Amplitude Modulation :-

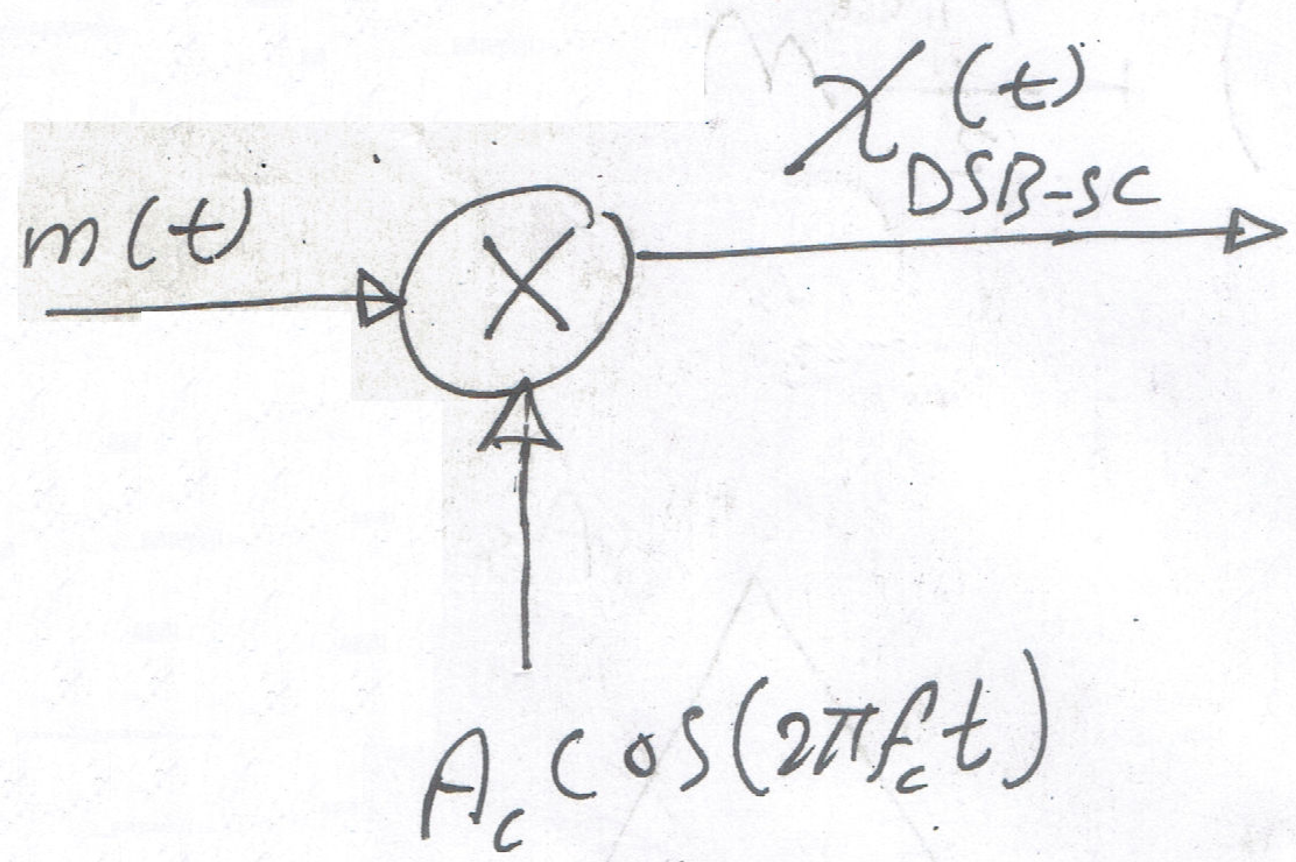
* In linear modulation, there are DSB-SC, AM-modulation, & SSB modulation.

For the double side band DSB modulation:-

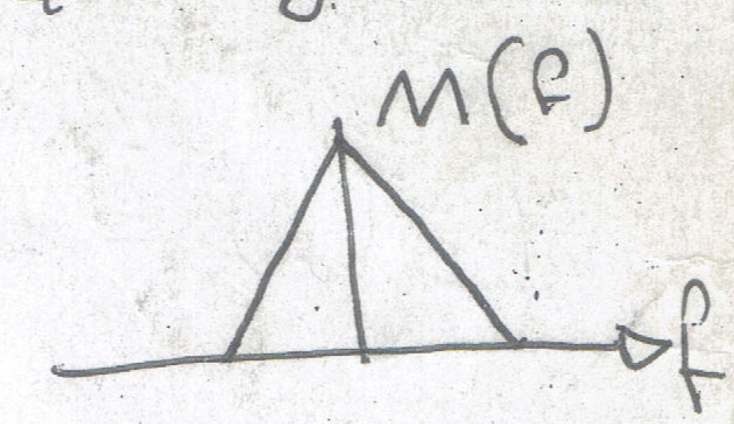
if the message is $m(t)$ and the carrier is $A_c \cos(2\pi f_c t)$, then

