Lecture #1.2

(ii) Analogue and Digital signals :-

* Analogue signal :- $x(t)$ is analogue if it is defined on every point, instance, in the continuous time signal and has any value over the time.

* Digital signal :- $x[n]$ is digital signal if it is discrete signal and has finite value at each point of the discrete time.

(iii) Real and Complex signals :-

* $x(t)$ or $x[n]$ are real signals if their values are only real.
* $x(t)$ or $x[n]$ are complex signals if their values are complex,

$$x(t) = x_{real}(t) + j\, x_{imag.}(t)$$

$$x[n] = x_{real}[n] + j\, x[n]_{imag.}$$

(iv) Deterministic and Random Signals :

* $x(t)$ or $x[n]$ are deterministic signals if all their values are specified for any given instant, in other words they are function of continuous or discrete time.

* $x(t)$ or $x[n]$ are random signals if any of their values take a random number at any given time instant, in other words, they are statistical signals.

(v) Even and Odd Signals :

Any signal is even if

$$x(t) = x(-t) \qquad (2)$$

$$x[n] = x[-n] \qquad (3)$$

or it is odd if

$$x(t) = -x(-t) \qquad (4)$$

$$x[n] = -x[-n] \qquad (5)$$

For instance, $x(t) = t$, $x[n] = n$ are odd signals, while $x(t) = t^2$, $x[n] = n^2$ are even signals.

Generally speaking, any continuous or discrete time signal involves two components, even and odd.

$$x(t) = x_e(t) + x_o(t) \quad \text{——} (6)$$

$$x[n] = x_e[n] + x_o[n] \quad \text{——} (7)$$

where

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right] \quad \text{——} (8)$$

$$x_e[n] = \frac{1}{2}\left[x[n] + x[-n]\right] \quad \text{——} (9)$$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right] \quad \text{——} (10)$$

$$x_o[n] = \frac{1}{2}\left[x[n] - x[-n]\right] \quad \text{——} (11).$$

EX. 6 Find the even and odd components of $x(t)=t^2+t$.

solution we need $x(-t)=(-t)^2+(-t)=t^2-t$

$$x_e(t)=\frac{1}{2}\left[x(t)+x(-t)\right]=\frac{1}{2}\left[t^2+\cancel{t}+t^2-\cancel{t}\right]=\frac{1}{2}\left[2t^2\right]$$

$\therefore\ x_e(t)=t^2$.

$$x_o(t)=\frac{1}{2}\left[x(t)-x(-t)\right]=\frac{1}{2}\left[t^2+t-(t^2-t)\right]$$

$$=\frac{1}{2}\left[\cancel{t^2}+t-\cancel{t^2}+t\right]=\frac{1}{2}\left[2t\right]$$

$\therefore\ x_o(t)=t$

Note: to prove the solution is correct,

$$x(t) \text{ original} = \underbrace{x_e(t)} + \underbrace{x_o(t)}$$

$$x(t)=t^2+t$$

EX. 7 Find the even and odd parts of

$$g[n] = [0, 1, 3, \underline{2}, 0, 4, 4].$$

$$\begin{array}{ccccccc} -3 & -2 & -1 & 0 & 1 & 2 & 3 \end{array}$$

Solution we need $g[-n]$

$$g[n] = [0, 1, 3, \underline{2}, 0, 4, 4]$$

$$g[-n] = [4, 4, 0, \underline{2}, 3, 1, 0]$$

$$g_e[n] = \frac{1}{2}\left[g[n] + g[-n]\right]$$

$$= \frac{1}{2}\begin{bmatrix} 0, & 1, & 3, & \underline{2}, & 0, & 4, & 4 \\ + & + & + & + & + & + & + \\ 4, & 4, & 0, & \underline{2}, & 3, & 1, & 0 \end{bmatrix}$$

$$= \frac{1}{2}[4, 5, 3, \underline{4}, 3, 5, 4]$$

$$g_e[n] = [2, 2.5, 1.5, \underline{2}, 1.5, 2.5, 2]$$

$$g_o[n] = \frac{1}{2}\begin{bmatrix} 0, & 1, & 3, & \underline{2}, & 0, & 4, & 4 \\ - & - & - & - & - & - & - \\ 4, & 4, & 0, & 2, & 3, & 1, & 0 \end{bmatrix}$$

$$g_o[n] = \frac{1}{2}[-4, -3, 3, \underline{0}, -3, 3, 4] = [-2, -1.5, 1.5, \underline{0}, -1.5, 1.5, 2]$$

