Phase-Locked Loop (PLL)

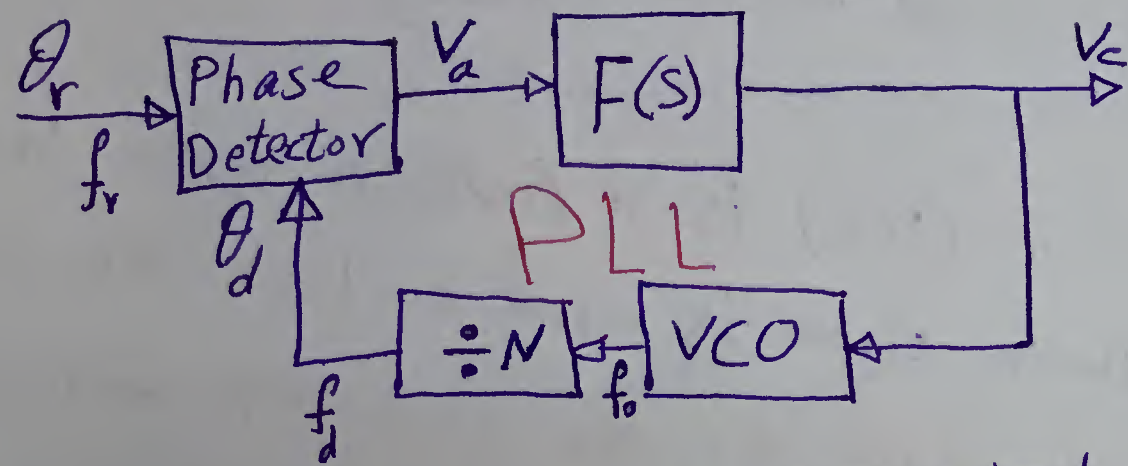
* A phase-Locked Loop (PLL) is a feedback system in which the feedback signal is used to Lock the output frequency and phase to the frequency and phase of an input signal

* The input waveform can be of many different types, including sinusoidal or digital.

* PLL used for the first time in 1932 for the synchronuous detection of radio signals.

* Next PLL used by NASA in 1960's in its satellite programs.

* PLL can be analoge or Digital.



* The Phase detector generates an output signal that is a function of the difference between the two signals.

* The detector output is filtered and the dc component of the error signal is applied to the Voltage-controlled oscillator (VCO).

* The signal fed back to the PD is the VCO output frequency divided by N.

* The VCO control voltage $V_c(t)$ forces the VCO to change frequency in the direction that reduces the difference between the input frequency and the divider output frequency.

* If the two frequencies are sufficiently close, the PLL feedback mechanism forces the two-PD input frequencies to be equal, and the VCO is 'Locked' with the incoming frequency.

$\therefore f_r = f_d$ where f_r is the reference signal
f_d is the second i/p to PD
or, divider output frequency.

$$\boxed{f_d = \frac{f_o}{N}} \quad \text{OR} \quad \boxed{f_o = N f_d}$$

*If a divider is not used, $N = 1$.

$$\boxed{V_a = K_d(\theta_r - \theta_d)} \quad \text{———} (1)$$

K_d :- is the phase detector gain factor. (V/rad)

*The VCO output frequency deviates from its free-running frequency by an increment of frequency

$$\Delta\omega = K_o V_c \quad \text{———} (2)$$

where: V_c is the input voltage to VCO,
K_o is the VCO gain factor rad/sec/Vol.
or the conversion factor

$\therefore$ The output frequency is :

$$\omega_o = \omega_c + \Delta\omega = \omega_c + k_o V_c$$

where : ω_c is the free running frequency of the VCO.

EX.1 The output frequency of a VCO changes from 100 KHz when a change of 0.5 V occurs at the VCO input. Calculate the conversion gain factor k_o.

