Angle Modulation

$$x(t) = A_c \cos(2\pi f_c t + \theta_0) \quad \text{———— (1)}$$

or $x(t) = A_c \cos(\omega_c t + \theta_0)$ ———— (1)

Let $\phi = \omega_c t + \theta_0$ ———— (2)

$$\frac{d\phi}{dt} = \frac{d\omega_c t}{dt} + \frac{d\theta_0}{dt}$$

$$\boxed{\omega_c = \frac{d\phi}{dt}}$$

Lets say ω_c = instantinuous angular frequency

$\therefore \quad \omega_c = \omega_i$

$\therefore \quad \phi = \int \omega_i \, dt$ ———— (3)

Now ϕ is time-dependent, thus if ϕ varies with the message, the carrier signal is then angle modulated

$$x(t) = A_c \cos(\phi) \quad \text{———— (4)}$$

Angle Modulation

Frequency Modulation (FM)

- frequency of the carrier is varied ~~with~~ according to the message signal

Phase Modulation (PM)

- Phase angle of the carrier is varied according to the message signal

* FM & PM are better than amplitude modulation such as noise reduction, and efficient use of power.

* Disadvantages of FM & PM are increased bandwidth and use of complex circuits.

<u>Applications</u>

1. Radio broadcasting,
2. Two way mobile Radio,
3. Microwave communication,
4. TV sound transmission,
5. Cellular radio, and
6. Satellite Communication.

Phase Modulation

we know $x(t) = A_c \cos(\omega_c t + \theta_0)$

or $x(t) = A_c \cos(\phi)$

where $\phi = \omega_c t + \theta_0$

* Neglecting θ_0, we get, total phase angle of unmodulated carrier is

$$\phi = \omega_c t$$

* In PM, ϕ changed-Linearly with the message.

→ Denoting ϕ_i as the instantinuous phase angle,

$$\phi_i = \omega_c t + K_p m(t) \quad (10)$$

K_p is the constant of phase sensitivity.

$$\therefore \boxed{x_{PM}(t) = A_c \cos[\omega_c t + K_p m(t)]} \quad (12)$$

Frequency Modulation :

* The carrier frequency will changed according to the message signal,
* The carrier frequency will deviate linearly according to the message signal.

→ The instantinuous frequency is

$$\boxed{\omega_i = \omega_c + K_f\, m(t)} \quad (13)$$

K_f is constant of the frequency sensitivity.

Hence, $\phi_i = \int \left[\omega_c + k_f\, m(t)\right] dt$

$$= \omega_c t + k_f \int m(t)\, dt$$

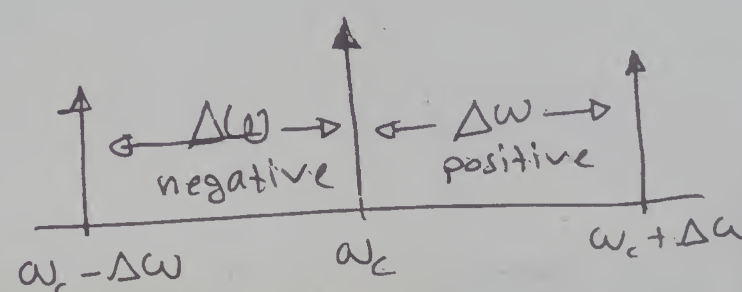
Thus :

$$\boxed{x_{FM}(t) = A_c \cos\left[\omega_c t + k_f \int_0^t m(t)\, dt\right]} \quad (14)$$

* The maximum change in ω_i (instantinuous angular freq.) from the carrier frequency ω_c is called the frequency deviation ($\Delta\omega$)

* The deviation is due to the message, thus, the deviation maybe +ve or −ve.

$\therefore \quad \Delta\omega = \left| k_f \, m(t) \right|_{max}$



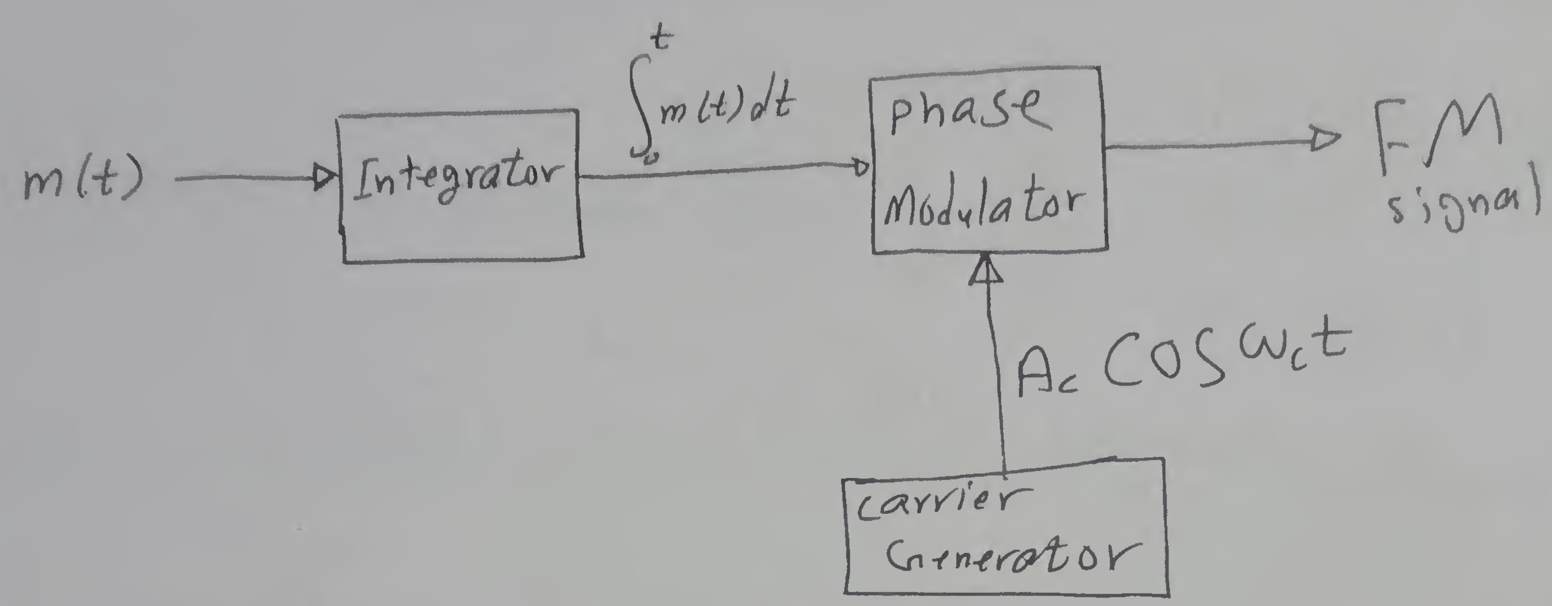
Relationship between FM & PM :-

- Angle modulation is $S(t) = A_c \cos \phi_i$
- For PM $\quad S_{PM}(t) = A_c \cos[\omega_c t + k_p x(t)]$
- For FM $\quad S_{FM}(t) = A_c \cos\left[\omega_c t + k_f \int_0^t m(t)\, dt\right]$

