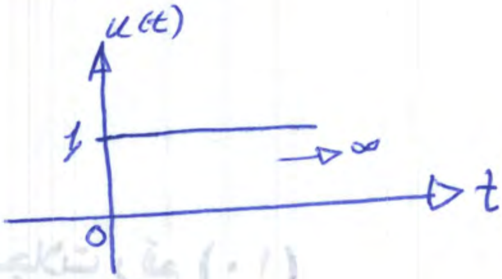
Lecture #4

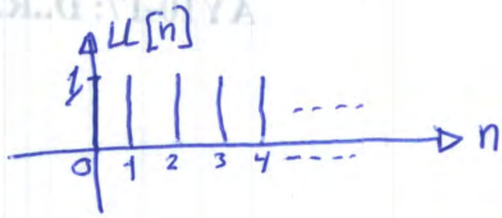
Basic signal Types

1) The unit step Function $u(t)$:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

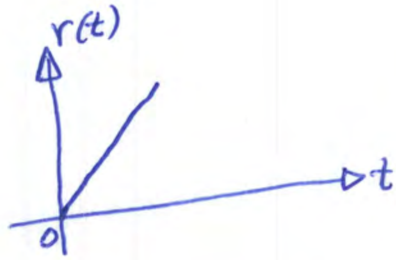


$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

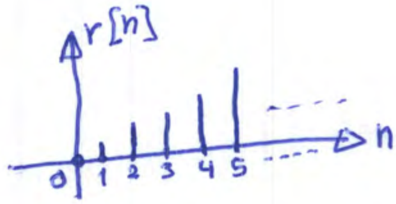


2) The unit ramp Function $r(t)$:

$$r(t) = t\,u(t) = \begin{cases} t & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



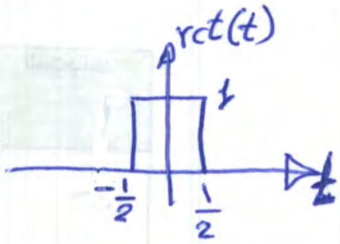
$$r[n] = n\,u[n] = \begin{cases} n & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



3) The unit Rectangular function :

unit rectangular function $rct(t)$ or $rect(t)$ or $\Pi(t)$.

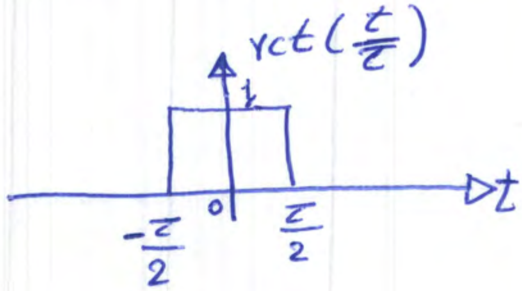
$$rct(t) = u\left(t+\tfrac{1}{2}\right) - u\left(t-\tfrac{1}{2}\right) = \begin{cases} 1 & -\frac{1}{2} \leq t \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$



where the width of the pulse is $\frac{1}{2} - (-\frac{1}{2}) = 1$.

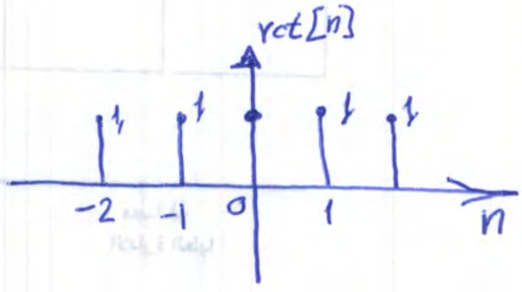
in general, a pulse of width $= \tau$ can be expressed as

$$rct\left(\frac{t}{\tau}\right) = \begin{cases} 1 & |t| \leq \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$



$$= u\left(t+\frac{\tau}{2}\right) - u\left(t-\frac{\tau}{2}\right)$$

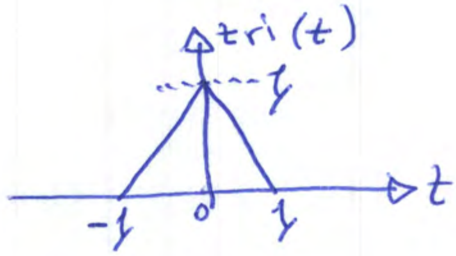
$$rct\left(\frac{n}{2N+1}\right) = \begin{cases} 1 & |n| \leq N \\ 0 & |n| > N \end{cases}$$



