0.9. The flow being upwards and the difference in elevations of throat section and entrance section of the venture meter is 30 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. Take Cd = 0.98 and calculate: -

i-The discharge of oil

ii-The pressure difference between the entrance and throat sections.

Solution:



$$i-\ \ Q = u_2 A_2 = Cd\sqrt{\frac{2R(\rho_m-\rho)g}{\rho}}\ \frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}$$

$$= 0.98\sqrt{\frac{2(0.25)(12700)9.81}{900}}\left[\frac{0.3^2[(\pi/4)(0.15)^2]}{\sqrt{0.3^4-0.15^4}}\right] \quad = 0.1488\ m^3/s$$

ii- Applying Bernoulli's equation at points 1 and 2

$$\frac{P_1}{\rho g}+\frac{u_1^2}{2g}+\cancel{z_1}=\frac{P_2}{\rho g}+\frac{u_2^2}{2g}+z_2$$

$$\frac{P_1-P_2}{\rho g}=z_2+\frac{u_2^2-u_1^2}{2g}$$

$u_1 = 0.1488/(\pi/4\ 0.3^2) = 2.1$ m/s,

$u_2 = 0.1488/(\pi/4\ 0.15^2) = 8.42$ m/s

$P_1 - P_2 = 900\ (9.81)\ [0.3 + (8.422 - 2.12)/2(9.81)]$

= 32.5675 kPa

but $P_1 - P_2 = 0.25\ (13600–900)(9.81)$

= 31.1467 kPa

% error = 4.36 %

Home Work

P.7.1

A Venturi meter with a 15 cm I.D. at inlet and 10 cm I.D. at throat is laid with its axis horizontal and is used for measuring the flow of oil of sp.gr. = 0.9. The oil-mercurry differential manometer shows a gauge difference of 20 cm. If Cd = 0.98, calculate the discharge of oil.

Ans. Q = 0.06393 m^3/s

P.7.2

A horizontal Venturi meter (160mm x 80mm) used to measure the flow of oil of sp.gr. = 0.8. Determine the deflection of oil-mercury gauge, if discharge of oil is 50 lit/s.

Ans. R = 29.6 cm Hg

P.7.3

A Venturi meter has an area ratio (9:1), the larger diameter being 30 cm. During the flow the recorded pressure head in larger section is 6.5 m and that at throst 4.25 m. If Cd = 0.99, compute the discharge through the meter.

Ans. Q = 0.052 m^3/s

P.7.4

A Venturi meter is fitted to 15 cm diameter pipeline conveying water inclined at 60° to the horizontal. The throat diameter is 5 cm and it is placed higher than the inlet side. The difference of pressure between the throat and the inlet which are 0.9 m apart is equivalent to 7.5 cm of mercury. Calculate the discharge if Cd = 0.98.

Ans. Q = 0.00832 m^3/s

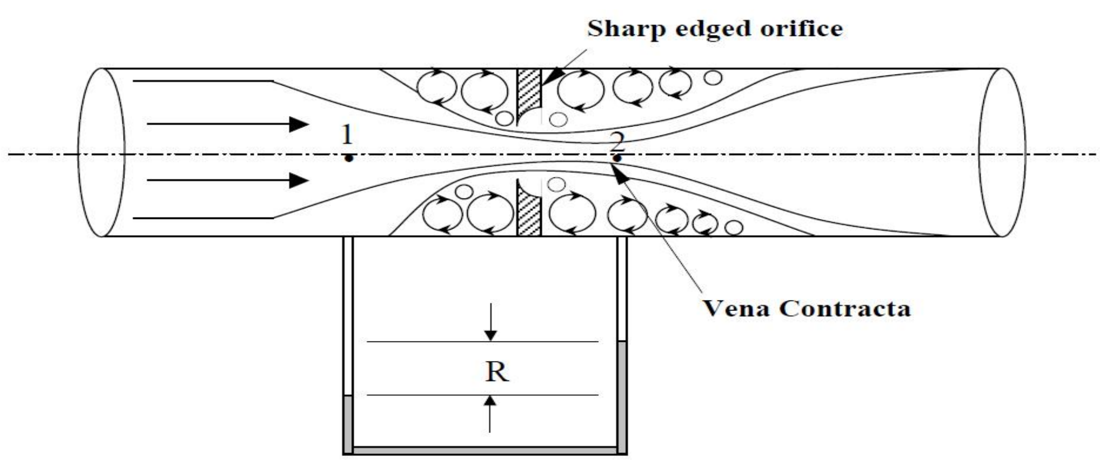
P.7.5

Find the throat diameter of a Venturi meter when fitted to a horizontal pipe 10 cm diameter having a discharge of 20lit/s. The differential U-tube mercury manometer, shows a deflection giving a reading of 60 cm, Cd = 0.98. In case, this Venturi meter is introduced in a vertical pipe, with the water flowing upwards, find the difference in the reading of mercury gauge. The dimensions of pipe and Venturi meter remain unaltered, as well as the discharge through the pipe.