the modulated DSB-SC signal is

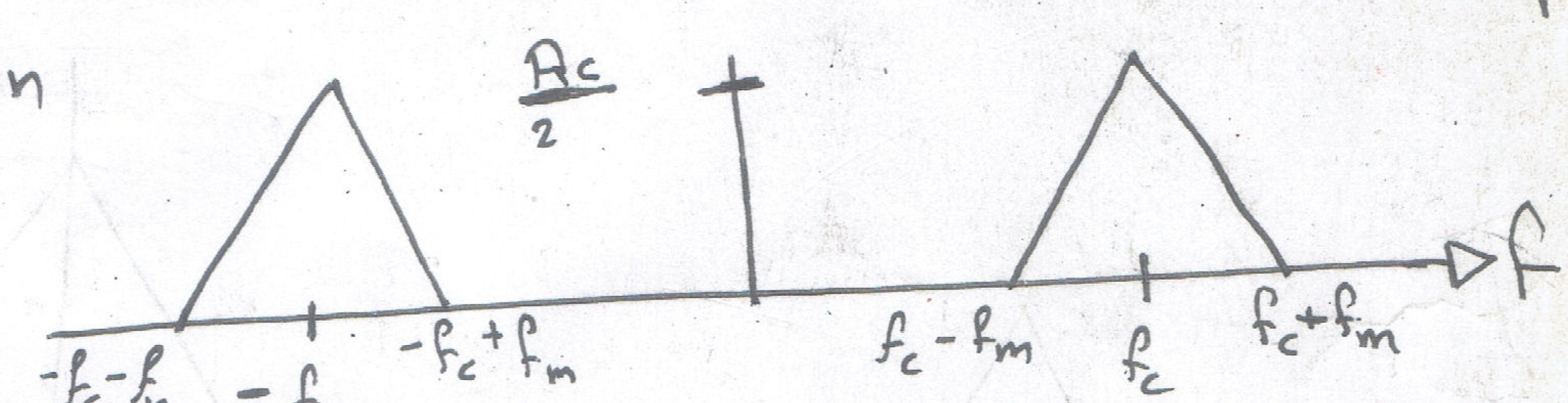
$$x_{DSB-SC}(t) = A_c m(t) \cos(2\pi f_c t) \quad \text{——— (1)}$$



