Noise in Analog Communication Systems

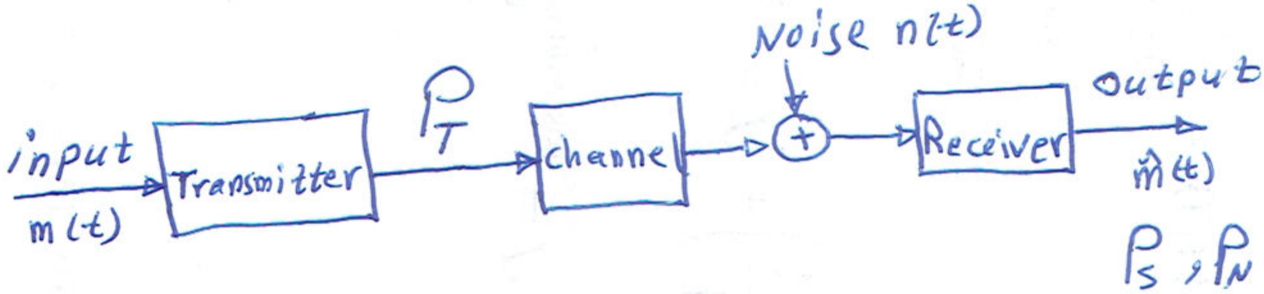
1. What will be the performance of the different modulation methods in the presence of noise?

2. Which modulation method is better?

3. How to measure the performance?

To measure the performance of analog modulation methods, we use the SNR

The general analog communication system is shown below



where P_T is the transmitted power

P_S is the output received power

P_N is the output noise power

Note : The transmitted power P_T is

1. Limited by equipment capability, cost, government restrictions, interference with other channels, etc.
2. The higher it is, the more the received power (P_S), the higher the SNR.

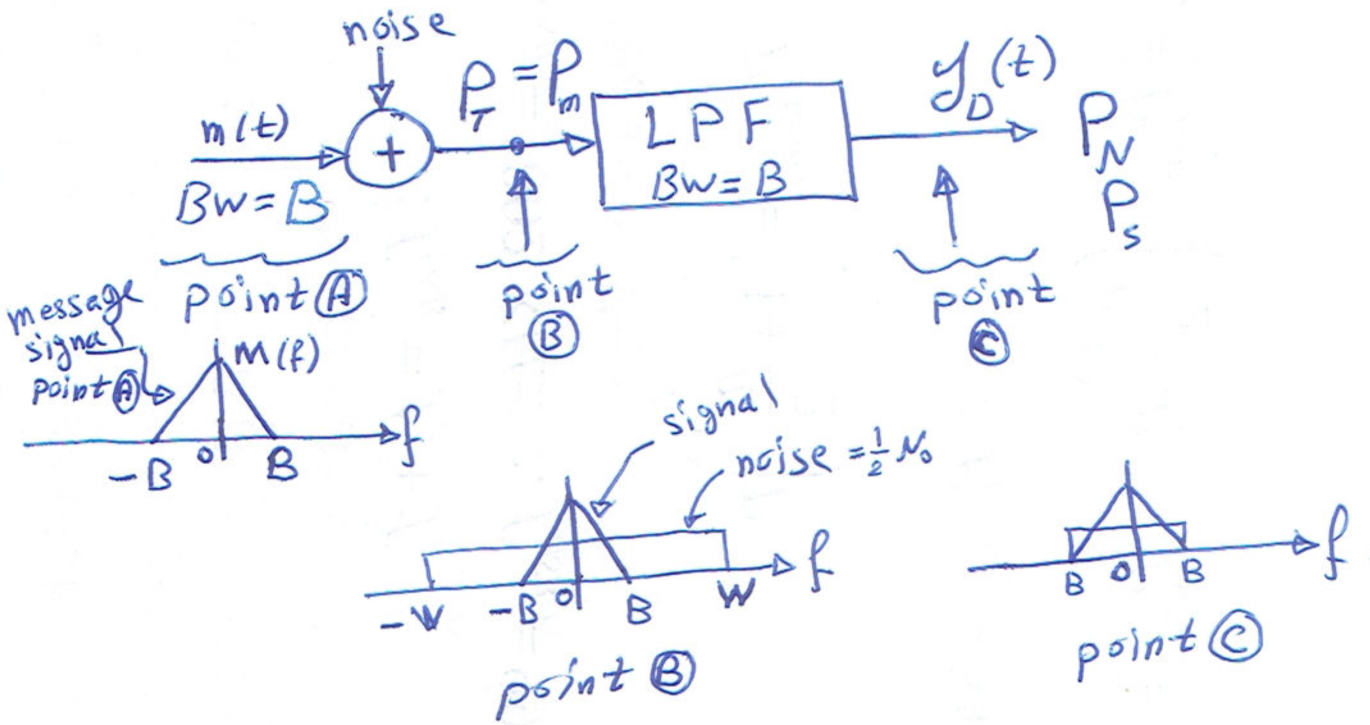
The best way to measure the performance is by comparing the baseband SNR with the obtained SNR of the system after the modulation/demodulation

Baseband : we mean without modulation.

so $P_T = P_{message}$

we use Additive white noise with power spectral density (PSD) $= N_o/2$.