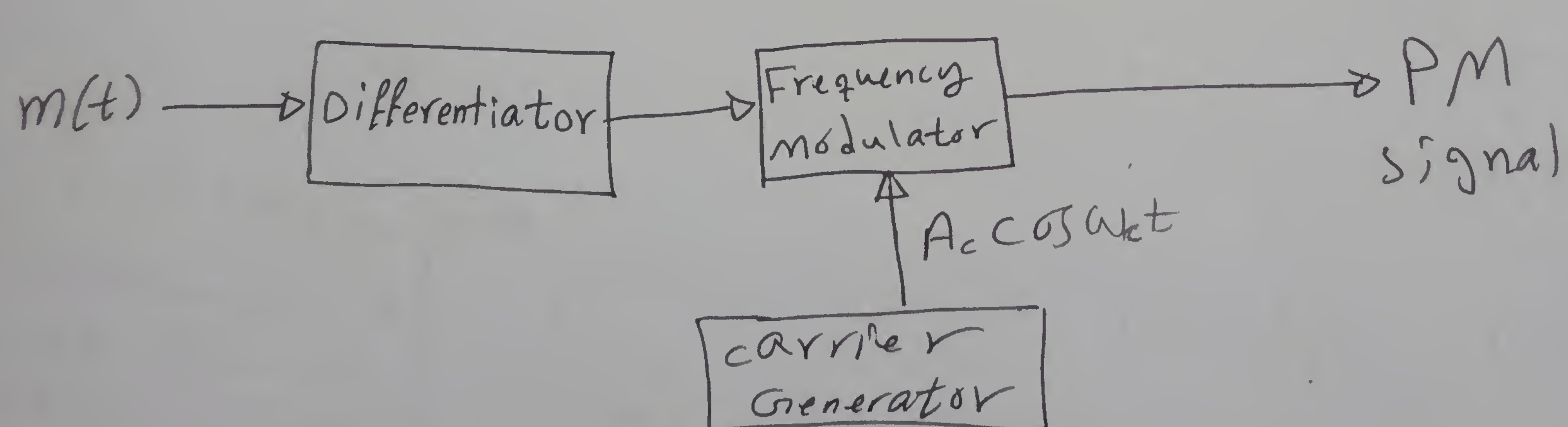
related to each other, because in both cases there is a variation in total phase angle.

Hence, FM can be obtained using PM or PM can be obtained using FM.

Generating FM From PM



Generating PM from FM



Single Tone FM:

$$S(t)_{FM} = A_c \cos\left[\omega_c t + k_f \int_0^t m(t)\,dt\right]$$

Assuming the message signal is

$$m(t) = V_m \cos\omega_m t$$

$$\therefore \phi_i = \int \omega_i \, dt = \int \left[\omega_c + k_f V_m \cos\omega_m t\right] dt \quad \text{--- (i)}$$

But $\Delta\omega = |k_f\, m(t)|_{max} = k_f\, |m(t)|_{max}$

$$\therefore \quad \Delta\omega = k_f V_m$$

$$\therefore \omega_i = \omega_c + \Delta\omega \cos\omega_m t$$

$$\therefore \phi_i = \int \left[\omega_c + \Delta\omega \cos\omega_m t\right] dt$$

$$= \omega_c t + \frac{\Delta\omega}{\omega_m} \sin\omega_m t$$

$\frac{\Delta\omega}{\omega_m} \rightarrow m_f$ = modulation index

$$= \omega_c t + m_f \sin\omega_m t$$

$$\therefore \boxed{S(t)_{FM} = A_c \cos\left[\omega_c t + m_f \sin\omega_m t\right]}$$ Single-Tone FM signal.

~~Third Year Exam 2015/2016~~

~~31/1/2016~~

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Ex. 1 A single-tone FM is represented by the voltage equation as:

$$v(t) = 12 \cos(6\times10^8 t + 5 \sin 1250\, t)$$

Determine the following :-

(i) carrier frequency (ii) modulating frequency (iii) modulation index (iv) maximum deviation (v) what power will this FM wave dissipate in 10 Ω resistor?

Solution :- The standard expression for a single-tone FM is

$$v(t) = A \cos(\omega_c t + m_f \sin \omega_m t)$$

we have given

$$v(t)\ 12 \cos(6\times10^8 t + 5 \sin 1250\, t)$$

$$\therefore\ \omega_c = 6\times10^8 \text{ rad/sec.}$$

(1) $\therefore\ f_c = 95.5 \text{ MHz}.$

$$\omega_m = 1250 \text{ rad/sec}$$

(2) $f_m = 199 \text{ Hz}$

(3) $m_f = 5$

(4) $m_f = \dfrac{\Delta\omega}{\omega_m} = \dfrac{\Delta f}{f_m}$

$$\Delta f = m_f f_m = 995 \text{ Hz}$$

(5) The power dissipated is $P = \dfrac{V_{rms}^2}{R} = \dfrac{\left(\frac{12}{\sqrt{2}}\right)^2}{10} = \dfrac{72}{10} = 7.2 \text{ W.}$

EX.2 A 107.6 MHz carrier signal is frequency modulated by a 7 KHz sine wave. The resaltant FM signal has a frequency deviation of 50 KHz. Determine the following:

① The carrier swing of the FM signal. ② The highest & lowest frequencies attained by the modulated signal. ③ modu. index of the FM wave.

Solution

$f_c = 107.6 \text{ MHz}$.

$f_m = 7 \text{ KHz}$.

$\Delta f = 50 \text{ KHz}$.

① Carrier swing $= 2 * \text{frequency deviation} = 2 \times 50 = 100 \text{ KHz}$.

②

$$f_H = f_c + \Delta f = 107.6 \text{ MHz} + 50 \text{ KHz}$$

$$f_H = 107.65 \text{ MHz}$$

$$f_L = f_c - \Delta f = 107.6 \text{ MHz} - 50 \text{ KHz}$$

$$f_L = 107.55 \text{ MHz}$$

③ Modulation index

$$m_f = \frac{\Delta f}{f_m} = \frac{50 \times 10^3}{7 \times 10^3} = 7.143.$$

EX. 3 Determine Δf and carrier swing of FM signal which has a resting frequency of 105 MHz and upper frequency of 105.007 MHz. Also find the lowest frequency reached.

solution

$f_c = 105$ MHz

$f_H = 105.007$ MHz

$\Delta f = f_H - f_c = 105.007 - 105 = 7$ kHz.

carrier swing $= 2 \times \Delta f = 14$ kHz.

$f_L = f_c - \Delta f = 104.993$ MHz.

EX. 4 what is m_f of FM signal of carrier swing = 100 kHz when the modulating signal has frequency = 8 kHz?

Solution

+ carrier swing = 100 kHz $= 2\Delta f \rightarrow \Delta f = 50$ kHz
+ $f_m = 8$ kHz

$$m_f = \frac{\Delta f}{f_m} = \frac{50}{8} = 6.25$$

Technical Considerations for FM Stations :

* The amount of Δf depends on amplitude (loudness) of the message.

Louder sound $\longrightarrow$ greater deviation

Lower Sound $\longrightarrow$ Lower deviation

* But it is agreed internationally that maximum $\Delta f = 75$ kHz for FM broadcast stations and 25 kHz for TV sound.

* The commercial FM broadcasting in 88–108 MHz.
* Since maximum deviation is 75 kHz, hence the channel width is $75 \times 2 = 150$ kHz, allowing 25 kHz guard band on either sides, the channel width becomes 200 kHz.

* In FM, the highest audio frequency transmitted is 15 kHz.
* The percent modulation : in FM it refers to the ratio of actual frequency deviation $\Delta f_{actu.}$ to the maximum allowable frequency deviation. Thus, 100% modulation corresponds to 75 kHz in FM broadcast band OR 100% modulation corresponds to 25 kHz for TV sound.

Percent Modulation $\boxed{M = \dfrac{\Delta f_{actual}}{\Delta f_{max}}}$

Ex.4 An FM transmission has a frequency deviation of 20 KHz.

① Determine the percent modulation of this signal if it is broadcasted in the 88–108 MHz band.

② calculate the percent modulation if this signal is broadcasted as the audio portion of a television broadcast.