Ans. d_2 = 0.04636 m, and the same reading in case II i.e. 60 cm Hg

7.2.2.2 Orifice Meter

The primary element of an orifice meter is simply a flat plate containing a drilled located in a pipe perpendicular to the direction of fluid flow as shown in Figure;



At point 2 in the pipe the fluid attains its maximum mean linear velocity u2 and its smallest cross-sectional flow area A2. This point is known as ***"the vena contracta".*** *It is occurs at about one-half to two pipe diameters downstream from the orifice plate.*

Because of relatively the large friction losses from the eddies generated by the expanding jet below vena contracta, the pressure recovery in orifice meter is poor.

- From continuity equation $A_1u_1 = A_2u_2 \Rightarrow u_1 = (A_2/A_1)\, u_2$

- From Bernoulli's equation between points 1 and 2

$$\frac{P_1}{\rho g} + \frac{u_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{u_2^2}{2g} + \cancel{z_2}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{u_2^2 - u_1^2}{2g} = \frac{u_2^2}{2g}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] = \frac{u_2^2}{2g}\left[\frac{A_1^2 A_2^2}{A_1^2}\right]$$

But **$C_c = A_2/A_o \Rightarrow A_2 = C_cA_o$**

C_c: coefficient of contraction [0.6 – 1.0] common value is 0.67

A_2: cross-sectional area at vena contracta

A_o: cross-sectional area of orifice

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$$\Longrightarrow \quad \frac{P_1 - P_2}{\rho} = \frac{u_2^2}{2}\left[1 - \left(\frac{C_C A_0}{A_1}\right)^2\right] = \frac{u_2^2}{2}\left[\frac{A_1^2 - (C_C A_0)^2}{A_1^2}\right]$$

Using a coefficient of discharge Cd to take into account the frictional losses in the meter and of parameters C_c, α_1, and α_2. Thus the velocity at orifice or the discharge through the meter is;

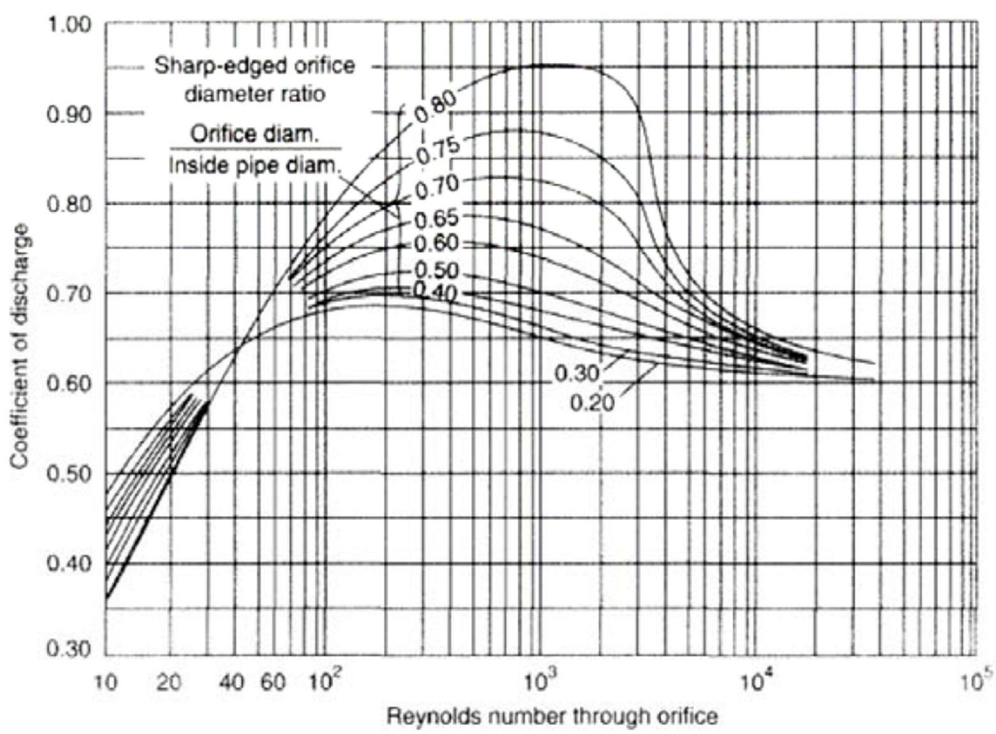
$$Q = Cd\sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)\left[\frac{A_0^2}{1 - (A_0/A_1)^2}\right]} = Cd\sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)}\,\frac{A_1 A_0}{\sqrt{A_1^2 - A_0^2}}$$

$$Or\ Q = Cd\sqrt{2g\Delta h}\left[\frac{A_0^2}{1 - (A_0/A_1)^2}\right] = Cd\sqrt{2g\Delta h}\,\frac{A_1 A_0}{\sqrt{A_1^2 - A_0^2}}$$

$$Or\ Q = Cd\sqrt{\frac{2R(\rho_m - \rho)g}{\rho}\left[\frac{A_0^2}{1 - (A_0/A_1)^2}\right]} = Cd\sqrt{\frac{2R(\rho_m - \rho)g}{\rho}}\,\frac{A_1 A_0}{\sqrt{A_1^2 - A_0^2}}$$

$$\dot{m} = Q\rho, \qquad G = u\rho = \frac{\dot{m}}{A} \qquad Re_0 = \frac{\rho u_0 d_0}{\mu}$$

For Reo > 10^4 Cd = 0.61 And for Reo > 10^4 Cd From Figure below



The holes in orifice plates may be ***concentric***, ***eccentric*** or ***segmental*** as shown in Figure. Orifice plates are prone to damage by erosion.

