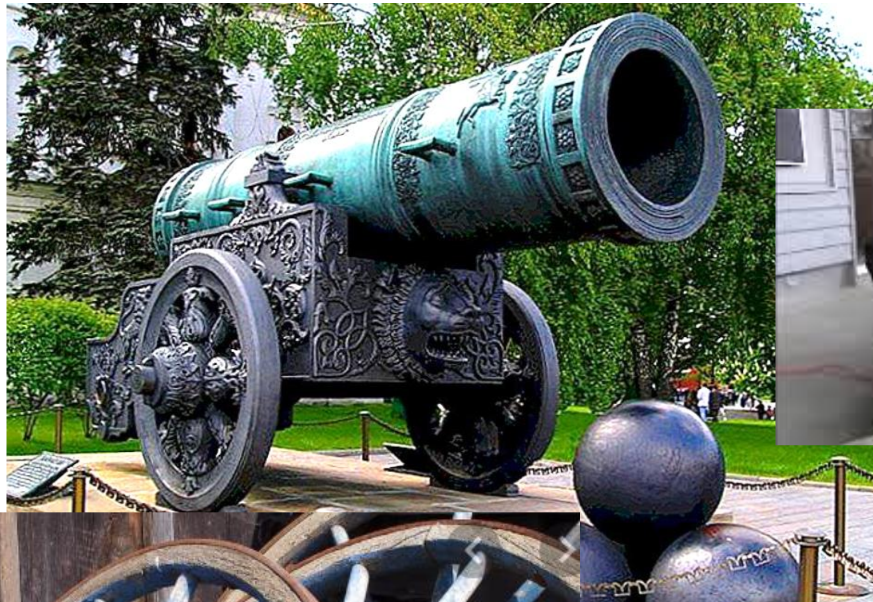
Pre-stressed Concrete Beams

ا.د.خطاب / ديالى

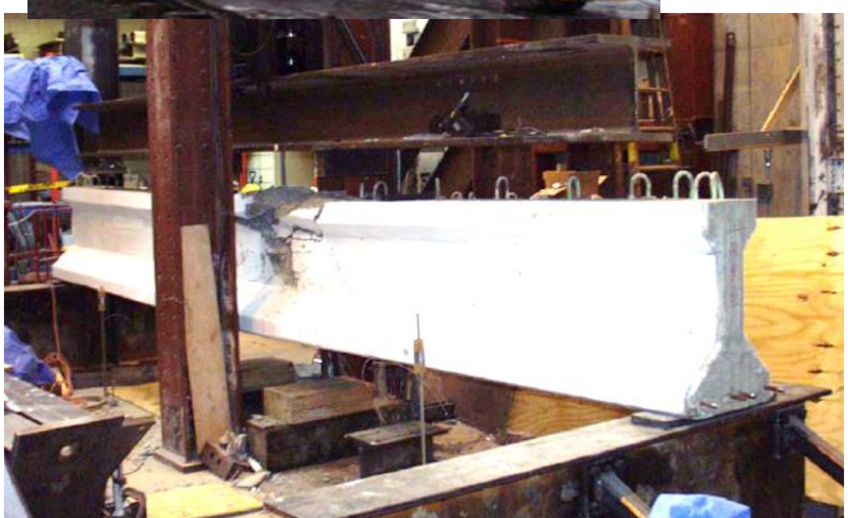
Prestressed reinforced concrete beams

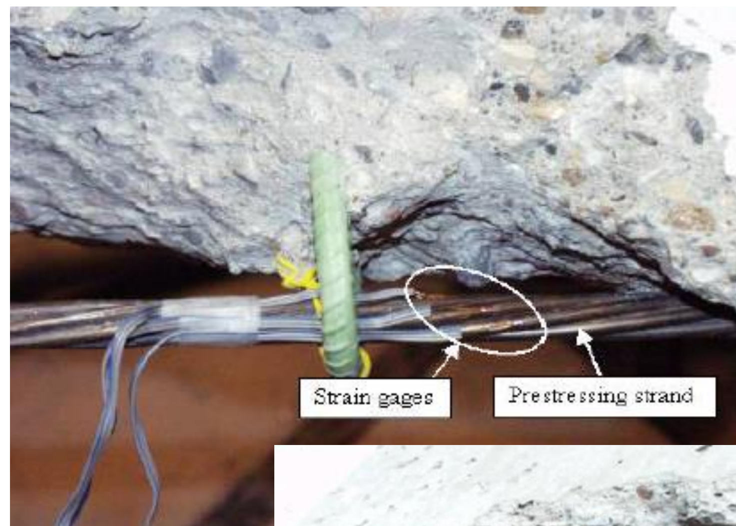
"Prestressed" means a stress that acts even though no external applied loads are acting. The principle of prestressing has been used for centuries, for example, canons, wooden barrels wheels …etc.

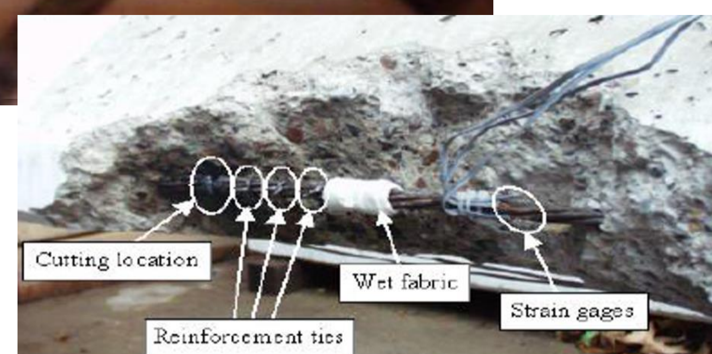


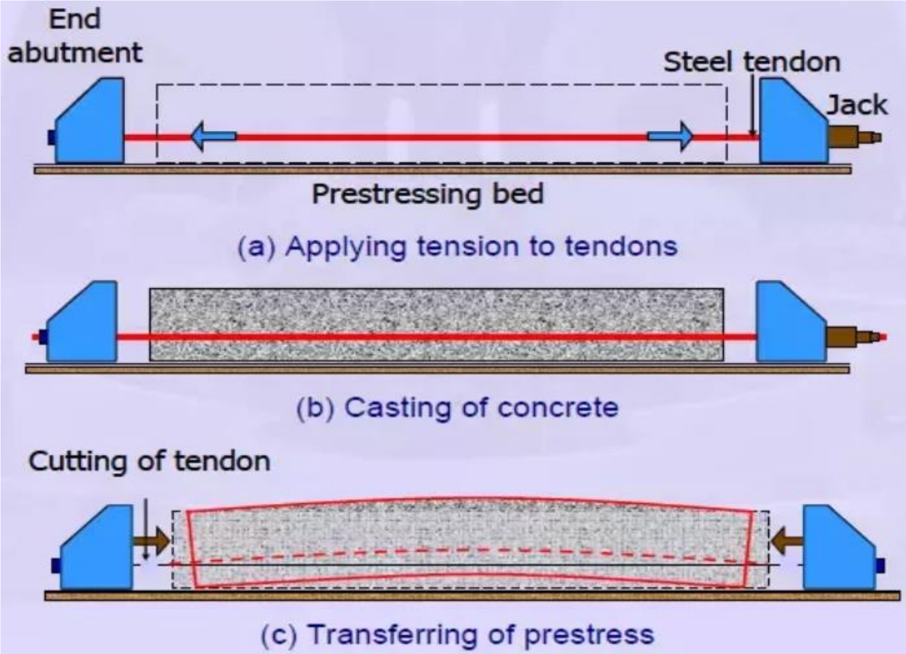
Production

Mild shear reinf.