The baseband communication system can be configured as shown below



$P_T = P_m = P$ is the average power of the transmitted signal.

$P_N \Rightarrow$ is the average power of the noise

Thus; at point C: average noise power at the receiver is $P_N = 2B \times \frac{N_0}{2} = BN_0$

$\therefore P_T = P_m = P_s$: if we neglected the losses in the propagation channel

and since $P_N = BN_o$

$$\therefore SNR_{baseband} = \frac{P_T}{P_N} = \frac{P_T}{N_o B}$$

OR

$$\boxed{SNR_{base} = \frac{P_T}{N_o B}}$$

we will compare our modulation methods with this baseband value.

However : To enhance this SNR, we do one or two or all of the following ;

1. increase P_T
2. Reduce Bandwidth
3. Design low noise receiver (less N_o).

Linear Modulations

(1) $s(t)_{AM} = [A_c + m(t)] \cos(2\pi f_c t)$ [AM]

For synchronous detection (coherent), we multiply the received signal by the local carrier

$$\therefore\ s(t)_{AM} \times \cos(2\pi f_c t) = [A_c + m(t)] \cos^2(2\pi f_c t)$$

$$= [A_c + m(t)] \left\{ \frac{1}{2} + \frac{1}{2} \cos(2\pi 2 f_c t) \right\}$$

After LPF: the recovered signal is

$\frac{1}{2}[m(t) + A_c]$. Finally we get $m(t)$

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(2) $s(t)_{DSB\text{-}SC} = A_c\, m(t) \cos(2\pi f_c t)$ [DSB-SC]

Recovery of $m(t)$ is only by synchronous detection.

Noise in DSB-SC

$$x(t) = s(t) + n(t)$$

* $s(t) = A_c m(t) \cos(2\pi f_c t)$
* $x(t)$ is the received signal.

$$= s(t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

$$= \left[A_c m(t) + n_c(t)\right]\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

For synchronous detection: we multiply by $2\cos(2\pi f_c t)$

$$\therefore \; y_{Detec}(t) = 2\cos(2\pi f_c t)\cdot x(t)$$

$$= A_c m(t)\left[1 + \cos(4\pi f_c t)\right] + n_c(t)\left[1 + \cos(4\pi f_c t)\right]$$

$$- n_s(t)\sin(4\pi f_c t)$$

* After LPF: we get $\boxed{\tilde{y} = \underbrace{A_c m(t)}_{\text{signal}} + \underbrace{n_c(t)}_{\text{noise}}}$

$$P_s = A_c^2 P \quad , \quad P_N = \int_{-B}^{B} N_0 \, df = 2N_0 B$$

$$\therefore \; SNR_o = \frac{P_s}{P_N}$$

$$SNR_o = \frac{A_c^2 P}{2N_0 B}$$

note that transmitted power is $\frac{A_c^2 P}{2}$

$$\therefore \; \boxed{SNR_o = \frac{P_T}{N_0 B} = SNR_{DSB\text{-}SC}}$$

$$SNR_{DSB\text{-}SC} = SNR_{base}$$

Noise in AM signal (DSB-WC)

7

$$x(t) = [A_c + m(t)] \cos(2\pi f_c t) + n(t)$$

$$= [A_c + m(t) + n_c(t)] \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$$

1 For synchronucous detection :

$$y(t) = A_c[1 + \cos(4\pi f_c t)] + m(t)[1 + \cos(4\pi f_c t)]$$

$$+ n_c(t)[1 + \cos(4\pi f_c t)] - n_s(t) \sin(4\pi f_c t)$$

* after LPF : $\boxed{\tilde{y}(t) = A_c + m(t) + n_c(t)}$

* Signal power at the output is :

