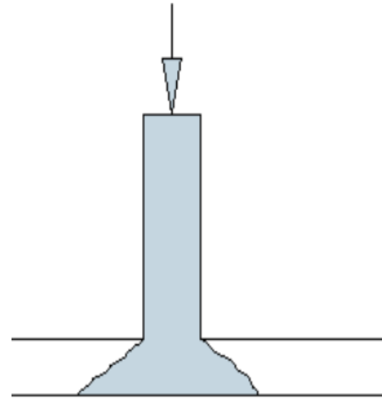
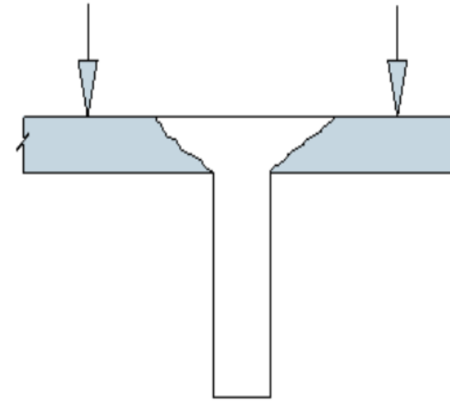
Shear in Slabs

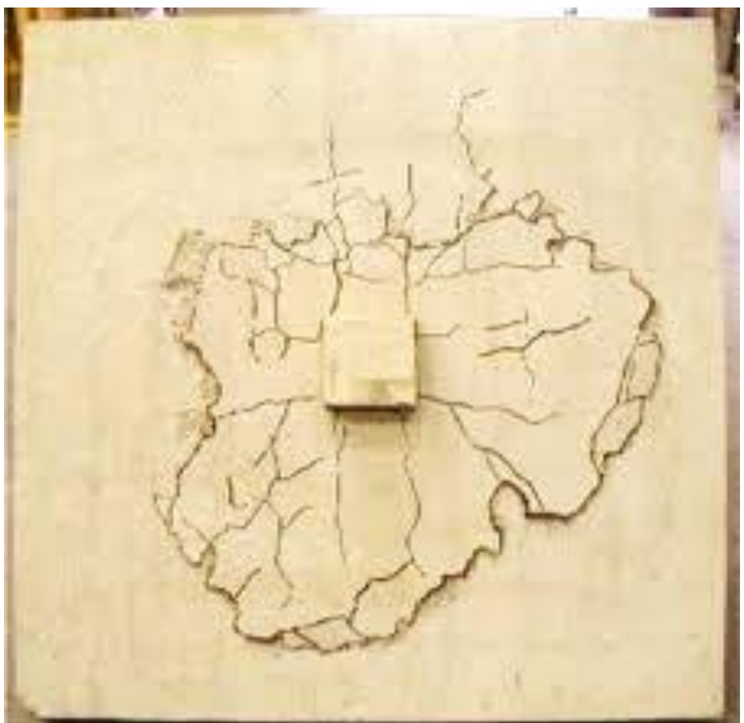
Prof. Dr. Khattab Saleem Abdul-Razzaq

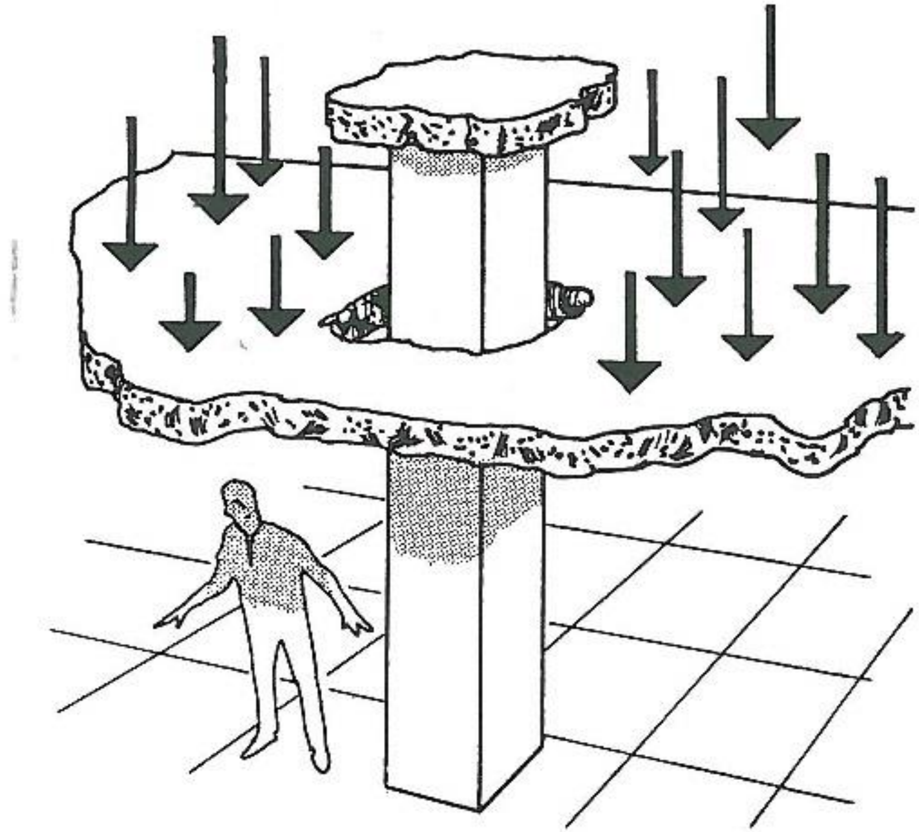


Column on Slab



Slab on Column



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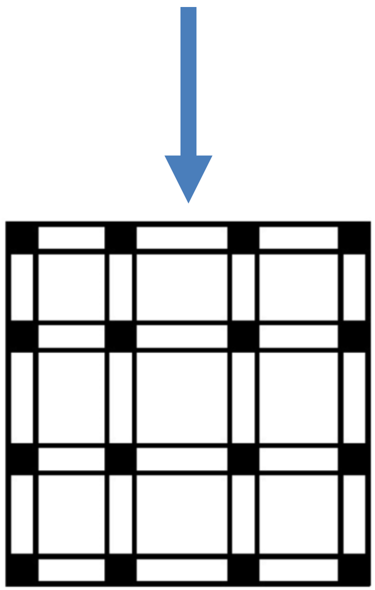
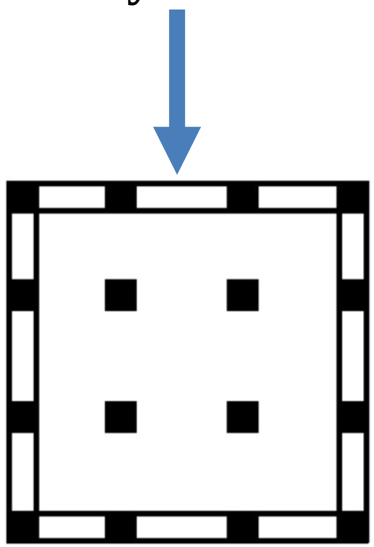
There are two types of shear in rc slabs:

1. One-way shear only

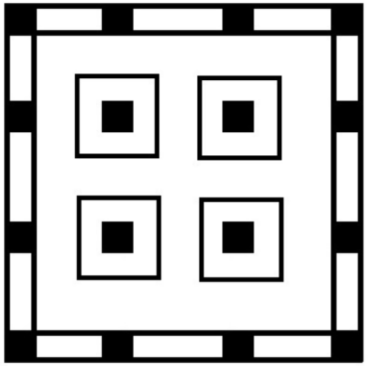


rc slab with beams between all supports, with edge beam

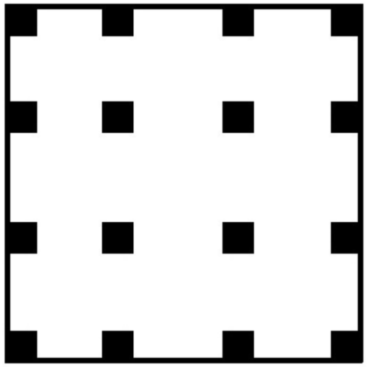
2- Both 1-way and 2-way shear



rc slab without internal beams, with edge beam



rc slab with drop panels, with edge beam



rc flat plate slab

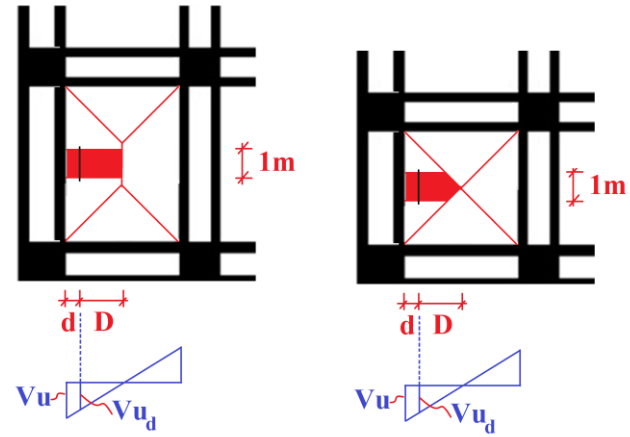
The **flat slab** includes either drop panels or column capitals at columns.
The **flat plate slab** is just a flat plate!!!

1. one-way shear

$$Vu_d = Wu * D$$

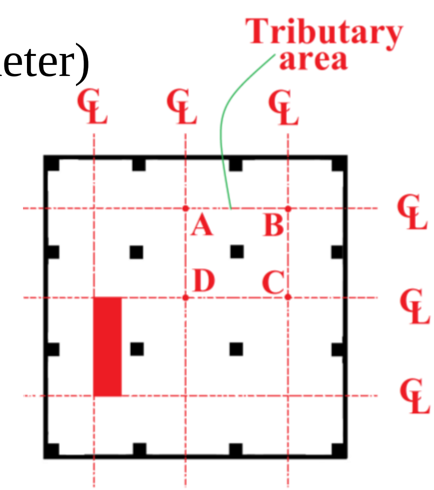
$$\emptyset Vc = \frac{0.75}{6}\sqrt{f'c}\, b\, d$$

$$if\ \emptyset Vc \ \geq \ Vu_d \quad o.k$$



Vu=max shear at the face of support
Vud= shear at d from the support face
Wu= ultimate load (1.2D+1.6L)
D= distance from span centre to d (in short direction)
φVc= factored concrete shear resistance (without reinf.)
φ= shear reduction factor=0.75
d= effective depth (h - 20mm concrete cover - 0.5 bar diameter)

b=1m



$$\boldsymbol{if\ \emptyset Vc \ < \ Vu_d \quad not\ o.k, so:}$$

- Increase slab thickness
- Increase f'c

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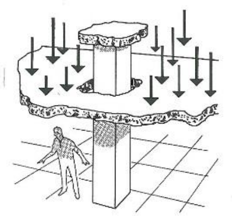
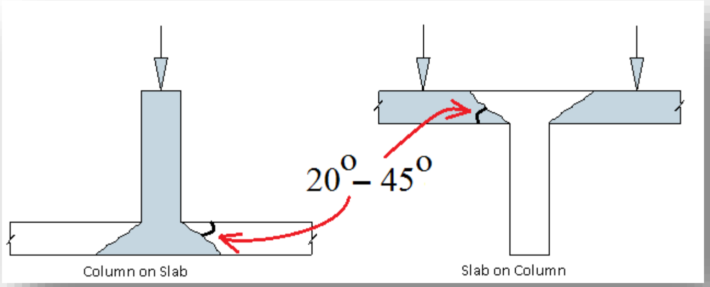
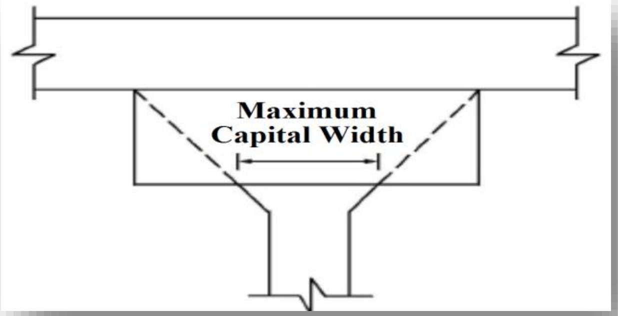
2. Two-way shear (Punching shear)

$$if\ \emptyset Vc \geq Vu_d \quad o.k$$



$$Vc = min\begin{Bmatrix} \frac{\lambda}{3}\sqrt{f'c}\, b_o\, d \\ \left(1+\frac{2}{\beta}\right)\frac{\lambda\sqrt{f'c}}{6} b_o\, d \\ \left(2+\frac{\alpha_s}{\beta_o}\right)\frac{\lambda\sqrt{f'c}}{12} b_o\, d \end{Bmatrix}$$





$\lambda = 1\ for\ normal\ weight\ concrete$

$b_o = Circumference\ of\ the\ critical\ section$

$d = average\ effective\ depth$

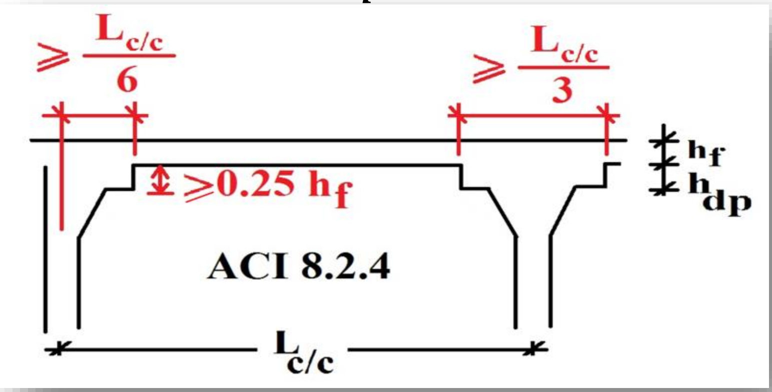
$\alpha_s = 40\ for\ int., 30\ for\ ext., and\ 20\ for\ corner\ column.$

$\beta_o = {b_o}/{d}$

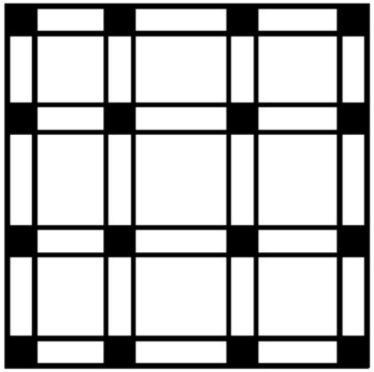
$\beta = longer\ to\ shorter\ section\ dimensions\ for\ col., or\ col. capital.$

$\boldsymbol{if\ \emptyset Vc < Vu_d \quad not\ o.k, so}$:

- Increase column section dimensions
- Increase slab thickness
- Increase f'c
- Add col. capital
- Use drop panel, in case of flat plate.
- Add reinforcement.