Solution we have given $\Delta f = 20\ KHz$.

① percent modulation for an FM wave is

$$M = \frac{\Delta f_{actual}}{\Delta f_{max}} * 100\%$$

the maximum Δf in FM is 75 KHz

$$\therefore \quad M = \frac{20}{75} * 100\% = 26.67\%$$

② the maximum Δf for the FM audio portion of a TV broadcast is 25 KHz

$$\therefore \quad M = \frac{20 * 10^3}{25 * 10^3} * 100\%$$

$$M = 80\%$$

Types of FM modulation :-

* There are two types of FM signals :-

① Narrow band FM ② wide band FM.

Narrowband FM :- we know

$$S(t)_{FM} = A_c \cos\left[\omega_c t + k_f \int_0^t x(t)\,dt\right]$$

let $y(t) = \int_0^t x(t)\,dt$

$$\therefore S(t)_{FM} = A_c \cos\left[\omega_c t + k_f\, y(t)\right]$$

now in phasor form

$$S(t)_{FM} = Re\left[A_c\, e^{j[\omega_c t + k_f y(t)]}\right]$$

or

$$C_{FM}(t) = A_c\, e^{j[\omega_c t + k_f y(t)]}$$

* The condition of narrowband is $\boxed{k_f\, y(t) \ll 1}$

$$\therefore e^{jk_f y(t)} \simeq 1 + j k_f\, y(t)$$

$$\therefore C_{FM}(t) = A_c e^{j\omega_c t} \cdot e^{jk_f y(t)}$$

$$= A_c \left[1 + jk_f y(t)\right] e^{j\omega_c t}$$

But $s(t)_{FM}$ = Real part of $C_{FM}(t)$

$$\therefore \boxed{s(t)_{FM} = A_c \cos\omega_c t - A_c k_f y(t) \sin\omega_c t}$$

narrowband PM :

using the same procedure ~~for~~ of the narrowband F.M,

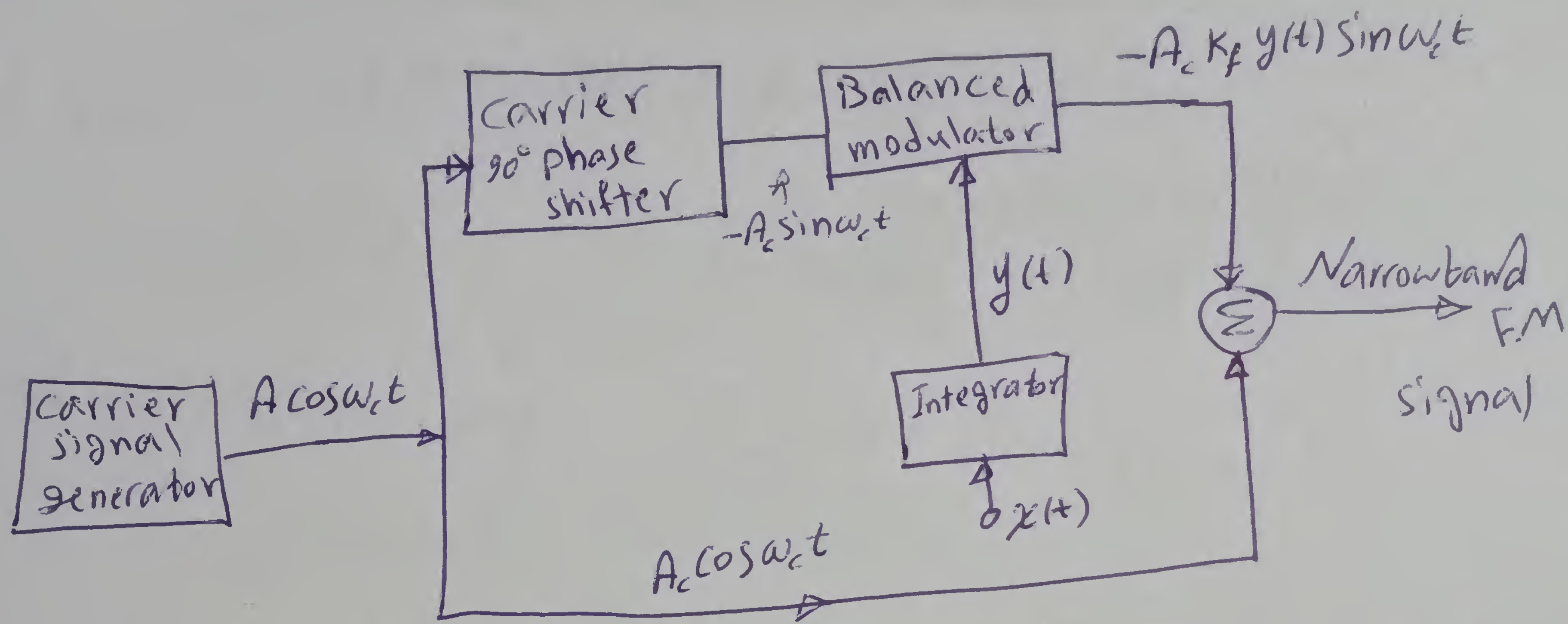
the PM signal (narrowband) will be

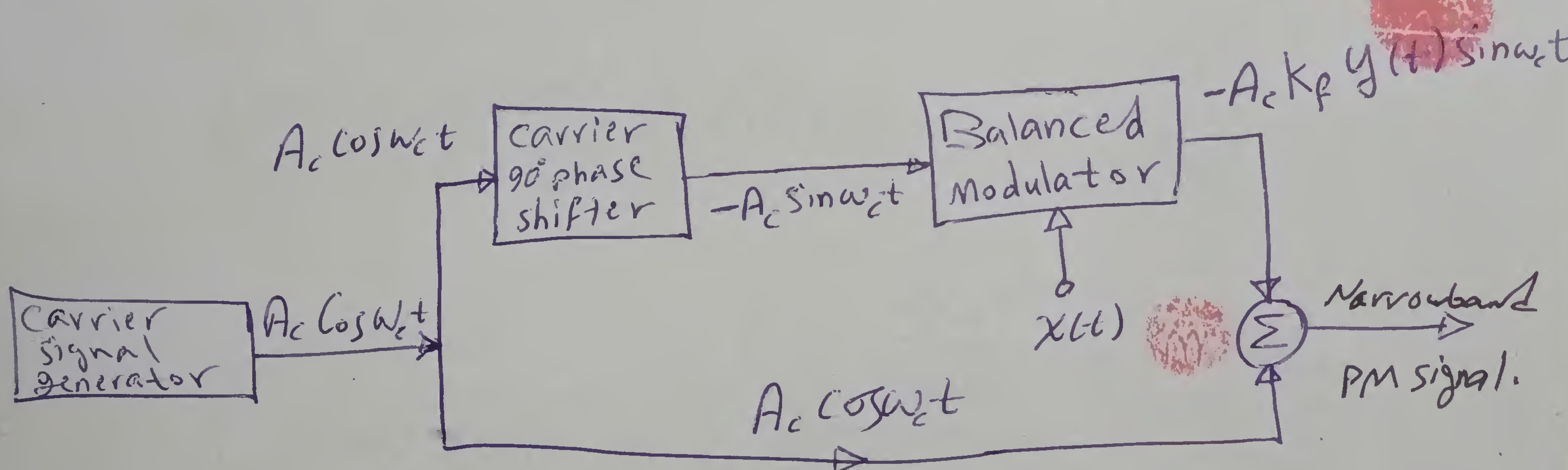
$$\boxed{s(t)_{PM} = A_c \cos\omega_c t - A k_p x(t) \sin\omega_c t}$$