$P_s = P$, noise power $P_N = 2N_0 P$

$$\therefore \boxed{SNR_o = \frac{P}{2N_0 B} = SNR_{AM}}$$

Transmitted power $P_T = \frac{A_c^2}{2} + \frac{P}{2} = \frac{A_c^2 + P}{2}$

$$SNR_{base} = \frac{A_c^2 + P}{2N_0 B}$$

Hence

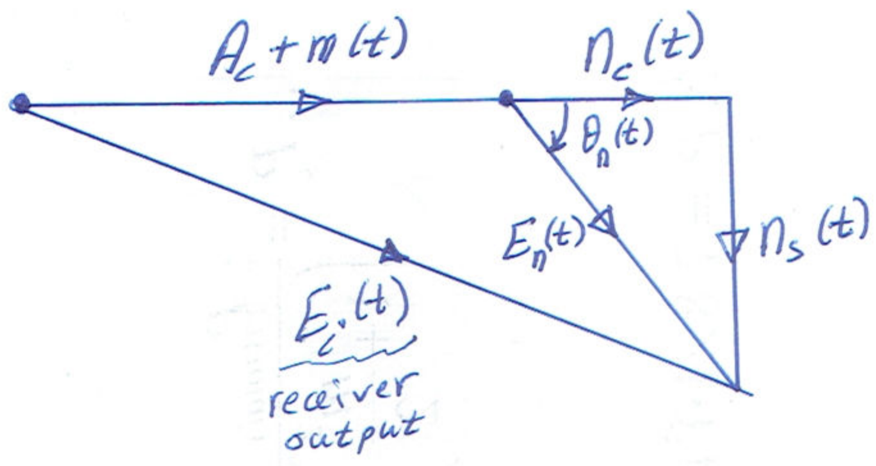
$$SNR_{AM} = \frac{P}{A_c^2 + P} \times SNR_{base}$$

where $\frac{P}{A_c^2+P} < 1$

$\therefore$ Conventional AM with synchronuous detection is <u>worse</u> than that of a baseband system

(2) For Envelope Detection

the received signal can be represented using the phasor diagram:-



$y(t) =$ envelope of $x(t)$

$$y(t) = \sqrt{[A_c + m(t) + n_c(t)]^2 + n_s^2(t)}$$

① very complicated signal.

* This equation is very complicated, we to use limiting cases, in order to put it in the form of noise + message

* we need the approximation.

Small Noise.

① First approximation is by considering

$$n(t) \ll [A_c + m(t)]$$

then $n_s(t) \ll [A_c + m(t) + n_c(t)]$

then $y(t) \approx A_c + m(t) + n_c(t)$

therefore:

$$\boxed{SNR_o = \frac{P}{2N_o B} \approx SNR_{envelope}}$$

comparing it with baseband SNR_{base}:

$$SNR_{env} \simeq \frac{P}{A_c^2 + P} \times SNR_{base}$$

2 Second approximation is by considering large Noise.

$$n(t) >> [A_c + m(t)]$$

Eq. (1) becomes

$$y^2(t) = [A_c + m(t) + n_c(t)]^2 + n_s^2(t)$$

$$y^2(t) \cong \underbrace{[n_c^2(t) + n_s^2(t)]}_{\downarrow}\left[1 + \frac{2[A_c + m(t)]\, n_c(t)}{n_c^2(t) + n_s^2(t)}\right]$$

$$y^2(t) = E_n^2(t)\left(1 + \frac{2[A_c + m(t)]\, n_c(t)}{E_n^2(t)}\right)$$

using the phasor diagram and $\sqrt{1+x} \approx 1 + \frac{x}{2}$ $x \ll 1$

$$\therefore \boxed{y(t) \approx E_n(t) + [A_c + m(t)]\cos(\theta_n(t))}$$

received Signal X noise

$\therefore$ the noise is multiplicative in this case, then we lost the signal.

Noise in SSB signal

$$S(t)_{SSB} = \frac{A_c}{2} m(t) \cos(2\pi f_c t) \pm \frac{A_c}{2} \tilde{m}(t) \sin(2\pi f_c t)$$

* where $\tilde{m}(t)$ is the Hilbert transform of $m(t)$
* $+$ for LSB $\quad -$ for USB
* Power of $\tilde{m}(t)$ = Power of $m(t)$
* Average power then $A_c^2 P/4$

we will analyse the lower side band (LSB)

→ $x(t) = S_{LSSB}(t) + n(t)$ or $x(t) = S(t) + n(t)$

→ Using coherent Detection

$$y(t) = x(t)\, 2\cos(2\pi f_c t)$$

$$y(t) = \left[\frac{A_c}{2} \tilde{m}(t) + n_s(t)\right] \sin(4\pi f_c t)$$

$$+ \left[\frac{A_c}{2} m(t) + n_c(t)\right] + \left[\frac{A_c}{2} m(t) + n_c(t)\right] \cos(4\pi f_c t)$$