Solution

$$k_o = \frac{\Delta\omega}{\Delta V} = \left(\frac{150\,KHz - 100\,KHz}{0.5\,V}\right)2\pi = \left(\frac{50\,KHz}{0.5\,V}\right)2\pi$$

$$k_o = 2\pi(100\,KHz)/Volt$$

OR

$$\left[\omega_o = \omega_c + k_o V_c\right] \div 2\pi$$

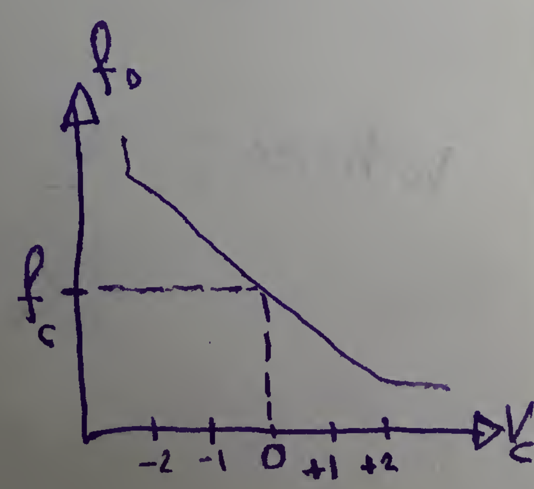
$$f_o = f_c + \frac{k_o V_c}{2\pi}$$

$$k_o = \frac{150-100}{0.5}\,KHz/V = 100\,KHz/V$$

NOTE :

* Increasing V_c, decreases f_o,

* Decreasing V_c, increases f_o.



$\therefore$ $f_o = f_c \pm \Delta f$

* The VCO operation can be described as:-

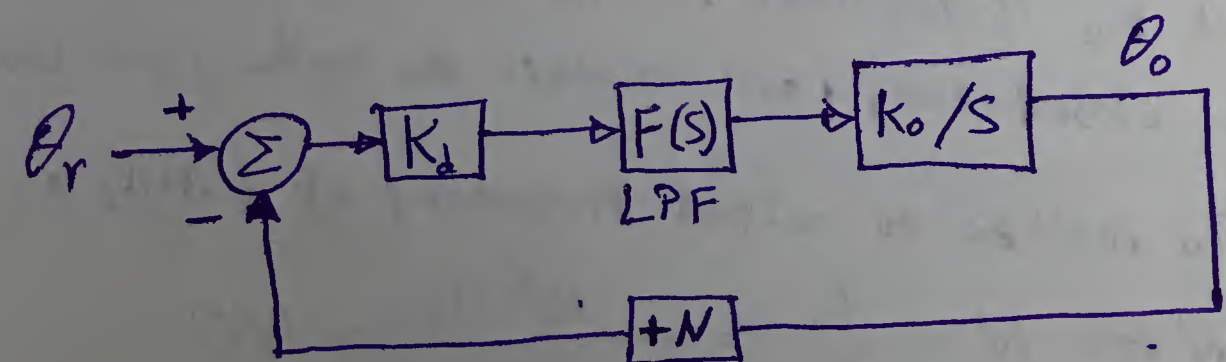
$$\Delta\omega = \frac{d\theta_o}{dt} = K_o V_c \quad \text{—————— (3)}$$

$$\rightarrow f_d = \frac{f_o}{N}$$

Since phase is the time integral of frequency,

$$\theta_d = \frac{\theta_o}{N}$$

∴ The PLL can be represented by the linear model shown below:-



$$\frac{\theta_o(S)}{\theta_r(S)} = \frac{K_o K_d F(S)/S}{1 + K_d K_o F(S)/(NS)} = \frac{G(S)}{1 + G(S)/N} \quad \text{—— (4)}$$

* → If NO LPF, then

$$\frac{\theta_o}{\theta_r} = \frac{K_d K_o}{S + K_d K_o / N} = \frac{N K_v}{S + K_v}$$

which is equivalent to the transfer function of a simple LPF with a dc gain of N and a bandwidth equal to K_v &

where : $$K_v = \frac{K_d K_o}{N}$$

* This type of PLL is referred to as first-order Loop since it can be described by a first order differential equation and is also of type I.