Prove: $g[n] = g_e[n] + g_o[n] = [4, 1, 3, \underline{2}, 0, 4, 4]$ ✓

Exercise #6 Find the even and odd components for each of the following signals (resolve the signals to the even and odd parts):

a) $x(t) = 2\cos(\omega t) - 3\sin(\omega t)$.

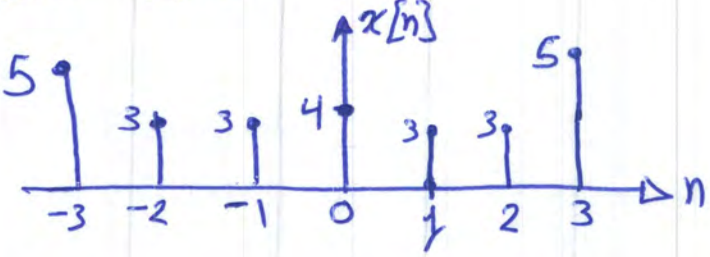
b) $x[n] = 4n^2 - 3n + 1$

c) $x(t) = e^{j\omega t}$

d) $x[n] = [\underline{0}, 0, 0, 1]$.

e) $x\{n\} = \{2, 2, 2, \underline{4}\}$

f) $x[n]$ is drawn below



Exercise #7 Resolve $H[n]$ in exercise #3 to its even and odd components.

Exercise #8 find the even and odd parts of $x(t) = t\cos(\omega t)$.

(vi) Periodic and Aperiodic Signals :-

If any signal (continuous or discrete) repeats itself every certain period of time, then it is periodic, otherwise the signal is aperiodic or nonperiodic.

Thus,

$$x(t+T_0) = x(t) \quad \text{——— (12)}$$

$\therefore$ $x(t)$ is periodic, where T_0 is the fundamental period. In other words, the signal $x(t)$ repeats itself every T_0 seconds.

On the other hand $x[n]$ is periodic if

$$x[n+KN_0] = x[n] \quad \text{——— (13)}$$

where $x[n]$ repeats itself every N_0 samples, for any integer K.

Notes :- (1) The sampled version of the periodic $x(t)$ may not produce periodic discrete time signal.

(2) The summation of two continuous time signals may not be periodic.

(3) The summation of two discrete time periodic signals is always periodic.

Example 1.6

Consider the analog signal $x(t) = \cos(2\pi t + \theta)$, $-\infty < t < \infty$. Determine the value of θ for which $x(t)$ is even and for which it is odd. If $\theta = \pi/4$ is $x(t) = \cos(2\pi t + \pi/4)$, $-\infty < t < \infty$, even or odd?

Solution

The reflection of $x(t)$ is $x(-t) = \cos(-2\pi t + \theta)$, then:

(i) $x(t)$ is even if $x(t) = x(-t)$ or

$$\cos(2\pi t + \theta) = \cos(-2\pi t + \theta) = \cos(2\pi t - \theta)$$

or $\theta = -\theta$, that is when $\theta = 0$, or π. Thus, $x_1(t) = \cos(2\pi t)$ as well as $x_2(t) = \cos(2\pi t + \pi) = -\cos(2\pi t)$ are even.

(ii) For $x(t)$ to be odd, we need that $x(t) = -x(-t)$ or

$$\begin{aligned}\cos(2\pi t + \theta) &= -\cos(-2\pi t + \theta) = \cos(-2\pi t + \theta \pm \pi)\\ &= \cos(2\pi t - \theta \mp \pi)\end{aligned}$$

which can be obtained with $\theta = -\theta \mp \pi$ or $\theta = \mp \pi/2$. Indeed, $\cos(2\pi t - \pi/2) = \sin(2\pi t)$ and $\cos(2\pi t + \pi/2) = -\sin(2\pi t)$ both are odd.