assume the message in frequency domain is a triangular function

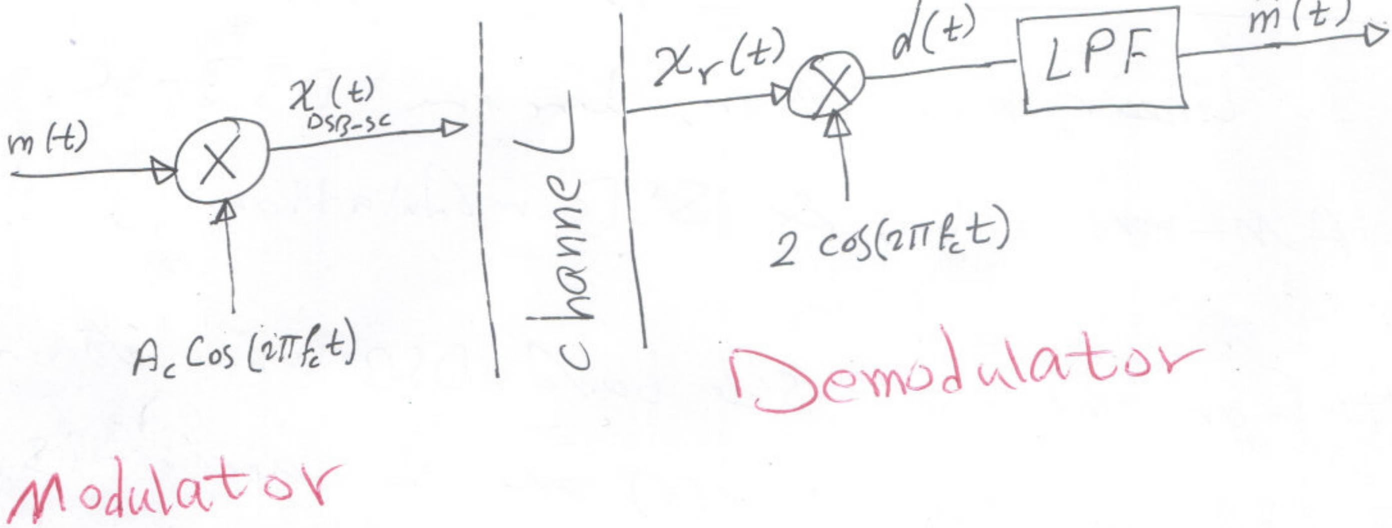


then



Thus, $X_{DSB-SC}(f) = \frac{A_c}{2} M(f-f_c) + \frac{1}{2} A_c M(f+f_c)$

DSB-SC Demodulation

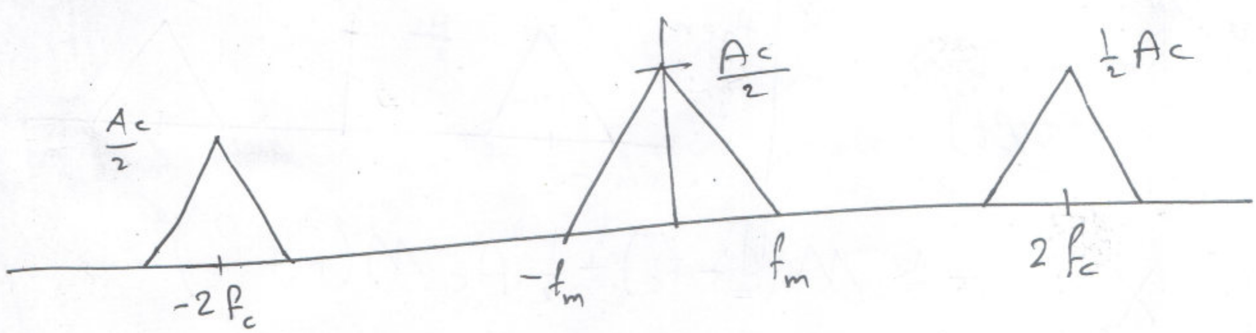


Assuming an ideal channel

$$d(t) = \left[A_c\, m(t) \cos(2\pi f_c t)\right] 2\cos(2\pi f_c t)$$

$$d(t) = A_c\, m(t) + A_c\, m(t) \cos(2\pi (2f_c) t)$$

$$D(f) = A_c M(f) + \frac{A_c}{2} M(f - 2f_c) + \frac{A_c}{2} M(f + 2f_c)$$



A_c can be adjusted since it is simply a gain only. AGC amplifier can adjust it as an example.

In Double Side Band modulation, the required bandwidth (see the plot of the spectrum on page 2) is double the message's frequency f_m.

$$\therefore BW_{DSB-SC} = 2 f_m$$

At the Demodulator side, the carrier should be identical to that in the transmitter (Modulation). Moreover, the phase also should be exactly synchronous.

II if neither the frequency nor the phase are correct, there will be a major problem in the process of message recovery.

* Let the carrier is imperfect as $c(t) = 2\cos[2\pi f_c t + \theta(t)]$, then, where $\theta(t)$ is time-varying phase error

$$d(t) = A_c m(t)\cos\theta + A_c m(t)\cos[2\pi f_c t + \theta(t)]$$

$\hat{m}(t) = m(t)\cos(\theta(t))$ —— if $\theta(t)$ slowly varying or constant then, $\cos(\theta(t))$ appears as fixed or time varying attenuation factor

- If $\theta(t) = \Delta f t$

and $m(t) = \cos(2\pi f_m t)$ then

$$\hat{m}(t) = \frac{1}{2}\left[\cos[2\pi(f_m - \Delta f)t] + \cos[2\pi(f_m + \Delta f)t]\right]$$

thus, the received message $\hat{m}(t)$ is a sum of two tones.

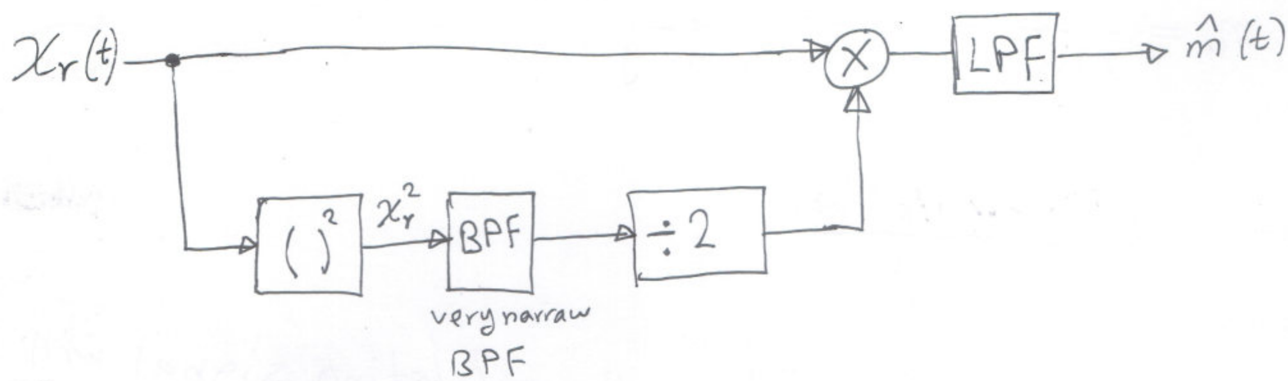
Hence, it is very important to provide a perfect synchronized carrier in the receiver. Then we perform the Coherent demodulation

Many methods are available to recover the carrier from the transmitted (modulated) signals such as the;

1. square Law.
2. phase Locked Loop (PLL).

we will see here the first method, square Law, while the PLL will be discussed Later in details.

By squaring the received signal $\hat{m}(t)$, we can extract the carrier completely as follows:-



$$x_r^2(t) = A_c^2\, m^2(t) \cos^2(2\pi f_c t)$$

$$= \frac{1}{2} A_c^2\, m^2(t) + \frac{1}{2} A_c^2\, m^2(t) \cos\left[2\pi (2 f_c)\, t\right]$$

Amplitude Modulation :- [used in Radio broadcasting & TV picture]

amplitude modulation is simply $\boxed{\text{DSB + carrier}}$

Thus,

$$x_{AM}(t) = A_c\, m(t) \cos(2\pi f_c t) + A_c \cos(2\pi f_c t)$$

$$X_{AM}(f) = \frac{A_c}{2}\left[M(f+f_c) + M(f-f_c)\right] + \underbrace{\frac{A_c}{2}\left[\delta(f+f_c) + \delta(f-f_c)\right]}_{\text{pure carrier}}$$