EX. 2 A frequency synthesizer uses PLL to synthesize a 1 MHz signal from a 25 KHz reference frequency. Assume there is no filtering included, and $K_d = 2\,V/rad$ and K_o is 100 Hz/V. Find the transfer function of the closed-Loop, and calculate the synthesizer bandwidth.

Solution : To realize an output frequency of 1 MHz,

$$f_o = f_d N \Rightarrow N = \frac{f_o}{f_d} = \frac{10^6\ Hz}{25 \times 10^3\ Hz} = 40$$

$$\frac{\theta_o}{\theta_r} = \frac{K_d K_o / S}{1 + K_d K_o / (SN)} = \frac{K_d K_o}{S + K_d K_o / N}$$

$$\frac{\theta_o}{\theta_r} = \frac{(2 \times 100) 2\pi}{S + (2 \times 100 \times 2\pi)/40} = \frac{16000\pi}{40S + 16000\pi}$$

* The synthesizer bandwidth $K_v = \frac{K_d K_o}{N}$

$$K_v = \frac{2 \times 100}{40} = 5\ Hz.$$
$$= 10\ rad/s$$

End of solution

* Normally, the loop will also contain a filter to filter out undesirable components from the phase detector, and to provide further control over the loop's frequency response.

* Let the filter is a simple LPF,

$$F(s) = \left(\frac{s}{\omega_L} + 1\right)^{-1}$$

then,

$$\frac{\theta_o(s)}{\theta_r(s)} = \frac{N K_v}{s(s/\omega_L + 1) + K_v} = \frac{N}{s^2/\omega_n^2 + (2\xi/\omega_n)s + 1} \quad \text{—(5)}$$

where :

$$K_v = \frac{K_d K_o}{N} \quad \text{—(6)}$$

$$\omega_n^2 = K_v \omega_L \quad \text{—(7)}$$

$$2\xi = \frac{\omega_n}{K_v} = \left(\frac{\omega_L}{K_v}\right)^{1/2} \quad \text{—(8)}$$

Equation (5) is the general form of the second-order Low-Pass transfer function.

The magnitude of the steady-state frequency response is

$$\left|\frac{\theta_o}{\theta_r}(j\omega)\right| = \frac{N}{\left[\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2\right]^{1/2}} \quad \text{--- (9)}$$