When $\theta = \pi/4, x(t) = \cos(2\pi t + \pi/4)$ is neither even nor odd according to the above. ■

Example 1.7

Consider the signal

$$x(t) = \begin{cases} 2\cos(4t) & t > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find its even and odd decomposition. What would happen if $x(0) = 2$ instead of 0, i.e., when we let $x(t) = 2\cos(4t)$ at $t \geq 0$, and zero otherwise? Explain.

Solution

The signal $x(t)$ is neither even nor odd given that its values for $t \leq 0$ are zero. For its even-odd decomposition, the even component is given by

$$x_e(t) = 0.5[x(t) + x(-t)] = \begin{cases} \cos(4t) & t > 0 \\ \cos(4t) & t < 0 \\ 0 & t = 0 \end{cases}$$

and the odd component is given by

$$x_o(t) = 0.5[x(t) - x(-t)] = \begin{cases} \cos(4t) & t > 0 \\ -\cos(4t) & t < 0 \\ 0 & t = 0 \end{cases}$$

which when added give $x(t)$. See Figure 1.6.

If $x(0) = 2$, we have

$$x_e(t) = 0.5[x(t) + x(-t)] = \begin{cases} \cos(4t) & t > 0 \\ \cos(4t) & t < 0 \\ 2 & t = 0 \end{cases}$$

while the odd component is the same. The even component has a discontinuity at $t=0$ in both cases. ■

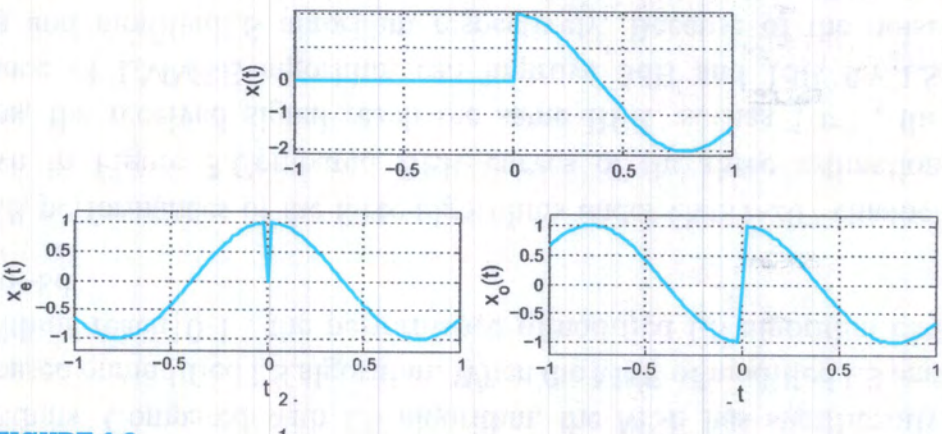


FIGURE 1.6

Even and odd components of $x(t)$ ($x(0) = 0$). Notice that in this case $x_e(0) = 0$. The value $x_o(0)$ is always zero.

(4) If the signal is constant (D.C.), $x(t) = d.c.$, then the fundamental period is not defined.

The fundamental period T_0 or N_0 represents the frequency of the periodic signal.

* Befor we go ahead, we need to know the Greatest Common Divisor and the least Common Multiple.