Conclusion for narrowband FM & PM :

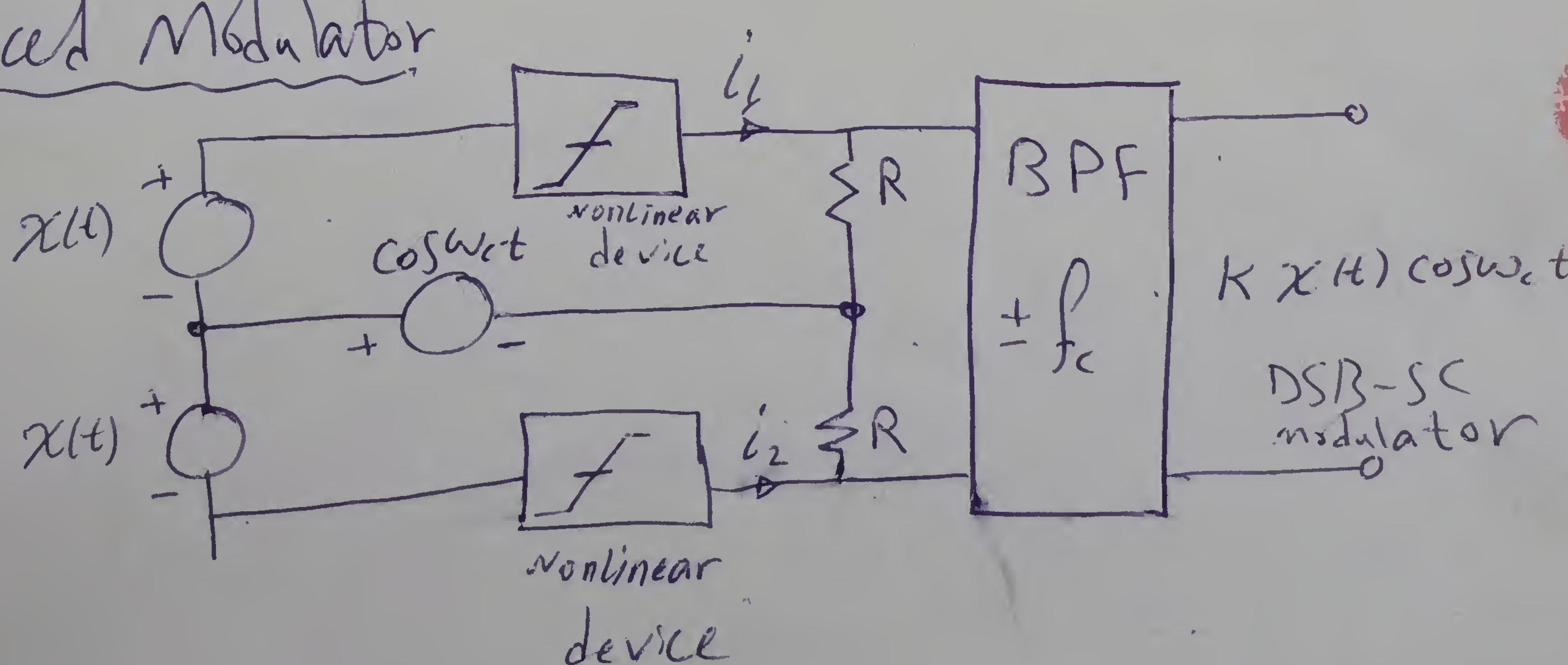
- It can be seen that the expressions for narrowband FM & PM are similar to the AM signal, thus, the bandwidth is almost similar or same as that of AM.

Generation of Narrowband FM & PM :





Balanced Modulator



Single-Tone Narrowband FM :

$$S(t)_{NFM} = A_c \cos\omega_c t - A_c k_f\, y(t) \sin\omega_c t$$

and $y(t) = \int x(t)\, dt$

let $x(t) = V_m \cos\omega_m t$ (the message signal)

$$\therefore\ y(t) = \int V_m \cos\omega_m t\, dt = \frac{V_m}{\omega_m} \sin\omega_m t$$

$$\therefore\ S(t)_{NFM} = A_c \cos\omega_c t - A_c \underbrace{k_f \frac{V_m}{\omega_m}}_{m_f} \sin\omega_m t \sin\omega_c t$$

but $m_f = \dfrac{k_f V_m}{\omega_m}$

$$\therefore\ \boxed{S(t)_{NFM} = A_c \cos\omega_c t - A_c\, m_f \sin\omega_m t \sin\omega_c t}$$

Wideband FM :

$m_f \geq 5$

* if m_f is Large ⟶ Large number of sidebands produced.

* To understand those complicated analysis, a message of single tone sinusoid will be used.

$$S(t) = A\cos(\omega_c t + m_f \sin\omega_m t)$$

The phasor form of $s(t)$ is

$$C_{FM}(t) = A e^{j\omega_c t} \underbrace{e^{j m_f \sin\omega_m t}}_{\text{periodic with period } \frac{1}{f_m}}$$

* thus $e^{j m_f \sin\omega_m t}$ is periodic with period $\frac{1}{f_m}$

* $e^{j m_f \sin\omega_m t}$ can be expanded using Fourier series :

$$e^{j m_f \sin\omega_m t} = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_m t} \qquad \text{for } -\frac{1}{2f_m} \leq t \leq \frac{1}{2f_m}$$

$$C_n = f_m \int_{-\frac{\pi}{\omega_m}}^{\frac{\pi}{\omega_m}} e^{j(m_f \sin\omega_m t)} e^{-jn\omega_m t}\, dt$$

$x = \omega_m t$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(m_f \sin x - nx)}\, dx$$

$\therefore\ C_n = J_n(m_f)$

where $J_n(m_f)$ is the Bessel function of n^{th} order of the first kind.

$$\therefore\ e^{jm_f \sin\omega_m t} = \sum_{n=-\infty}^{\infty} J_n(m_f)\, e^{jn\omega_m t}$$

Thus

$$C_{FM}(t) = A e^{j\omega_c t} \sum_{n=-\infty}^{\infty} J_n(m_f)\, e^{jn\omega_m t}$$

$$C_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(m_f)\, e^{j(\omega_c + n\omega_m)t}$$

from which

$$\boxed{S(t)_{FM} = A \sum_{n=-\infty}^{\infty} J_n(m_f) \cos(\omega_c t + n\omega_m t)}$$

*Properties of Bessel function :-

① $J_n(m_f) = J_{-n}(m_f)$ n is even
$J_n(m_f) = -J_{-n}(m_f)$ n is odd

② For small values of m_f :- $J_0(m_f) \simeq 1$, $J_1(m_f) \simeq \frac{m_f}{2}$, $J_n(m_f) \simeq 0$ $n > 1$