* **After LPF** :

$$y(t) = \frac{A_c}{2} m(t) + n_c(t)$$

since we have :

① signal power $= \frac{A_c^2 P}{4}$

② Noise power for $n_c(t) = N_o B$

$\therefore$ $$\boxed{SNR_{SSB} = \frac{A_c^2 P}{4 N_o B}}$$

comparing it with baseband SNR :

we see that

$$SNR_{SSB} = SNR_{DSB\text{-}SC} = SNR_{base}$$

Figure of Merit : $\boxed{F = \frac{SNR_o}{SNR_{base}}}$

① For AM coherent $F = \frac{P}{A_c^2 + P}$ which is < 1

② For AM envelope low noise

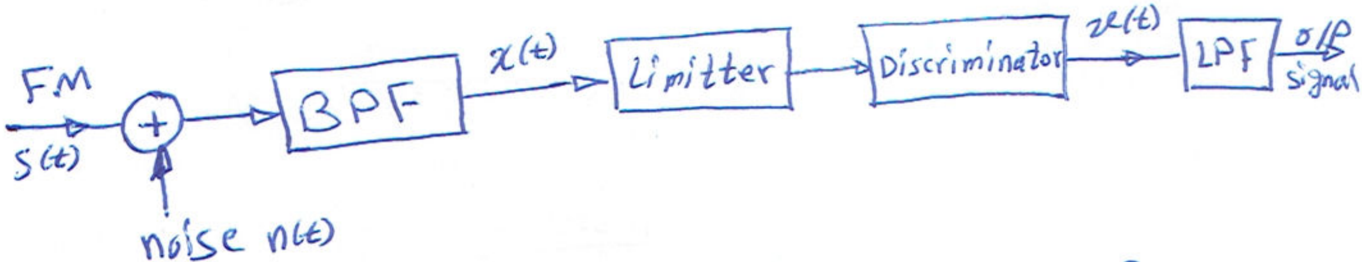
$F = 1$

③ For AM envelope Large noise F is poor

④ For DSB-SC coherent $F = 1$

⑤ For SSB coherent $F = 1$

Noise in FM signal



BPF : removes unwanted signals outside $f_c \mp \frac{B_T}{2}$.

Limitter : to remove amplitude variation.

Discriminator : recovers the message signal.

LPF : has bandwidth = bandwidth of the message signal.

* For high SNR, noise output and message signal are approximately independent of each other.

* we can compute the signal power for the case without noise, and accept that the result holds for the case with noise too.

Instantaneous frequency of the input signal:

$$f_i = f_c + k_f\, m(t)$$

output of discriminator:

$$k_f\, m(t)$$

$\therefore$ output signal power:

$$P_s = k_f^2\, P$$

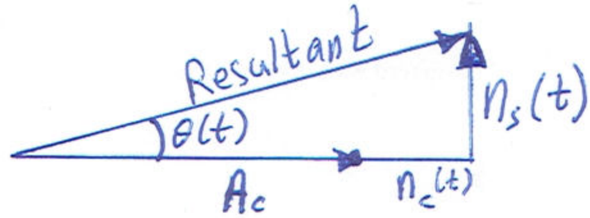
where P is the average power of the message signal.

* in the presence of additive noise,

$$x(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right] + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t).$$

we see that we have only carrier and noise, hence

$$\tilde{x}(t) = A_c \cos(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$



Instantaneous phase: $\theta_i(t) = \tan^{-1} \dfrac{n_s(t)}{A_c + n_c(t)}$

For Large carrier power (A_c is Large):-

$$\theta_i(t) = \tan^{-1} \frac{n_s(t)}{A_c} \simeq \frac{n_s(t)}{A_c}$$

* Discriminator output $= f_i(t)$, then

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = \frac{1}{2\pi A_c} \frac{dn_s(t)}{dt}$$

* The Discriminator output in the presence of noise and signal

$$k_f\, m(t) + \frac{1}{2\pi A_c} \frac{dn_s(t)}{dt}$$

From Fourier theory;

$$x(t) \xleftrightarrow{F.T.} X(f)$$

$$\frac{d}{dt} x(t) \xleftrightarrow{F.T.} j2\pi f\, X(f)$$

This means that the differentiation with respect to time $\Rightarrow$ passing the signal through a system with transfer function of $H(f) = j2\pi f$

then, the power spectral density of the signal after the discriminator is

$$S_o(f) = |H(f)|^2 S_i(f)$$

where $S_i(f)$ is the PSD of the i/p signal.