Example -7.9-

An orifice meter consisting of 10 cm diameter orifice in a 25 cm diameter pipe has Cd = 0.65. The pipe delivers oil of sp.gr. = 0.8. The pressure difference on the two sides of the orifice plate is measured by mercury oil differential manometer. If the differential gauge is 80 cm Hg, find the rate of flow.

Solution:

$$Q = Cd\sqrt{\frac{2R(\rho_m - \rho)g}{\rho}}\;\frac{A_1A_0}{\sqrt{A_1^2 - A_0^2}}$$
$$= 0.65\sqrt{\frac{2(0.8)(13600-800)(9.81)}{800}}\left[\frac{(\pi/4)(0.1)^2(0.25)^2}{\sqrt{[(0.25)^4-(0.1)^4]}}\right]$$
$$= 0.08196\ m^3/s$$

Example -7.10-

Water flow through an orifice meter of 25 mm diameter situated in a 75 mm diameter pipe at a rate of 300 cm^3/s, what will be the difference in pressure head across the meter μ = 1.0 mPa.s.

Solution:

Q = 300 x 10-6 $m^3/s \Rightarrow$ u = (300 x 10-6 m3/s) / (π/4 0.25^2) = 0.611 m/s

$$Q = Cd\sqrt{2g\Delta h}\ \frac{A_1 A_0}{\sqrt{A_1^2 - A_0^2}}$$

$$Re_0 - \frac{\rho u_0 d_0}{\mu} - \frac{1000(0.611)(0.025)}{1*10^{-3}} - 1.528*10^4 \rightarrow C_d - 0.61$$

$$300*10^{-6}\ m^3/s - 0.61\sqrt{2(9.01)(\Delta h)}\left[\frac{(\pi/4)(0.025)^2(0.075)^2}{\sqrt{[(0.075)^4-(0.025)^4]}}\right]$$

$$\sqrt{\Delta h} = 0.2248 \rightarrow \Delta h = 0.05\,m\,H_2O = 50\,mm\,H_2O$$

Example -7.11-

Water flow at between 3000-4000 cm^3/s through a 75 mm diameter pipe and is metered by means of an orifice. Suggest a suitable size of orifice if the pressure difference is to be measured with a simple water manometer. What approximately is the pressure difference recorded at the maximum flow rate? Cd = 0.6.

Solution:

The largest practicable height of a water manometer is 1.0 m

The maximum flow rate = $4 \times 10^{-3}\ m^3/s$

$$Q = Cd\sqrt{2g\Delta h}\ \frac{A_1 A_0}{\sqrt{A_1^2 - A_0^2}} \Rightarrow 4*10^{-3}\ m^3/s = 0.6\sqrt{2(9.81)1.0}\left[\frac{(\pi/4)(0.05)^2(d_0)^2}{\sqrt{[(0.05)^4 - d_0{}^4]}}\right]$$

$$\frac{d_0{}^2}{\sqrt{[(0.05)^4 - d_0{}^4]}} = 0.7665 \rightarrow d_0^4 = 3.67*10^{-6} - 0.5875\, d_0^4$$

$\Rightarrow$ do = 0.039 m = 39 mm

$(P_1 - P_2) = \Delta h\, \rho g = 1.0\,(1000)(9.81) = 9810$ Pa.

7.2.2.3 The Nozzle

The nozzle is similar to the orifice meter other than that it has a converging tube in place of the orifice plate, as shown in below. The velocity of the fluid is gradually increased and the contours are so designed that almost frictionless flow takes place in the converging portion; the outlet corresponds to the vena contracta on the orifice meter. The nozzle has a constant high coefficient of discharge (ca. 0.99) over a wide range of conditions because the coefficient of contraction is unity, though because the simple nozzle is not fitted with a diverging cone, the head lost is very nearly the same as with an orifice. Although much more costly than the orifice meter, it is extensively used for metering steam. When the ratio of the pressure at the nozzle exit to the upstream pressure is less than the critical pressure ratio ωc, the flow rate is independent of the downstream pressure and can be calculated from the upstream pressure alone.