4) The Triangular Pulse Function :

triangular function $tri(t)$ or $\Lambda(t)$

$$tri(t) = \begin{cases} 1+t & -1<t<0 \\ 1-t & 0<t<1 \\ 0 & \text{otherwise} \end{cases}$$

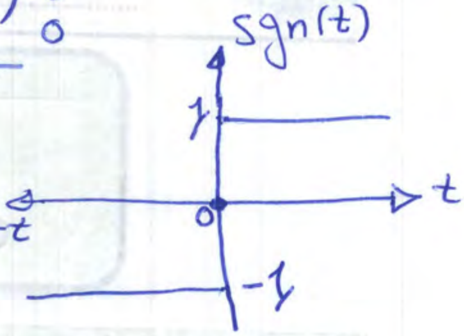


In general, a triangular signal of width τ is

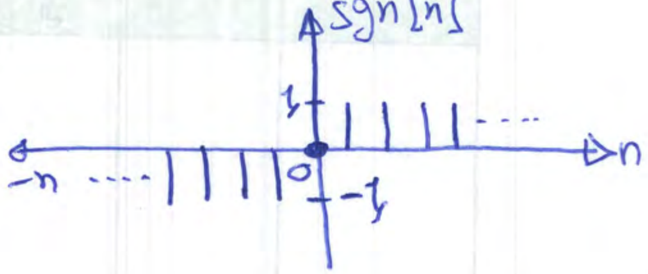
$$tri\left(\frac{t}{\tau}\right) = \begin{cases} 1-\dfrac{|t|}{\tau} & -\tau<t<\tau \\ 0 & \text{otherwise} \end{cases} = \begin{cases} 1+\dfrac{t}{\tau} & -\tau<t<0 \\ 1-\dfrac{t}{\tau} & 0<t<\tau \\ 0 & \text{otherwise} \end{cases}$$

5) Signum Function $sgn(t)$:

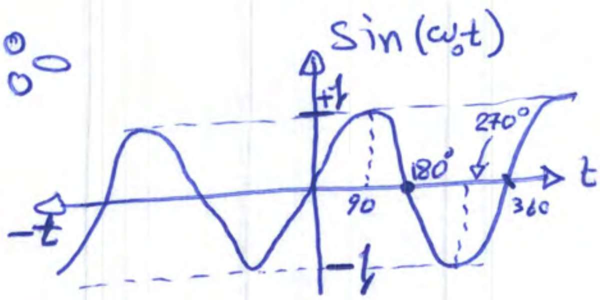
$$sgn(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$



$$sgn[n] = \begin{cases} 1 & n > 0 \\ 0 & n = 0 \\ -1 & n < 0 \end{cases}$$



6) Sinusoidal Function:

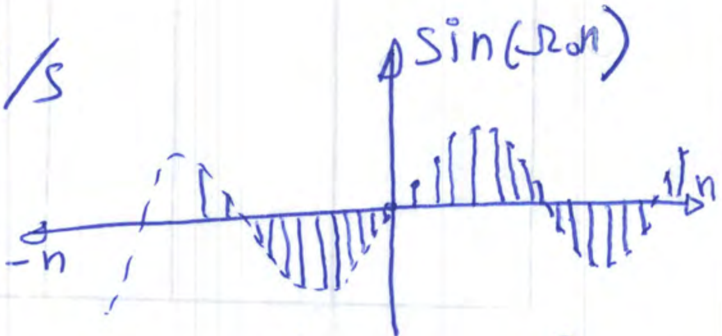


$$x(t) = \sin(\omega_0 t + \phi)$$

$$x[n] = \sin(\Omega_0 n + \phi)$$

$$\omega_0 = 2\pi f_0 \quad rad/s$$

$$\Omega_0 = 2\pi F_0$$



* Discrete time sinusoid is periodic only if the fraction $\frac{\Omega_0}{2\pi}$ is rational number.