* In this case the Bandwidth $BW = 2 f_m$

Modulation Index

modulation index is also called the modulation depth.
or degree of modulation
or modulation factor

$$m_a = \frac{|m(t)|_{max}}{\text{Maximum carrier Amplitude}}$$

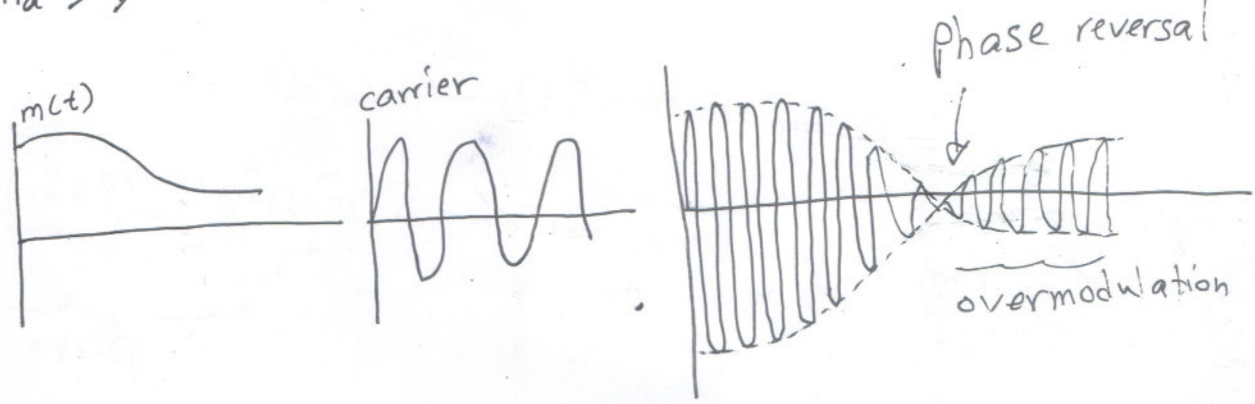
$$\therefore \quad m_a = \frac{|m(t)|_{max}}{A_c}$$

* Note: the baseband or the modulating signal will be **preserved** in the envelope of the AM signal only if we have

$$\boxed{|m(t)|_{max} \leq A_c}$$

In other words, $\boxed{m_a \leq 1}$

* if $m_a > 1$ → overmodulation [signal distortion]



Single Tone Amplitude Modulation

* So far, we have considered that the message or the baseband or the modulating signal is random.
* Lets be more specific: Assume the message is single-tone, in other words, let the message be a single carrier sinusoid signal:-

$$m(t) = V_m \cos(\omega_m t) \qquad \boxed{\omega_m = 2\pi f_m}$$

if the carrier signal is $c(t) = A_c \cos(\omega_c t)$ $\boxed{\omega_c = 2\pi f_c}$

$$\therefore \boxed{x_{AM}(t) = \underbrace{A_c\left[1 + \frac{V_m}{A_c}\cos(\omega_m t)\right]}_{\text{Envelope part}} \underbrace{\cos(\omega_c t)}_{\text{carrier}}}$$

Hence, the modulation index $\boxed{m_a = \frac{V_m}{A_c}}$

Therefore,

$$\boxed{x_{AM}(t) = A_c\left[1 + m_a \cos(\omega_m t)\right]\cos(\omega_c t)}$$

* Lets see the frequency components of the AM-signal, which is a single-tone in the current case.

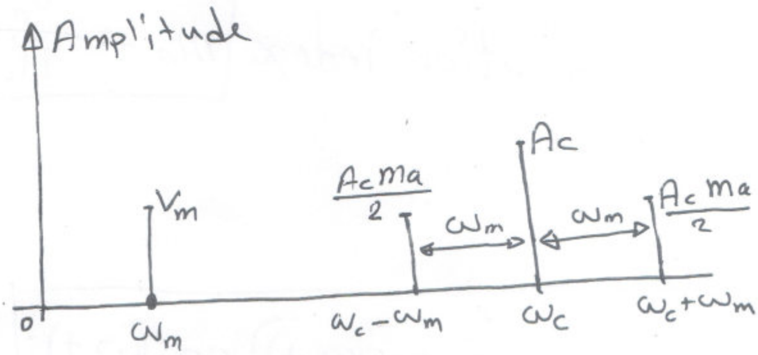
$$x_{AM}(t) = A_c\left[1 + m_a \cos(\omega_m t)\right]\cos(\omega_c t)$$

$$= A_c \cos(\omega_c t) + A_c m_a \cos(\omega_m t)\cos(\omega_c t)$$

$$= A_c \cos(\omega_c t) + \frac{A_c m_a}{2}\left[2\cos(\omega_m t)\cos(\omega_c t)\right]$$

$$= \underbrace{A_c \cos(\omega_c t)}_{\text{first frequency}} + \frac{A_c m_a}{2}\underbrace{\cos(\omega_c + \omega_m)t}_{\text{second frequency}} + \frac{A_c m_a}{2}\underbrace{\cos(\omega_c - \omega_m)t}_{\text{third frequency}}$$