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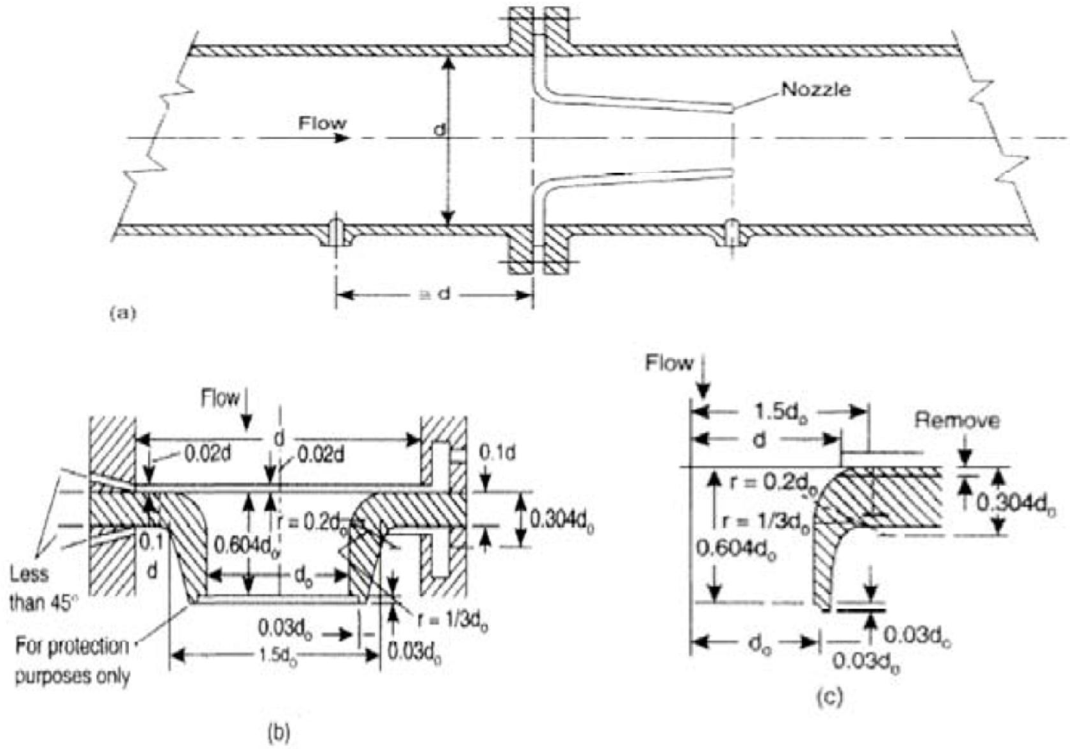


Figures of nozzle (a) General arrangement (b) Standard nozzle (Ao/A1) is less than 0.45. Left half shows construction for corner tappings. Right half shows construction for piezometer ring (c) Standard nozzle where (Ao/A1) is greater than 0.45

7.2.3 Variable Area Meters - Rotameters

In the previous flow rates the area of constriction or orifice is constant, and *the pressure drop is dependent on the rate of the flow (due to conversions between the pressure energy with kinetic energy). Float*

In the Rotameter *the drop in pressure is constant and the flow rate is function of the area of constriction. When the fluid is flowing the float rises until its weight is balanced by the up thrust of the fluid. Its position then indicting*

the rate of flow.

Force balance on the float

Gravity force = up thrust force + Pressure force

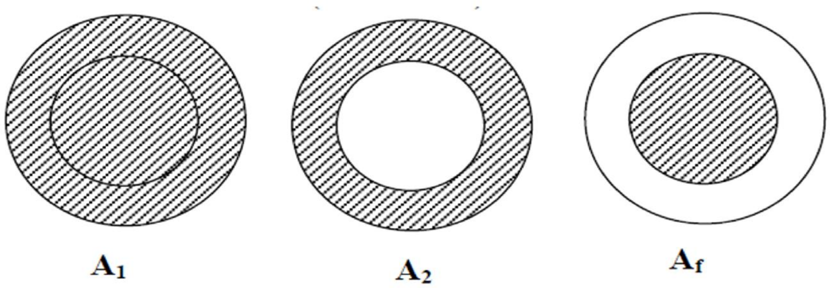
Vf ρf g = Vf ρg + (−ΔP) Af

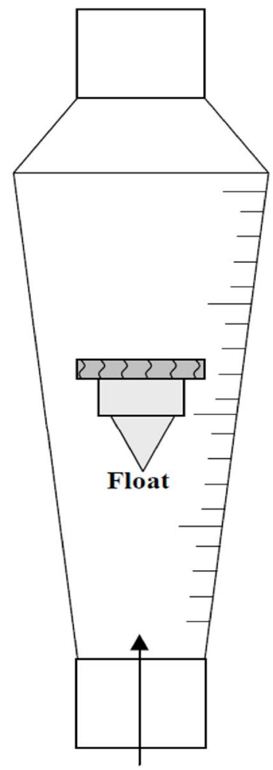
$$Q = Cd \sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)} \frac{A_1 A_0}{\sqrt{A_1^2 - A_0^2}}$$

$$Q = Cd \sqrt{\frac{2V_f g \left(\rho_f - \rho\right) g}{\rho A_f}} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

A_1 : cross-section area of the tube when the float arrived.

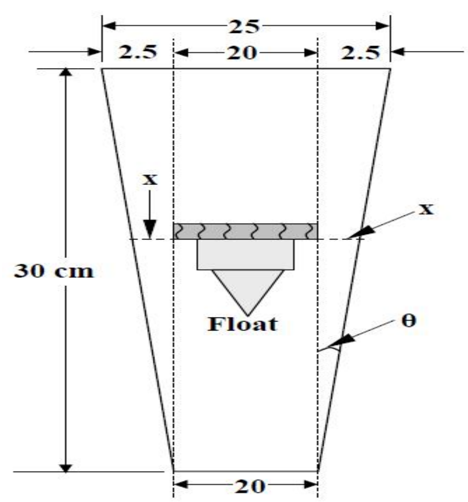
A_2 : cross-section area of the annulus (flow area).