* The Greatest Common Divisor (GCD) can be explained by examples:

$54 = 1 \times 54$
$54 = 2 \times 27$
$54 = 2 \times 3 \times 9$
$54 = 6 \times 9$
$54 = 3 \times 18$

The divisors of 54 are

$\boxed{1, 2, 3, 6, 9, 18, 27, 54}$

Common between are $\boxed{1, 2, 3, 6}$

greatest $= 6$

$24 = 1 \times 24$
$24 = 2 \times 12$
$24 = 3 \times 8$
$24 = 4 \times 6$

The divisors of 24 are

$\boxed{1, 2, 3, 4, 6, 8, 12, 24}$

$\therefore \boxed{GCD(54, 24) = 6}$

Ex. Find the greatest common divisor of 30, 36, 24.

solution

$$24 = 1\times 24 = 2\times 12 = 2\times 3\times 4 = 6\times 4 = 8\times 3 = 2\times 2\times 2\times 3$$

1, 2, 3, 4, 6, 8, 12, 24

$$30 = 1\times 30 = 5\times 6 = 2\times 15 = 3\times 10$$

1, 2, 3, 5, 6, 10, 15, 30

$$36 = 1\times 36 = 2\times 18 = 3\times 12 = 6\times 6 = 4\times 9$$

1, 2, 3, 4, 6, 9, 12, 18, 36

(1, 2, 3, 6)

$\therefore GCD(30, 36, 24) = 6$

Ex: Find The Least Common Multiple (LCM) of 4 and 6

solution

$4, \overset{+4}{8}, \overset{+4}{\underline{12}}, \overset{+4}{16}, 20, \overset{+4}{\underline{24}}, \overset{+4}{28}, \overset{+4}{32}, \underline{36}, 40, 44, \underline{48}, \ldots$

$6, \overset{+6}{\underline{12}}, \overset{+6}{18}, \overset{+6}{\underline{24}}, \overset{+6}{30}, \underline{36}, 42, \underline{48}, \ldots$

In commone : 12, 24, 36, 48, 60, ---

Least is $\boxed{12}$

$\therefore LCD(4,6) = 12$

Ex. Find the LCD of 21, 6.

Solution $21, \overset{+21}{\underline{42}}, \overset{+21}{63}, 84, 105 \ldots$

$6, 12, 18, 24, 30, 36, \underline{42}, 48, 54 \ldots$

$\therefore LCD(21,6) = 42$

Ex. $LCD(4,5)$, solution $4, 8, 12, 16, \underline{20}, 24, 28, 32, \ldots$

$5, 10, 15, \underline{20}, 25, 30, 35, 40, \ldots$

$\therefore LCD(4,5) = 20$

Back to periodicity

* The Fundamental frequency of a periodic signal is the GCD of all frequencies of the signal.

* The fundamental period of a periodic signal is the LCM of all periods of the signal.

Ex. 8 Find the fundamental frequency of the signal $x(t) = \cos\left(\frac{10}{3}\pi t\right) + \sin\left(\frac{5\pi}{4} t\right)$, and the fundamental period.

solution

$$\omega_1 = \frac{10\pi}{3} \rightarrow f_1 = \frac{\omega_1}{2\pi} = \frac{\frac{10\pi}{3}}{2\pi} = \frac{10\pi}{3} \cdot \frac{1}{2\pi} = \frac{5}{3}\ Hz,$$

$$\omega_2 = \frac{5\pi}{4} \rightarrow f_2 = \frac{\omega_2}{2\pi} = \frac{5\pi}{4} \cdot \frac{1}{2\pi} = \frac{5}{8}\ Hz$$

The fundamental frequency is the $GCD\left(\frac{5}{3}, \frac{5}{8}\right)$

$$f_0 = GCD\left(\frac{5}{3}, \frac{5}{8}\right) = GCD\left(\frac{40}{24}, \frac{15}{24}\right)$$

$15 = 1 \times 15 = 3 \times 5$, $40 = 1 \times 40 = 2 \times 20 = 8 \times 5 = 4 \times 10$

$GCD(15, 40) = 5$

$\therefore GCD\left(\frac{40}{24}, \frac{15}{24}\right) = \frac{5}{24}$ is the fundamental frequency