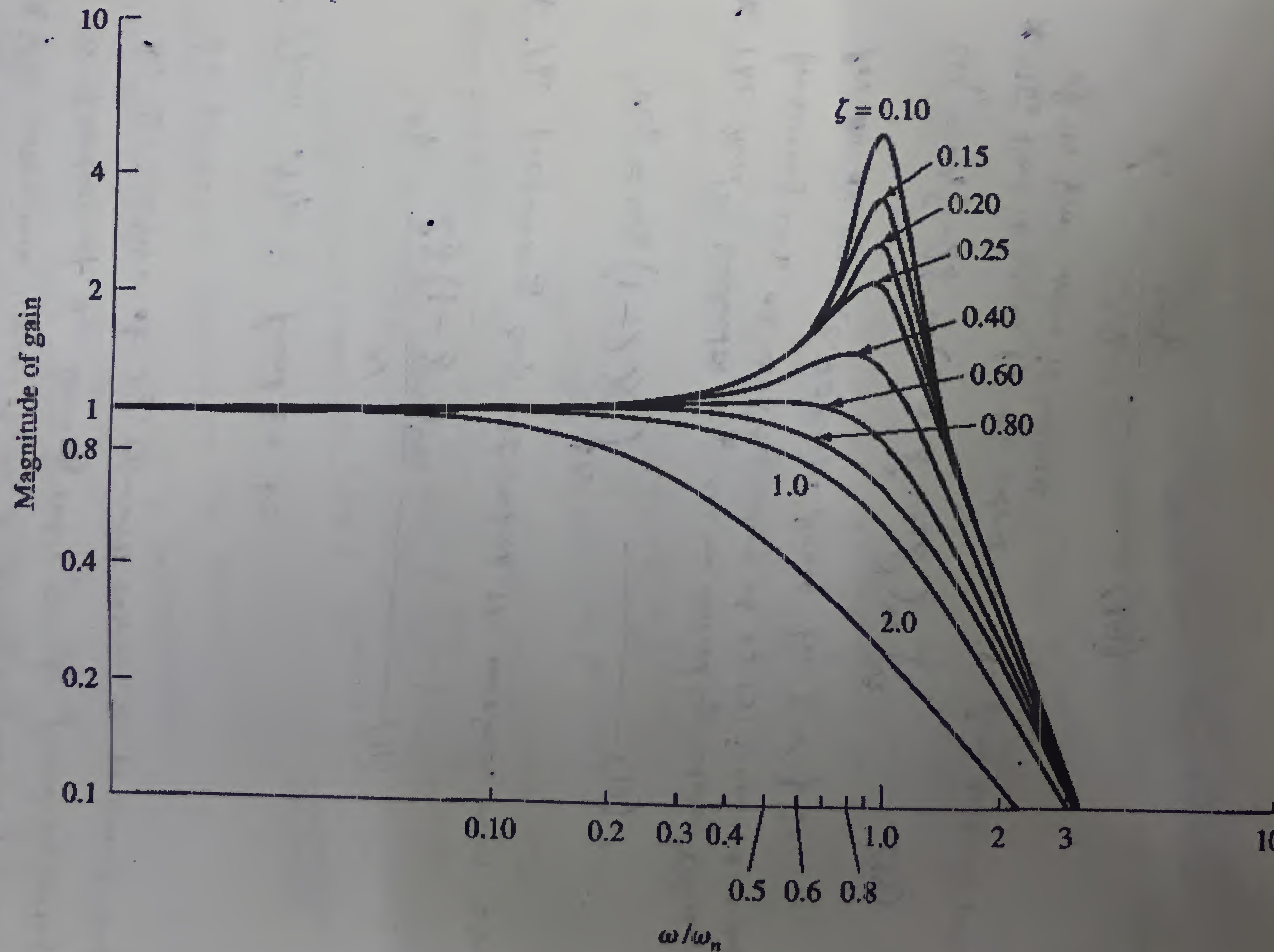
and the phase shift is

$$\arg\frac{\theta_o}{\theta_r}(j\omega) = \angle\frac{\theta_o}{\theta_r}(j\omega) = -\tan^{-1}\left[\frac{2\zeta\omega}{\omega_n\left(1-\frac{\omega^2}{\omega_n^2}\right)}\right] \quad \text{--- (10)}$$

* The function of Eq. (9) is plotted in the next page, for selected values of ζ.

* For $\zeta = 0.707$, the transfer function becomes the second-order "maximally flat" Butterworth response.

* For values of $\zeta < 0.707$, the gain exhibits peaking in the frequency domain.



Magnitude of gain
10
4
2
1
0.8
0.4
0.2
0.1
ζ = 0.10
0.15
0.20
0.25
0.40
0.60
0.80
1.0
2.0
0.10
0.2
0.3
0.4
0.5
0.6
0.8
1.0
2
3
10
ω/ω_n

* The maximum value of the frequency response M_p as a function of the damping ratio can be found by setting the derivative of Eq. (9) – with respect to frequency – equal to zero.

* Then M_p is found to be

$$\boxed{M_p = \frac{N}{2\zeta(1-\zeta^2)^{1/2}}} \quad (11)$$

* The frequency ω_p at which the maximum occurs is :

$$\boxed{\omega_p = \omega_n (1-2\zeta^2)^{1/2}} \quad (12)$$

* The 3-dB bandwidth ω_h can be derived by solving for the frequency ω_n at which the value of Eq. (9) is equal to 0.707 for the dc gain (0.707N). Hence for $\zeta < 1$:

$$\omega_h = \omega_n [1-2\zeta^2 + (2-4\zeta^2+4\zeta^4)^{1/2}]^{1/2} \quad (13)$$

* The time it takes for the output to rise from 10 to 90 percent of its final value is called the rise time t_r.