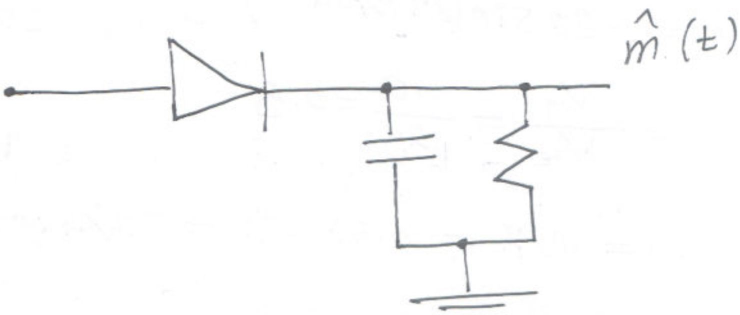
* the first frequency is a pure carrier with amplitude A_c,
* the second frequency is the upper sideband $(\omega_c + \omega_m)$ with amplitude $\frac{A_c m_a}{2}$,
* the third frequency is the lower sideband $(\omega_c - \omega_m)$ with amplitude $\frac{A_c m_a}{2}$.

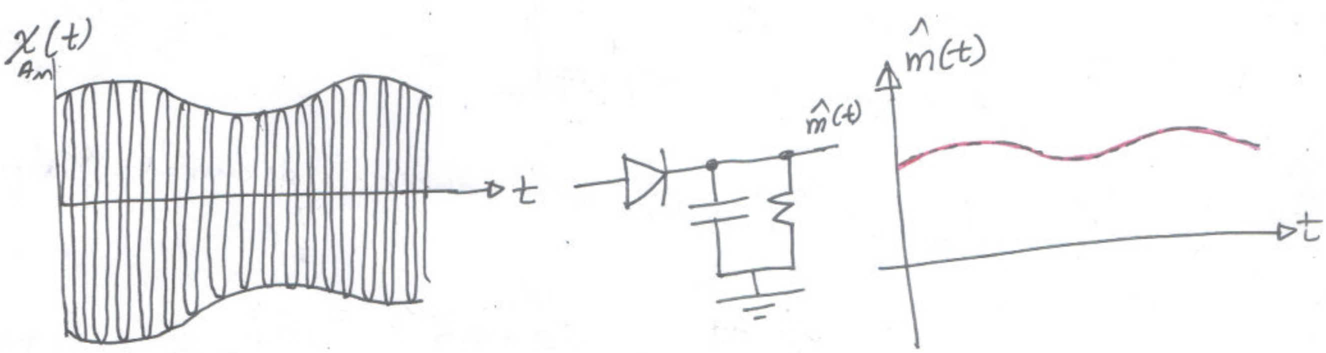


this plot is for single-sided frequency spectrum of the single-tone AM.

AM- Demodulation:

* Because of the AM-signal has a pure carrier frequency componentg a simple envelope detector can be employed to recover the baseband signal.





EX.1 A modulating signal $10\sin(2\pi 10^3 t)$ is used to modulate a carrier signal $20\sin(2\pi 10^4 t)$. Determine the modulation index, percentage modulation, frequencies of the sideband components and their amplitudes. What will be the bandwidth of the modulated signal?

Solution: we have given

$$m(t) = V_m \sin(2\pi f_m t)$$
$$m(t) = 10\sin(2\pi 1000 t)$$
$$c(t) = A_c \sin(2\pi f_c t)$$
$$c(t) = 20\sin(2\pi 10{,}000 t)$$

$f_m = 1000\ Hz$

$V_m = 10$

$f_c = 10{,}000\ Hz$

$A_c = 20$

$\therefore$ The modulation index $m_a = \dfrac{V_m}{A_c} = \dfrac{10}{20} = 0.5$

The percentage modulation $= m_a\% = 0.5 * 100\% = 50\%$.

The upper sideband $f_{USB} = f_c + f_m = 11{,}000\ Hz = 11\ KHz$.

The Lower sideband $f_{LSB} = f_c - f_m = 9000\ Hz = 9\ KHz$.

The amplitude of the upper and Lower sidebands are:

$$A_{USB} = A_{LSB} = \frac{A_c m_a}{2} = \frac{20 * 0.5}{2} = \frac{10}{2} = 5 \text{ volts.}$$

The Bandwidth $BW_{AM} = 2f_m = 2 * 1000 = 2\ KHz$.

Power Contents in AM signal:

The general mathematical expression for the AM signal is simply by summing the pure carrier with the DSB-SC signals.

$$\therefore \quad x_{AM}(t) = A_c \cos(\omega_c t) + m(t)\cos(\omega_c t)$$

* there are two power components in $x_{AM}(t)$; the carrier power (P_c) & the sideband power (P_s).