NOW $\circ\circ$

$$\text{PSD of } \frac{d n_s(t)}{dt} = |j2\pi f|^2 \times \text{PSD}\{n_s(t)\}$$

PSD of $n_s(t) = N_o$ within band $\pm \frac{B_T}{2}$

Hence

$$\text{PSD of } \frac{d n_s(t)}{dt} = |j2\pi f|^2 N_o$$

$$\text{PSD of} \left\{ f_i(t) = \frac{1}{2\pi A_c} \frac{d n_s(t)}{dt} \right\} = \left(\frac{1}{2\pi A_c}\right)^2 |j2\pi f|^2 N_o = S_D(f)$$

AFTER LPF $\circ\circ$ (LPF restricted to $\mp B$)

$$P_N = \int_{-B}^{B} S_D(f)\, df = \frac{2 N_o B^3}{3 A_c^2}$$

$\downarrow$

Note from the last expression :

$$P_N \propto \frac{1}{A_c^2}$$

i.e., high carrier power, less noise power
low carrier power, high noise power

this is called quieting effect

However

$$\boxed{SNR_o = \frac{3 A_c^2 k_f^2 P}{2 N_o B^3} = SNR_{FM}}$$

comparing it with base band :
transmitted power of FM $= P_T = \frac{A_c^2}{2}$

$$\therefore SNR_{FM} = \frac{3 k_f^2 P}{B^2} SNR_{base} = 3\beta^2 \frac{P}{m_p^2} SNR_{base}$$

this is valid for large carrier power.

Just like AM envelope detector, FM detector has threshold effect, which is when the signal power $= 10 \times$ noise power :

$$\frac{A_c^2}{2 N_0 B_T} = 10$$

Note that

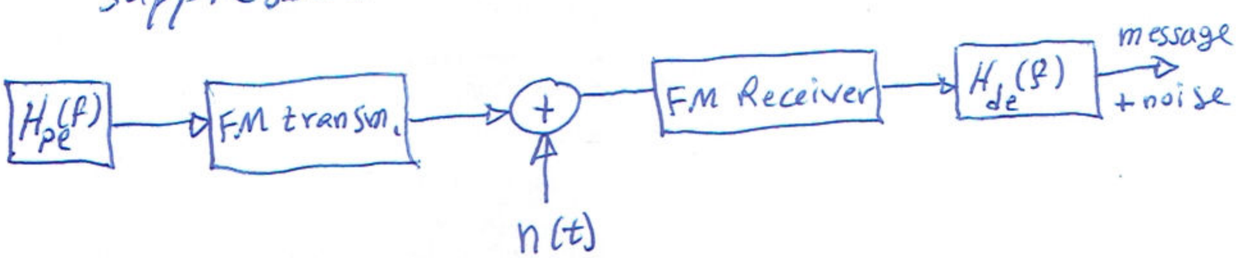
$B_T = 2 f_m (\beta + 1)$ [Carson's rule]

also note that the PSD of the noise at the o/p of the detector $\propto f_c^2$

Thus, to increase SNR_{FM} : we use

$$\boxed{\text{Pre-emphasis and de-emphasis}}$$

where the message unchanged and the high frequency components of noise are suppressed.



$H_{pe}(f)$: is a HPF used to artificially emphasize the high frequency components of the message prior to modulation, and hence, before noise introduced.

$H_{de}(f)$: is a LPF used to de-emphasize the high frequency components of the message at the receiver, and restore the original PSD of the message signal.

That is : $H_{pe}(f) \propto f$ and $H_{de}(f) \propto \frac{1}{f}$

this operation improves the output SNR by around 13 dB.

Lets compare Linear and non-Linear modulation schemes :

For fair comparison; we have to assume the following :

we assume :

1. $m(t) = A_m \cos(2\pi f_m t)$ → single tone
2. $B = f_m$ → message bandwidth
3. modulation index of AM $m_a = 1$
4. For FM system: $\beta = 5$ (which is the real in practical commercial transmission) and $\Delta f = 75\,KHz$ with $B = 15\,KHz$.

Thus :

$$SNR_{DSBSC} = SNR_{base}$$

$$SNR_{AM} = \frac{1}{3} SNR_{base}$$

$$SNR_{FM} = \frac{3}{2}\beta^2 SNR_{base} = \frac{75}{2} SNR_{base}$$

We conclude :

* the performance of AM SNR is 4.8 dB worse than baseband.
* The DSB-SC performance is identical to baseband as well as for SSB but with $B_T = f_m$ not $2f_m$.
* the FM performance is 15.7 dB better than baseband, where $B_T = 2(\beta+1)B = 12B$ i.e., $= 12 f_m$. If pre- and de-emphasis used, the SNR increased by 13 dB.