Lecture #2

Amplitude and Phase Line Spectrum

Recall the following identity [Eular's identity]

$$\boxed{e^{\pm j\theta} = \cos(\theta) \pm j\sin(\theta)}$$

where $\text{Real}\{e^{\pm j\theta}\} = \cos(\theta)$

and $\text{Imaginary}\{e^{\pm j\theta}\} = \sin(\theta)$

* Line or Magnitude spectrum can shows the frequency contents of a sinusoidal signals.

$$x(t) = \underbrace{A}_{\text{amplitude}} \cos(\underbrace{\omega_o}_{\text{freq.}} t + \underbrace{\phi}_{\text{Phase}})$$

$$\boxed{\omega_o = 2\pi f_o} \quad \text{or} \quad \boxed{\omega_o = \frac{2\pi}{T_o}}, \boxed{f_o = \frac{1}{T_o}}$$

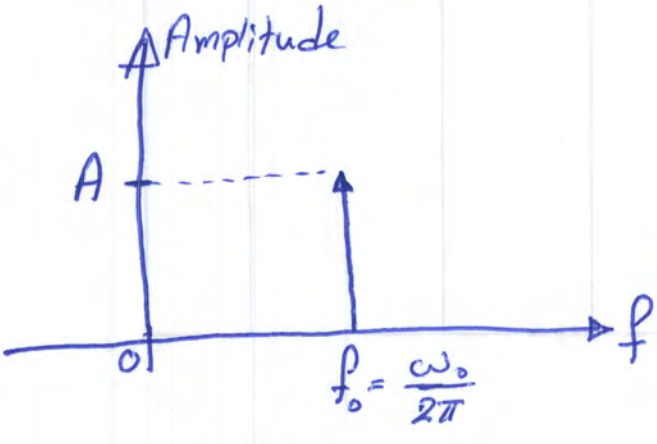
PDF created with pdfFactory Pro trial version www.pdffactory.com

Using Euler's identity

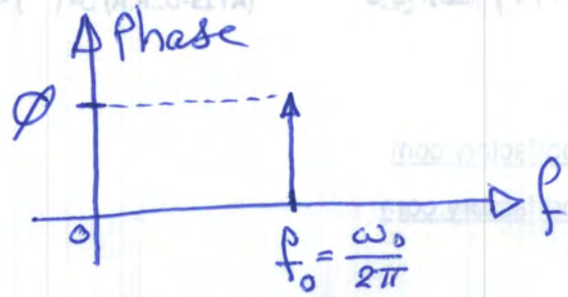
$$A\cos(\omega_0 t+\phi) = A\,\mathrm{Re}\left\{e^{j(\omega_0 t+\phi)}\right\}$$

$$= \mathrm{Re}\left\{A e^{j\phi} e^{j\omega_0 t}\right\}$$

The line or magnitude or amplitude spectrum is shown below



The phase line spectrum is



* To plote the spectrum, convert all sinusoids to cosine terms for simplicity.