$\boxed{T_0 = \frac{1}{f_0} = \frac{24}{5}\,s} = LCM\{$

EX. 9 Find the fundamental period of $x[n] = \cos\left(\frac{\pi n}{18}\right) + \sin\left(\frac{10\pi n}{24}\right)$

solution we have $\omega_1 = \frac{\pi}{18} = \frac{2\pi}{36} = \frac{2\pi}{N_{01}}$

$$\omega_2 = \frac{10\pi}{24} = \frac{2\pi 5}{24} = 2\pi \frac{5}{24} = \frac{2\pi 5}{N_{02}}$$

$\therefore N_{01} = 36$, $N_{02} = 24$

$$\begin{array}{r|l} 2 & 24, \; 36 \\ 2 & 12, \; 18 \\ 2 & 6, \; 9 \\ 3 & 3, \; 9 \\ 3 & 1, \; 3 \\ & 1, \; 1 \\ \hline 72 & \end{array}$$

$\therefore LCM(36, 24) = 72$

$\therefore N_0 = 72$

EX. 10 is $x[n] = \cos\left(\frac{5\pi n}{13}\right) + \sin\left(\frac{8\pi n}{39}\right)$ periodic? find the fundamental period.

solution

$$\omega_1 = \frac{5\pi}{13} = \frac{2\pi 5}{26} \rightarrow N_{01} = 26$$

$$\omega_2 = \frac{8\pi}{39} = \frac{2\pi 4}{39} \rightarrow N_{02} = 39$$

$$\begin{array}{r|l} 2 & 26, \; 39 \\ 13 & 13, \; 39 \\ 3 & 1, \; 3 \\ \hline 78 & 1, \; 1 \end{array}$$

$N_0 = LCM(26, 39) = 78.$

Ex. 11 Determine the fundamental period of $x[n] = 5\sin\left(\frac{5\pi}{6}n\right) + 13\cos\left(\frac{3\pi}{4}n\right) + 25\sin\left(\frac{\pi}{3}n\right)$.

Solution

$$\omega_1 = \frac{5\pi}{6} = \frac{2\pi\, 5}{12} \rightarrow N_{01} = 12.$$

$$\omega_2 = \frac{3\pi}{4} = \frac{2\pi\, 3}{8} \rightarrow N_{02} = 8.$$

$$\omega_3 = \frac{\pi}{3} = \frac{2\pi}{6} \rightarrow N_{03} = 6$$

2	12, 8, 6
2	6, 4, 3
2	3, 2, 3
3	3, 1, 3
24	1, 1, 1

$$N_0 = LCM(12, 8, 6) = 24.$$

Note: For any Two signals $x_1(t)$ and $x_2(t)$ or $x_1[n]$ and $x_2[n]$, they can be combined to produce the signal $y(t)$ or $y[n]$ as

$$y(t) = x_1(t) + x_2(t)$$

or

$$y[n] = x_1[n] + x_2[n]$$

$y[n]$ or $y(t)$ is periodic if

$$\boxed{\frac{T_1}{T_2} = \frac{m}{n} = \text{rational number}}$$

or

$$\boxed{\frac{N_1}{N_2} = \frac{m}{n} = \text{rational number}}$$

On the other hand: as we stated in note #1 on page #17, the discrete time signal, which is obtained by sampling periodic signal, may not be periodic. Hence, discrete time signal can be periodic if its discrete frequency is rational number

$$\boxed{\frac{f}{f_0} = \text{rational number}}$$

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EX. 12 Is $x[n] = \cos(2\pi F_0 n)$ periodic if $F_0 = 0.32$? Find the fundamental period.

Solution $x[n] = \cos(2\pi 0.32\, n) = \cos\left(2\pi \frac{32}{100} n\right)$,

$F_0 = \frac{32}{100} = 0.32$ is a rational number

$\therefore$ $x[n]$ is periodic.

$$\omega_0 = \frac{2\pi 32}{100} = \frac{2\pi 4}{25} \rightarrow \boxed{N_0 = 25}$$

fundamental period.