③ $\sum_{n=-\infty}^{\infty} J_n^2(m_f) = 1$

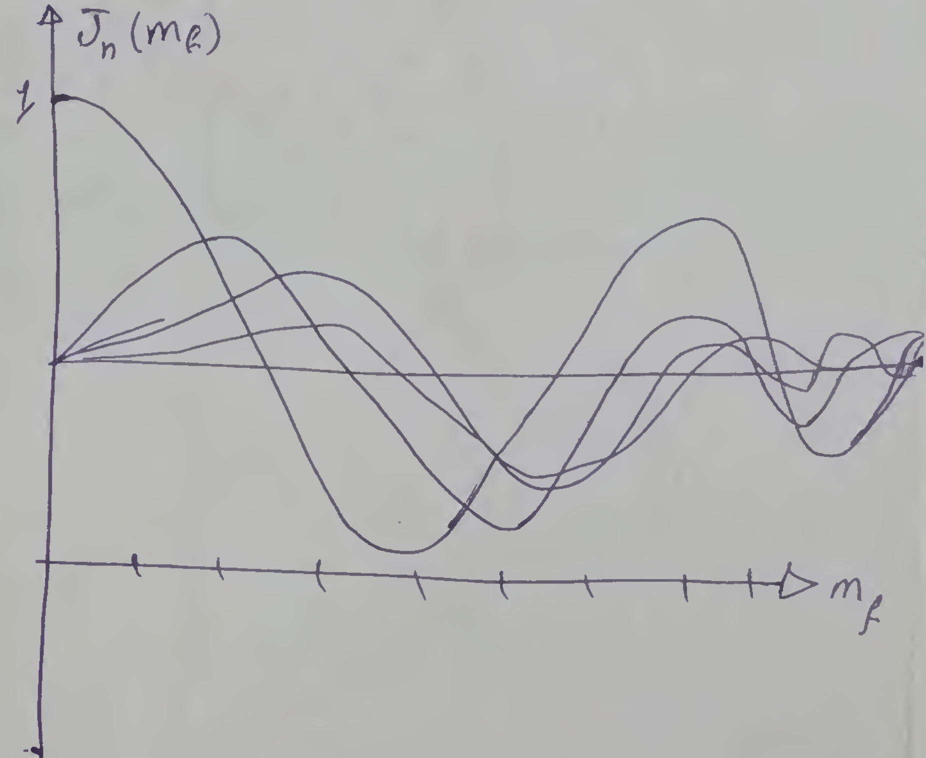
using the first property:

$$s_{FM}(t) = A J_0(m_f)\cos\omega_c t + A J_1(m_f)\left[\cos(\omega_c+\omega_m)t - \cos(\omega_c-\omega_m)t\right]$$

$$+ A J_2(m_f)\left[\cos(\omega_c+2\omega_m)t + \cos(\omega_c-2\omega_m)t\right]$$

$$+ A J_3(m_f)\left[\cos(\omega_c+3\omega_m)t - \cos(\omega_c-3\omega_m)t\right]$$

$$+ \cdots$$



* we conclude from the last expression of $s_{FM}(t)$

① infinite Bandwidth (theoretically)

because of the infinite number of sidebands

② For small m_f (less than 0.6)

- thus there is only the carrier term and one pair of sidebands.
- this case equivalent to Narrowband FM.

$$s_{FM}(t) = A J_0(m_f)\cos\omega_c t + A J_1(m_f)\left[\cos(\omega_c+\omega_m)t - \cos(\omega_c-\omega_m)t\right].$$

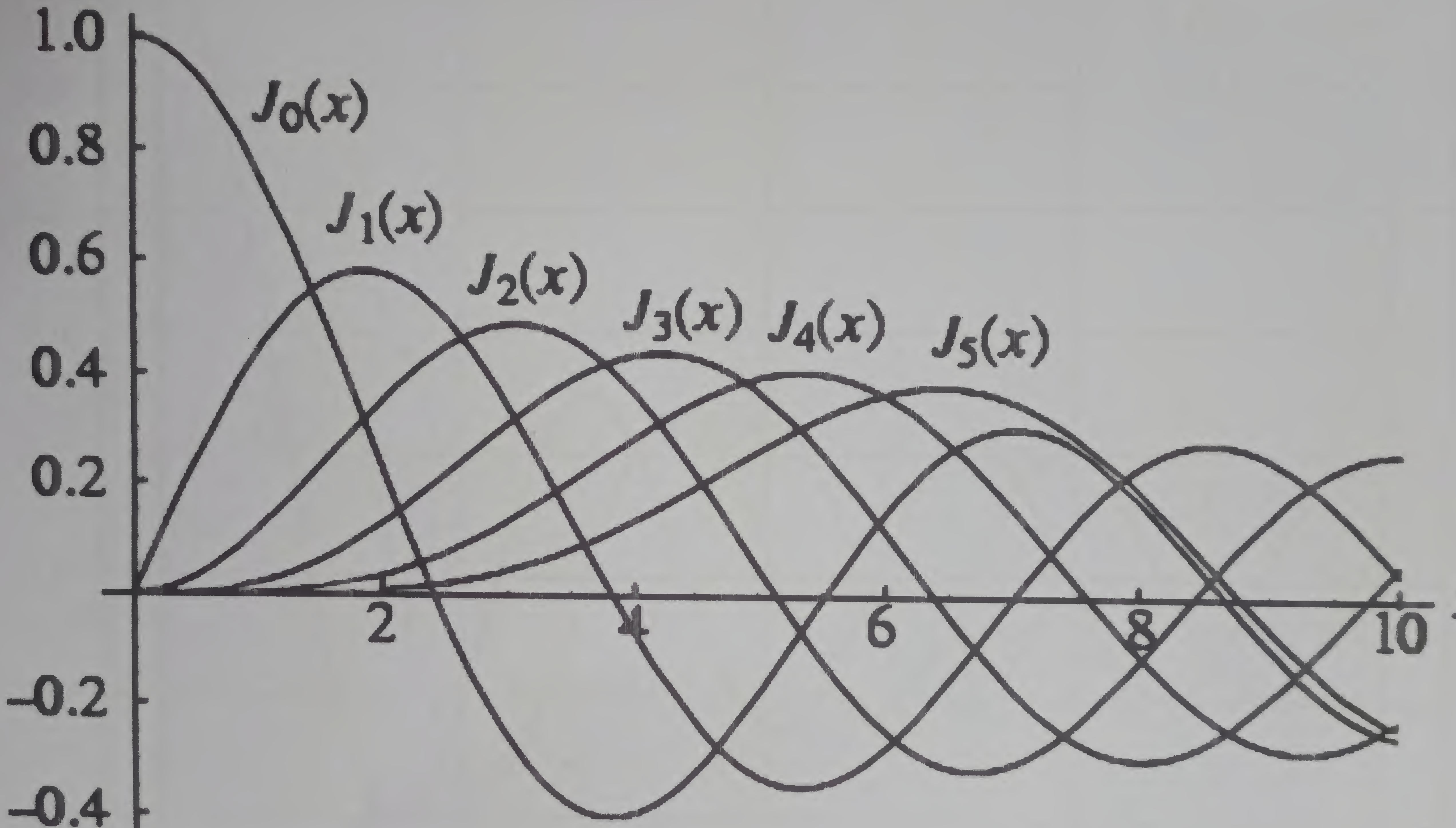
③ If $m_f > 1$ to be ≈ 2.4 or 5.2, $J_0(m_f) = 0$,

$\therefore$ carrier power $= 0$, all power carried by the sidebands ~~modulated~~ ~~signal~~ Hence efficiency $= 100\%$