Strain gages
Prestressing strand


Cutting location
Wet fabric
Strain gages
Reinforcement ties



Pre

1-the prestressed tendons are tensioned

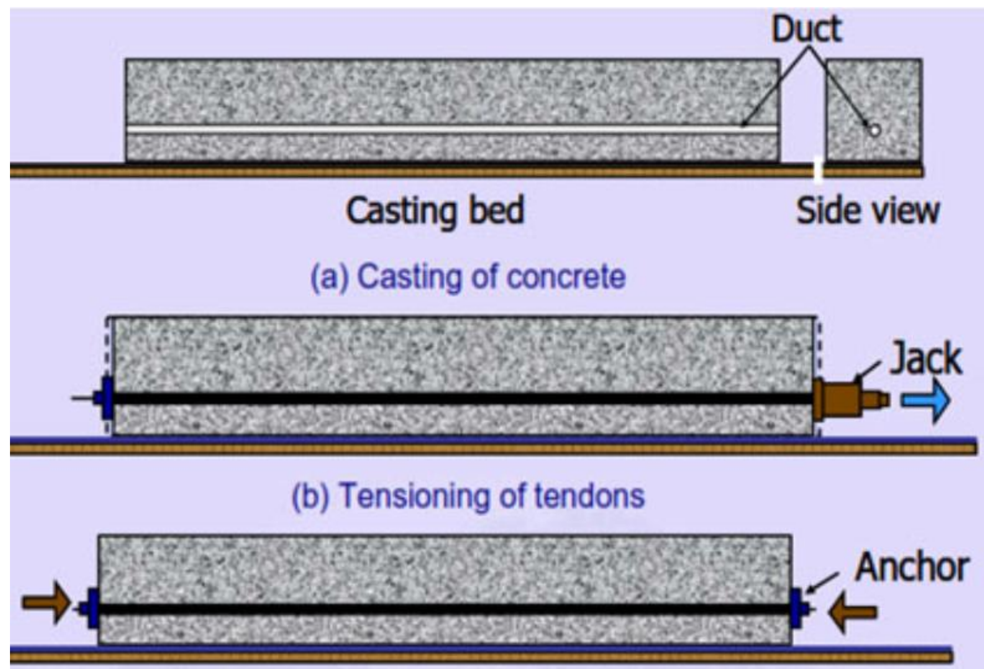
2-Pouring concrete.

3- After the concrete had hardened sufficiently, the tendons were cut and the prestressing force is suddenly **<u>transmitted</u>** to the concrete by bond.

Post

1-Pouring concrete with ducts.

2- After the concrete had hardened sufficiently, the tendons are stretched (the prestressing force is gradually **<u>transmitted</u>** to the concrete by end bearing (not by bond)).



Advantages of Prestressing:

1-Crackfree under service loads (no corrosion + effective section).

2-More accommodation of both shrinkage and creep.

3-Shear strength is more consistent than in nonprestressed ones.

4-Higher ability to absorb energy (impact resistance).

5-Higher fatigue resistance.

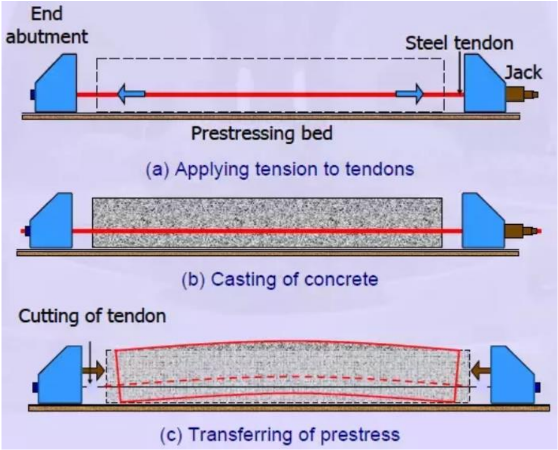
6-Higher live load.

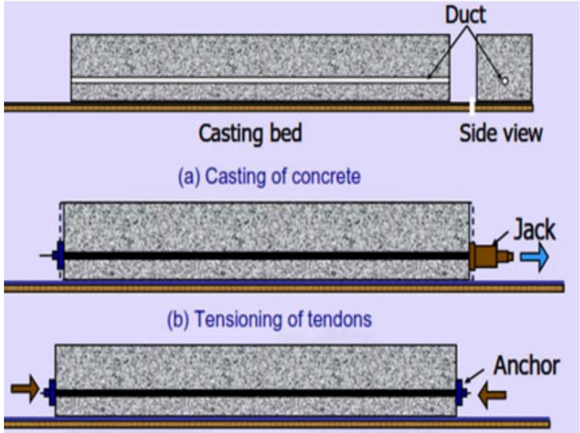
7-Less deflection.

8-Larger span/depth ratio (for example, nonprestressed slabs 1/28, while prestressed can be 1:45).

9-Higher quality and quantity.

10-More repetitive





ا.د.خطاب / ديالى

Disadvantages of prestressing

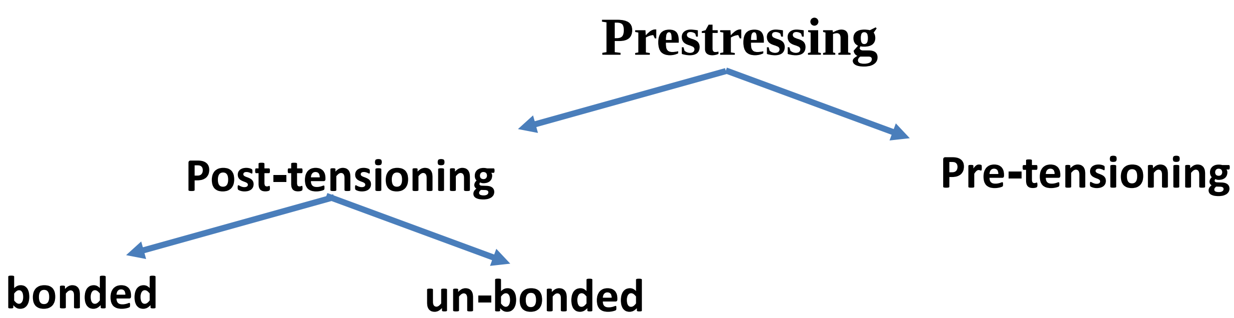
1-Higher cost

2-Complicated formwork.

3-Anchorage and plates are required.

4-Close control.

5-Significant force losses



bonded, aluminium, steel, or other metal sheathing conduits. steel tubing or rods or rubber cores that are cast in the concrete and removed later. Cement grout is injected into the duct for bonding. The grout is also useful in protecting the steel from corrosion.

unbonded, greased to facilitate tensioning and to protect them from corrosion.

Concrete 4 to 8 ksi (28 to 56 MPa)

In this type of structures, high strength concrete is required because:

1-High bond strength is required in pre-tensioning construction.

2-High bearing strength is required to resist the high bearing stresses at ends in post-tensioning construction.

3-High *E* is required to reduce the initial strain values and the strain values of creep.

Properties of Prestressing Steel

1) Higher strength

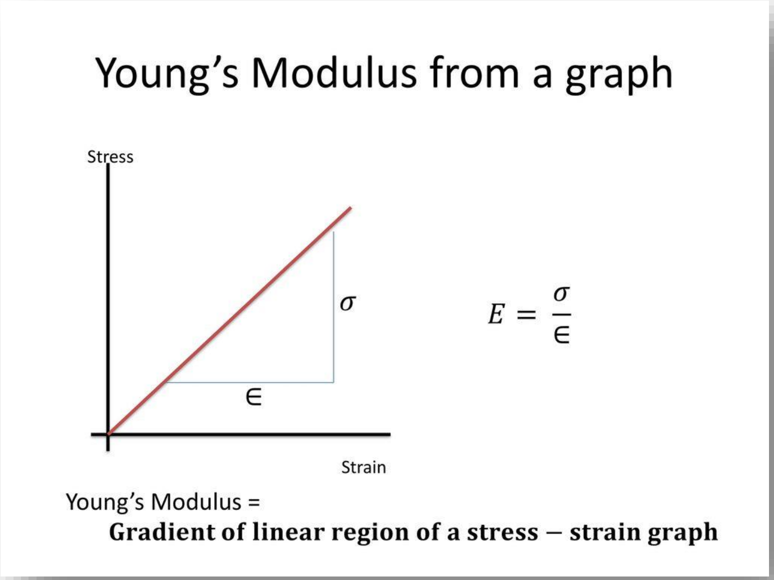
2) More ductility

3) More bendability

4) Higher bond

5) Lower relaxation

6) Less corrosion.



1 ksi=6.89476 MPa

Forms of Pre-stressing Steel

Bars. Bars are available in the following sizes: 10, 12, 16, 20, 22, 25, 28 and 32 mm.

type	Diameter in mm	Tensile strength f_{pu}	Yield Strength f_{py}
Plain	19-35	1035 MPa	85%f_{pu}
Deformed	15-36	1035 MPa	80%f_{pu}

Wires. A pre-stressing wire is a single unit made of steel. The nominal diameters of the wires are 2.5, 3.0, 4.0, 5.0, 7.0 and 8.0 mm. The different types of wires are as follows: 1) Plain wire: No indentations on the surface. 2) Indented wire: There are circular or elliptical indentations on the surface.

Cables. A group of tendons form a pre-stressing cable. The cables are used in bridges.

Strands. 1) Two-wire strand, 2) Three-wire strand, 3) Seven-wire strand.