$$t_r = \frac{2.2}{\omega_h} \quad (14)$$

Ex. 3 A phase-Locked loop with closed Loop band-width $K_v = 10\pi$ rad/s. What value of LPF should be used so that the closed-Loop system approximates a second-order Butterworth filter? Determine the corresponding system rise time.

solution

For Butterworth filter, the damping ratio $\zeta = 0.707$

$$2\zeta = \frac{\omega_n}{K_v} = \left(\frac{\omega_L}{K_v}\right)^{1/2} \Rightarrow 2(0.707) = \left(\frac{\omega_L}{10\pi}\right)^{1/2}$$

$\therefore$ The required Low-pass filter bandwidth is $\boxed{\omega_L = 20\pi \text{ rad/s}}$

the bandwidth of the closed-Loop system is

$$\omega_h = \omega_n\left[1-2\zeta^2 + (2-4\zeta^2+4\zeta^4)^{1/2}\right]^{1/2} \Leftarrow \zeta = 0.707 = \frac{1}{\sqrt{2}}$$

$$\therefore \omega_h = \omega_n\left[1-2\left(\frac{1}{\sqrt{2}}\right)^2 + \left(2-4\left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{2}}\right)^4\right)^{1/2}\right]^{1/2}$$

$$= \omega_n\left[1-\frac{2}{2} + \left(2-\frac{4}{2}+\frac{4}{4}\right)^{1/2}\right]^{1/2}$$

$$= \omega_n\left[1-1+(2-2+1)^{1/2}\right]^{1/2}$$

$$= \omega_n[1]$$

$$\therefore \omega_h = \omega_n = (K_v\,\omega_L)^{1/2} = 14.14\pi \text{ rad/s}$$

then the corresponding system rise time is

$$t_r = \frac{2.2}{\omega_h} = 49.4 \text{ ms}$$

Phase Detectors: (PD)

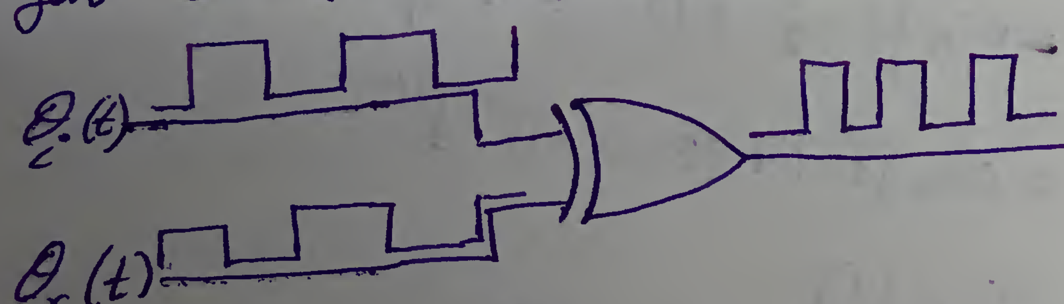
* PD is also called Phase comparator (PC).
* PLLs performance vary depending on the PD type.
* In PLL, different types of PDs can be used, such as
  1. Digital PD,
  2. Analog mixer, or multiplier,
  3. Sampling phase Detector.

* We shall focus on Digital PD only.

⟹ The output of alogic circuit PD is a constant-amplitude pulse whose width is proportional to the phase difference between the two input signals (which can be either analog or digital).

* The X-OR gate serves as a PD.

X	y	Z
0	0	0
0	1	1
1	0	1
1	1	0

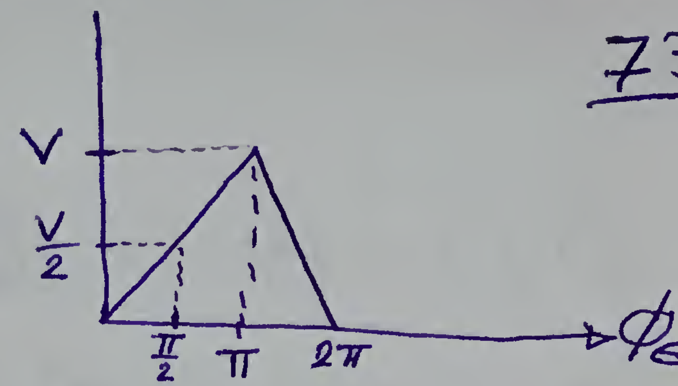


* The phase error is

$$\boxed{\phi_\epsilon = \frac{\tau}{T} 2\pi} \qquad (15)$$

where τ is the time difference between the two signals.
T is the period of the input signals.