* The carrier power $P_c = \frac{1}{2\pi}\int_0^{2\pi} A_c^2 \cos^2(\omega_c t)\, dt = \frac{A_c^2}{2}$

* The sideband power $P_s = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2}\left[2\cos^2(\omega_c t)\right] m^2(t)\, dt$

filter-out

$$P_s = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} m^2(t)\, dt + \frac{1}{2\pi}\int_0^{2\pi} \cancel{m^2(t)\cos(2\omega_c t)}\, dt$$

$$\therefore \quad P_s = \frac{1}{2\pi}\int_0^{2\pi} \left[\frac{1}{2} m^2(t)\right] dt$$

* However, P_s is the contribution of the USB and the LSB,

Hence,

$$P_{LSB} = P_{USB} = \frac{P_s}{2}$$

* $\therefore$ total power $P_t = P_c + P_s$

* NOW, the transmission efficiency η will be

$$\boxed{\eta = \frac{P_s}{P_t}}$$

and the percentage efficiency is

$$\boxed{\eta = \frac{P_s}{P_t} * 100\,\%}$$

* For single-tone AM-signal,

$$P_s = \frac{1}{2}\,\frac{V_m^2}{2}$$

$$\boxed{P_s = \frac{V_m^2}{4}}$$

$$\therefore\ P_t = P_c + P_s = \frac{A_c^2}{2} + \frac{1}{4} V_m^2 = \frac{A_c^2}{2}\left[1 + \frac{1}{2}\left(\frac{V_m}{A_c}\right)^2\right],\quad \text{since } m_a = \frac{V_m}{A_c}$$

$$\therefore\ P_t = \frac{A_c^2}{2}\left[1 + \frac{1}{2} m_a^2\right]$$

or $\boxed{P_t = P_c\left[1 + \frac{m_a^2}{2}\right]}$ for single-tone AM

EX. A 400 watts carrier is modulated to a depth of 75%, find the total power in the amplitude-modulated wave. Assume the modulating signal to be a sinusoidal one.

Solution we have given the carrier power $\boxed{P_c = 400\ w}$ and
we have given the modulation index $\boxed{m_a = 0.75}$.
we also given that the modulating signal is sinusoidal,
this represents the modulated signal is single-tone AM.

$$P_t = P_c + P_s = \frac{A_c^2}{2} + \frac{1}{4}V_m^2 = \frac{A_c^2}{2}\left[1 + \frac{1}{2}\left[\frac{V_m^2}{A_c^2}\right]\right]$$

$$P_t = P_c\left[1 + \frac{m_a^2}{2}\right]$$

$$\therefore P_t = 400\left[1 + \frac{(0.75)^2}{2}\right] = 512.5\ w.$$

EX. An AM broadcast radio transfers radiates 10k watts of power, if the modulation percentage is 60%, calculate how much of this is the carrier power.

Solution $P_t = P_c\left[1 + \frac{m_a^2}{2}\right]$, we have given $P_t = 10\ kw$
$m_a = 0.6$

$$10,000 = P_c + \frac{P_c}{2}(0.6)^2$$

$$20,000 = 2P_c + 0.36\,P_c$$

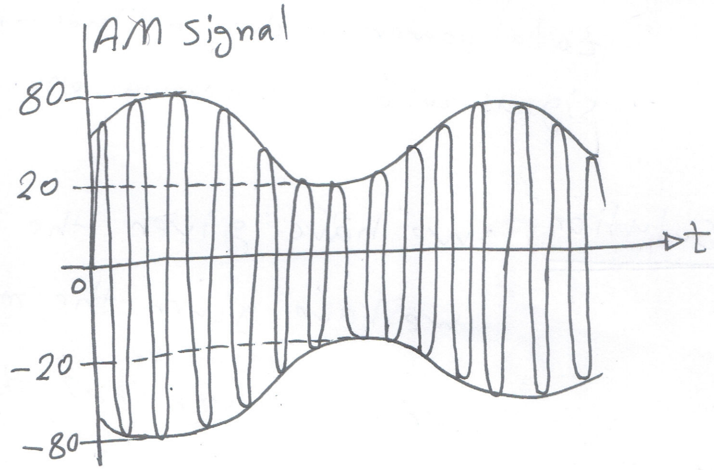
$$2.36\,P_c = 20,000$$

$$P_c = 8474.6\ w$$

or

$$P_c = 8.475\ kw$$

Ex. Determine the modulation index of the signal shown below.



Solution

modulation Index

$$m_a = \frac{V_m}{A_c} = \frac{max[P\text{-}P] - min[P\text{-}P]}{max[P\text{-}P] + min[P\text{-}P]}$$

$$max(P\text{-}P) = 2(80) = 160$$

$$min(P\text{-}P) = 2(20) = 40$$

$$\therefore m_a = \frac{160-40}{160+40} = \frac{120}{200} = 0.6$$

EX. A carrier wave is represented by expression $V_c(t) = 10 \sin\omega_c t$. Draw the waveform of an AM wave for $m_a = 0.5$, and 1 Hz modulating signal.

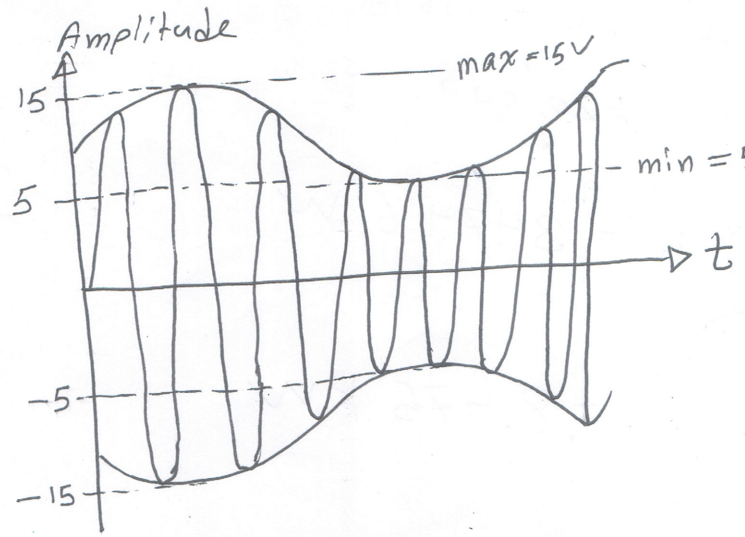
Solution Given $A_c = 10$ Volts. or $V_c(t) = 10 \sin\omega_c t$