Grade	Diameter in mm	Tensile strength f_{pu}	Yield Strength f_{py}
Grade 250 (1725 MPa)	6.35-15.24	1725 MPa	85%f_{pu}, except 90%f_{pu} for relaxation strand
Grade 270 (1860 MPa)	9.53-15.24	1860 MPa	

Tendons. A group of strands or wires are placed together to form a pre-stressing tendon. The tendons are used in post-tensioned members. The following figure shows the cross section of a typical tendon. The strands are placed in a duct which may be filled with grout after the post-tensioning operation is completed.

Note: Steel:

Full prestressing: only pre or post stressed steel

Partial prestressing: mix of mild and pre or post stressed steel

Typical Stages of loading:

<u>1-Initial stage:</u>

1-a) Prestressing bed stage:

- Full prestressing force (Pi):

$f_{si} \leq 94\% f_{py} < 0.8 f_{pu} <$ manufacturer recommendation

1-b) Applying beam own weight

- Full prestressing force (Pi)
- Mg

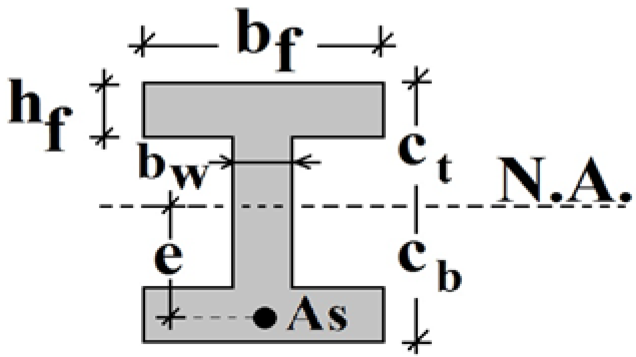
<u>2-Service stage</u>:

2-a) Prestress loss (Pe): install in bridge

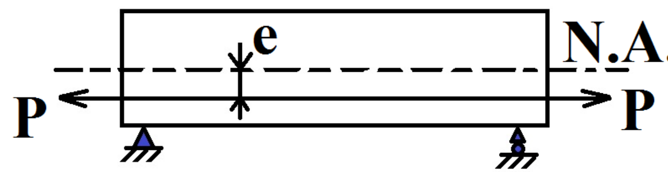
- Loss happened in prestressing force

2-b) Applying full load (Pe):

- Loss happened in prestressing force
- Wg+Ws=Mg+ML+MD



Stresses in Prestressed Concrete

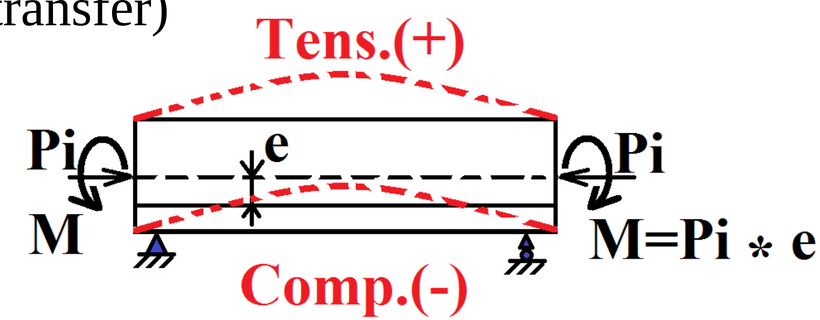


1-Initial Stage: (immediately after prestress transfer)

- initial prestress force (Pi)

$$fi_{top} = \frac{-Pi}{A} + \frac{Pi * e * c_t}{I}$$

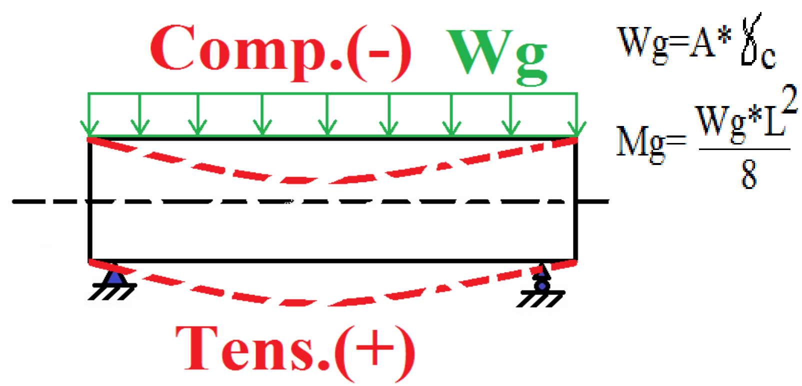
$$fi_{bot} = \frac{-Pi}{A} - \frac{Pi * e * c_b}{I}$$



- applying beam own-weight (Wg):

$$fi_{top} = \frac{-Pi}{A} + \frac{Pi * e * c_t}{I} - \frac{Mg * c_t}{I}$$

$$fi_{bot} = \frac{-Pi}{A} - \frac{Pi * e * c_b}{I} + \frac{Mg * c_b}{I}$$

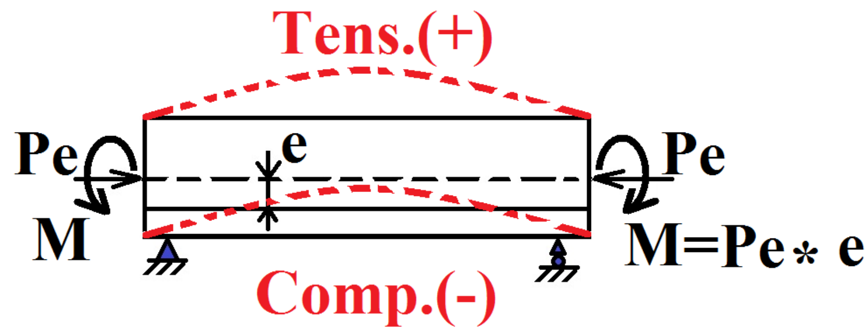


2-Service stage: (after prestress loss)

Pe=prestressing force after losing <Pi

$$fe_{top} = \frac{-Pe}{A} + \frac{Pe * e * c_t}{I}$$

$$fe_{bot} = \frac{-Pe}{A} - \frac{Pe * e * c_b}{I}$$

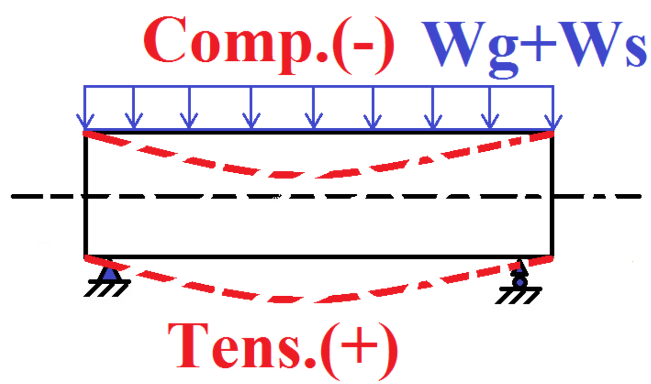


applying full service load (Ws):

Ws=WD+WL

$$fe_{top} = \frac{-Pe}{A} + \frac{Pe * e * c_t}{I} - \frac{Mg * c_t}{I} - \frac{Ms * c_t}{I}$$

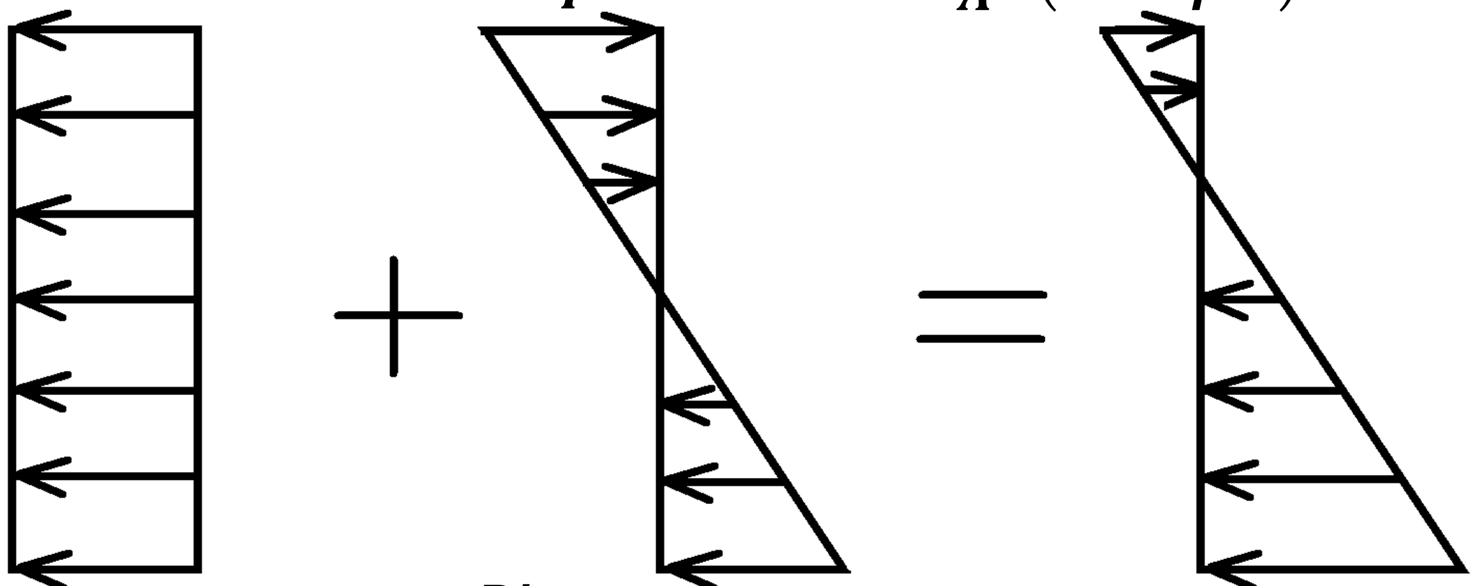
$$fe_{bot} = \frac{-Pe}{A} - \frac{Pe * e * c_b}{I} + \frac{Mg * c_b}{I} + \frac{Ms * c_b}{I}$$