* The average error is shown here



* The output is a maximum :

⇒ the gate output is high at all times, when the two signals are 180° out of phase.

* There are two values of phase error ϕ_E :

1)- One value corresponds to negative Loop gain,
2)- Second value corresponds to positive Loop gain

For the positive value of Loop gain, the closed-Loop system is unstable, and the error will adjust itself to the phase error corresponding to a negative-feedback Loop.

Voltage Controlled Oscillator (VCO)

A voltage controlled oscillator vco is a free-running oscillator whose frequency of operation is controlled by an external dc bias voltage.

PLL Applications

Many applications for the PLL such as:-

i) Frequency Modulation,
ii) Frequency demodulation, detection,
iii) tracking,
iv) Synthesis applications.

PLL Operation

* Important parameters of PLL must be defined to understand its operation,

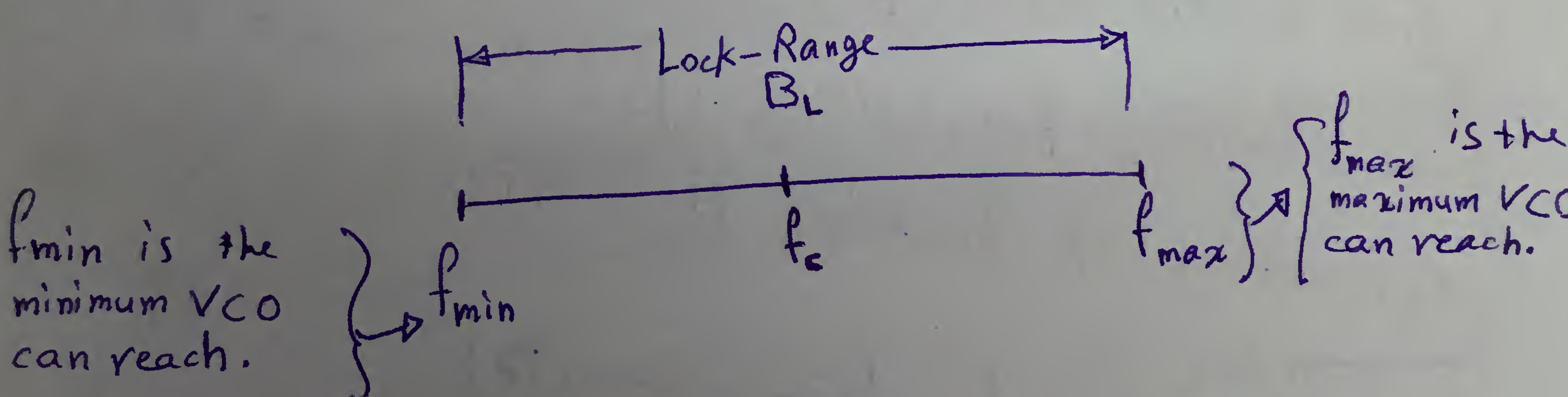
Lock Range: The range of frequencies over which a PLL can maintain phase Lock is called its Lock range. The Lock-range width, designated B_L, can be specified as:

$$\boxed{B_L = f_{max} - f_{min}} \qquad (16)$$

* f_{max} and f_{min}: maximum & minimum frequencies over which phase Lock can maintained.

* The factors Limiting the Lock-range include the maximum frequency deviation of the VCO and the dc voltage range of the phase detector output.

* The Lock-range is independent of the LPF frequency response, because when the PLL is in phase lock, the difference frequency $f_r - f_o = 0$.