1. Slab with beams between all supports: (only 1-way shear)



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Wu= ultimate load (1.2D+1.6L)

D= distance from span centre to d (in short direction)

φVc= factored concrete shear resistance (without reinf.)

φ= shear reduction factor=0.75

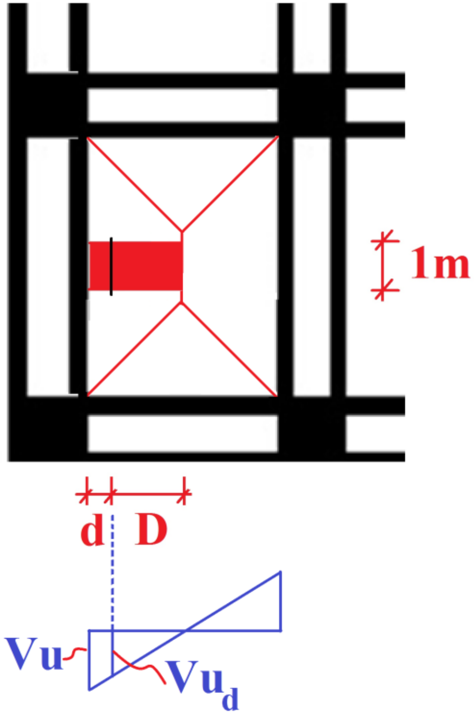
d= effective depth (h - 20mm concrete cover - 0.5 bar diameter)

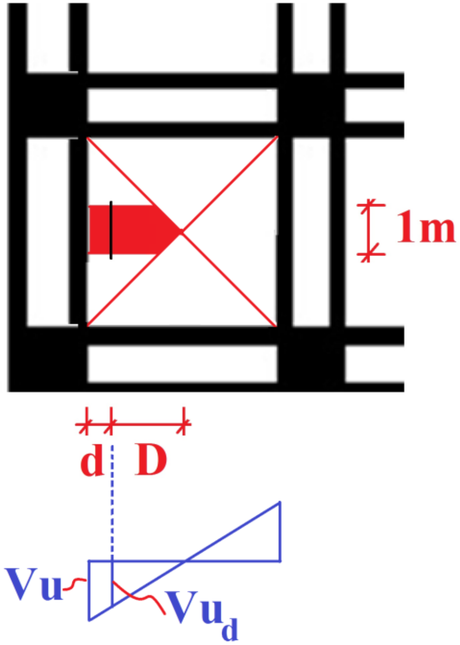
b=1m

$$Vu_d = Wu * D$$

$$\emptyset Vc = \frac{0.75}{6} \sqrt{f'c}\, b\, d$$

$$if\ \emptyset Vc \geq Vu_d \quad o.k$$



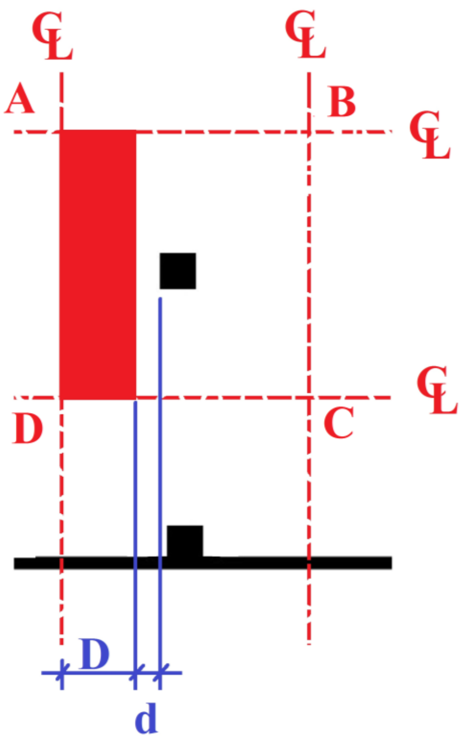


V = force
v = stress

2. Slabs without beams between supports: (both 1&2-way shear)

shear at centrelines=zero

2.1 One-way shear (wide beam shear) in slabs without beams between supports:



$$Vu_d = Wu * D$$

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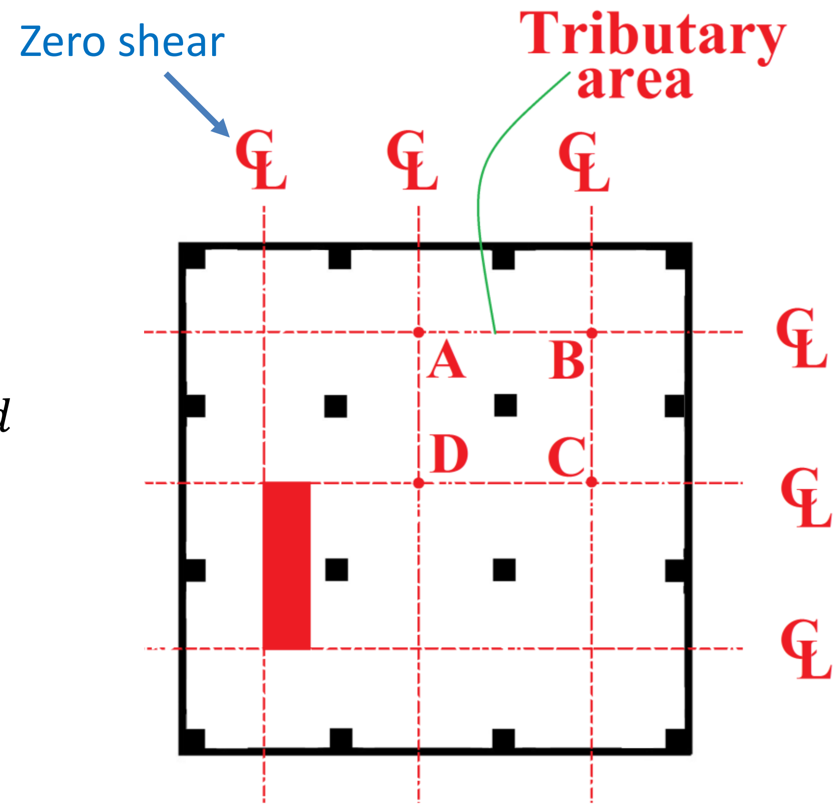
$$if\ \emptyset Vc \geq Vu_d \quad o.k$$

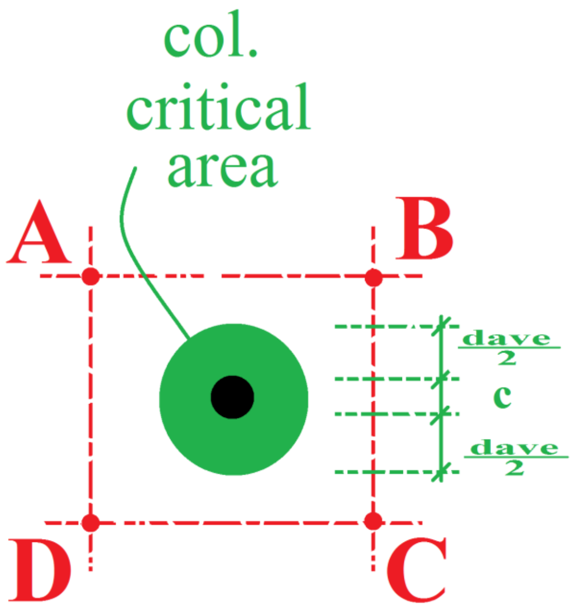


Notes:

- Convert circular column to equivalent square one in 1-way.
- Use (d_{ave}=h-20-bar diam.) for 2-way and d for 1-way.