Flexural Analysis

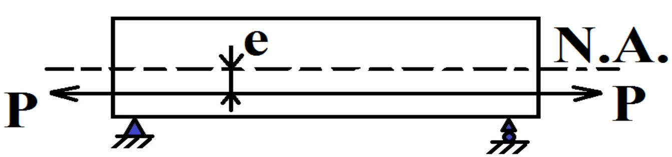
1-Initial Prestressing Force (Pi): a-wire cut

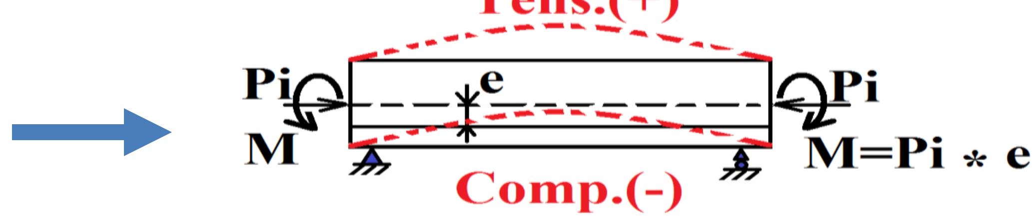
$$\frac{-Pi}{A} \qquad + \frac{Pi * e * c_t}{I} \qquad \frac{-Pi}{A}\left(1 - \frac{e * c_t}{r^2}\right)$$



$$\frac{-Pi}{A} \qquad - \frac{Pi * e * c_b}{I} \qquad \frac{-Pi}{A}\left(1 + \frac{e * c_b}{r^2}\right)$$

$$r = \sqrt{\frac{I}{A}}$$

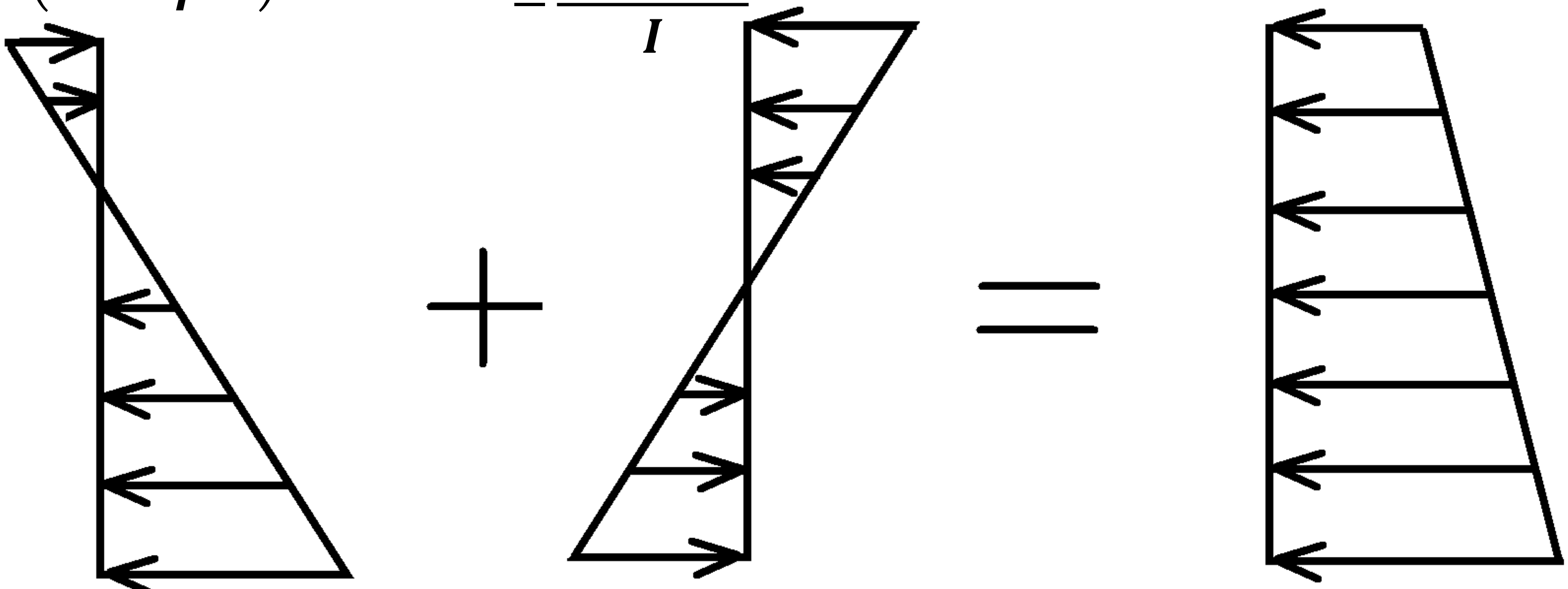




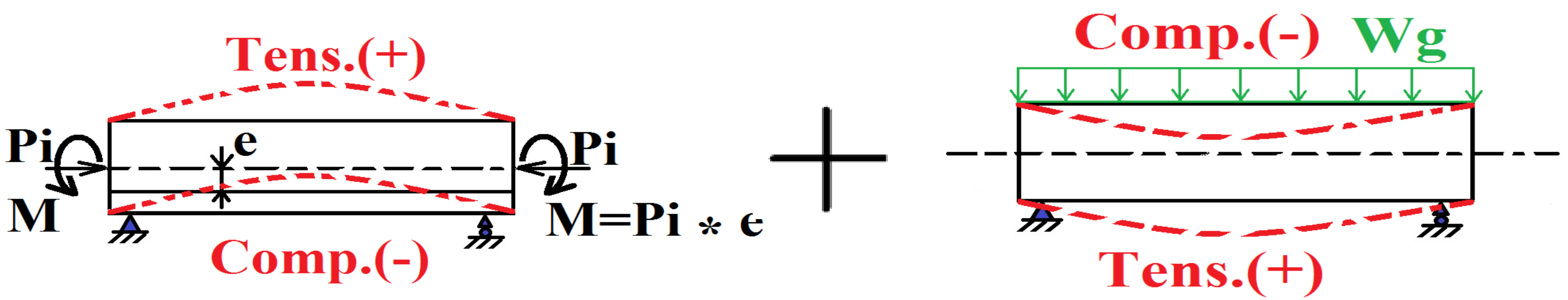
ا.د.خطاب / ديالى

b-applying beam own-weight (Pi): 

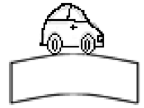
$$\frac{-Pi}{A}\left(1-\frac{e*c_t}{r^2}\right) \qquad -\frac{Mg*c_t}{I} \qquad -\frac{Pi}{A}\left(1-\frac{e*c_t}{r^2}\right)-\frac{Mg*c_t}{I}$$



$$\frac{-Pi}{A}\left(1+\frac{e*c_b}{r^2}\right) \qquad +\frac{Mg*c_b}{I} \qquad -\frac{Pi}{A}\left(1+\frac{e*c_b}{r^2}\right)+\frac{Mg*c_b}{I}$$