for example $\boxed{\sin(\omega t) = \cos(\omega t - 90^\circ)}$

* The negative amplitude can be converted to positive

for instance $\boxed{-A\cos(\omega t) = A\cos(\omega t \mp 180^\circ)}$

$\mp$ have the same effect

Ex. 21 plote the magnitude and phase line spectrum of the signal

$$x(t) = 7 - 10\cos(40\pi t - 60^\circ) + 4\sin(120\pi t)$$

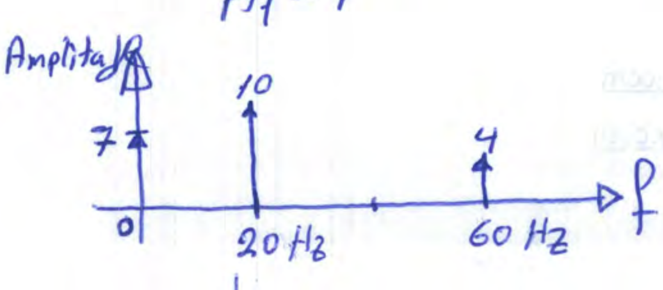
solution

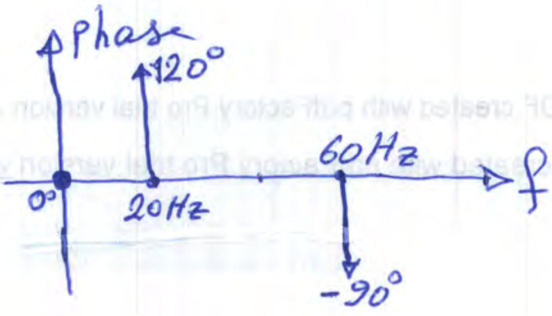
$$x(t) = \underbrace{7\cos(2\pi 0 t)} + \underbrace{10\cos(40\pi t - 60^\circ + 180^\circ)} + \underbrace{4\cos(120\pi t - 90^\circ)}$$

$f_1 = 0\ Hz$ $\quad$ $f_2 = 20\ Hz$ $\quad$ $f_3 = 60\ Hz$

$\phi_1 = 0^\circ$ $\quad$ $\phi_2 = 120^\circ$ $\quad$ $\phi_3 = -90^\circ$

$A_1 = 7$ $\quad$ $A_2 = 10$ $\quad$ $A_3 = 4$





EX. 22 plote the various possible spectra of the signal

$$x(t) = 5\cos(2\pi 10 t + 30^\circ) - 10\cos(2\pi 20 t + 60^\circ)$$

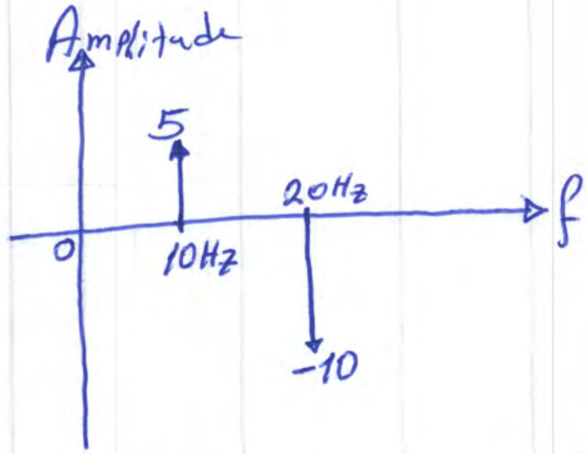
Solution case #1

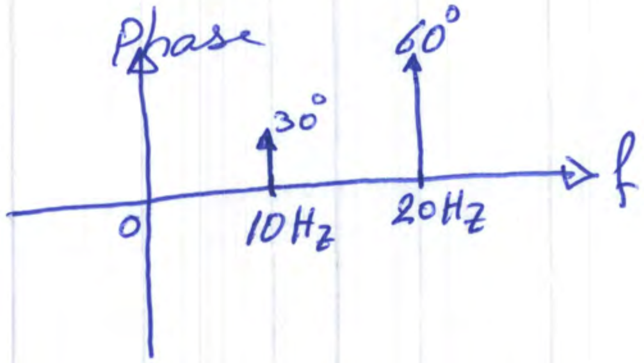
$$x(t) = \underbrace{5\cos(2\pi 10 t + 30^\circ)} - \underbrace{10\cos(2\pi 20 t + 60^\circ)}$$

$A_1 = 5$ $\quad$ $A_2 = -10$

$f_1 = 10\,Hz$ $\quad$ $f_2 = 20\,Hz$

$\phi_1 = 30^\circ$ $\quad$ $\phi_2 = 60^\circ$





case #2 $\quad x(t) = \underbrace{5\cos(2\pi 10 t + 30^\circ)} + \underbrace{10\cos(2\pi 20 t - 120^\circ)}$

$A_1 = 5$, $f_1 = 10\,Hz$ $\quad$ $A_2 = 10$, $f_2 = 20\,Hz$

$\phi_1 = 30^\circ$ $\quad$ $\phi_2 = -120^\circ$

Double or Two sided Spectrum:

Double sided spectrum is used to plot frequency contents of both positive and negative frequencies.