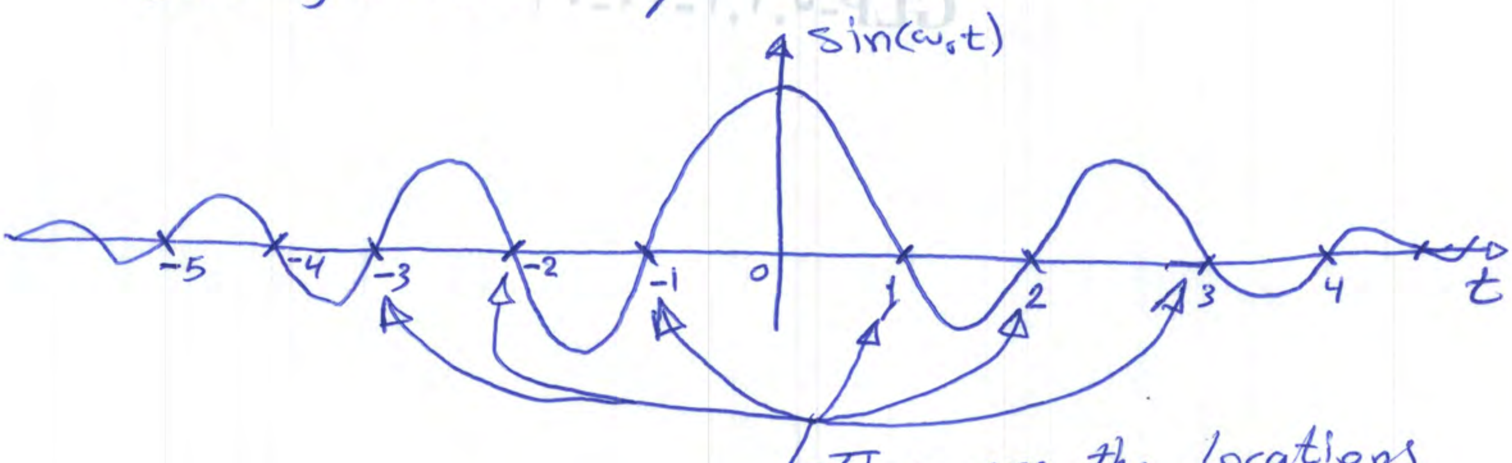
7) The sinc Function: sinc(t)

sinc ≡ Filter function ≡ Sampling function

$$\boxed{\text{sinc}(\omega_o t) = \frac{\sin(\omega_o t)}{\omega_o t}} \quad CT$$

$$\boxed{\text{sinc}(\Omega_o n) = \frac{\sin(\Omega_o n)}{\Omega_o n}} \quad DT$$

sinc function = 1 at n or t = 0



These are the locations where $\sin(\omega t) = 0$

For example $\text{sinc}(t) = \dfrac{\sin(\pi t)}{\pi t}$ crosses zero at $t = \mp 1, \mp 2, \mp 3 \ldots$

8) The Exponential Function e^{st} :

$$x(t) = e^{st} \quad \text{where} \quad \underbrace{s = \sigma + j\omega_0}_{\text{complex frequency}}$$

$$\therefore \; x(t) = e^{(\sigma + j\omega_0)t} = e^{\sigma t}\left[\cos(\omega_0 t) + j\sin(\omega_0 t)\right]$$

$$x[n] = e^{(\sigma + j\Omega_0)n} = e^{\sigma n}\left[\cos(\Omega_0 n) + j\sin(\Omega_0 n)\right]$$

* if $\omega_0 = \text{Zero} \rightarrow x(t) = e^{\sigma t}$ and $x[n] = e^{\sigma n}$

then the exponential is Real function decay up when $\sigma > 0$ or decay down when $\sigma < 0$