Example -7.12-

A rotameter tube of 0.3 m long with an internal diameter of 25 mm at the top and 20 mm at the bottom. The diameter of float is 20 mm, its sp.gr. is 4.8 and its volume is 6 cm^3. If the coefficient of discharge is 0.7, what will be the flow rate water when the float is half way up the tube?



Solution:

$A1 = \pi/4\, d_1^2$, $d_1 = d_f + 2x$

To find x

1-0.25/30 = x/15, ⇒ x = 0.125 cm

2- tan (θ) = 0.25 / 30 = x/15, ⇒ x = 0.125 cm

⇒ $d_1 = 2 + 2(0.125) = 2.25$ cm

⇒ $A_1 = \pi/4\,(0.0225)2 = 3.976 \times 10\text{-}4$ m2

$A_2 = A_1 - A_f = 3.976 \times 10\text{-}4 - \pi/4\,(0.02)^2$

$= 8.345 \times 10\text{-}5\ m^2$

$$Q = Cd\sqrt{\frac{2V_f g\left(\rho_f - \rho\right)g}{\rho A_f}}\ \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$= 0.7\sqrt{\frac{26*10^{-6}(9.81)(4.8-1)}{(\pi/4)(0.0)^2}}\ \frac{(3.976*10^{-4})(8.345*10^{-5})}{\sqrt{\left[(3.976*10^{-4})^2 - \left(8.345*10^{-5}\right)^2\right]}}$$

$$= 7.13*10^{-5}\ m^3/s$$

Example -7.13-

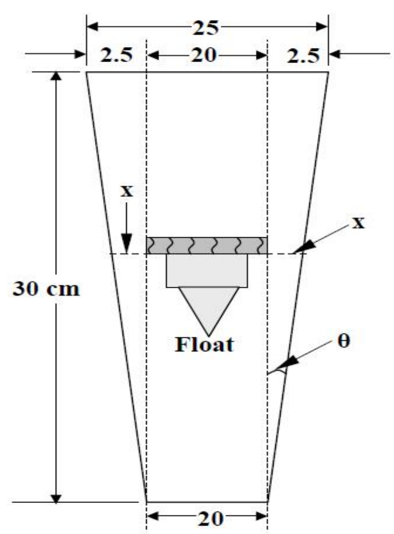
A rotameter has a tube of 0.3 m long, which has an internal diameter of 25 mm at the top and 20 mm at the bottom. The diameter of float is 20 mm, its effective sp.gr. is 4.8 and its volume is 6.6 cm^3. If the coefficient of discharge is 0.72, what height will the float be when metering water at 100 cm^3/s?

Solution:

$$Q = 10^{-4}\ m^3/s = Cd\sqrt{\frac{2V_f g\left(\rho_f - \rho\right)g}{\rho A_f}}\ \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$\frac{A_1 A_2}{\sqrt{A_1^2 \quad A_2^2}} = 1.10976*10^{-4} = \frac{A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

$$\text{Assume that } 1 - \left(\frac{A_2}{A_1}\right)^2 = 0 \qquad i.e\ A_2 = 0$$



⇒ A2 = 1.10976 x 10-4 m^2, A1 = A2 + Af

⇒ A1 = 4.2513 x 10-4 m^2

Correct the assumption $\sqrt{1-\left(\frac{A_2}{A_1}\right)^2} = 0.965$

⇒ A2 = 0.965 (1.10976 x 10^{-4} m^2) = 1.0713 x 10^{-4} m^2

A1 = A2 + Af ⇒ A1 = 4.213 x 10^{-4} m^2

Re-correct the last value

$\sqrt{1-\left(\frac{A_2}{A_1}\right)^2} = 0.967$ – – – – – – – – – close enough

⇒ d1 = (A1 / π/4)$^{0.5}$ = 0.02316 m = 2.316 cm

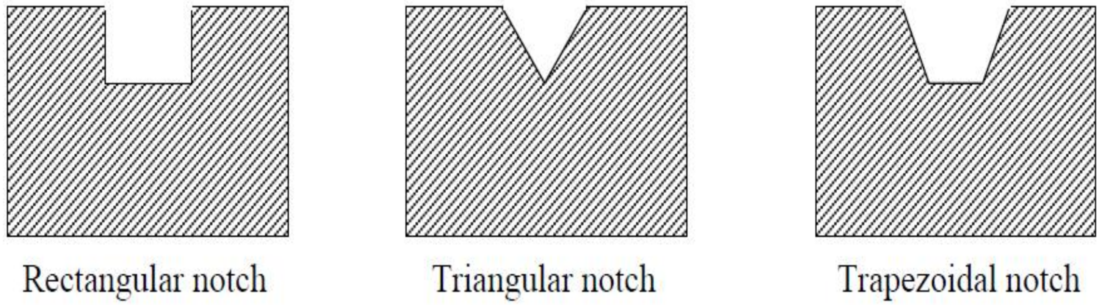
d1 = 2x + df ⇒ x = (0.02316 – 0.02) / 2 = 0.0016 m = 0.16 cm