suppose $z = e^{j\theta} = \cos(\theta) + j\sin(\theta)$ —— (1)

complex conjugate of $z = z^*$

$$z^* = e^{-j\theta} = \cos(\theta) - j\sin(\theta) \quad \text{—— (2)}$$

$$z + z^* = 2\cos(\theta) = 2\,Re\{z\}$$

$$\therefore\ Re[z] = \frac{1}{2}(z + z^*) \quad \text{—— (3)}$$

For the signal $x(t) = A\cos(\omega_o t + \phi)$

Let $z = A\,e^{j(\omega_o t + \phi)}$ then $z^* = A\,e^{-j(\omega_o t + \phi)}$

then using Eq. (3)

$$Re[A\,e^{j(\omega_o t + \phi)}] = \frac{A}{2}e^{j(\omega_o t + \phi)} + \frac{A}{2}e^{-j(\omega_o t + \phi)}$$

OR

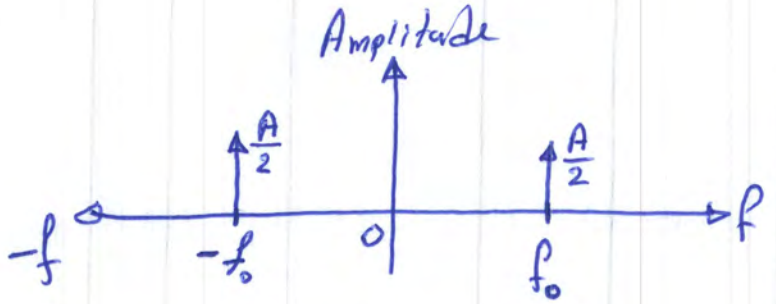
$$\boxed{A\cos(\omega_o t + \phi) = \frac{A}{2}e^{j(\omega_o t + \phi)} + \frac{A}{2}e^{-j(\omega_o t + \phi)}} \quad \text{—— (4)}$$

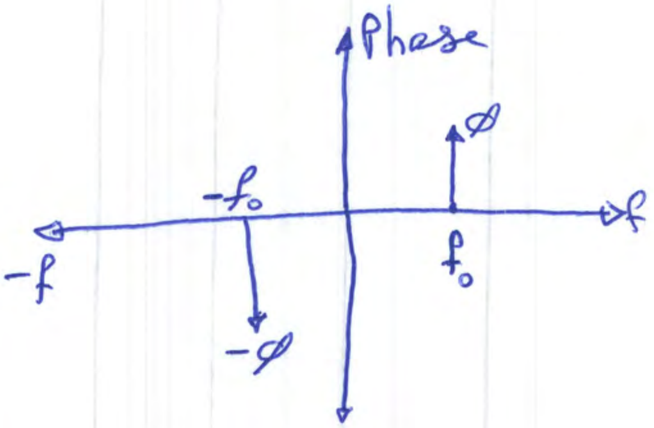
Hence :

$$A\cos(\omega_0 t+\phi)=\underbrace{\frac{A}{2}e^{j(\omega_0 t+\phi)}}+\underbrace{\frac{A}{2}e^{-j(\omega_0 t+\phi)}}$$

Amplitude $= \frac{A}{2}$
Frequency $= \frac{\omega_0}{2\pi}$ Hz
phase $= \phi$

Amplitude $= \frac{A}{2}$
Frequency $= \frac{\omega_0}{2\pi}$ Hz
Phase $= -\phi$





Note : Double sided spectrum

1. Amplitude has even symmetry
2. phase has odd symmetry

EX. 23 Draw the double sided spectrum of the signal

$$x(t) = 7 + 10\cos(2\pi 20 t + 120^\circ) + 4\cos(2\pi 60 t - 90^\circ).$$

Solution $x(t)$ can be re-written as

$$x(t) = 7\cos(2\pi 0 t + 0^\circ) + 10\cos(2\pi 20 t + 120^\circ) + 4\cos(2\pi 60 t - 90^\circ)$$

we know that $A\cos(\omega_0 t + \varphi) = \frac{A}{2} e^{j\omega_0 t} e^{j\varphi} + \frac{A}{2} e^{-j\omega_0 t} e^{-j\varphi}$