EX. 13 Is $x[n] = \sin(2\pi\sqrt{3}/3\, n)$ periodic? If it is periodic, Find the fundamental period.

solution $\omega_0 = \frac{2\pi\sqrt{3}}{3} = 2\pi \frac{\sqrt{3}}{3} = 2\pi \frac{1}{\sqrt{3}} \rightarrow f_0 = \frac{1}{\sqrt{3}}$ is irrational number

$\therefore$ $x[n]$ is not periodic.

EX. 14 Determine wether $g[n] = \cos\left(\frac{\pi n}{8}\right)$ periodic or aperiodic and find the fundamental period.

solution $g[n] = \cos\left(\frac{\pi}{8} n\right) \rightarrow \omega_o = \frac{\pi}{8} = \frac{2\pi}{16} \rightarrow F_o = \frac{1}{16}$

$F_o = \frac{1}{16} = 0.0625$ rational number

$\therefore g[n]$ is periodic.

$N_o = 16$ is the fundamental period.

EX. 15 Find if $y[n] = \cos(1.1\pi n) + \sin(0.7\pi n)$ is periodic, then calculate the fundamental period.

solution $\omega_1 = 1.1\pi = 2\pi f_1 \rightarrow f_1 = \frac{1.1}{2} = 0.55 = \frac{55}{100} = \frac{11}{20}$ rational number

$\omega_2 = 0.7\pi = 2\pi f_2 \rightarrow f_2 = \frac{0.7}{2} = 0.35 = \frac{35}{100} = \frac{7}{20}$ rational number

$\therefore y[n]$ periodic because both its terms are periodic

$N_o = LCM(20, 20) = 20$

Exercise #9 Determine whether or not each of the following signals is periodic. If the signal is periodic, determine its fundamental period.

1) $x(t) = \cos(t + \frac{\pi}{4})$. 2) $x(t) = \sin(\frac{2\pi}{3} t)$,
3) $x(t) = \cos(\frac{\pi}{3} t) + \sin(\frac{\pi}{4} t)$, 4) $x(t) = \cos(t) + \sin(\sqrt{2}\, t)$.
5) $x(t) = \sin^2(t)$, 6) $x(t) = e^{j(\frac{\pi}{2} t - 1)}$,
7) $x[n] = e^{j\frac{\pi}{4} n}$, 8) $x[n] = \cos(\frac{n}{4})$,
9) $x[n] = \cos(\frac{\pi n}{3}) + \sin(\frac{\pi n}{4})$, 10) $x[n] = \cos^2(\frac{\pi n}{8})$

Exercise #10 for the signal $f(t) = \left|\sin\left(\frac{-5\pi}{8} t + \frac{\pi}{2}\right)\right|$, determine if it is periodic or not, then if it is periodic, find the fundamental period.

Exercise #11 Given $f(t) = \sin(\frac{6\pi}{7} t) + 2\cos(\frac{3t}{5})$, estimate if it is periodic, if it is periodic, find its fundamental period.

Exercise #12 Given $f(t) = e^{j\frac{3\pi t}{8}} + e^{\frac{\pi t}{86}}$, is it periodic? If so, find the fundamental period.

Exercise #13 Check the periodicity of $h(t) = \cos(\pi t)\sin(3\pi t)$, and find the fundamental period.

Exercise #14 is $g(t) = \begin{cases} \cos(10\pi t) & -12 \le t \le 12 \\ 0 & \text{elsewhere} \end{cases}$ periodic? why?

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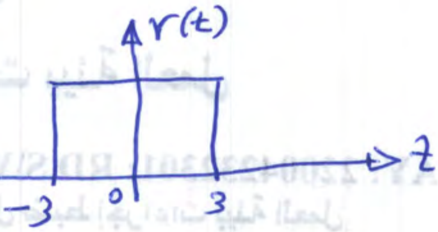
Exercise # 15 Given the signal shown in Exe. 15, Find its periodicity.



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Exercise # 16 $x(t) = 4t^3 \quad |t| \leq 1$, is $x(t)$ periodic? Find its period.

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Exercise # 17 $h(t) = \begin{cases} 5 & -2 \leq t \leq 2 \\ 0 & 2 < |t| \leq 4 \end{cases}$ with repeatition every 8 seconds. Is $h(t)$ periodic? Calculate the fundamental period.