0.25/30 = 0.16/ L ⇒ L = 19.2 cm

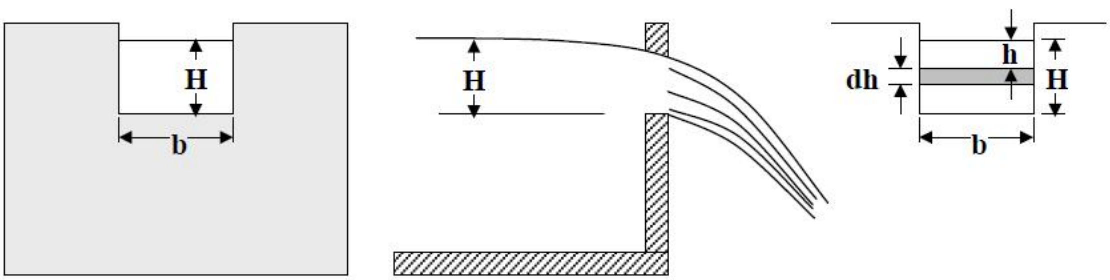
7.2.4 The Notch or Weir

The flow of liquid presenting a free surface (open channels) can be measured by means of a weir. The pressure energy converted into kinetic energy as it flows over the weir, which may or may not cover the full width of the stream, and a calming screen may be fitted before the weir. Then the height of the weir crest gives a measure of the rate of flow. The velocity with which the liquid leaves depends on its initial depth below the surface.

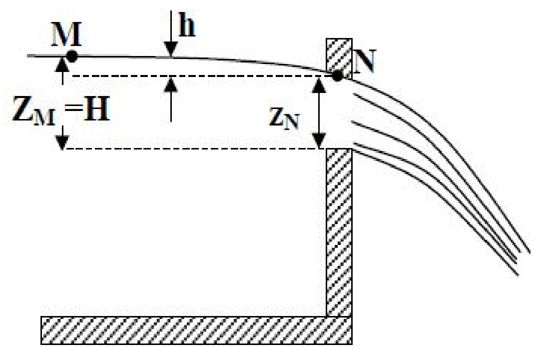
Many shapes of notch are available of which three shapes are given here as shown in Figures,



7.2.4.1 Rectangular Notch



H: height of liquid above base of the notch
h: depth of liquid from its level
b: width or length of notch
Consider a horizontal strip of liquid of thickness (dh) at depth (h).
The theoretical velocity of liquid flow through the strip $= \sqrt{2gh}$



To prove this equation applies Bernoulli's equation between points M and N as shown in Figure;

$$\frac{P_M}{\rho g} + \frac{u_M^2}{2g} + z_M - \frac{P_N}{\rho g} + \frac{u_N^2}{2g} + z_N$$

The cross sectional area of flow at point M is larger than that at notch (point N), then ($u_M \approx 0$)

$P_M = P_N = P_o$ atmospheric pressure

$$z_M - z_N - \frac{u_N^2}{2g} \therefore u_N = \sqrt{2gh}$$

The area of the strip **dA = b.dh** The discharge through the strip

$$\rightarrow \int_0^Q dQ = C_d\sqrt{2g}\int_0^H h^{1/2}\,dh \Rightarrow Q = C_d b\sqrt{2g}\,\frac{H^{3/2}}{3/2} = \frac{2}{3}C_d\, b\,\sqrt{2g}\; H^{3/2}$$

Example -7.14-

A rectangular notch 2.5 m wide has a constant head of 40 cm, find the discharge over the notch where Cd = 0.62

Solution:

$$Q = \frac{2}{3}C_d\, b\,\sqrt{2g}\; H^{3/2}$$ **= 2/3 (0.62) (2.5) (2 x 9.81) $(0.4)^{3/2}$ =1.16 m^3/s**

Example -7.15-

A rectangular notch has a discharge of 21.5 m3/min, when the head of water is half the length of the notch. Find the length of the notch where Cd = 0.6.

Solution:

$$Q = \frac{2}{3}C_d\, b\,\sqrt{2g}\; H^{3/2}$$ ⇒ **21.5/60 = 2/3 (0.6) (b) $(2 \times 9.81)^{0.5}$ $(0.5\ b)^{3/2}$**

⇒ $b^{5/2}$ = 0.572 ⇒ b = $(0.572)^{2/5}$ = 0.8 m

7.2.4.2 Triangular Notch

A triangular notch is also called a *V-notch.*

H: height of liquid above base of the apex of the notch.