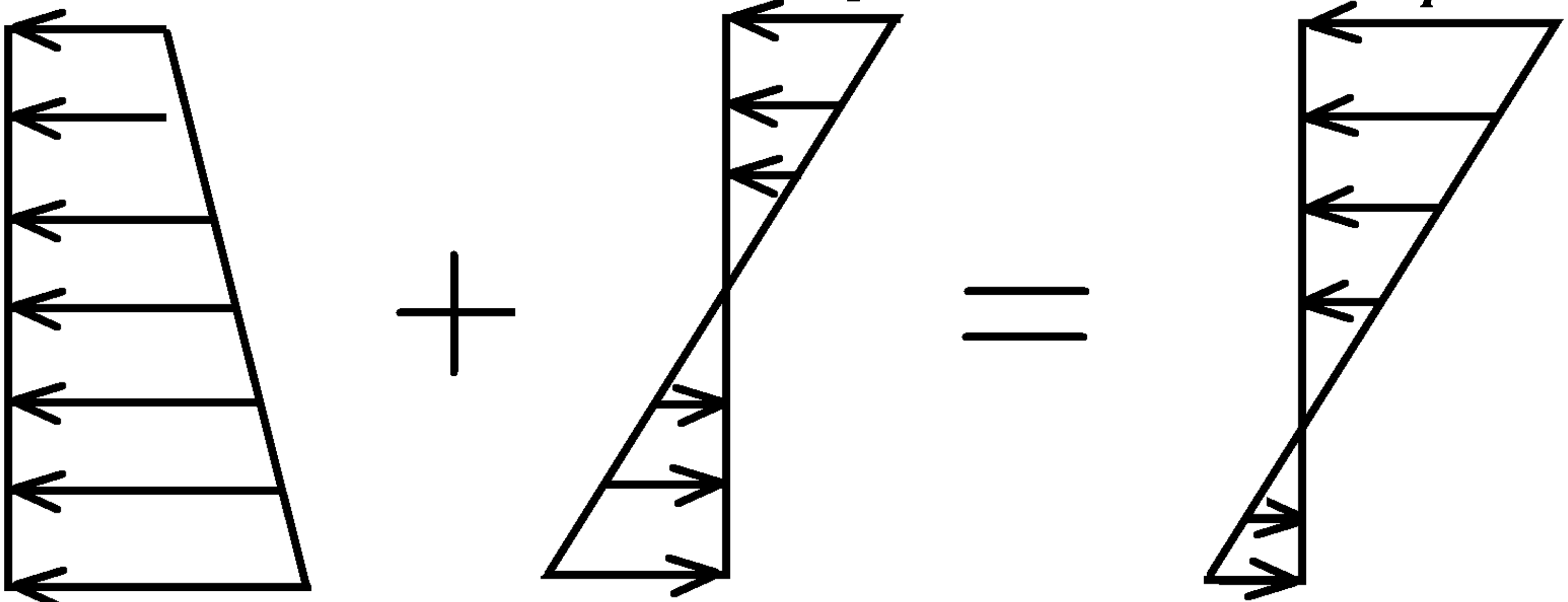
2- Applying full load (Pe):



$$-\frac{Pe}{A}\left(1-\frac{e*c_t}{r^2}\right)-\frac{Mg*c_t}{I}$$

$$-\frac{(MD+ML)*c_t}{I}$$

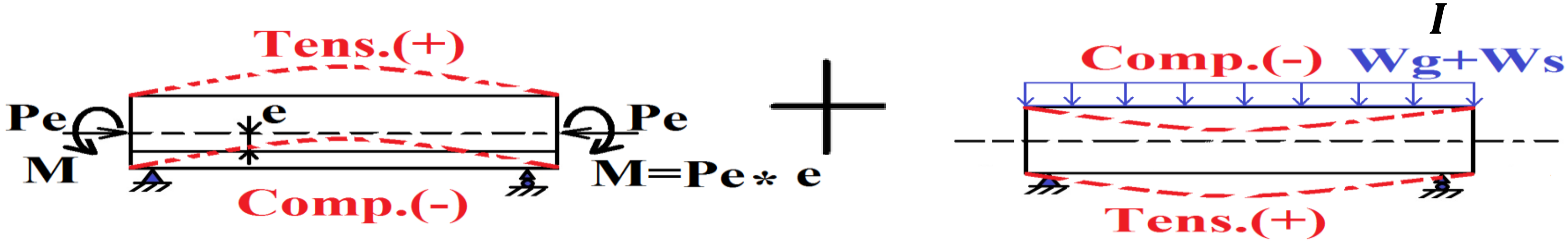
$$-\frac{Pe}{A}\left(1-\frac{e*c_t}{r^2}\right)-\frac{(Mg+MD+ML)*c_t}{I}$$



$$-\frac{Pe}{A}\left(1+\frac{e*c_b}{r^2}\right)+\frac{Mg*c_b}{I}$$

$$+\frac{(MD+ML)*c_b}{I}$$

$$-\frac{Pe}{A}\left(1+\frac{e*c_b}{r^2}\right)+\frac{(Mg+MD+ML)*c_b}{I}$$



ACI
Permissible stresses

- **Steel**
  - immediately after prestress loss: $0.82 fp_y \le 0.74 fp_u$
  - Post tensioning tendons at anchorage: $0.7 fp_u$
  - due to prestressing jacking force: $0.94 fp_y \le 0.8 fp_u$
- **Concrete**
  - Immediately after prestress transfer (before losses)
    - tension
      - at ends: $0.50\sqrt{f'_{ci}}$
      - at any location except ends: $0.25\sqrt{f'_{ci}}$
    - compression
      - at ends: $0.70 f'_{ci}$
      - at any location except ends: $0.60 f'_{ci}$
  - At service load (after losses)
    - tension
      - at ends: $0.5\sqrt{f'c}$
      - at any location except ends: $0.25\sqrt{f'c}$
    - compression
      - at ends
      - at any location except ends
        - Prestress plus sustained load: $0.45 f'c$
        - Prestress plus total load: $0.6 f'c$

Permissible Stresses

Concrete compressive stress limits immediately after transfer of prestress

Location	Concrete compressive stress limits
End of simply-supported members	$0.70f_{ci}'$
All other locations	$0.60f_{ci}'$

Concrete tensile stress limits immediately after transfer of prestress, without additional bonded reinforcement in tension zone

Location	Concrete tensile stress limits
Ends of simply-supported members	$0.50\sqrt{f_{ci}'}$
All other locations	$0.25\sqrt{f_{ci}'}$

Concrete compressive stress limits at service loads

Load condition	Concrete compressive stress limits
Prestress plus sustained load	$0.45f_c'$
Prestress plus total load	$0.60f_c'$

Sustained loads are self-weight + finishes
Service load: live load + cast-in-situ concrete

Serviceability requirements for prestressed and nonprestressed members

	Prestressed			
	Class U	Class T	Class C	Nonprestressed
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 24.5.2.2	Gross section 24.5.2.2	Cracked section 24.5.2.3	No requirement
Allowable stress at transfer	24.5.3	24.5.3	24.5.3	No requirement
Allowable compressive stress based on uncracked section properties	24.5.4	24.5.4	No requirement	No requirement
Tensile stress at service loads 24.5.2.1	$\leq 0.62\sqrt{f'_c}$	$0.62\sqrt{f'_c} < f_t \leq 1.0\sqrt{f'_c}$	No requirement	No requirement
Deflection calculation basis	24.2.3.8, 24.2.4.2 Gross section	24.2.3.9, 24.2.4.2 Cracked section, bilinear	24.2.3.9, 24.2.4.2 Cracked section, bilinear	24.2.3, 24.2.4.1 Effective moment of inertia
Crack control	No requirement	No requirement	24.3	24.3
Computation of Δf_{ps} or f_s for crack control	—	—	Cracked section analysis	$M/(A_s \times \text{lever arm})$, or $2/3 f_y$
Side skin reinforcement	No requirement	No requirement	9.7.2.3	9.7.2.3

Classification of prestressed flexural members based on f_t

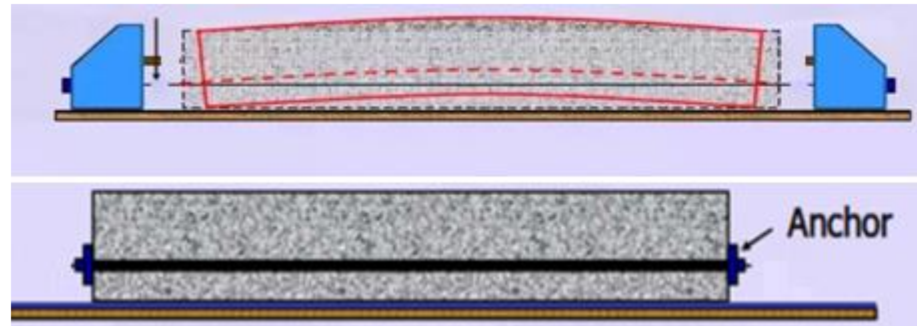
Assumed behavior	**Class**	**Limits of f_t**
uncracked	U	$f_t \leq 0.62\sqrt{f'_c}$ *
Transition between uncracked and cracked	T	$0.62\sqrt{f'_c} < f_t \leq 1.0\sqrt{f'_c}$
cracked	C	$f_t > 1.0\sqrt{f'_c}$