(vii) Energy and power Signals

→ The signal is energy type if its energy exists

$$0 < E < \infty$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad \text{Joules (J)}$$

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 \quad \text{Watts (W)}$$

→ The signal is power type if its power exists

$$0 < P < \infty$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

* if Neither energy nor power are exist, the signal type neither energy nor power type.

* The periodic signal is power type if its energy over one period is exists.

Ex. 16 is $x(t) = t$ where $0 \leq t \leq 2$ energy or power signal? Calculate its value.

solution since the signal $x(t)$ is not periodic starts at 0 and ends at 2, then

$$E_x = \int_0^2 |t|^2 dt = \frac{t^3}{3}\Big|_0^2 = \left[\frac{8}{3}\right] - \left[\frac{0}{3}\right]$$

$$E_x = \frac{8}{3} \text{ J}.$$

Ex. 17 Calculate the suitable measure (energy or power) for the signal $x(t) = 4t^3$, $|t| \leq 2$.

solution $x(t) = 4t^3 \quad -2 \leq t \leq 2$ $\therefore$ the signal is not periodic, hence it is energy type signal

$$E_x = \int_{-2}^{2} |4t^3|^2 dt = \int_{-2}^{2} 16\, t^6 dt = \frac{16}{7}\left[t^7\right]_{-2}^{2}$$

$$E_x = \frac{16}{7}\left[2^7 - (-2)^7\right] = \frac{16}{7}\left[128 + 128\right] = \frac{4096}{7} \text{ J}$$

$$\boxed{E_x = 585.143 \text{ J}}$$

Ex. 18 classify $x[n] = \left(\frac{1}{4}\right)^n \quad n \geq 0$ according to the power or energy types.

Solution lets find first the energy E_x

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=0}^{\infty} \left|\left(\tfrac{1}{4}\right)^n\right|^2 = \sum_{n=0}^{\infty} \left(\frac{1}{16}\right)^n = \text{geometric series}$$

$$\therefore E_x = \frac{1}{1-\frac{1}{16}} = \frac{16}{15} \, J.$$

since the energy is exists then the signal is energy type and its power $P = 0$.

Ex. 19 Is $x[n] = e^{10jn} \quad n \geq 0$ power or energy signal? Find its value.

solution $x[n] = e^{j10n}$ is periodic with $F_o = 10$ $\therefore$ it is power signal

$$P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1^2 = \lim_{N \to \infty} \frac{N+1}{2N+1}$$

$\Rightarrow$ Using L'Hopital's rule $\quad \dfrac{\frac{d(N+1)}{dN}}{\frac{d(2N+1)}{dN}} = \dfrac{1}{2}$

$$\therefore \boxed{P_x = \frac{1}{2} W}$$

Exercise #18 Find the energy of $x(t)=e^{-t}$, $t \geq 0$.

Exercise #19 is $x(t)=Ae^{j\omega_0 t}$ energy or power signal? Calculate its value.

Exercise #20 For the sequence $x[n]=\begin{cases} 2^{-n} & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$

Determine if it is power or energy signal.

Hint: $\sum_{n=-\infty}^{\infty}(a)^n = \frac{1}{1-a}$ when $a<1$.

Exercise #21 Classify each of the following signals as an energy signal or power signal by calculating the energy E or the power P (A, θ, ω, and τ are real positive constants).

a) $x_1(t) = A|\sin(\omega t+\theta)|$. b) $x_2(t)=\frac{A\tau}{\sqrt{\tau+jt}}$ where $j=\sqrt{-1}$

c) $x_3(t)=At^2 e^{-t/\tau}$ $t \geq 0$.