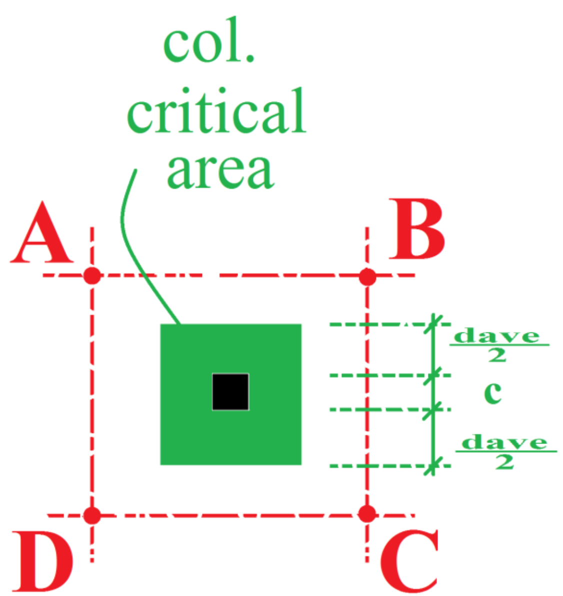
2.2 Two-way shear in slabs without beams between supports:

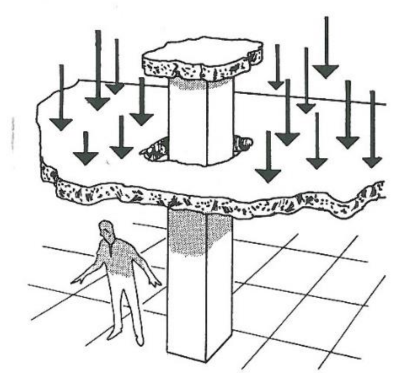
2.2.1 <u>Without</u> drop panel:

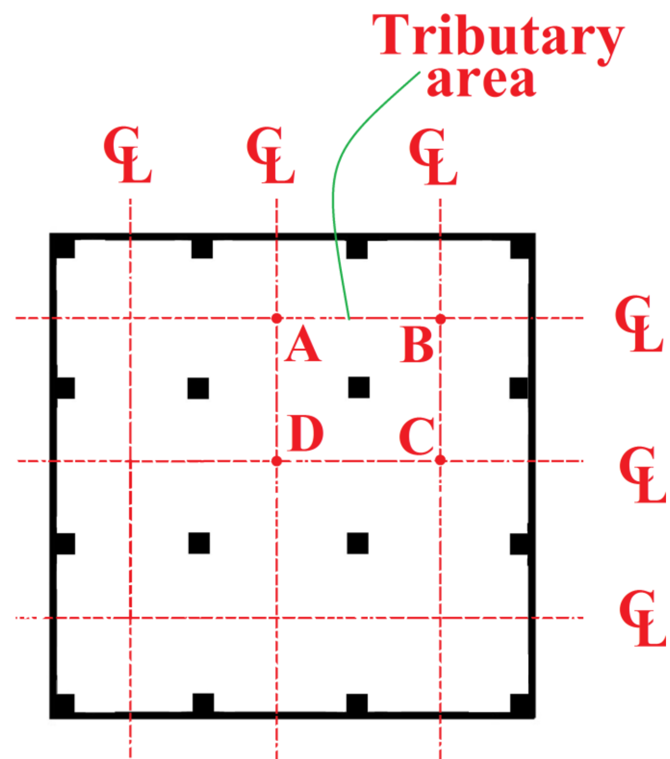


$$Vu_p = Wu\,[ABCD - col.\,critical\ area]$$

$$if\ \emptyset Vc \geq Vu_d \quad o.k$$

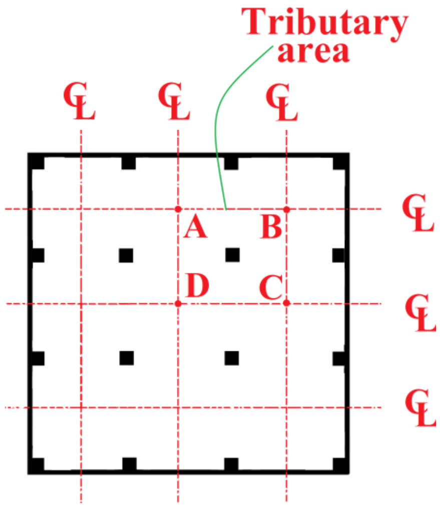






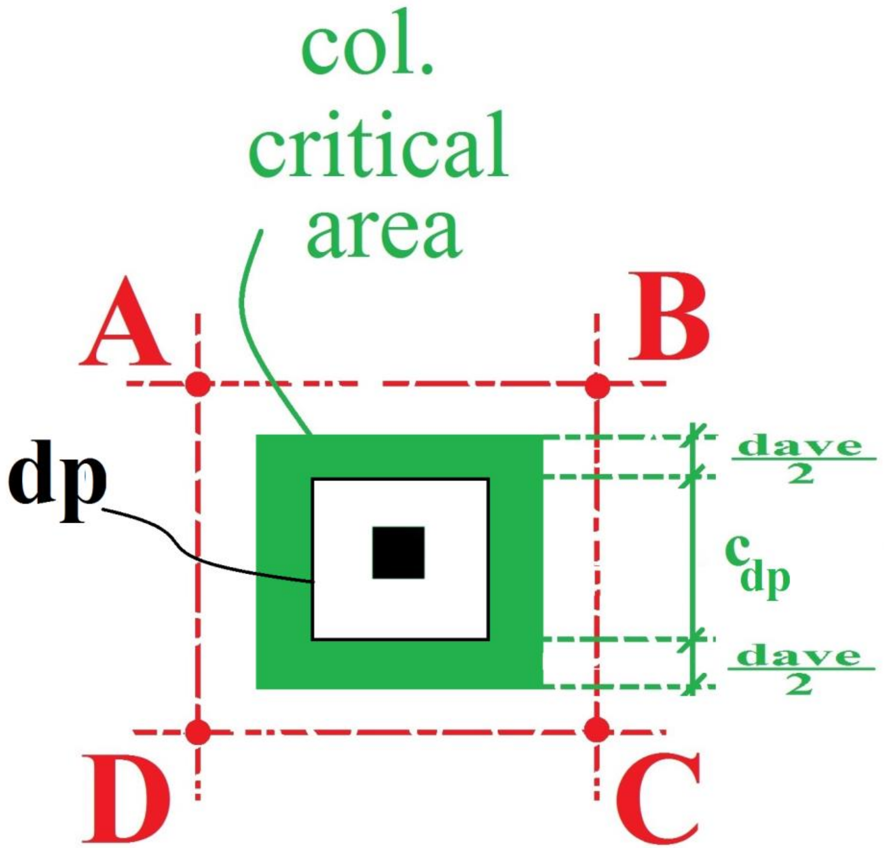
2.2.2 <u>With</u> drop panel:

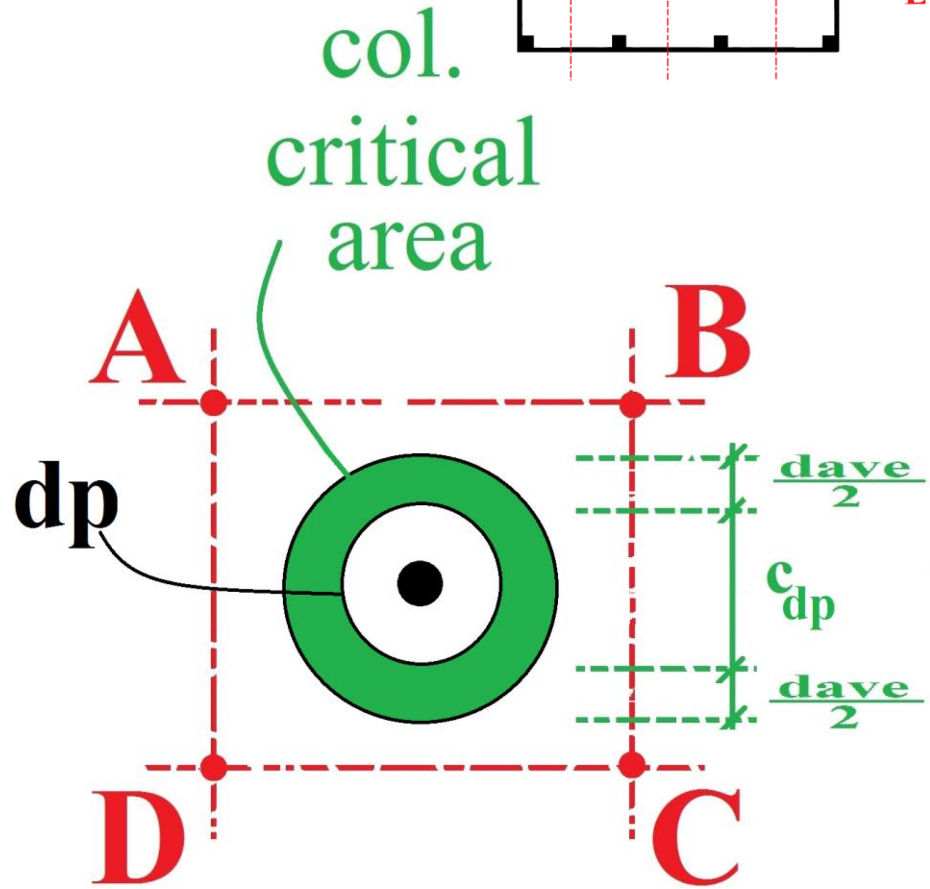
2.2.2.1 <u>About slab</u>:



$$Vu_p = Wu\,[ABCD - col.\,critical\,area]$$

$$if\ \emptyset Vc \geq Vu_d \quad o.k$$

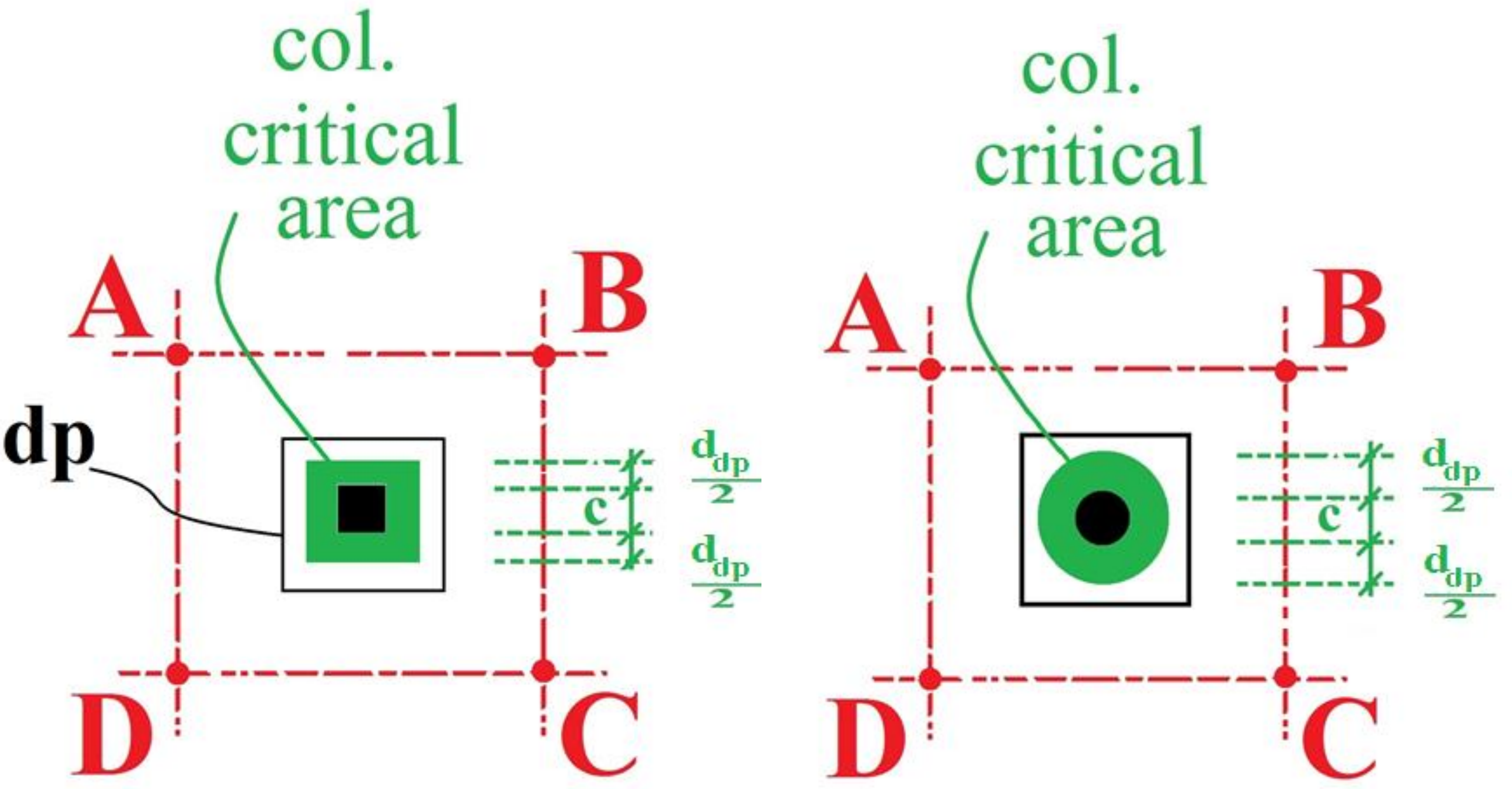




2.2.2.2 __About drop panel__:

$$Vu_p = Wu\ [ABCD - col.\,critical\ area]$$

$$if\ \emptyset Vc\ \geq\ Vu_d \quad o.k$$



Moments applied on columns and walls:

1-Internal column and walls (ACI 318-14, 8.10.7.2):

$$M_{sc} = 0.07[q_{DU} + 0.5\, q_{LU})l_2 l_n^2 - q'_{DU}\, l'_2 l'^2_n]$$

where

q_{DU}=factored dead load applied on longer span

q_{LU}=factored live load applied on longer span

q'_{DU}=factored dead load applied on shorter span

l_2 = strip width in the longer span

l'_2= strip width in the shorter span

l_n =clear in the longer direction

l'_n =clear in the shorter direction

> **8.10.7.2** At an interior support, columns or walls above and below the slab shall resist the factored moment calculated by Eq. (8.10.7.2) in direct proportion to their stiffnesses unless a general analysis is made.
>
> $$M_{sc} = 0.07[(q_{Du} + 0.5q_{Lu})\ell_2\ell_n{}^2 - q_{Du}'\ell_2'(\ell_n')^2] \quad (8.10.7.2)$$
>
> where q_{Du}', ℓ_2', and ℓ_n' refer to the shorter span.