$$m_a = \frac{V_m}{A_c} \Rightarrow V_m = m_a * A_c = 10 * 0.5 = 5 \text{ Volts}$$

Hence $V_{max} = A_c + V_m = 10 + 5 = 15$ volts.

and

$$V_{min} = A_c - V_{min} = 10 - 5 = 5 \text{ Volts}$$



Quadrature-Amplitude Modulation (QAM)

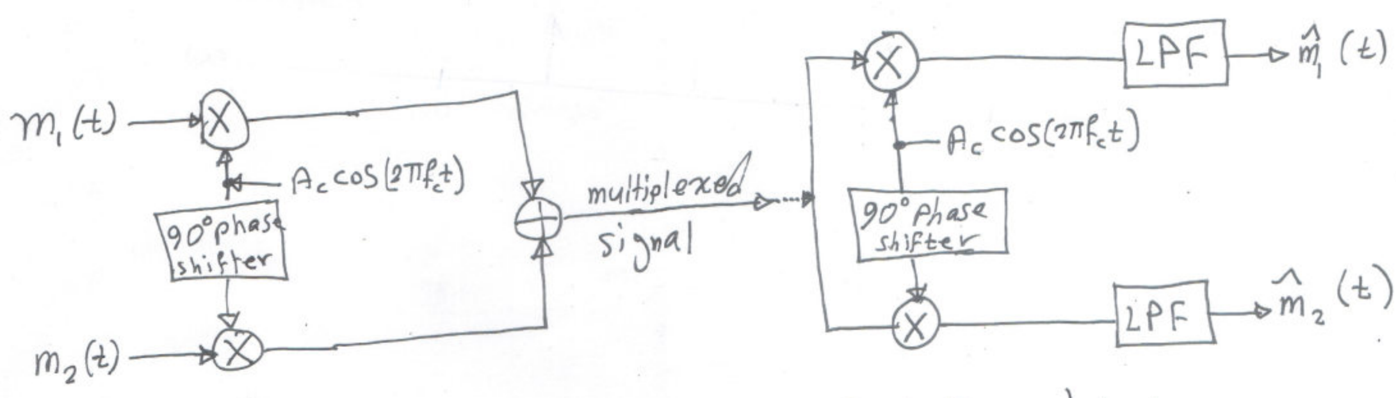
- Also called Quadrature Carrier Multiplexing
- In this modulation scheme, two DSB-SC can hold two different messages on the same carrier.
- Therefore, this scheme is bandwidth-conservation scheme.
- QAM is used for digital modulation of analoge carriers in data modems to convey data through the public telephone network. Also, QAM used for digital satellite communications systems.

* Thus, two messages $m_1(t)$ & $m_2(t)$ using one f_c as follows

$$x_{QAM}(t) = m_1(t) A_c \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t)$$