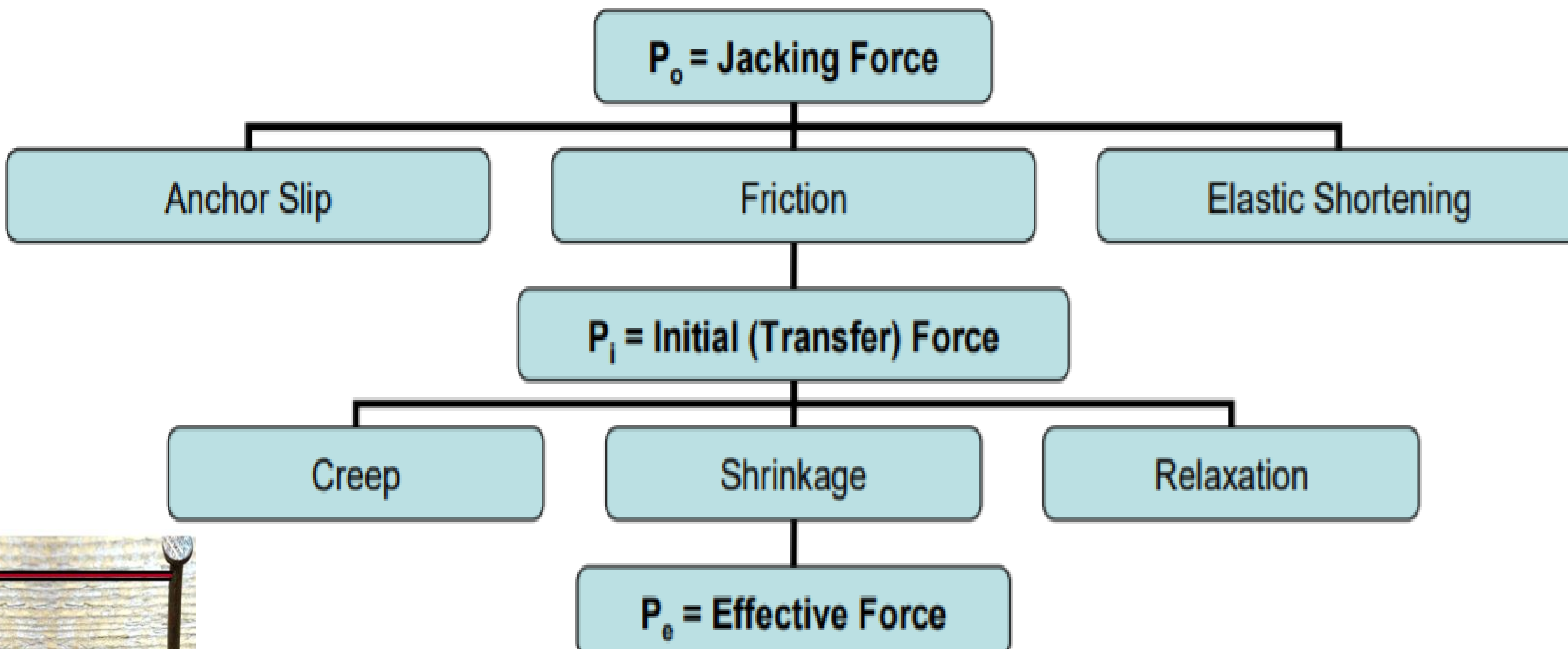
***for prestressed 2-way slabs**

Detailed Loss of Prestressing

The initial prestressing force (Pi) will be reduced to effective prestressing force (Pe) due to the following reasons:

1-Elastic shortening of Concrete (ES)
2-Creep of Concrete (CR)
3-Shirnkage of Concrete (SH)
4-Steel Relaxation (RE)
5-Anchorage Slip loss (ANC)





1-Lump Sum Prestressing Losses

Approximate lump sum values for average steel and concrete properties and for average curing conditions are presented in the following table:

	Case	Pre-tensioning%	Post-tensioning
1	Elastic shortening of Concrete (ES)	4%	1%
2	Creep of Concrete (CR)	6%	5%
3	Shrinkage of Concrete (SH)	7%	6%
4	Steel Relaxation (RE)	8%	8%
	Total	**25%**	**20%**

2-Detailed Losses

1. Elastic shortening of Concrete (ES)

$$ES = K_{es}\ E_s\ \frac{f_{cir}}{E_{ci}} \qquad MPa$$

where:

$$f_{cir} = \frac{Pi}{A} + \frac{Pi *\ e^2}{A} - \frac{Mg * e}{I}$$

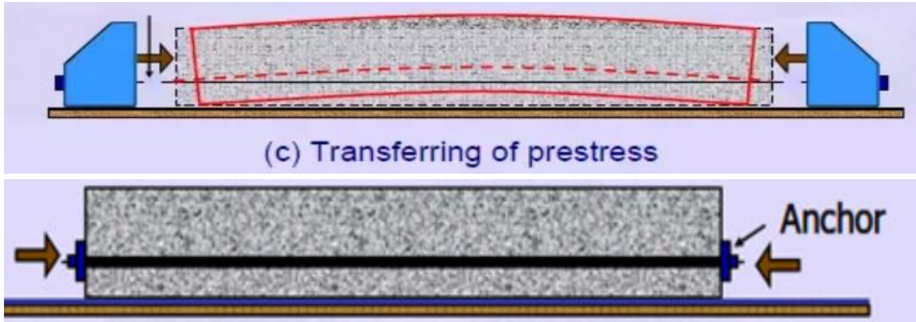
K_{es}=1 for pre-tensioned members

K_{es}=0.5 for post-tensioned members

E_{ci}= 4700√f'ci

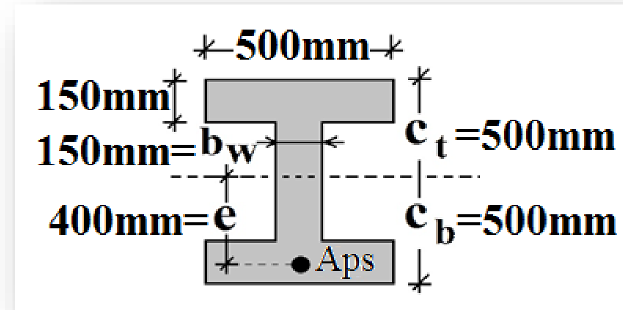
E_s=200000 MPa

Pi=fpi*Aps



2-Creep of Concrete (CR)

$$CR = K_{cr} \frac{E_s}{E_c} (f_{cir} - f_{cds}) \qquad MPa$$



where:

$$f_{cds} = \frac{M_d * e}{I}$$

K_{cr}=2 for pre-tensioned members

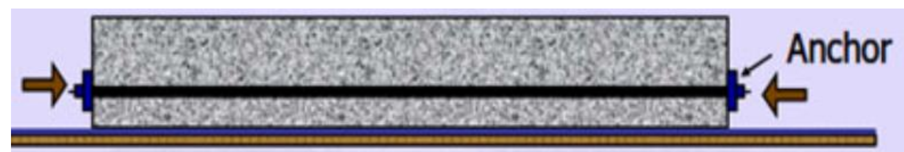
K_{cr}=1.6 for post-tensioned members

E_c= 4700√f'ci

E_s=200000 MPa

Pi=fpi*Aps

E_c= 4700√f'ci
E_s=200000 MPa
Pi=fpi*Aps



3-Shirnkage of Concrete (SH)

$$SH = 8.2 * 10^{-6}\ K_{sh}\ E_s \left(1 - \frac{0.06}{25} * \frac{V}{S}\right)(100 - RH) \qquad MPa$$

where:

K_{sh}=1 for pre-tensioned members

K_{sh}= Table for post-tensioned members

RH=relative humidity

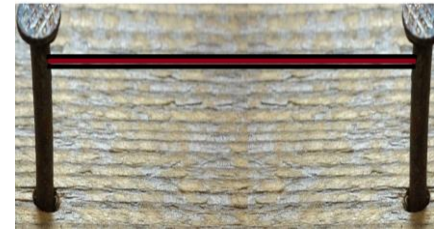
Es=200000 MPa

V/S=Volume / Surface area of evaporation

Days after curing end to prestress application	1	3	5	7	10	20	30	60
K_{sh}	0.92	0.85	0.8	0.77	0.73	0.64	0.58	0.45