Note: If the spans on both sides of the column are equal, and the strip has the same width (l_2):

$$M_{sc} = 0.035\, q_{LU} l_2\, l_n^2$$

2-External columns and walls:

The moment that is transferred from the external slabs to the external supports = total external negative moment of the design strip (i.e. before distribution to column and middle strips).

Note: the moments are distributed between the lower and the upper columns by dividing according to (EI/L)

Transfer of Moments at Columns

- We previously studied that shear stresses are distributed uniformly around the circumference b_o....
- But: if the column is under unbalanced moments on both sides, the hypothesis of uniform distribution will not be accurate…
- Part of the moment will be transferred as shear, added to one side and subtracted from the other side ...
- Moments transfer from slab to column through:

1-Flexure (M_{uf}) + 2-Shear (M_{uv})

Distribution of unbalanced Moments

$M_{uf} = \gamma_f \, Mu$ (ACI 318-14, 8.4.2.3.2)

$M_{uv} = \gamma_v \, Mu = (1 - \gamma_f) \, Mu$

$\gamma_f = 0.6$ for square column, i.e., $\gamma_v = 0.4$

More specifically:

$$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}}$$

8.10.7.3 The gravity load moment to be transferred between slab and edge column in accordance with 8.4.2.3 shall not be less than $\mathbf{0.3}M_o$.

γ_f = factor used to determine the fraction of M_{sc} transferred by slab flexure at slab-column connections

Where b_1 and b_2 are critical section width, parallel and perpendicular to the analysis direction, respectively.

According to ACI 318-14, 8.10.7.3, transferred moment from slab to edge column ≥ 30%M_o.

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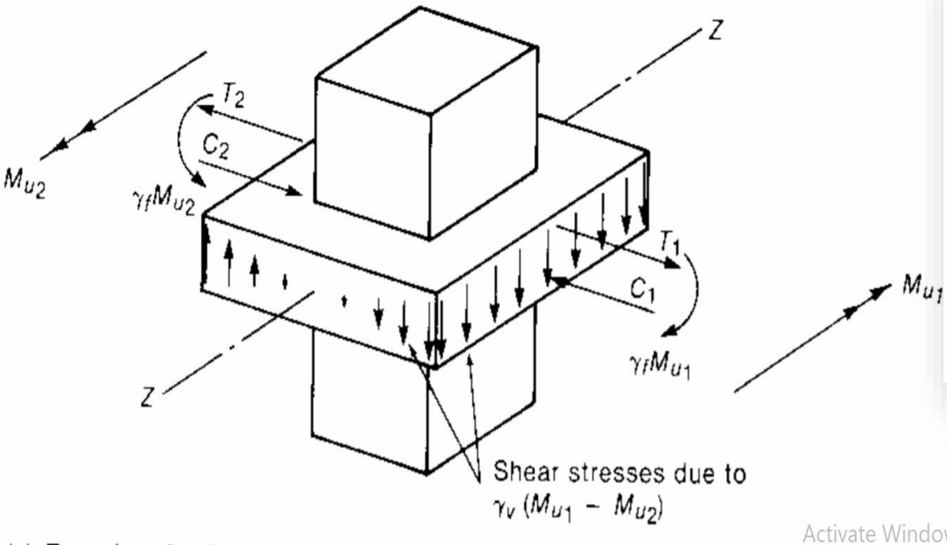
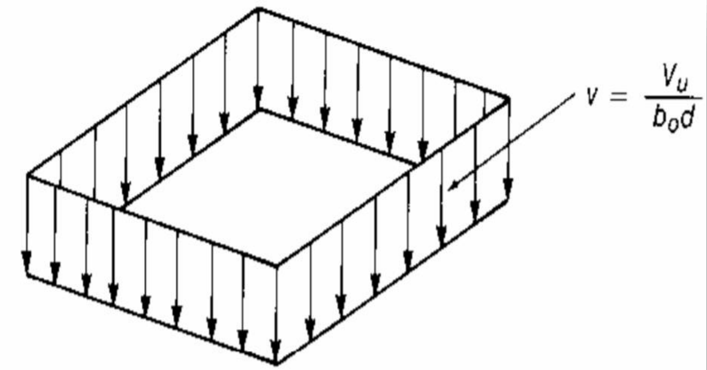
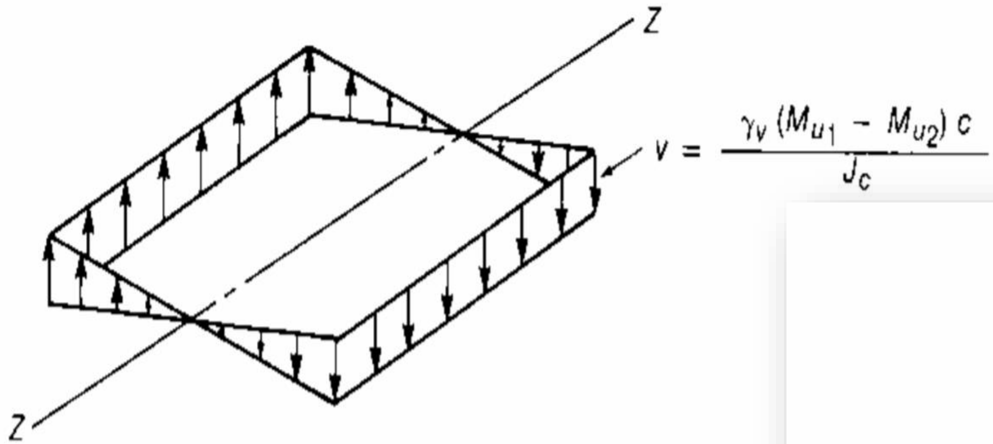


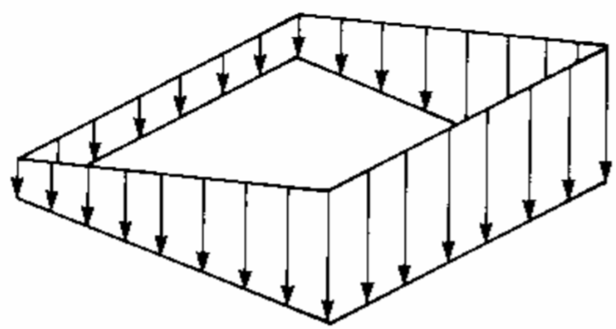
(a) Transfer of unbalanced moments to column.