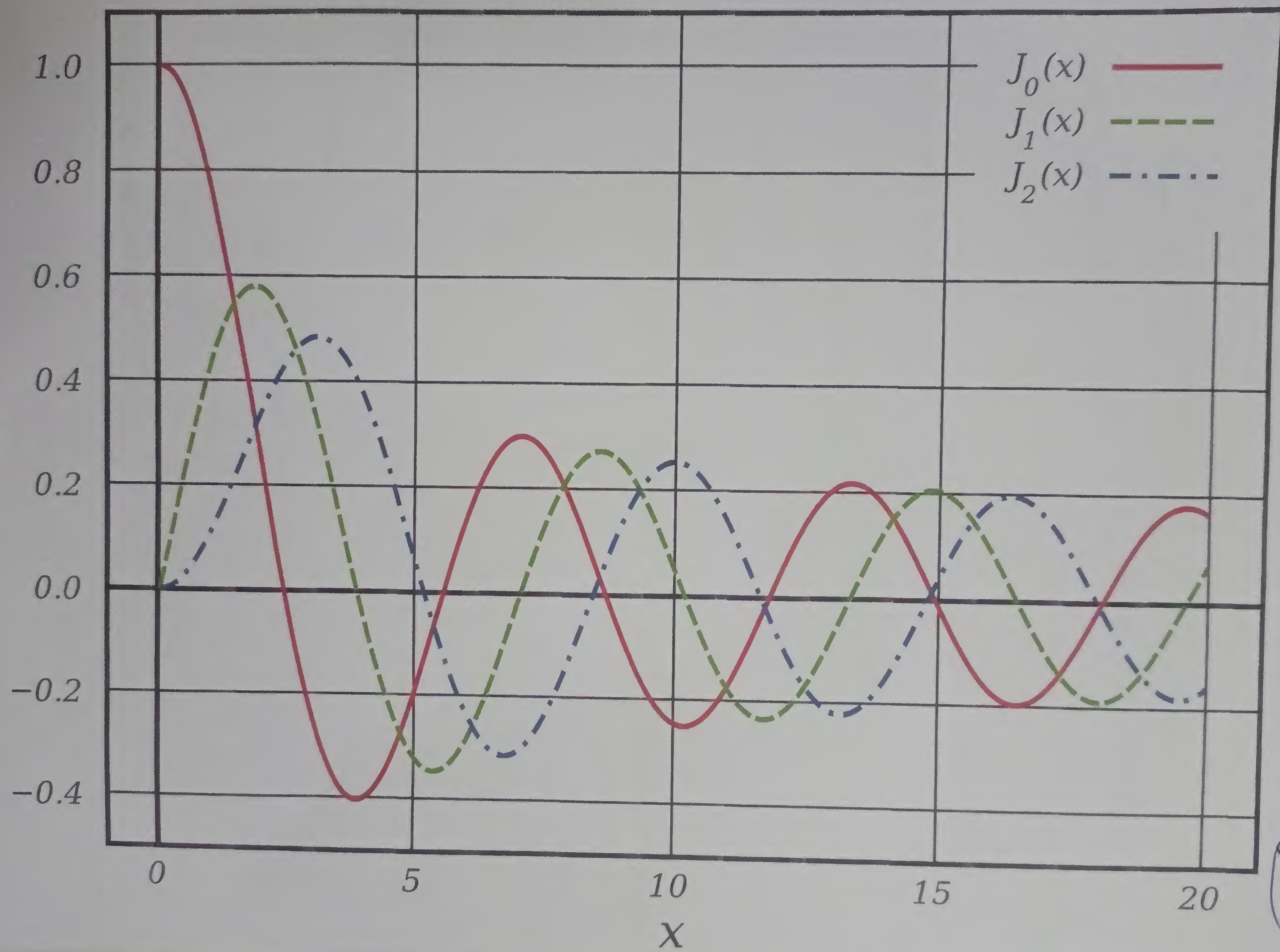
$$\therefore \underset{AM}{33} \leq \eta_{FM} \leq \underset{100\%}{DSB\text{-}SC}$$

Thus: According to CCIR (Consultative committee for International Radio), there are some regulations:-

1. Maximum modulation Frequency = 15 KHz, (message)
2. Maximum frequency deviation $\Delta f = 75$ KHz,
3. Frequency Stability of the carrier is ± 2 KHz.
4. Allowable bandwidth per channel = 200 KHz.



37-a



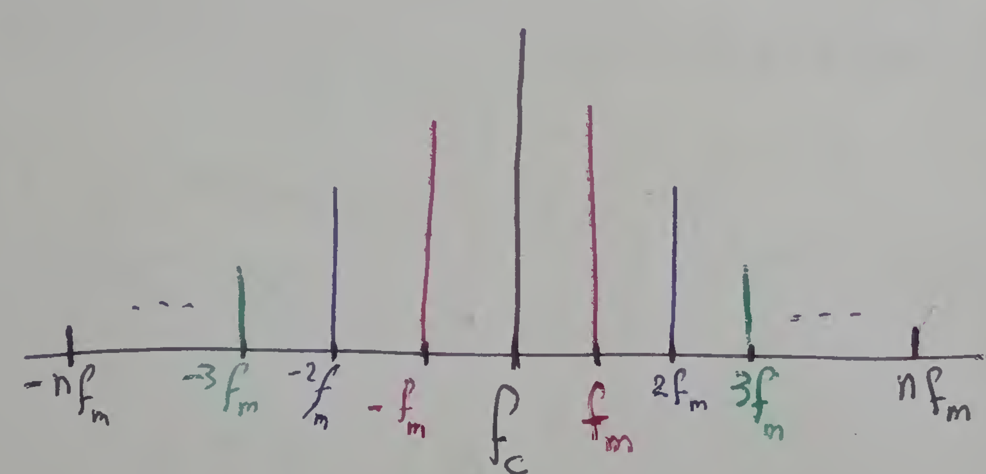
37-b

FM signal Bandwidth :

we have shown that FM Bandwidth depends on the modulation index m_f.

$$
\begin{aligned}
S(t)_{FM} = {} & A J_0(m_f)\cos\omega_c t + A J_1(m_f)\left[\cos(\omega_c+\omega_m)t - \cos(\omega_c-\omega_m)t\right] \\
& + A J_2(m_f)\left[\cos(\omega_c+2\omega_m)t + \cos(\omega_c-2\omega_m)t\right] \\
& + A J_3(m_f)\left[\cos(\omega_c+3\omega_m)t - \cos(\omega_c-3\omega_m)t\right] \cdots
\end{aligned}
$$

:



frequency spectrum of FM signal for n significant sidebands.

: Thus, to calculate the bandwidth, we need to consider n sidebands.

* In this case, n must not be very Large, we can consider the sideband of amplitude which is 1% of the unmodulated carrier's amplitude (A_c).

In other words sideband amplitude $= 1\%\, A_c$ to be considered.

Carson's Rule :

As a rule of thumb, for single-tone FM signal, and as an approximation :

$$B_{FM} = 2(\Delta\omega + \omega_m)$$

since $\Delta\omega = m_f \omega_m$

$$B_{FM} = 2(m_f \omega_m + \omega_m)$$

$$\therefore \boxed{B_{FM} = 2\omega_m (m_f + 1)}$$

can be used if $m_f \geq 5$

Two cases can be considered :

① $\Delta\omega \ll \omega_m \rightarrow m_f \ll 1$ (Narrowband FM)

$$\boxed{B_{FM} = 2\omega_m}$$

similar to AM signal

② $\Delta\omega \gg \omega_m \longrightarrow m_f \gg 1$ (wideband FM)

$$\boxed{B_{FM} = 2\Delta\omega}$$

EX. Find the bandwidth of a commercial FM transmission if the frequency deviation = 75 KHz and modulated by a 15 KHz signal.

Solu. $BW = 2(\Delta f + f_m) = 2(75+15) = 180 KHz$

EX. Determine the bandwidth of a narrowband FM signal which is generated by a 4 KHz audio signal modulating a 125 MHz carrier.

Solu. since it is a narrowband FM, $BW = 2f_m = 8 KHz$.

EX. the maximum deviation allowed in an FM broadcast system is 75 KHz. If the modulating signal is a single-tone sinusoid of 8 KHz, determine the bandwidth of the FM signal. What will be the bandwidth when the modulating signal amplitude is doubled?

Solu. we have given $\Delta f = 75 KHz$, $f_m = 8 KHz$.

$BW = 2(\Delta f + f_m) = 2(75+8) = 166 KHz$.