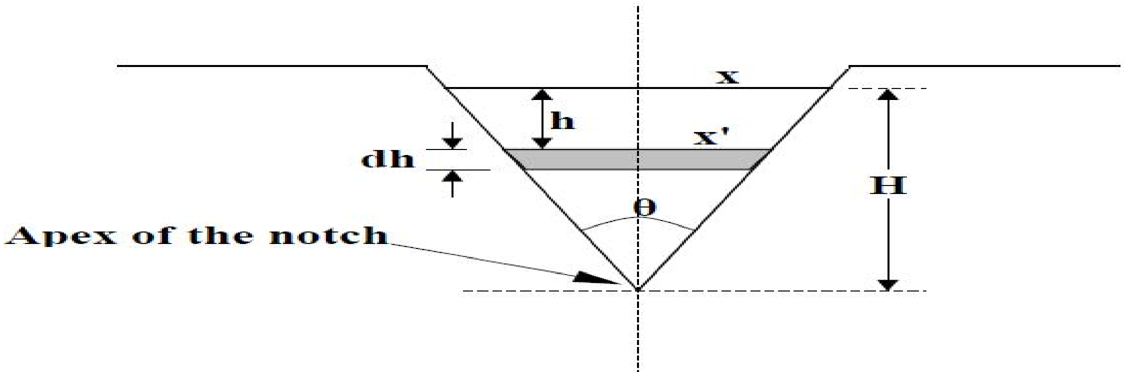
θ: Angle of the notch.

tan (θ/2) = x / H = x' / (H-h)

The width of the notch at liquid surface = 2x = 2H tan(θ/2)

The width of the strip = 2x' = 2(H-h) tan(θ/2)

The area of the strip = 2x' dh = 2(H-h) tan(θ/2)dh



The theoretical velocity of water through the strip = $\sqrt{2gh}$

The discharge over the notch dQ = u. dA= $C_d(\sqrt{2gh})[2(H-h)\tan(\vartheta/2)dh]$

$$\int_0^Q dQ = 2C_d \tan(\theta/2)\sqrt{2g}\int_0^H \left(Hh^{1/2} - h^{3/2}\right) dh$$

$$Q = 2C_d \tan(\theta/2)\sqrt{2g}\left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$\therefore Q = \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\ H^{5/2}$$

If Cd = 0.6 and θ =90^0 ⇒ Q = 1.417 H$^{5/2}$

Example -7.16-

During an experiment in a laboratory, 50 liters of water flowing over a right-angled notch was collected in one minute. If the head of still is 50mm. Calculate the coefficient of discharge of the notch.

Solution:

$$Q = \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\ H^{5/2}$$

Q= 50 lit/min (m^3/1000lit)(min/60s) = 8.334 x 10^{-4} m^3/s

⇒ Cd = (8.334 x 10^{-4})/[(8/15)$(2x9.81)^{0.5}$ tan(θ/2)$(0.05)^{5/2}$]

⇒ Cd = 0.63

Example -7.17-

A rectangular channel 1.5 m wide is used to carry 0.2 m^3/s water. The rate of flow is measured by placing a 90^o V-notch weir. If the maximum depth of water is not to exceed 1.2 m, find the position of the apex of the notch from the bed of channel. Cd = 0.6.

Solution:

Q = 1.417 $H^{5/2}$ ⇒ $H^{5/2}$ = (0.2 m^3/s)/1.417 ⇒ H = 0.46 m

The maximum depth of water in channel = 1.2 m

H is the height of water above the apex of notch.

Apex of triangular notch is to be kept at distance = 1.2 –0.46

= 0.74 m from the bed of channel.

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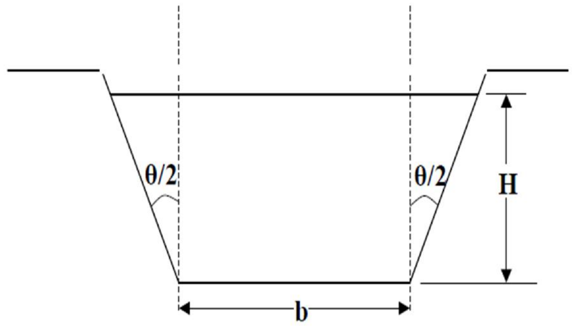
7.2.4.3 Trapezoidal Notch

A trapezoidal notch is a combination of a rectangular notch and triangular notch as shown in Figure;



Discharge over the trapezoidal notch,
Q=[Discharge over the rectangular notch+ Discharge over the triangular notch]

$$\therefore Q = \frac{2}{3} C_{d1} b \sqrt{2g}\; H^{3/2} + \frac{8}{15} C_d \tan(\theta/2) \sqrt{2g}\; H^{5/2}$$

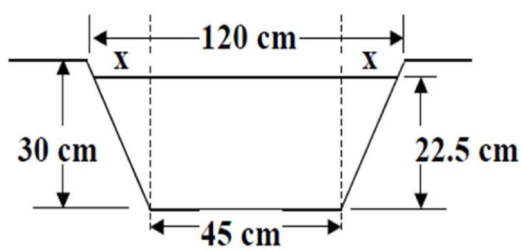
Example -7.18-

A trapezoidal notch 120 cm wide at top and 45 cm at the bottom has 30 cm height. Find the discharge through the notch, if the head of water is 22.5 cm. Cd1 = Cd2 = 0.6.