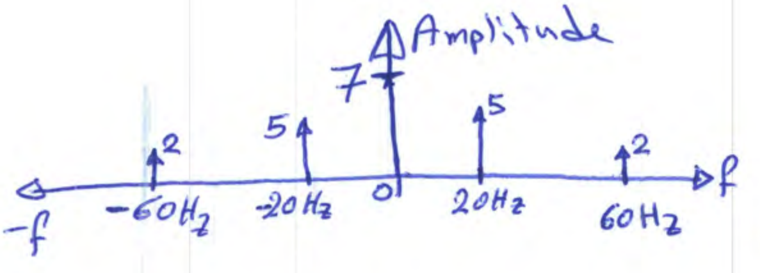
$$\therefore x(t) = \underbrace{7\cos(2\pi 0 t + 0^\circ)}_{\text{D.C. signal unchanged}} + \frac{10}{2} e^{j2\pi 20 t} e^{j120^\circ} + \frac{10}{2} e^{-j2\pi 20 t} e^{-j120^\circ}$$

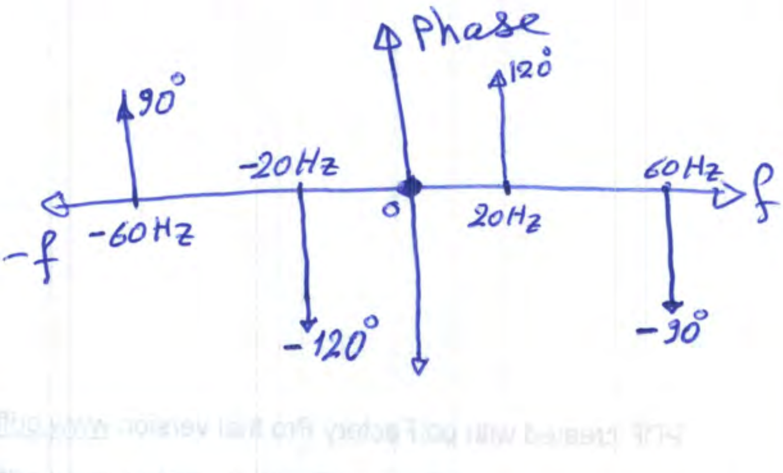
$$+ \frac{4}{2} e^{j2\pi 60 t} e^{-j90} + \frac{4}{2} e^{-j2\pi 60 t} e^{j90^\circ}$$

OR

$$x(t) = (A_0 = 7, f_0 = 0\,Hz, \varphi_0 = 0^\circ) + (A_1 = 5, f_1 = 20, \varphi_1 = 120^\circ) + (A_{-1} = 5, f_{-1} = -20\,Hz, \varphi_{-1} = -120^\circ)$$

$$+ (A_2 = 2, f_2 = 60\,Hz, \varphi_2 = -90^\circ) + (A_{-2} = 2, f_{-2} = -60\,Hz, \varphi_{-2} = 90^\circ)$$





Exercise #24 Constract and plot two sided and one side spectrums of $x(t) = -3 - 4\sin(30\pi t)$.

Exercise #25 Represent the signal $x(t) = 12\sin(\omega_0 t - 25^\circ)$ where $\omega_0 = 2000\pi$ in double and single sided spectrums and plot them.

Exercise #26 Write and plot the double sided spectram of the signal $x(t) = 4[1 + 0.5\cos(100\pi t)]\cos(4000\pi t)$.

Exercise #27 Find and plot the double sided spectram of the signal $x(t) = 4[1 + \cos(50\pi t)\sin(50\pi t)]\cos(3000\pi t)$.

Exercise #28 plote the double sided spectram of the signal $x(t) = 16\sin(500\pi t)\cos(10000\pi t)$.

Exercise #29 sketch the single- and double-sided spectram of the signal $x(t) = 2\sin(10\pi t - \frac{\pi}{6})$.

Exercise #30 sketch the single and double sided spectram of the signal $x(t) = 22\cos(10\pi t - \frac{\pi}{3}) - 2\sin(20\pi t - \frac{\pi}{6})$.