OR

$$x_{QAM}(t) = m_1(t) A_c \sin(2\pi f_c t) + A_c m_2(t) \cos(2\pi f_c t)$$



QAM-Modulator QAM-Demodulator

which

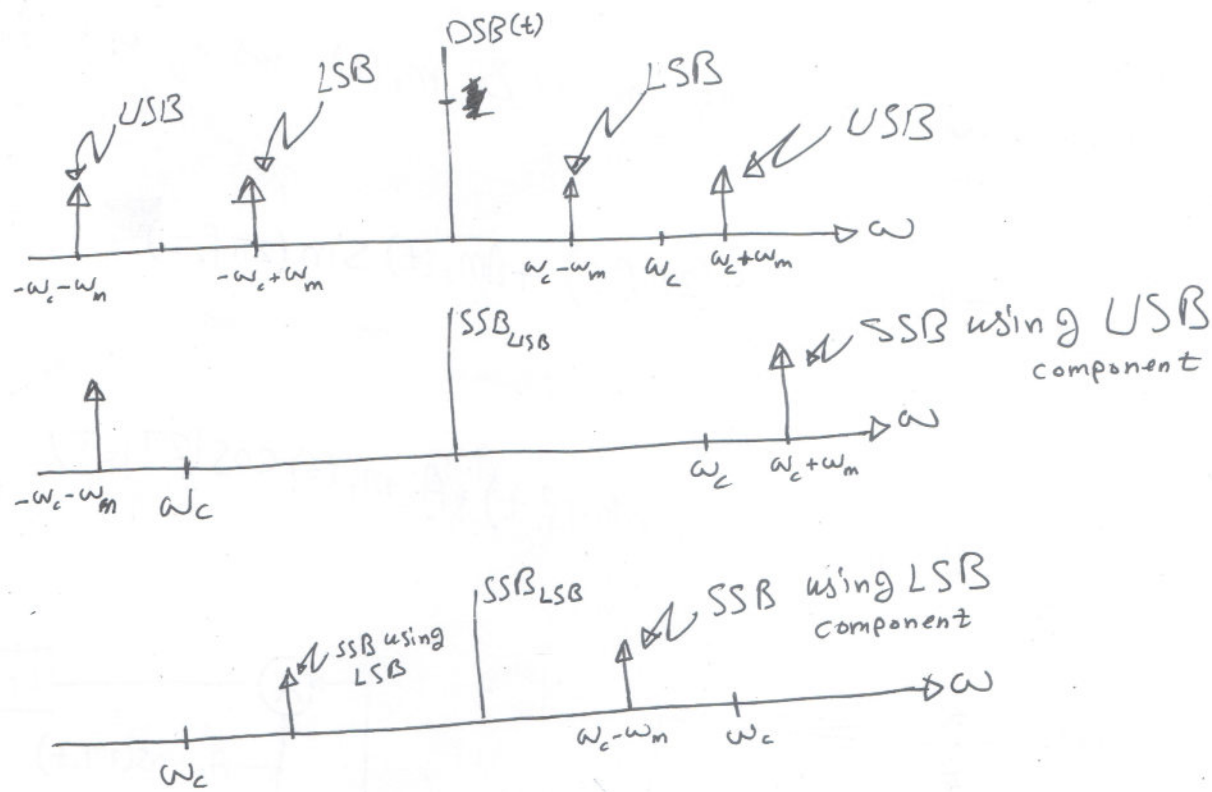
Single Side Band (SSB) Modulation

* We have seen that DSB transmits LSB & USB which they are holding the same information message. Thus, it is not necessary to transmit both sidebands.

* If one sideband (SSB) was transmitted, the occupied bandwidth of the DSB-SC will be halved.

$$\therefore BW_{SSB} = \frac{BW_{DSB}}{2} = \frac{2f_m}{2} = f_m \quad Hz.$$

However, this bandwidth saving is accompanied by a considerable increase in complexity.



However, the SSB signal can be expressed as

$$x_{SSB}(t) = m(t)\cos\omega_c t \pm \tilde{m}(t)\sin\omega_c t$$

where $\tilde{m}(t)$ is the same as $m(t)$ but with its phase shifted by $-\frac{\pi}{2}$.

Hence,

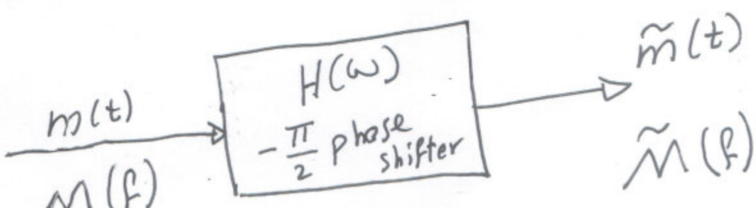
* For selecting the Lower sideband of the DSB-SC,

$$x_{LSSB}(t) = m(t)\cos\omega_c t + \tilde{m}(t)\sin\omega_c t$$

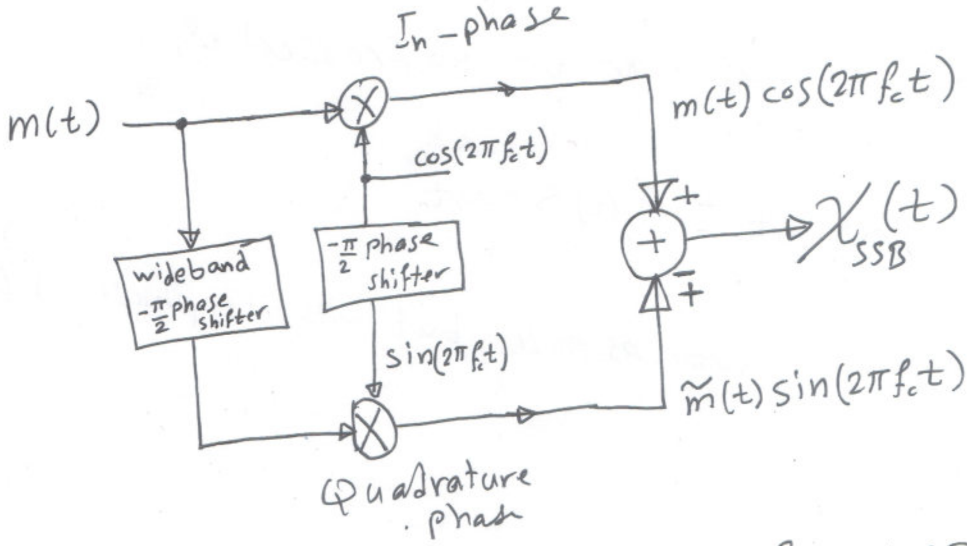
* For selecting the upper sideband of the DSB-SC

$$x_{USSB}(t) = m(t)\cos\omega_c t - \tilde{m}(t)\sin\omega_c t$$

* $\tilde{m}(t)$ can be obtained using Hilpert transform



* SSB finds applications in mobile radio communications, telemetry, military, navigation and amature radio. Many of these applications are point to point communication applications.



where (+) for LSB & (−) for USB