Solution:

x = (120+45)/2 = 37.5 cm

tan(θ/2) = x/30 = 37.5/30 = 1.25



$$\therefore Q = \frac{2}{3} C_{d1} b \sqrt{2g}\; H^{3/2} + \frac{8}{15} C_d \tan(\theta/2) \sqrt{2g}\; H^{5/2}$$

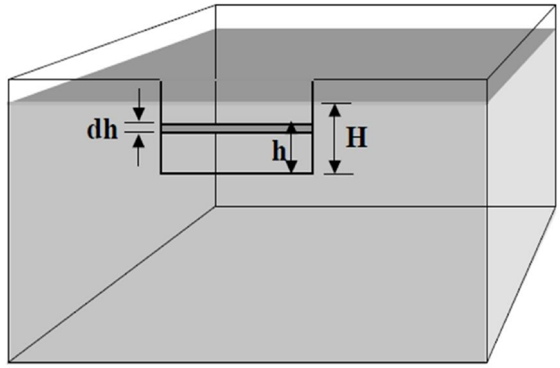
Q =2/3(0.6)(0.45)(2 x 9.81)$^{0.5}$ 0.225)$^{3/2}$ +8/15(0.6) (2 x 9.81)$^{0.5}$ (1.25)(0.225)$^{5/2}$

= 0.1276 m^3/s

7.3 Unsteady State Problems

Example -7.19-

A reservoir 100 m long and 100 m wide is provided with a rectangular notch 2m long. Find the time required to lower the water level in the reservoir from 2 m to 1 m. Cd = 0.6.



Solution:

Let, at some instant, the height of the water above the base of the notch be (h) and the liquid level fall to small height (dh) in time (dt). The volume of water discharged in time (dt)

is: dV = - A dh, A= 100 x 100 = 10^4 m^2

$$Q = \frac{dV}{dt} = \frac{2}{3} C_d b \sqrt{2g}\ h^{3/2} = -A\ dh/dt$$

$$\int_0^T dt = \frac{-A}{(2/3) C_d b \sqrt{2g}} \int_{H_1}^{H_2} h^{-3/2} dh$$

$$T = \frac{3}{2} \frac{-A}{C_d b \sqrt{2g}} \left[\frac{h^{-1/2}}{-1/2} \right]_{H_1}^{H_2}$$

$$= \frac{3A}{C_d b \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right] = \frac{3 * 10^4}{0.6(2)\sqrt{2 * 9.81}} \left[\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} \right]$$

$$= 1653.1\ sec = 27.33\ min$$

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Example -7.20-

A tank 25 m long and 15 m wide is provided with a right-angled V-notch. Find the time required to lower the level in the tank from 1.5 m to 0.5 m. Cd = 0.62.

Solution:

Let, at some instant, the height of the liquid above the apex of the notch be (h) and a small volume of the liquid (dv) flow over the notch in a small interval of time (dt), reducing the liquid level by an amount (dh) in the tank.

dV = - A dh, A= 25 x 15 = 375 m^2

$$Q = \frac{dV}{dt} = \frac{8}{15} C_d \sqrt{2g}\, tan\,(\theta/2) h^{5/2} = -A\ dh/dt$$

$$\int_0^T dt = \frac{-A}{(8/15) C_d \sqrt{2g}\, tan\,(\theta/2)} \int_{H_1}^{H_2} h^{-5/2} dh \Rightarrow T = \frac{8}{15} \frac{-A}{C_d \sqrt{2g}\, tan\,(\theta/2)} \left[\frac{h^{-3/2}}{-3/2}\right]_{H_1}^{H_2}$$

$$= \frac{5}{4} \frac{A}{C_d \sqrt{2g}\, tan\,(\theta/2)} \left[\frac{1}{\sqrt{H_2^3}} - \frac{1}{\sqrt{H_1^3}}\right]$$

$$= \frac{5}{4} \frac{375}{0.6\sqrt{2 * 9.81 * 1}} \left[\frac{1}{\sqrt{0.5^3}} - \frac{1}{\sqrt{1.5^3}}\right] = 390\ sec$$

$$= 6.3\ min$$

Home Work

P.7.6

A wier 8 m length is to be built across a rectangular channel to discharge a flow of 9 m3/s. If the maximum depth of water on the upstream side of weir is to be 2 m, what should be the height of the weir? Cd = 0.62.

Ans. 1.277 m

P.7.7

A rectangular notch 1 m long and 40 cm high is discharging water. If the same quantity of water be allowed to flow over a 90^o V-notch, find the height to which water will rise above the apex of notch. Cd = 0.62.

Ans. Q = 464 lit/s, H = 63.1 cm

P.7.8

Water flow over a right angled V-notch under a constant head of 25 cm. 1- Find the discharge. 2- Using principles of geometric similarity find the head required for a flow of 1417.6 lit/s through the same notch. Cd = 0.62.

<u>Hint</u>

For similar notch $\frac{Q_1}{Q_2} = \frac{b_1}{b_2}\left[\frac{H_1}{H_2}\right]^{3/2}$ and for the same notch $\frac{Q_1}{Q_2} = \left[\frac{H_1}{H_2}\right]^{3/2}$

Ans. Q = 44.3 lit/s, H_2 = 1 m

P.7.9

A sharp-edge 90º V-notch is inserted in the side of a rectangular tank 3 m long and 1.5 m wide. Find how long it will take to reduce the head in tank from 30 cm to 7.5 cm if the water discharges freely over the notch and there is no inflow into the tank. Cd = 0.62.

Ans. T = 87s = 1min 27s