Now, we know $\Delta f = m_f f_m \rightarrow \Delta f = k_f V_m \cdot f_m$

$$\text{if } V_{m_{new}} = 2V_{m_{old}}$$

$$\therefore \Delta f = 2\underbrace{k_f V_m}_{m_f} f_m = 2m_f f_m$$

$$\therefore \Delta f_{new} = 2\Delta f_{old} = 2 * 75 = 150 KHz$$

$$\therefore BW_{FM} = 2(150+8) = 316 KHz.$$

PM Modulation (single-tone PM)

The instantinuous phase of PM is

$$\phi_i = \omega_c t + k_p x(t)$$

and for a single-tone modulating signal,

$$x(t) = V_m \cos\omega_m t$$

thus,

$$\phi_i = \omega_c t + \underbrace{k_p V_m}_{\theta_d} \cos\omega_m t$$

$$\phi_i = \omega_c t + \theta_d \cos\omega_m t$$

$\boxed{\theta_d = k_p V_m \quad m_p = \theta_d}$

θ_d is the phase deviation.

* modulation index of $PM = \theta_d = k_p V_m$

$$\therefore s(t)_{PM} = A\cos\phi_i$$

$$\therefore s(t)_{PM} = A\cos[\omega_c t + \theta_d \cos\omega_m t]$$

also, the instantinuous frequency ω_i

$$\omega_i = \frac{d\phi_i}{dt} = \frac{d}{dt}\left[\omega_c t + k_p V_m \cos\omega_m t\right]$$

$$\omega_i = \omega_c - \underbrace{K_p V_m \omega_m}_{\text{max. deviation}} \sin\omega_m t$$

$$\therefore \boxed{\Delta\omega_{PM} = K_p V_m \omega_m}$$ depends on ω_m

* A relation between K_f & k_p can be developed for equal bandwidth of FM & PM,

since

$$\Delta\omega_{FM} = k_f V_m$$

$$\therefore \boxed{K_f = k_p \omega_m}$$

using Carson's rule :

$$BW_{PM} \cong 2\Delta\omega \simeq 2K_p V_m \omega_m$$

Ex. A baseband signal $x(t) = 5\cos(2\pi \times 10^3 t)$ angle modulates a carrier signal $A_c \cos\omega_c t$.

1. Determine the modulation index and bandwidth for (a) FM (b) PM.
2. Find the change in the bandwidth and modulation index for both FM & PM if modulating frequency f_m is reduced to 5 KHz.

Assume $K_p = k_f = 15$ KHz/Volt.

Solution we have given $V_m = 5$ V & $f_m = 15$ KHz.

1. For (a) FM system:-

Frequency deviation $\Delta f = K_f V_m = 15 * 10^3 * 5 = 75$ KHz

$$\therefore \quad m_f = \frac{\Delta f}{f_m} = \frac{75}{15} = 5$$

$$BW_{fm} = 2(\Delta f + f_m) = 2(75 + 15) = 2 \times 90 = 180 \text{ KHz}$$

(b) For PM system

$$\Delta f = k_p V_m f_m = 15 \times 10^3 \times 5 \times 15 \times 10^3 = 1125 \text{ MHz}.$$

$$BW_{PM} = 2(\Delta f + f_m) \cong 2\Delta f = 2(1125 \text{ MHz}) = 2250 \text{ MHz}.$$

$$m_p = K_p V_m = 15 \times 10^3 \times 5 = 75{,}000$$

2. Now the modulating frequency f_m reduced to 5 KHz.

(a) For FM $\quad m_f = \frac{\Delta f}{f_m} = \frac{75}{5} = 15$

$$BW_{fm} = 2(\Delta f + f_m) = 2(75 + 5) = 2 \times 80 = 160 \text{ KHz}.$$

(b) For PM $\quad \Delta f = K_p V_m f_m = 15 \times 10^3 \times 5 \times 5 \times 10^3 = 375$ MHz.

$$BW = 2(\Delta f + f_m) \simeq 2\Delta f = 2 \times 375 = 750 \text{ MHz}.$$

$$m_p = K_p V_m = 15 \times 10^3 \times 5 = 75 \text{ KHz}.$$

EX. Determine the following ① carrier and modulating frequencies ② modulation index and maximum deviation ③ the power dissipated in a 5 Ω resistor.

for an FM signal given as $v = 10 \sin[5 \times 10^8 t + 4 \sin 1250 t]$.

Solution ① $f_c = \frac{5 \times 10^8}{2\pi} = 79.57 \text{ MHz}$

② $f_m = \frac{1250}{2\pi} = 199 \text{ Hz}$

③ $m_f = 4$

④ $\Delta f = m_f f_m = 4 \times 199 = 796 \text{ Hz}$.

⑤ $P = \frac{(\text{RMS } \cancel{\text{of}} \text{ value of FM wave})^2}{R} = \frac{\left[\frac{V_c}{\sqrt{2}}\right]^2}{R} = \frac{\left[\frac{10}{\sqrt{2}}\right]^2}{5} = 10 \text{ W}$.

EX. In an FM system, the modulating frequency $f_m = 1 \text{ KHz}$, the modulating voltage $V_m = 2$ volts and the deviation is 6 KHz. If the modulating voltage is raised to 4 volts, then, what will be the new deviation? voltage is further increased to 8 volts and the if the modulating voltage is further increased to 8 volts and the modulating frequency is reduced to 500 Hz, what will be the deviation?

Solution Given $f_m = 1 \text{ KHz}$, $V_m = 2 \text{ V}$, $\Delta f = 6 \text{ KHz}$

$\Delta f = K_f V_m \rightarrow K_f = \frac{\Delta f}{V_m} = \frac{6 \text{ KHz}}{2 \text{ V}} = 3 \text{ KHz/V}$

ⓐ when $V_m = 4 \text{ V} \rightarrow \Delta f = k_f V_m = 3 \frac{\text{KHz}}{\text{V}} \times 4 \text{ V} = 12 \text{ KHz}$

ⓑ For $V_m = 8 \text{ V}$ ~~and Δf~~ and $f_m = 500 \text{ Hz}$

$\Delta f = k_f V_m = 3 \frac{\text{KHz}}{\text{V}} \times 8 \text{ V} = 24 \text{ KHz}$

* Find also the modulation index in each case.

ⓐ For $\Delta f = 6 \text{ KHz}$ & $f_m = 1 \text{ KHz} \rightarrow m_f = \frac{\Delta f}{f_m} = 6$

ⓑ $\Delta f = 12 \text{ KHz}$ & $f_m = 1 \text{ KHz} \rightarrow m_f = \frac{12}{1} = 12$

ⓒ $\Delta f = 24 \text{ KHz}$ & $f_m = 0.5 \text{ KHz} \rightarrow m_f = \frac{24}{0.5} = 48$

Ex. what will be the bandwidth required for an FM signal if the modulating frequency is 1 kHz & the maximum deviation is 10 kHz? what is the bandwidth required for the corresponding DSB (AM) transmission?

Solution $BW = 2[\Delta f + f_m] = 2[10+1] = 22\ kHz$.

The corresponding AM bandwidth is

$BW_{AM} = 2f_m = 2\ kHz$.

$\boxed{\text{* Note :- Carson's rule is valid as Long as } m_f > 5 \text{ / important.}}$

Ex. A 20 MHz carrier is modulated by a 400 Hz modulating signal. The carrier voltage is 5 V and the maximum deviation is 10 kHz. Write down the mathematical expression for the FM and PM waves. If the modulating frequency is increased to 2 kHz keeping every thing else constant, write down the expression for the FM & PM waves. ~~[illegible]~~

Solution $\omega_c = 2\pi \times 20 \times 10^6 = 1.25 \times 10^8$ rad/sec.

$\omega_m = 2\pi \times 400 = 2513$ rad/sec.