* if $\sigma = 0 \rightarrow x(t) = \cos(\omega_0 t) + j\sin(\omega_0 t)$

$$x[n] = \cos(\Omega_0 n) + j\sin(\Omega_0 n)$$

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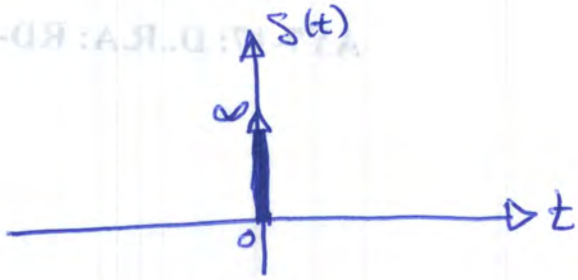
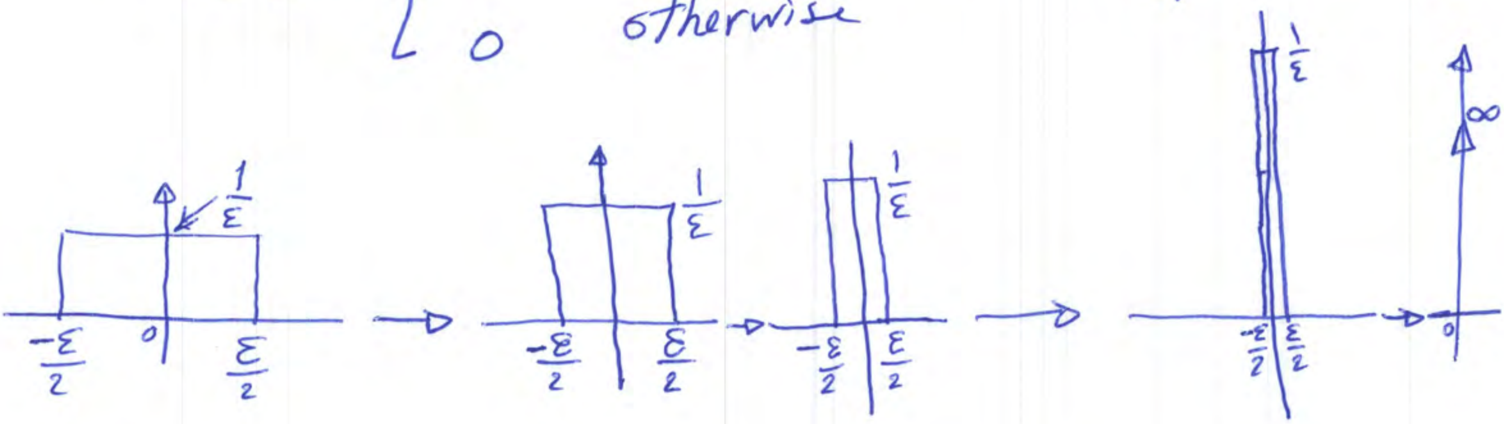
9) The Unit Impulse Function :

- Also known as delta function $\delta(t)$
- Also known as Dirac delta function $\delta(t)$
- Also known in DT as Kronecker delta function $\delta(t)$
- It is a generalized function by which any signal can be generated.

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$$



$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$



The direction where $\varepsilon \to 0$ $\qquad$ $Area = \frac{1}{\varepsilon} \times \varepsilon = 1$

1 * Impulse function is even $\delta(t) = \delta(-t)$

$$\delta[n] = \delta[-n]$$

2 * Area under $\delta(t)$ or $\delta[n]$ is always 1

$$Area = \int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

$$Area = \sum_{n=-\infty}^{n=\infty} \delta[n] = 1$$

3 * The time scale of $\delta(t)$ is

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

4 * multiplication by other function

$$x(t)\delta(t) = x(0)\delta(t)$$

and $\int_{-\infty}^{\infty} x(t)\delta(t)\, dt = x(0)$

5 * Integration of $\delta(t)$

$$\int_{-\infty}^{t} \delta(t)\, dt = u(t)$$

6 * the derivative of $u(t)$ is $\delta(t)$

$$\frac{d}{dt} u(t) = \delta(t)$$

However, in general delata function can be written as:-

$$\boxed{x(t) = A\,\delta(t - t_0)}$$

then

$$\int_{-T}^{T} A\,\delta(t - t_0)\, dt = \begin{cases} A & \text{for } -T < t_0 < T \\ 0 & \text{elsewhere} \end{cases}$$

and

$$\delta(at + b) = \frac{1}{a}\,\delta\left(t + \frac{b}{a}\right)$$

$$\phi(t)\,\delta(t - t_0) = \phi(t_0)\,\delta(t - t_0)$$

$$\int \phi(t)\,\delta(t - t_0)\, dt = \phi(t_0)$$

EX. 24 Simplify the following expressions: (i) $\frac{5-jt}{7+t^2}\delta(t)$; (ii) $\int_{-\infty}^{\infty}(t+5)\delta(t-2)\,dt$, (iii) $\int_{-\infty}^{\infty} e^{j\frac{\pi}{2}\omega+2}\delta(\omega-5)\,d\omega$

solution (i) $x(t)=\frac{5-jt}{7+t^2}\delta(t)=\left[\frac{5-jt}{7+t^2}\right]_{t=0}\delta(t) = \frac{5}{7}\delta(t)$.