* Lock-range is also called tracking-range.
* If the frequency deviation of the VCO which is $|\Delta\omega|$ or $|\Delta f|$ not equal to f_{max} or f_{min}, then the PLL can not track the input frequency.

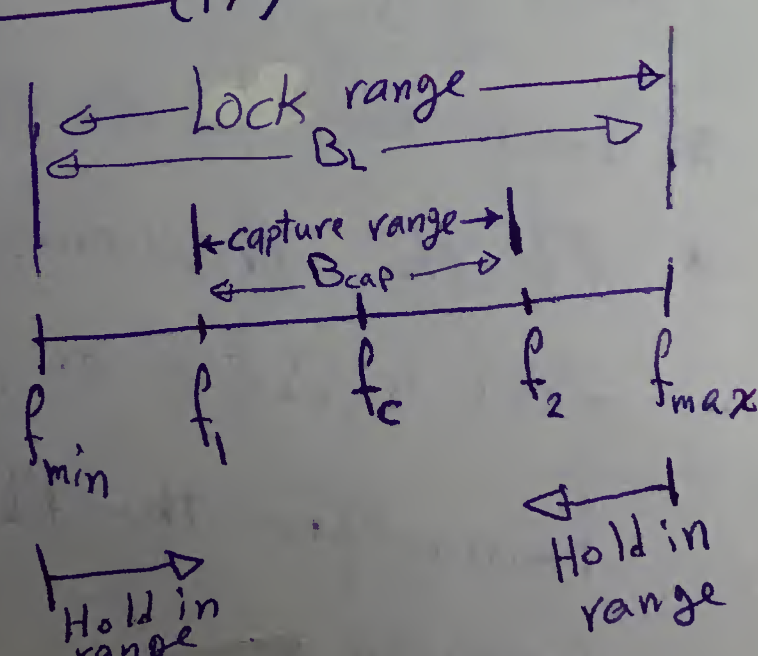
Capture Range :-

* The capture range width B_{cap}, is the range of frequencies over which a PLL can aquire Phase Lock when phase Lock does not yet exist.

⟹ In other words :

If PLL is not in Locked condition, the input signal to the PLL (f_r) must be close to the free running frequency (f_c) to start the operation of Locking the Loop.

$$\boxed{B_{cap} = f_2 - f_1} \quad (17)$$



Where :

f_1 : Lowest frequency the PLL can Lock onto.

f_2 : Highest frequency the PLL can Lock onto.

* Another words for the capture range :-

Capture Range : Range of input frequencies around the VCO center frequency f_c onto which the Loop will Lock when starting from unlocked condition.

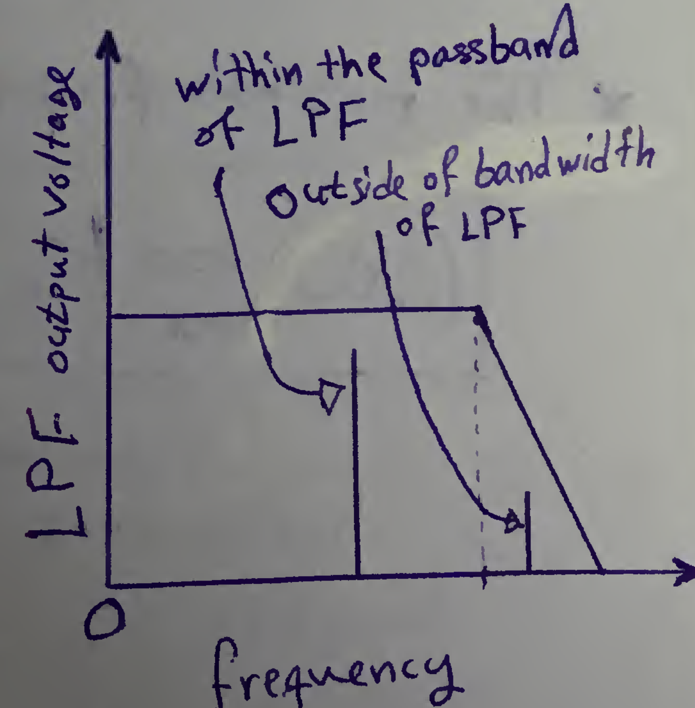
* In order for a PLL to acquire phase lock, the difference frequency $f_r - f_c$ must not exceed the bandwidth of the LPF.