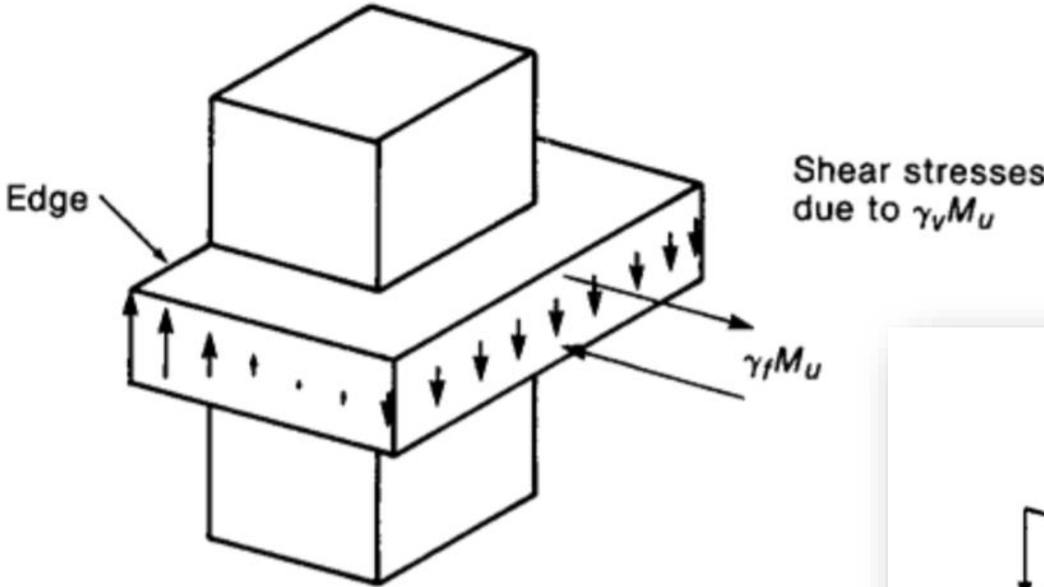
(b) Shear stresses due to V_u.



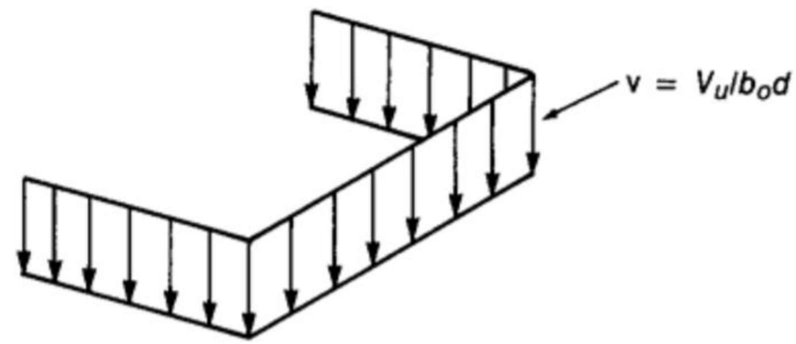
(c) Shear due to unbalanced moment.



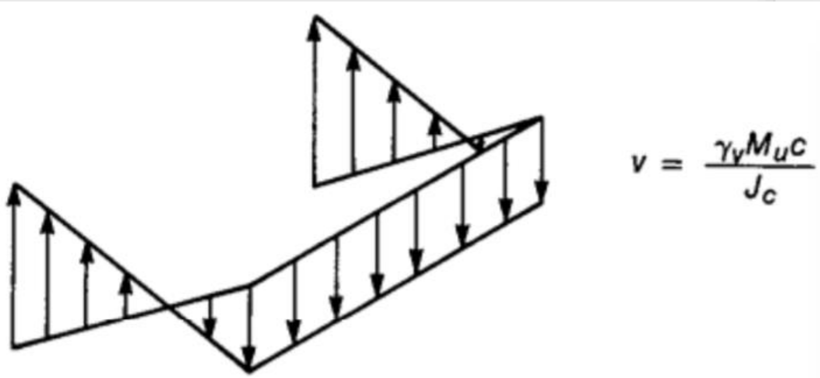
(d) Total shear stresses.



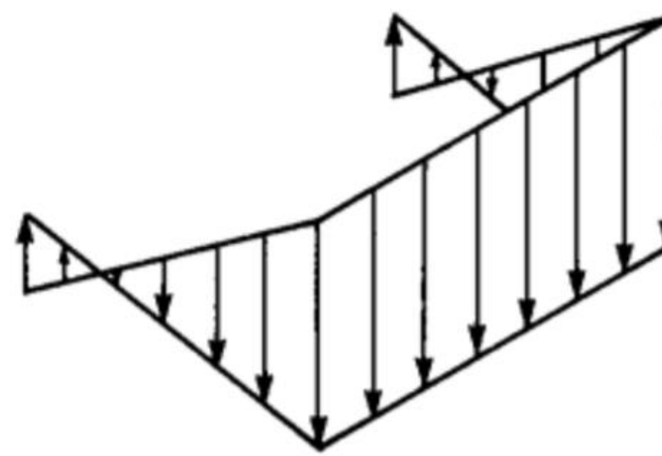
(a) Transfer of moment at edge column.



(b) Shear stresses due to V_u.



(c) Shear stresses due to M_u.



(d) Total shear stresses.

Modifications of Moment transfer ratios

$$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}}$$

Table 8.4.2.3.4—Maximum modified values of γ_f for nonprestressed two-way slabs

Column location	Span direction	v_{ug}	ε_t (within b_{slab})	Maximum modified γ_f
Corner column	Either direction	$\leq 0.5\phi v_c$	≥ 0.004	1.0
Edge column	Perpendicular to the edge	$\leq 0.75\phi v_c$	≥ 0.004	1.0
	Parallel to the edge	$\leq 0.4\phi v_c$	≥ 0.010	$\frac{1.25}{1+\left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} \leq 1.0$
Interior column	Either direction	$\leq 0.4\phi v_c$	≥ 0.010	$\frac{1.25}{1+\left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} \leq 1.0$

$$\boldsymbol{\varepsilon_t \geq 0.004} \qquad when$$

$$\boldsymbol{\rho \leq \rho_{max} = 0.85\,\beta_1 \frac{f'c}{fy} \frac{\varepsilon_u}{\varepsilon_u + 0.004}}$$

$\boldsymbol{\varepsilon_t}$ is reinforcement strain closest to the tension face in the effective slab width (b_{salb}).

b_{salb}=perpendicular dimension of column(c_2)+2(1.5h).

h=either slab thickness or drop panel thickness

$$\boldsymbol{\varepsilon_t \geq 0.01} \qquad when$$

$$\boldsymbol{\rho \leq 0.85\,\beta_1 \frac{f'c}{fy} \frac{\varepsilon_u}{\varepsilon_u + 0.01}}$$

Check slab after moment transfer (for flat plate and flat slabs)

A-Check shear stresses due to M_{uf}:

1. 1-Use DDM or EFM to find Msc applied on the column
2. Calculate Υ_f
3. Modifications of Moment transfer ratios (Table 8.4.2.3.4)
4. Calculate $M_{uf} = \Upsilon_f \, Mu$
5. Calculate b_{slab} (b_{salb}=perpendicular dimension of column(c_2)+2(1.5h).)
6. $\emptyset Mn = \emptyset \, \rho \, b_{slab} \, d^2 \, fy \, (1 - 0.59 \, \rho \, \frac{fy}{f'c})$