4-Steel Relaxation (RE)

$$RE = [K_{re} - J\,(SH + CR + ES)]C \qquad MPa$$



where:

Kre, J and C from tables:

Table 2 Values of K_{re} and J

Type of tendon[a]	K_{re} (MPa)	J
1860 MPa Grade stress-relieved strand or wire	138	0.15
1720 MPa Grade stress-relieved strand or wire	128	0.14
1655 MPa or 1620 MPa Grade stress-relieved wire	121	0.13
1860 MPa Grade low-relaxation strand	35	0.040
1720 MPa Grade low-relaxation wire	32	0.037
1655 MPa or 1620 MPa Grade low-relaxation wire	30	0.035
1000 MPa or 1100 MPa Grade stress-relieved bar	41	0.05

Table 2. Values of Coefficient *C*

f_{pi}/f_{pu}	Stress-relieved strand or wire	Stress-relieved bar or low-relaxation strand or wire
0.80		1.28
0.79		1.22
0.78		1.16
0.77		1.11
0.76		1.05
0.75	1.45	1.00
0.74	1.36	0.95
0.73	1.27	0.90
0.72	1.18	0.85
0.71	1.09	0.80
0.70	1.00	0.75
0.69	0.94	0.70
0.68	0.89	0.66
0.67	0.83	0.61
0.66	0.78	0.57
0.65	0.73	0.53
0.64	0.68	0.49
0.63	0.63	0.45
0.62	0.58	0.41
0.61	0.53	0.37
0.60	0.49	0.33

5-Anchorage Slip loss (ANC)

$$ANC = \Delta_{fs} = \frac{\Delta_a \ E_s}{L} \qquad MPa$$

where:

Δ_a=Anchorage deformation

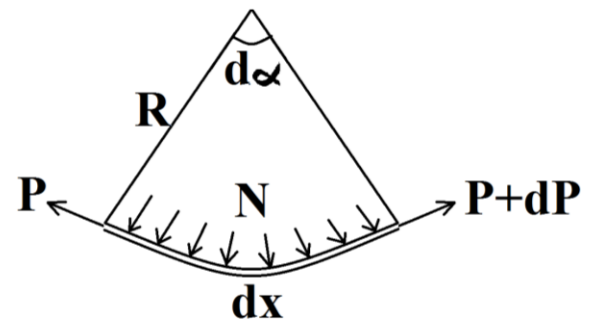
L =Length of tendon

E_s=200000 MPa

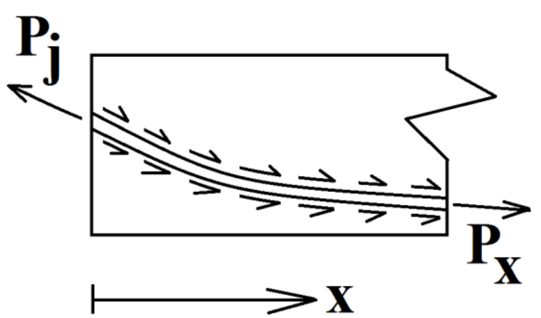
Friction Losses in Post-tensioned members

1-Wobble: means that, the tendon is not perfectly straight when it is stretched, but it is slightly twisted. To overcome that twisting, some additional force is needed.

2- Friction: due to the curvature of the tendon



$$P_x = P_j \ e^{-(k*l_x+\mu*\alpha_x)}$$



where:

Px=prestressing force evaluated at distance lx

Pj=prestressing force at jacking end

e=base of natural logarithms

k=wobble friction coefficient

lx=distance from jacking end of prestressing steel

μ=post-tensioned curvature friction coefficient

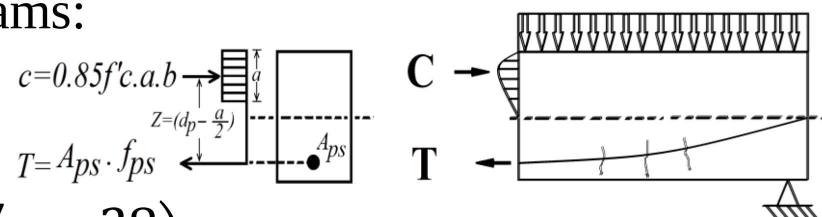
α_x=total angular change of tendon profile from tendon jacking end to point under considerations. αx is also equal to the change in the slope of the curved profile of tendon (the difference between the slopes at two points gives the value of the subtended angle α).

Flexural strength

The external moment from the applied loads is resisted by the internal force couple:

TZ=CZ

The strength of a pre-stressed beam can be predicted by the same methods developed for ordinary reinforced concrete beams:

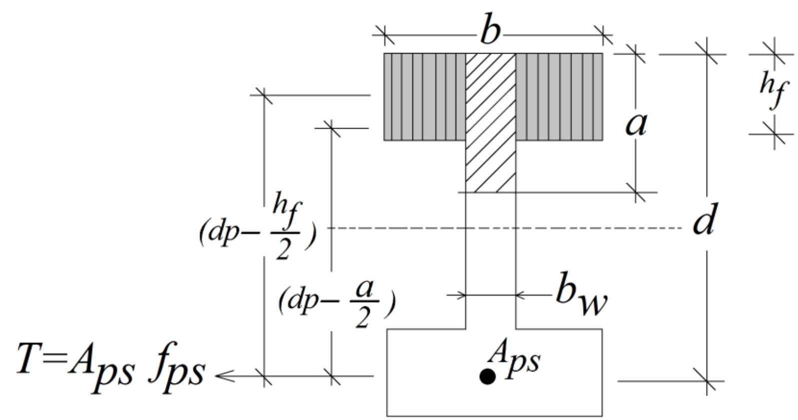


$a = \beta_1 \, C$

$\beta_1 = 0.85 \;\; for \;\; f'c \leq 28MPa$

$$for \;\; f'c > 28\,MPa \qquad \beta_1 = 0.85 - 0.05\left(\frac{f'c - 28}{7}\right) \geq 0.65$$

For rectangular cross section or flanged section such as (I or T) beams in which the stress block depth is equal to or less than the average flange thickness, the nominal flexural strength is:



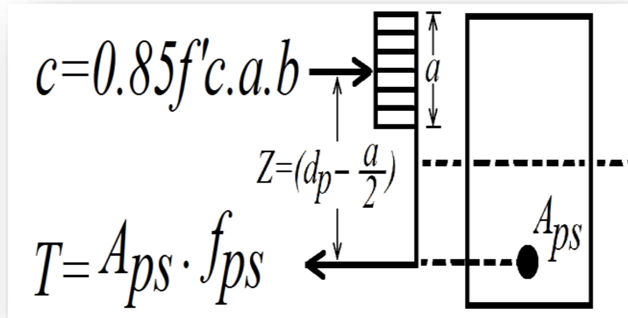
$$M_n = A_{ps}\, f_{ps}\, (d_p - \frac{a}{2})$$

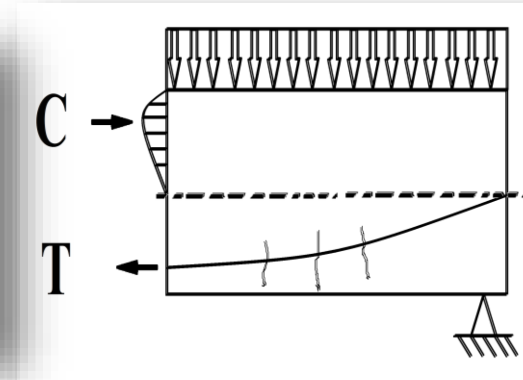
$$a = \frac{A_{ps}\, f_{ps}}{0.85\, f'c\, b}$$

$$or \qquad M_n = \rho_p f_{ps} b\, d_p^2 (1 - 0.588 \rho_p \frac{f_{ps}}{f'c})$$

Flexural design strength= ∅Mn where ∅: strength reduction factor = 0.9

If the stress block depth exceeds the average flange thickness the total prestressed tensile steel area is divided into two parts for computational purposes. The first part A_{pf} acting on the stress f_{ps} provides a tensile force to balance the compression in the overhanging parts of the flange thus