$m = m_f = m_p = \frac{\Delta f}{f_m} = \frac{10,000}{400} = 25$

$\therefore$ FM-wave: $s(t)_{FM} = 5 \sin[1.25 \times 10^8 t + 25 \sin 2513 t]$.

PM-wave: $s(t)_{PM} = 5 \sin\{1.25 \times 10^8 t + 25 \sin 2513 t\}$

now, the $f_m = 2\ kHz \rightarrow \omega_m = 2\pi \times 2000 = 12566.3$ rad/sec.

$m_f = \frac{10,000}{2000} = 5$

m_p is remains constant as it does not depend on the modulating frequency.

$\therefore m_p = 25$

Hence $\therefore$ $s(t)_{FM} = 5[1.25 \times 10^8 t + 5 \sin 12566 t]$

$s(t)_{PM} = 5[1.25 \times 10^8 t + 25 \sin 12566 t]$.

EX. A carrier wave of amplitude of 10 V and frequency 100 MHz is frequency modulated by a sinusoidal voltage. The modulating voltage has an amplitude of 5 V and frequency $f_m = 20$ kHz. The frequency deviation constant is 2 kHz/volt. Draw the frequency spectrum of FM wave. You have $J_0(0.5) \simeq 0.94$, $J_1(0.5) \simeq 0.24$, $J_2(0.5) \cong 0.03$

Solution modulating voltage $V_m = 5$ Volt

deviation constant $k_f = 2$ kHz/Volt

$\therefore$ frequency deviation $\Delta f = k_f \cdot V_m = 5 \times 2 \times 10^3 = 10^4$ Hz.

$$m_f = \frac{\Delta f}{f_m} = \frac{10^4}{20 \times 10^3} = 0.5.$$

Since we have $J_0 = 0.94$, $J_1 = 0.24$, $J_2 = 0.03$

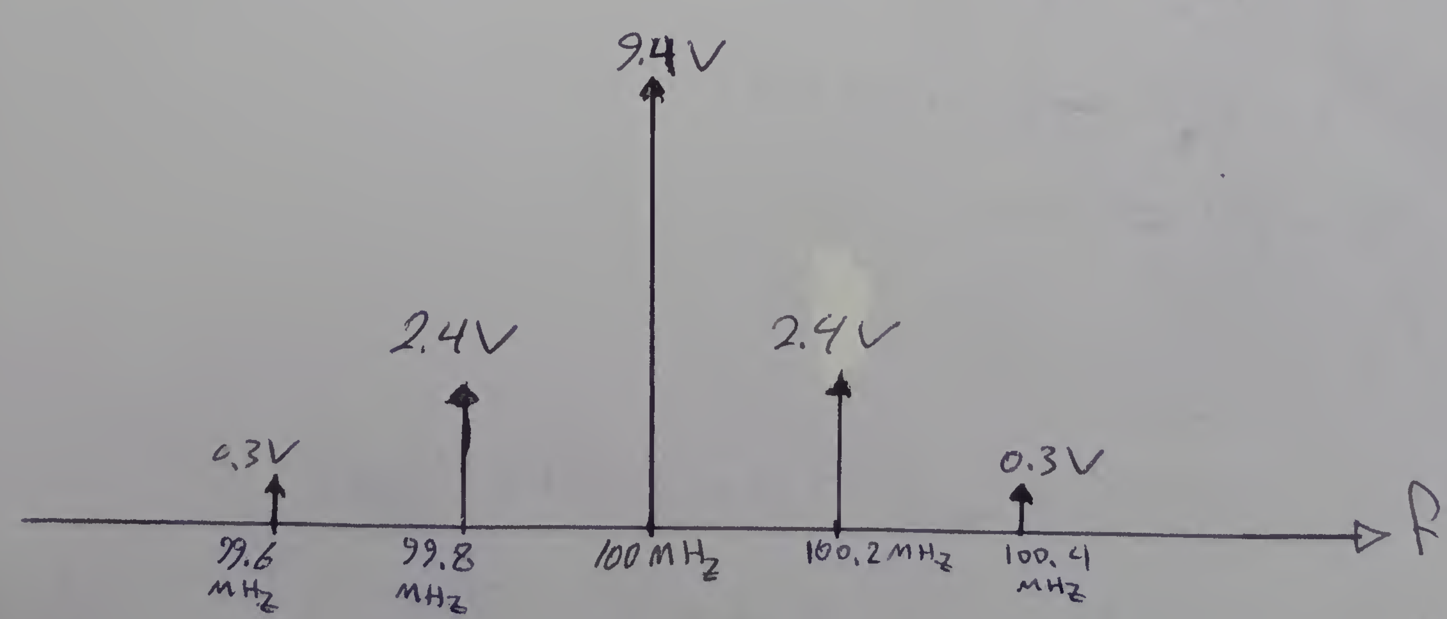
$\therefore$ carrier amplitude $= V_c J_0 = 10 \times 0.94 = 9.4$ volts.

Amplitude of the first pair of sidebands $= V_c J_1 = 10 \times 0.24 = 2.4$ Volts.

Amplitude of the second pair of sidebands $= V_c J_2 = 10 \times 0.03 = 0.3$ volts.

The first pair of sidebands $f_c \pm f_m = 100\text{ MHz} \pm 20\text{ kHz} = 100.2$ MHz & 99.8 MHz.

The second pair of sidebands $f_c \pm 2f_m = 100\text{M} \pm 40\text{ kHz} = 100.4$ MHz & 99.6 MHz.



EX. when the modulating frequency in an FM system is 400 Hz and the modulating voltage is 2.4 V, the modulation index is 60. calculate :-

① the maximum deviation ② what will be the modulation index when modulating frequency is reduced to 250 Hz and the modulating voltage is simultaneously raised to 3.2 V.

solution given $f_m = 400\ Hz$, $V_m = 2.4\ V$, $m_f = 60$.

ⓐ $\Delta f_{max} = m_f f_m = 24\ KHz$.

ⓑ $f_m = 250\ Hz$ & $V_m = 3.2\ V$,

$\Delta f = K_f V_m \Rightarrow 24\ KHz = K_f \times 2.4 \rightarrow K_f = 10\ KHz/volt$

$\therefore \Delta f = K_f V_m = 10 \times 3.2 = 32\ KHz$.

$$m_f = \frac{32 \times 10^3\ Hz}{250\ Hz} = 128.$$

EX. In An FM system, the audio frequency is 1 KHz and the audio voltage is 2 V. The deviation is 4 KHz. If the Audio voltage increased to 8 V and the audio frequency droped to 500 Hz, find the modulation index in each case and the corresponding bandwidth using Carson's rule.

Solution $f_{m1} = 1\ KHz$, $V_{m1} = 2\ V$, $\Delta f_1 = 4\ KHz$.

$f_{m2} = 0.5\ KHz$, $V_{m2} = 8\ V$.

$$\Delta f_1 = K_{f_1} V_{m1} \rightarrow K_{f_1} = \frac{\Delta f_1}{V_{m_1}} = \frac{4 \times 10^3}{2} = 2 \times 10^3\ Hz/V$$

$$\Delta f_2 = K_f V_{m2} = 2 \times 10^3 \times 8 = 16\ KHz.$$

$$m_{f_1} = \frac{\Delta f_1}{f_{m_1}} = \frac{4}{1} = 4$$

$$m_{f_2} = \frac{\Delta f_2}{f_{m2}} = \frac{16}{0.5} = 32.$$

$$BW_1 = 2[\Delta f_1 + f_{m_1}] = 2\{4+1\} = 10\ KHz.$$

$$BW_2 = 2\{\Delta f_2 + f_{m_2}\} = 2\{16+0.5\} = 32\ KHz$$