(ii) $$x(t)=\int_{-\infty}^{\infty}(t+5)\delta(t-2)\,dt = \int_{-\infty}^{\infty}\left[(t+5)\right]_{t=2}\delta(t-2)\,dt$$

$$= \int_{-\infty}^{\infty}(2+5)\delta(t-2)\,dt = 7\int_{-\infty}^{\infty}\delta(t-2)\,dt = 7.$$

(iii) $$x(t)=\int_{-\infty}^{\infty} e^{j\frac{\pi}{2}\omega+2}\delta(\omega-5)\,d\omega = \int_{-\infty}^{\infty} e^{j\frac{\pi}{2}(5)+2}\delta(\omega-5)\,d\omega$$

$$* \quad e^{j\frac{5\pi}{2}+2} = e^{j\frac{5\pi}{2}}e^{2} = je^{2}$$

$$\therefore \quad x(t) = \int_{-\infty}^{\infty} je^{2}\delta(\omega-5)\,d\omega = je^{2}\int_{-\infty}^{\infty}\delta(\omega-5)\,d\omega$$

$$x(t) = je^{2}.$$

EX. 25 For the DT sequence shown in Figure EX.24, express the sequence as a function of time shifted DT unit impulse functions.

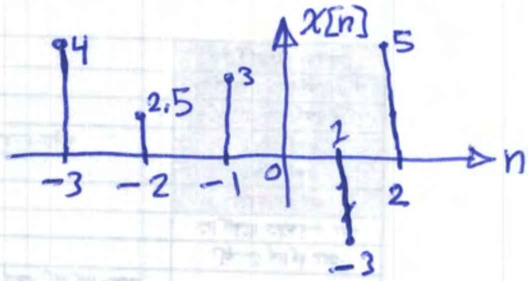


Figure EX.25

Solution

in the figure we have $x_1[n] = 4\delta(n+3)$

$x_2[n] = 2.5\,\delta(n+2)$

$x_3[n] = 3\,\delta(n+1)$

$x_4[n] = -3\,\delta(n-1)$

$x_5[n] = 5\,\delta(n-2)$

$\therefore$ $x[n]$ is summation of delta functions $x_1 \cdots x_5$

hence
$$x[n] = 4\delta(n+3) + 2.5\,\delta(n+2) + 3\delta(n+1) - 3\delta(n-1) + 5\delta(n-2)$$

* Another property of $\delta(t)$ function

$$\int_{-\infty}^{\infty} \delta^{(n)}(t-t_0)\,\phi(t)\,dt = (-1)^n \frac{d^n}{dt^n}\phi(t)\Big|_{t=t_0}$$

Ex. 26 Determine $\cos(t)\delta(t)$, $\cos(t)\delta(2t-3)$, and $\int_{-\infty}^{\infty} e^{-t}\,\delta'(t-1)\,dt$

Solution

1) $x(t) = \cos(t)\delta(t) = \cos(0)\delta(t) = \delta(t).$

2) $\cos(t)\delta(2t-3) = x(t) \Rightarrow x(t) = \frac{1}{2}\delta(t-\frac{3}{2})\cos(t)$

$$x(t) = \frac{\cos(\frac{3}{2})}{2}\delta(t-\frac{3}{2}) \simeq 0.035\,\delta(t-\frac{3}{2}).$$

3) $$x(t) = \int_{-\infty}^{\infty} e^{-t}\,\delta'(t-1)\,dt = (-1)\frac{d}{dt}e^{-t}\Big|_{t=1} = e^{-1}$$

Exercise #31 sketch the signal $x(t) = \delta(t) - 3\delta(t-1) - 3\delta(t-2) - 3\delta(t-3) + \delta(t+1) + \delta(t+2) + \delta(t+3) + u(t-4)$.

Exercise #32 sketch the signal $x[n] = 2\delta(n-1) - \delta(n-2) - \delta(n-3) + \delta[n] + \delta(n+4) + \delta(n+5)$.

Exercise #33 sketch $x(t) = \delta(2t-1) + \delta(4t) - \delta(t+4)$ and $x(t+1)$.

Exercise #34 sketch the following signals:

1) $x(t) + g(t)$ where $x(t) = 2\,\text{rect}(t/4)$, $g(t) = 3\,\text{rect}(t/2)$.

2) $x(t) = 2u(t) - 2u(t)$, $g(t) = 3u(t) - 2u(t)$.