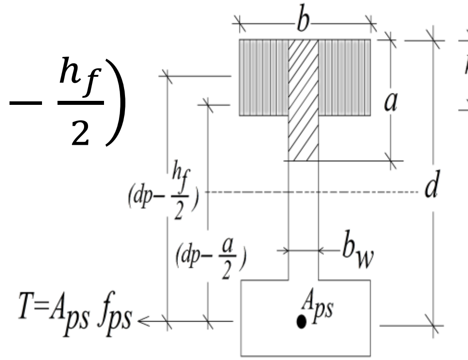
$$A_{pf} = 0.85\frac{f'c}{f_{ps}}(b - b_w)h_f$$

$$A_{pw} = A_{ps} - A_{pf}$$

A_{pw} provided tension to balance the compression in the web

$$M_n = A_{pw}\,f_{ps}\left(d_p - \frac{a}{2}\right) + A_{pf}\,f_{ps}\left(d_p - \frac{h_f}{2}\right)$$

or $$M_n = A_{pw}\,f_{ps}\left(d_p - \frac{a}{2}\right) + 0.85 * f'c * (b - b_w)h_f\left(d_p - \frac{h_f}{2}\right)$$



$$where \qquad a = \frac{A_{pw}\ f_{ps}}{0.85\,f'c\,b_w}$$

The design strength = ØMn where Ø is typically 0.9

(f_{ps}) the stress in the steel at failure may be taken equal to the following according to the ACI-code (2014), ch.24. If effective pre-stress in the steel fse ≥ 0.5 fpu:

a. For member with **bonded** tendons

$$f_{ps} = f_{pu}\left[1 - \frac{\gamma_p}{\beta_1}\left[\rho_p \frac{f_{pu}}{f'c} + \frac{d}{d_p}(\omega - \omega')\right]\right]$$

$$where \qquad \omega = \rho \frac{fy}{f'c} \omega' = \rho' \frac{fy}{f'c} \ \rho_p = \frac{A_{ps}}{b\, d_p}$$

b: width of compression face

β_1:the familiar relations between stress block depth and depth to the neutral axis

γp: is a factor that depends on the type of pre-stressing steel

γp = 0.55 for $f_{py}/f_{pu} \geq 0.80$ high strength bars

0.40 for $f_{py}/f_{pu} \geq 0.85$ ordinary strand

0.28 for $f_{py}/f_{pu} \geq 0.90$ low-relaxation strand

b- For members with **unbounded** tendons with

$$1 - \frac{span}{depth} \leq 35 \qquad f_{ps} = f_{se} + 70 + \frac{f'c}{100\, \rho_b}$$

$$f_{ps} \leq f_{py} \qquad f_{ps} \leq (f_{se}+420)$$

$$2 - \frac{span}{depth} > 35 \qquad f_{ps} = f_{se} + 70 + \frac{f'c}{300\, \rho_b}$$

$$f_{ps} \leq f_{py} \qquad f_{ps} \leq (f_{se}+210)$$

ACI-code requires the total tensile reinforcement must be adequate to support a factored load of at least 1.2*cracking load of beam

$\varphi Mn \geq 1.2Mcr$

To find (*Mcr*) the stress in the bottom fibre *fr* :

$$f_r = -\frac{Pe}{Ac} - \frac{Pe * e * C_2}{I_c} + \frac{M_{cr}\, C_2}{I_c}$$

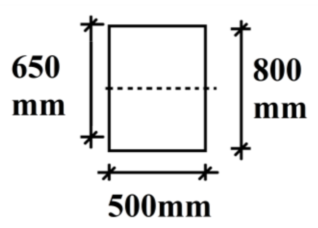
$$\therefore\ M_{cr} = f_r \frac{I_c}{C_2} + P_e\ e + \frac{P_e}{Ac}\left(\frac{I_c}{C_2}\right) = \frac{I_c}{C_2}\left(f_r + \frac{P_e * e * C_2}{I_c} + \frac{P_e}{Ac}\right)$$

Modulus of rapture of concrete $f_r = 0.62\sqrt{f'c}$

To control cracking in prestressed concrete member with unbounded reinforcement, some bonded reinforcement must be added in the form of non-pre-stressed reinforcement bars uniformly distributed over area of bonded reinforcement

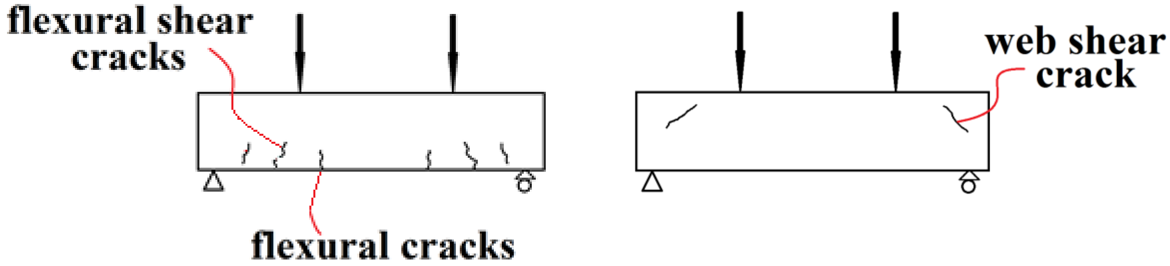
$$As = 0.004\ A_{ct}$$

Where A_{ct} area of that part of cross section between the flexural tensile face and the center of the gross concrete cross-section.



Shear in Prestressed Simply Supported Beams

Prestressed beam behaves like an ordinary concrete beam about failure, such as cracking:



$V_u \leq \varphi V_n$ where $\varphi = 0.75$

$V_n = V_c + V_s$

At a distance (h/2) from the face of support, the first critical section lies.

d = the distance from extreme compression fibre to centroid of prestressed and, non-prestressed longitudinal reinforcement if any, noting that **$d \geq 0.8h$**.

$$\rho_v = \frac{Av}{b_w * s} \qquad web\ reinforcement\ ratio$$

A_v = area for 2-legs

V_c = min $\{V_{ci}$ or $V_{cw}\}$ determined by flexure-shear cracking and web-shear cracking

$$V_{ci} = 0.05\lambda\sqrt{f'c}\, b_w\, d_p + V_d + \frac{V_i\, M_{cr}}{M_{max}} \qquad \text{where} \qquad V_{ci} \geq 0.14\sqrt{f'c}\, b_w\, d$$

$$M_{cr} = \frac{I}{C_2}(0.5\,\lambda\,\sqrt{f'c} + f_{pe} - f_d)$$

Values of M_{max} and Vi are calculated from the load combination maximum moment to occur at the section

V_d = shear force at section due to unfactored dead load

f_d = stress due to unfactored dead load at tension face of the section

f_{pe} = compressive stress at section face resulting from effective prestress force alone.

V_i = factored shear force at section due to externally applied loads occurring simultaneously with M_{max}

Web-shear cracks, start in the web due to high diagonal tension, and then spread diagonally both upward and downward.

$$V_{cw} = (0.29\sqrt{f'c} + 0.3f_{pc})b_w\, d_p + V_p$$

f_{pc}: the compression stress after losses at the centroid of concrete section; $V_p = P_e \sin(\theta)$

Approximate equation for prestressed member with $A_{ps}\, f_{pe} \geq 0.4\ (A_{ps}\, f_{pu} + As\ fy)$

$$V_c = (0.05\sqrt{f'c} + 4.8\frac{V_u\, d_p}{M_u})b_w\, d_p$$

$$0.17\sqrt{f'c}\, b_w d \leq V_c \leq 0.42\sqrt{f'c}\, b_w\, d$$

$$\frac{V_u\, d_p}{M_u} \leq 1.0$$

V_u and M_u: factored shear and moment at section considered resulting from total factored loads and M_d: moment because of unfactored dead load (moment related to f_d)

Required area of web reinforcement

$$V_s = \frac{(V_u - \emptyset V_c)}{\emptyset} \qquad\qquad V_s = \frac{Av * fy * d}{S}$$

$$\frac{(Vu - \emptyset Vc)}{\emptyset} = \frac{Av * fy * d}{S} \quad \rightarrow \quad Av = \frac{(Vu - \emptyset Vc)S}{\emptyset\, fy\;\, d}$$

$$S = \emptyset \frac{fy\;\, d\;\, Av}{Vu - \emptyset Vc}$$

Minimum web reinforcement: minimum area of shear reinforcement must be provided when Vu > 0.5 φVc minimum area is to be taken equal to the smaller of:

$$Av_{min} = 0.062\sqrt{f'c}\,\frac{b_w\, S}{f_{yt}} \geq \frac{0.35\, b_w\, S}{f_{yt}}$$

or:

$$Av_{min} = \frac{A_{ps}\, f_{pu}\, S}{S_v\, f_{yt}\, d}\sqrt{\frac{d}{b_w}}$$

f*yt*: specified yield strength (fy) of the transvers reinforcement (N/mm²)

Max Spacing

$$S \leq 0.75h \quad or \quad 600mm \qquad V_s \leq 0.33\sqrt{f'c}\, b_w\, d$$

$$S \leq \frac{0.75h}{2} \quad or \quad 300mm \qquad V_s > 0.33\sqrt{f'c}\, b_w\, d$$