* If $f_r - f_c$ exceeds the bandwidth of the LPF, Phase Lock can not acquired.

* If the bandwidth of the LPF $> f_r - f_c$, then Phase Lock can be achieved.

* If the bandwidth of the LPF $< f_r - f_c$, then Phase Lock cannot be achieved.

this is because the amplitude of the filter output voltage falls off sharply outside the bandwidth of the LPF.



Frequency Demodulation

*

* If the PLL is locked on an input frequency, the VCO control voltage is proportional to the VCO's frequency shift from its free-running frequency.

* If the input frequency f_r shifts, the control voltage shifts accordingly (provided the frequency shift is within the loop's tracking range).

* If the input signal is frequency-modulated, then the VCO control voltage will be the demodulated output.

* If the maximum phase detector output voltage is V volts, then the maximum control voltage applied to the VCO is KV volts, where K is the dc gain of the LPF.

* The maximum frequency deviation of the VCO is then

$$\boxed{(\Delta\omega)_{max} = K_o KV \quad \text{rad/s}}$$

* This of course, assumes that the VCO is designed to be linear over this frequency range.

* If the phase detector output can deviate between $\pm V$ volts, the tracking range (TR) will be:

$$\boxed{TR = 2(\Delta\omega)_{max} = 2K_o KV}$$

* This tracking range must be greater than the frequency deviation of the input signal.

* FM demodulation can then be obtained by setting the free-running frequency of the VCO equal to the center frequency of the input signal.

* This detection method assumes that the envelope of the input waveform has a constant amplitude.

* In many applications, an amplifier and amplitude Limiter are added before the PLL to ensure that this is the case.

Assignment: To the student, How to Demodulate the AM signal $v(t) = A_c[1 + m_a\, m(t)] \cos(\omega_c t)$ using PLL? Do all the mathematical analysis. You can use any reference.

EX.4 The output frequency of a VCO changes from 50 to 75 KHz when the dc input voltage changes by 0.5 V. Calculate the conversion gain.

Solution $K_o = \frac{\Delta f}{\Delta V} = \frac{75\,KHz - 50\,KHz}{0.5\,V} = 50\,KHz/V$

EX.5 A certain VCO has a natural frequency of 1 MHz with a dc input of 0 V. If the VCO has a conversion gain of 50 KHz/V, calculate the VCO output frequency for a voltage change of 1 V.

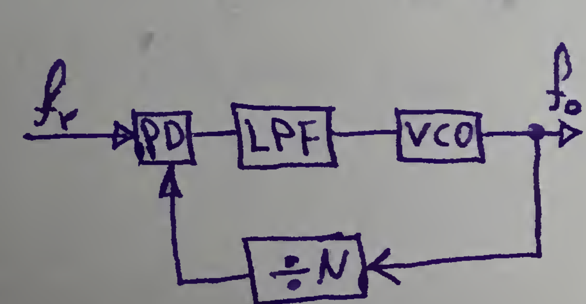
Solution $K_o = \frac{\Delta f}{\Delta V} \Rightarrow \Delta f = k_o\,\Delta V = 50{,}000\ Hz/V * 1\ V = 50\,KHz.$

The VCO frequency equals:

$$f_o = f_c \pm \Delta f = 1\,MHz \pm 50\,KHz$$

$\therefore f_o = 950\,KHz$ or $f_o = 1.05\,MHz$ depending on the transfer-function slope.

Problem 1: For the figure shown aside:



① what is f_o if $f_r = 25\,KHz$ and $N = 100$?
Ans. $f_o = 2.5\,MHz$.

② what is f_o if $f_r = 100\,KHz$ and $N = 35$?
Ans. $f_o = 3.5\,MHz$.

Problem 2: Use the datasheet of the Motorola CMOS IC MC14046B to design FM modulation and demodulation.