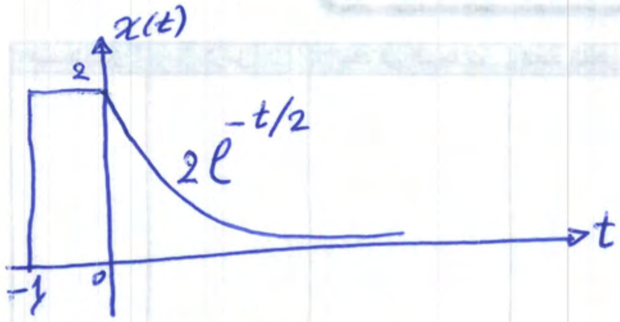
Exercise #22 consider the following DT sequence

$$f[K]=\begin{cases} e^{-0.5K} & K \geq 0 \\ 0 & K<0 \end{cases}$$

Determine if the signal is power or an energy signal.

Hint: $\sum_{K=0}^{\infty}(a)^K = \frac{1}{1-a}$ for $a<1$.

Exercise #23 Determine the suitable measures of the signals in Figure EX.23.



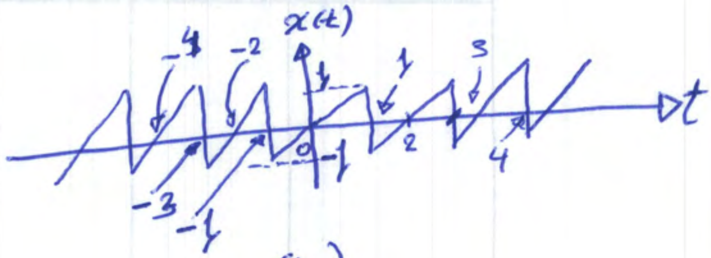


Figure EX.23

* Any sinusoidal periodic signal, it is periodic and power signal type, its power in general can be calculated as

$$x(t) = C\cos(\omega_0 t + \theta)$$

$$\boxed{P_x = \frac{C^2}{2}}$$

* If combination of two terms sinusoidal signal, it must be periodic before we decide it is power type signal. Thus

If $x(t) = C_1\cos(\omega_1 t + \theta_1) + C_2\cos(\omega_2 t + \theta_2)$ $\omega_1 \neq \omega_2$

is periodic then its power is

$$\boxed{P_x = \frac{C_1^2}{2} + \frac{C_2^2}{2}}$$

* The root mean square (rms) is

$$\boxed{rms = \frac{\sqrt{C_1^2 + C_2^2}}{2}}$$

* If the signal consists of sinusoidal periodic terms with d.c. term, then the signal power is

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(\omega_n t + \theta_n)$$

then

$$\boxed{P_x = C_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} C_n^2}$$

Ex. 20 show that if $\omega_1 = \omega_2$, the power of the periodic signal $x(t) = C_1 \cos(\omega_1 t + \theta) + C_2 \cos(\omega_2 t + \theta_2)$ is $\left[C_1^2 + C_2^2 + 2C_1 C_2 \cos(\theta_1 - \theta_2)\right]/2$ which is not equal to $\frac{C_1^2 + C_2^2}{2}$.

solution $P_x = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt = \frac{C_1^2}{2} + \frac{C_2^2}{2} + \frac{2}{T}\int_{-T/2}^{T/2} C_1 C_2 \cos(\omega_1 t + \theta_1)\cos(\omega_2 t + \theta_2)\, dt$

using $\cos(\alpha)\cos(\beta) = \frac{1}{2}\left\{\cos(\alpha+\beta) + \cos(\alpha-\beta)\right\}$

$$\therefore P_x = \frac{C_1^2}{2} + \frac{C_2^2}{2} + \frac{2C_1 C_2}{T\,2}\int_{-T/2}^{T/2}\left[\underbrace{\cos(2\omega t + \theta_1 + \theta_2)}_{\text{zero}} + \cos(\theta_1 - \theta_2)\right]dt$$

$$\therefore P_x = \frac{C_1^2}{2} + \frac{C_2^2}{2} + 2\,C_1 C_2 \cos(\theta_1 - \theta_2) = \frac{C_1^2 + C_2^2 + C_1 C_2 \cos(\theta_1 - \theta_2)}{2}$$