   Where ρ for perpendicular strip

7. if $M_{uf} \leq \varphi Mn$ ok, otherwise reinforcement should be added to resist the difference between M_{uf} and φMn

B-Check punching shear stresses due to M_{uv} and V_u:

1. 1-Use DDM or EFM to find Msc applied on the column
2. Calculate Vu at d/2
3. Calculate γ_f
4. Modifications of Moment transfer ratios (Table 8.4.2.3.4)
5. Calculate $M_{uv} = (1-\gamma_f) Mu$
6. Calculate J and c, in addition to (Ac = critical area = b_o * d) and (c'=b1-c)

J = Critical shear section characteristic

c, c'= distance from the (centre to the end) of the critical area

7. Calculate *vu1* $\left(vu1 = \frac{Vu}{Ac} + \frac{Muv \ \ c}{J}\right)$ and $\left(vu2 = \frac{Vu}{Ac} - \frac{Muv \ \ c\prime}{J}\right)$
8. Calculate φvc

$$Vc = min\begin{Bmatrix} \frac{\lambda}{3}\sqrt{f'c}\, b_o\, d \\ \left(1+\frac{2}{\beta}\right)\frac{\lambda\sqrt{f'c}}{6} b_o\, d \\ \left(2+\frac{\alpha_s}{\beta_o}\right)\frac{\lambda\sqrt{f'c}}{12} b_o\, d \end{Bmatrix}$$

9. If φvc ≥ vu1 then ok, otherwise

vu1≤φvv ok, otherwise additional strengthening is needed:

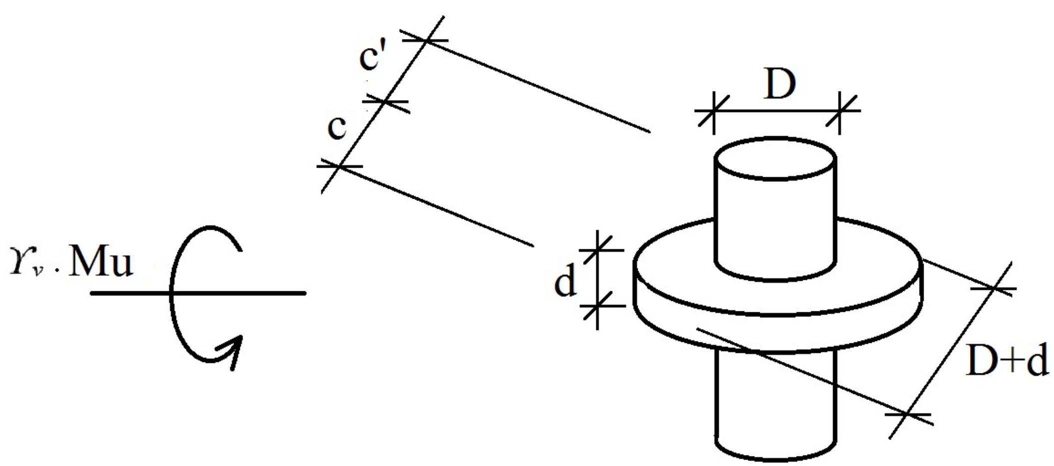
- Integral beam $vc_{cracked} = \frac{\lambda}{6}\sqrt{f'c}$, $vu_{max} = \frac{\emptyset\lambda}{2}\sqrt{f'c}$

  vu1≤vu,max ok, otherwise increase f'c or d
- Shear stud reinforcement $vc_{cracked} = \frac{\lambda}{4}\sqrt{f'c}$, $vu_{max} = \frac{2\emptyset\lambda}{3}\sqrt{f'c}$

  vu1≤vu,max ok, otherwise increase f'c or d

Note: for shear studs or integral beam stirrups, spacing will be:

$$S= \frac{Av*fy*d}{Vs} = \frac{Av*fy*d}{Vn-Vc} = \frac{\varphi Av*fy*d}{Vu-\varphi Vc} = \frac{(\varphi Av*fy*d)/(b_o\, d)}{(Vu-\varphi Vc)/(b_o\, d)} = \frac{\varphi Av*fy*d}{(Vu-\varphi Vc)/b_o}$$

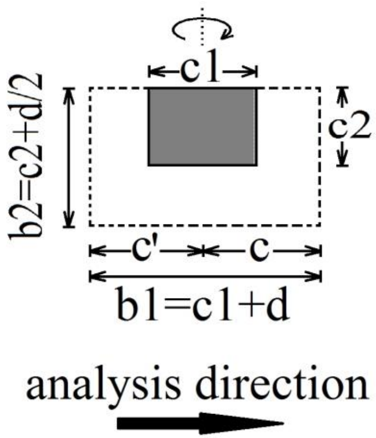


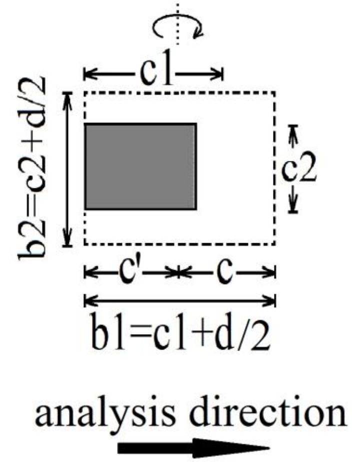
$$\mathbf{Ac=\pi(D+d)d}$$

$$\boldsymbol{c = c' = \frac{D+d}{2}}$$

$$\boldsymbol{\frac{J}{c} = \pi d(\frac{D+d}{2})^2 + \frac{d^3}{3}}$$

$$c = \frac{b1}{2}$$



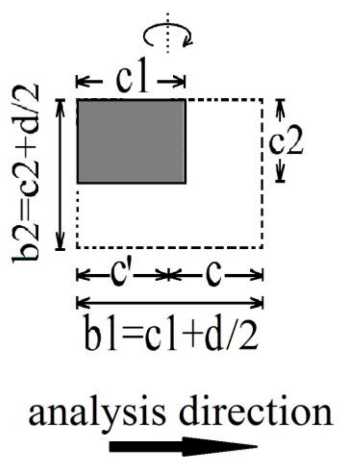


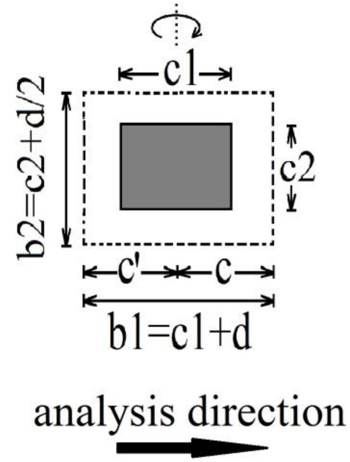
$$c = \frac{b1^2}{2b1 + b2}$$

$$\frac{J}{c} = \frac{b1 * d(b1 + 6b2) + d^3}{6}$$

$$\frac{J}{c} = \frac{2\, b1^2 * d(b1 + 2b2) + d^3(2b1 + b2)}{6\, b1}$$

$$c = \frac{b1^2}{2(b1 + b2)}$$





$$c = \frac{b1}{2}$$

$$\frac{J}{c} = \frac{b1^2 * d(b1 + 4b2) + d^3(b1 + b2)}{6\, b1}$$

$$\frac{J}{c} = \frac{b1 * d(b1 + 3b2) + d^3}{3}$$